COMPOSITE FUNCTION DOMAIN & RANGE

I. **(solved example)** Let $f(x) = \cos x$ for $0 \le x \le 2\pi$, and let $g(x) = \ln x$ for all $x > 0$. Let $S(x)$ be the composition of $g(x)$ with $f(x)$; that is, $S(x) = g(f(x))$.

A. Find the domain of $S(x)$.

Domain of $f(x)$: $[0, 2\pi]$
Range of $f(x)$: $[-1, 1]$
Domain of $g(x)$: $(0, \infty)$ Is the range of $f(x)$ **ALL** contained in the domain of $g(x)$? **NO.**
What is missing from the domain of $g(x)$ that is included in the range of $f(x)$? $[-1, 0]$
So, I must exclude numbers from the domain of $f(x)$ so that I only get numbers that fit into the domain of $g(x)$. For this problem, I must exclude from the domain of $f(x)$ all numbers that give result in the interval $[-1, 0]$.

Since $f(x) = \cos x$, $\cos x$ gives results $[-1, 0]$ on the interval of $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

So the composite domain of $S(x)$ is $\left[0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right]$.

If the range of the inside function is ALL contained in the domain of the outside function, the composite domain is the domain of the inside function; however, when the range of the inside function is NOT ALL contained in the domain of the outside function, the above procedure must be used to find the composite domain.

B. Find the range of $S(x)$.

The composite range of $S(x)$ is $(-\infty, 0]$. You may also think about the graph of $S(x)$. Basically, since $S(x) = \ln(\cos x)$ and the composite domain is $\left[0, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right]$, you are taking the natural log of numbers between 0 and 1.

II. Let $f(x) = \ln(x^2)$ for all $x > 0$, and let $g(x) = e^{2x}$ for all $x \ge 0$. Let $H(x)$ be the composition of $f(x)$ with $g(x)$; that is, $H(x) = f(g(x))$. Let $K(x)$ be the composition of $g(x)$ with $f(x)$; that is, $K(x) = g(f(x))$.

A. Find the domain of $H(x)$ and write an expression for $H(x)$ that does not contain the exponential function. Find the range of $H(x)$.

B. Find the domain of $K(x)$ and write an expression for $H(x)$ that does not contain the exponential function. Find the range of $K(x)$.

C. Find an expression for $f^{-1}(x)$, where $f^{-1}(x)$ denotes the inverse function of $f(x)$, and find the domain of $f^{-1}(x)$.