PRE-CALCULUS: by Finney, Demana, Watts and Kennedy

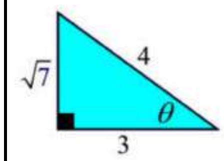
Chapter 4: Trigonometric Functions 4.2: Trigonometric Functions of Acute Angles

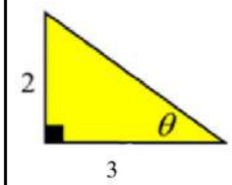
What you'll Learn About

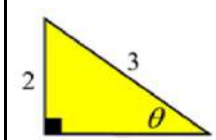
- Right Triangle Trigonometry/ Two Famous Triangles
- Evaluating Trig Functions with a calculator/Applications of right triangle trig

The six trigonometric functions

Find the values of all six trigonometric functions.







Assume that θ is an acute angle in a right triangle satisfying the given conditions. Evaluate the remaining trigonometric functions.

A) $\sin\theta = \frac{4}{9}$

B) $\cos\theta = \frac{2}{9}$

C) $\tan\theta = \frac{4}{9}$

D) $\cot\theta = \frac{2}{9}$

E) $\csc\theta = \frac{10}{7}$

F) $\sec\theta = \frac{4}{3}$

Evaluate using a calculator. Make sure your calculator is in the correct mode. Give answers to 3 decimal places and then draw the triangle that represents the situation.

A) $\sin 53^\circ$

B) $\cos\frac{2\pi}{5}$

C) $\tan 154^\circ$

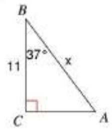
D) $\cot\frac{\pi}{9}$

E) $\csc 220^\circ$

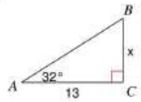
F) $\sec\frac{8\pi}{5}$

Solve the triangle for the variable shown.

9)



10)



Solve the triangle ABC for all of its unknown parts. Assume C is the right angle.

$\alpha = 40^\circ$ a = 10

Solve the triangle ABC for all of its unknown parts. Assume C is the right angle.

β=62   a = 7

Example 6: From a point 340 feet away from the base of the Peachtree Center Plaza in Atlanta, Georgia, the angle of elevation to the top of the building is 65°. Find the height of the building.

What you'll Learn About

- Trig functions of any angle/Trig functions of real numbers
- Periodic Functions/The Unit Circle

Point P is on the terminal side of angle θ. Evaluate the six trigometric functions for θ.

A) (5, 4) B) (-3, 4)

C) (-2, -5) D) (-4, -1)

E) (0, -3) F) (3, 0)

Determine the sign (+ or -) of the given value without the use of a calculator.

A) $\sin 53^\circ$ B) $\cos \frac{2\pi}{5}$

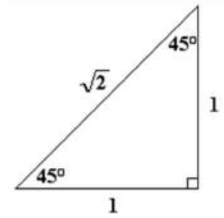
C) $\tan 154^\circ$ D) $\cot \frac{\pi}{9}$

E) $\csc 220^\circ$ F) $\sec \frac{8\pi}{5}$

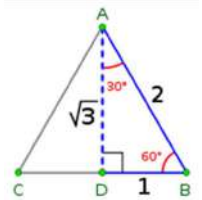
Evaluate without using a calculator

A) Find $\sin\theta$ and $\tan\theta$ if $\cos\theta = \frac{3}{4}$ and $\cot\theta < 0$

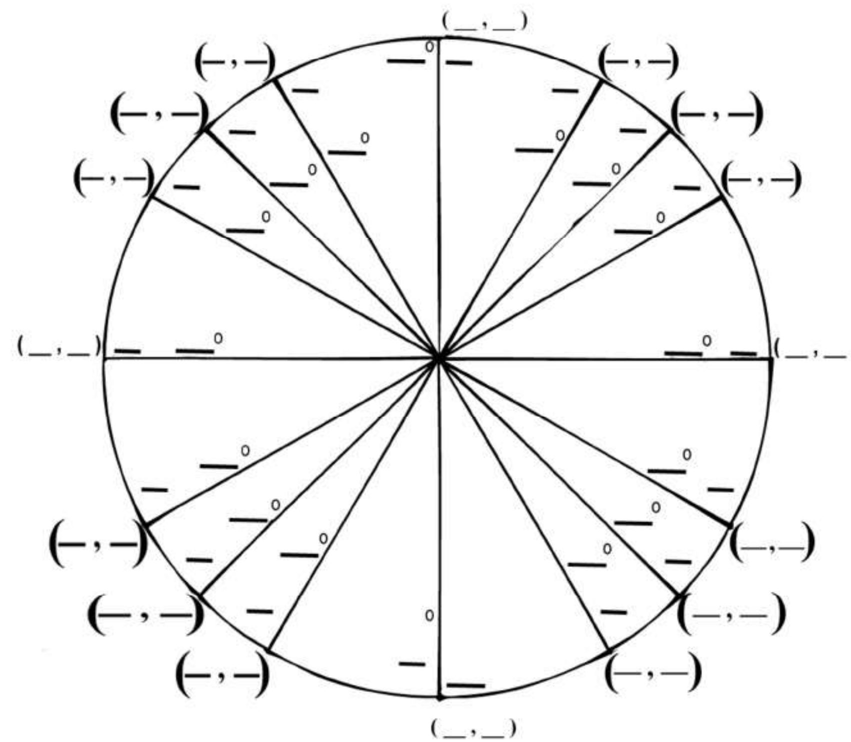
B) Find $\sec\theta$ and $\csc\theta$ if $\cot\theta = \frac{-6}{5}$ and $\sin\theta > 0$



45 – 45 - 90 Triangle

	30 – 60 – 90 Triangle

Unit Circle, Fill in the blank



Evaluate without using a calculator by using ratios in a reference triangle.

A) $\sin 120^{\circ}$

B) $\cos \frac{2\pi}{3}$

C) $\tan \frac{13\pi}{4}$

D) $\cot \frac{-13\pi}{6}$

E) $\csc \frac{7\pi}{4}$

F) $\sec \frac{23\pi}{6}$

Find sine, cosine, and tangent for the given angle.

A) 90°

B) $-\frac{\pi}{2}$

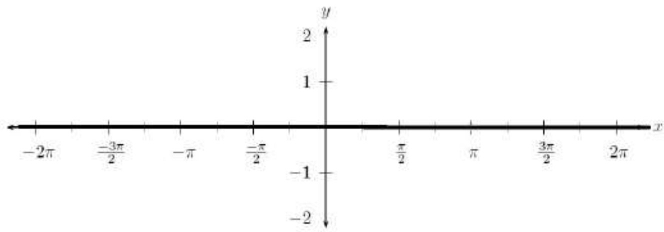
C) 6π

D) $\frac{-7\pi}{2}$

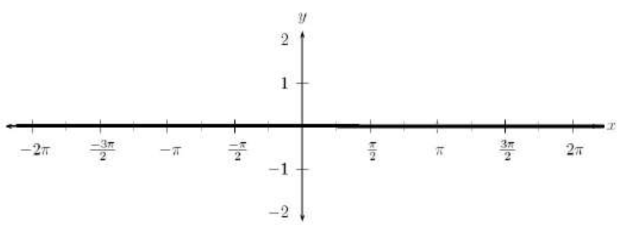
What you'll Learn About

- Inverse Trigonometric Functions and their Graphs

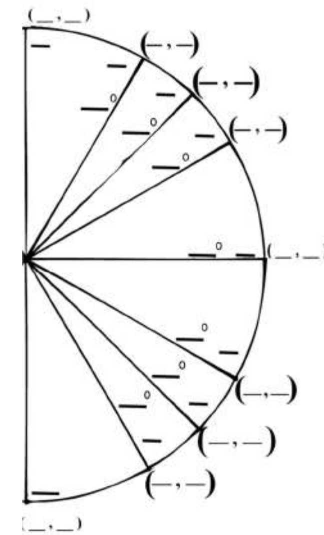
The graph of y = sin x



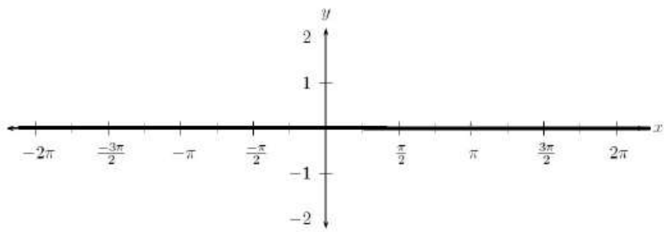
The graph of $y = \sin^{-1} x = \arcsin x$



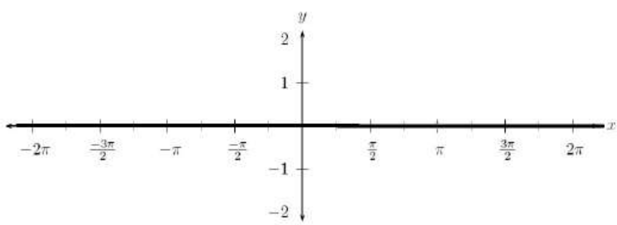
The Unit Circle and Inverse Functions



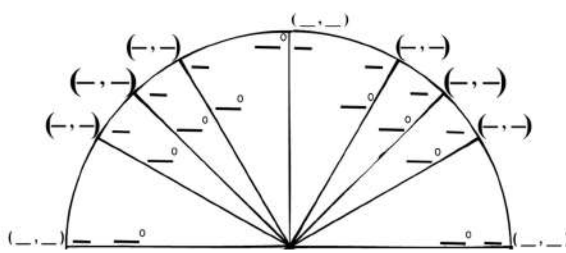
The graph of y = cos x



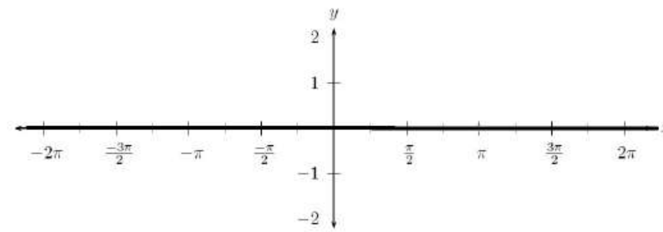
The graph of $y = \cos^{-1} x = \arccos x$



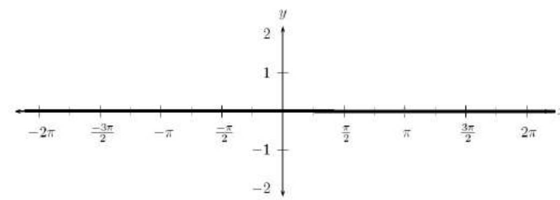
The Unit Circle and Inverse Functions

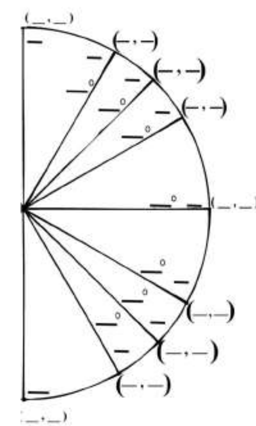


The graph of y = tan x



The graph of $y = \tan^{-1} x = \arctan x$





Find the exact value

$A)\ \cos^{-1}\frac{\sqrt{3}}{2}$ $B)\ \cos^{-1}\frac{1}{2}$ $C)\ \cos^{-1}\left(\frac{-1}{2}\right)$

$D)\ \sin^{-1}\frac{-\sqrt{3}}{2}$ $E)\ \sin^{-1}\frac{1}{2}$ $F)\ \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$G)\ \tan^{-1}(1)$ $H)\ \tan^{-1}(\sqrt{3})$ $I)\ \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$J)\ \cos^{-1}(0)$ $K)\ \sin^{-1}(-1)$ $L)\ \tan^{-1}(0)$

Use a calculator to find the approximate value in degrees. Draw the triangle that represents the situation.

$A)\ \arccos(.456)$ $B)\ \arcsin(-.456)$

$C)\ \arctan(-5.768)$

Use a calculator to find the approximate value in radians. Draw the triangle that represents the situation.

$A)\ \arcsin(.456)$ $B)\ \arccos(-.456)$

$C)\ \arctan(-5.768)$

Find the exact value without a calculator.

$A)\ \sin\left(\cos^{-1}(1/2)\right)$

$B)\ \cos\left(\tan^{-1}(0)\right)$

$C)\ \tan\left(\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$

$D)\ \sin\left(\tan^{-1}\left(-\sqrt{3}\right)\right)$

$E)\ \cos^{-1}\left(\sin\left(\frac{\pi}{4}\right)\right)$

$F)\ \sin^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right)$

1. $\cos\left(\sin^{-1}\frac{1}{\sqrt{2}}\right) =$

2. $\sin\left(\tan^{-1}\left(\frac{\sqrt{2}}{2}\right)\right) =$

3. $\cot\left(\cos^{-1}\frac{-1}{4}\right) =$

4. $\csc\left(\sin^{-1}\frac{-2}{3}\right) =$

What you'll Learn About

Solve each trigonometric equation for θ on the interval $[0,2\pi]$. Then give a formula for all possible angles that could be a solution of the equation.

A) $\sin\theta = \frac{\sqrt{2}}{2}$

B) $\cos\theta = \frac{-1}{2}$

C) $\sin\theta = 1$

D) $\cos\theta = 0$

E) $\tan\theta = \sqrt{3}$

F) $\tan\theta = -1$

Solve each trigonometric equation for θ on the interval $[0,2\pi]$.

A) $\cos 2\theta = \frac{1}{2}$

B) $\sin 3\theta = \frac{1}{2}$

C) $\cos\frac{\theta}{3} = \frac{\sqrt{3}}{2}$

D) $\tan\left(\frac{\theta}{2} + \frac{\pi}{3}\right) = 1$

E) $\sin\theta = .4$

F) $\cos\theta = -.2$

A) $\sqrt{2}\cos\theta - 1 = 0$

B) $\sqrt{3}\csc\theta - 2 = 0$

C) $4\sin^2\theta - 1 = 0$

D) $(3\cot^2\theta - 1)(\cot^2\theta - 3) = 0$

E) $3\tan^2\theta - 1 = 0$

F) $\cos^2\theta = 3\sin^2\theta$

G) $2\cos^2\theta + \cos\theta = 0$

H) $2\sin\theta\cos\theta = \cos\theta$

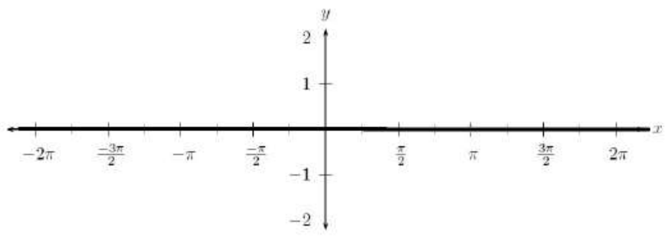
I) $\csc^2\theta - \csc\theta = 2$

J) $\sin^3\theta = \sin\theta$

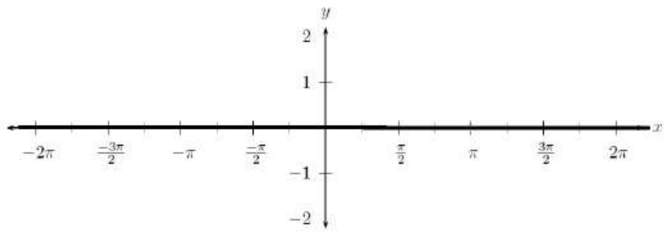
What you'll Learn About

- The basic waves revisited/Sinusoids and Transformations
- Modeling

The graph of y = sin x



The graph of y = cos x



Find the amplitude of the function and use the language of transformations to describe how the graph of the function is related to the graph of y = sinx

A) y = 3sinx B) $y = \frac{3}{4}\sin x$ C) y = - 5sinx

Find the amplitude of the function and use the language of transformations to describe how the graph of the function is related to the graph of y = sinx

A) y = 2cosx B) $y = \frac{5}{3}\cos x$ C) y = - 2cosx

Find the period of the function and use the language of transformations to describe how the graph of the function is related to the graph of y = cosx

A) $y = \cos(2x)$ B) $y = \cos\frac{x}{2}$ C) $y = \cos\left(\frac{-3x}{4}\right)$

Graph 1 period of the function without using your calculator.

A) $y = 3\sin\frac{x}{2}$ $y = 5\sin(-2x)$

Find the coordinates of the maximum, minimum, and zeros of the function on the interval $[-2\pi, 2\pi]$.

A) $y = 4\sin x$ $y = -2\cos\left(\frac{x}{3}\right)$

Graph 1 period of the function without using your calculator.

A) $y = -2\cos\frac{3\pi x}{4}$ $y = 3\sin\left(\frac{\pi x}{6}\right)$

Determine the phase shift for the function and the sketch the graph.

A) $y = \cos\left(x - \frac{\pi}{6}\right)$ B) $y = \sin\left(x + \frac{\pi}{3}\right)$

Determine the vertical shift for the function and the sketch the graph.

A) $y = \cos x - 2$ B) $y = \sin x + 3$

Determine the vertical shift and phase shift of the function and then sketch the graph

A) $y = \cos\left(x + \frac{\pi}{6}\right) - 1$ B) $y = \sin\left(x - \frac{\pi}{3}\right) + 2$

$Amp = A = \frac{Max - Min}{2}$

$Vertical = (C) = \frac{\text{Max} + \text{Min}}{2}$

$period = p$

$Horizontal$ Stretch/Shrink

$\text{B} = \frac{2\pi}{\text{p}}$

How to choose an appropriate model based on the behavior at some given time, T.

y = A cos B(t – T) + C
if at time T the function attains a maximum value

y = -A cos B(t – T) + C
if at time T the function attains a minimum value

y = A sin B(t – T) + C
if at time T the function halfway between a minimum and a maximum value

y = -A sin B(t – T) + C
if at time T the function halfway between a maximum and a minimum value

State the Amplitude and period of the sinusoid, and relative to the basic function, the phase shift and vertical translation.

A) $y = 3\sin\left(x - \frac{\pi}{4}\right) + 2$ B) $y = -2\cos\left(3x - \frac{\pi}{4}\right) - 4$

C) $y = 5\sin 4\pi x + 6$

$Amp = A = \frac{Max - Min}{2}$ $Vertical = (C) = \frac{Max + Min}{2}$ $period = p$ *Horizontal* Stretch/Shrink $B = \frac{2\pi}{p}$ How to choose an appropriate model based on the behavior at some given time, T. y = A cos B(t – T) + C if at time T the function attains a maximum value y = -A cos B(t – T) + C if at time T the function attains a minimum value y = A sin B(t – T) + C if at time T the function halfway between a minimum and a maximum value y = -A sin B(t – T) + C if at time T the function halfway between a maximum and a minimum value	Construct a sinusoid with the given amplitude and period that goes through the given point. A) Amp: 4, period 4π, point (0, 0) B) Amp: 2.5, period $\frac{\pi}{5}$, point (2, 0)

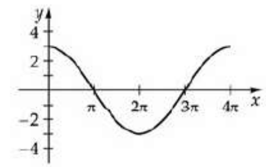
	Example 7: Calculating the Ebb and Flow of Tides One particular July 4th in Galveston, TX, high tide occurred at 9:36 am. At that time the water at the end of the 61st Street Pier was 2.7 meters deep. Low tide occurred at 3:48 p.m, at which time the water was only 2.1 meters deep. Assume that the depth of the water is a sinusoidal function of time with a period of half a lunar day (about 12 hrs 24 min) a) Model the depth, D, as a sinusoidal function of time, t, algebraically then graph the function. b) At what time on the 4th of July did the first low tide occur. c) What was the approximate depth of the water at 6:00 am and at 3:00 pm? d) What was the first time on July 4th when the water was 2.4 meters deep?

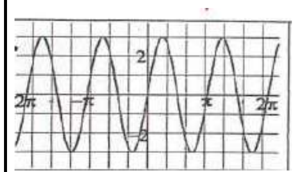
80) Temperature Data: The normal monthly Fahrenheit temperatures in Helena, MT, are shown in the table below (month 1 = January)

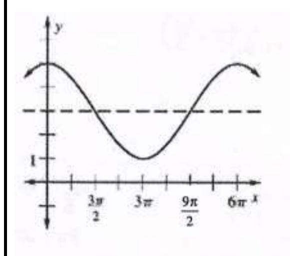
Model the temperature T as a sinusoidal function of time using 20 as the minimum value and 68 as the maximum value. Support your answer graphically by graphing your function with a scatter plot.

M	1	2	3	4	5	6	7	8	9	10	11	12
T	20	26	35	44	53	61	68	67	56	45	31	21

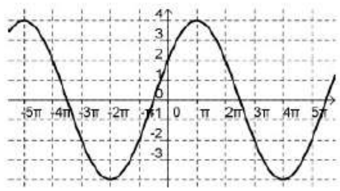
Determine the sinusoidal model from the graph(No phase shift)

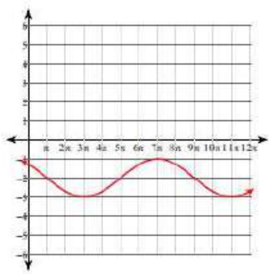






Determine the sinusoidal model from the graph(phase shift)

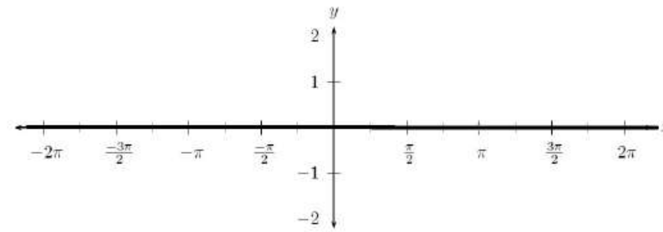




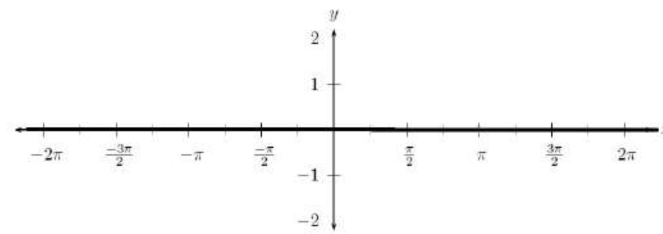
What you'll Learn About

- The graphs of the other 4 trig functions

The graph of y = csc x



The graph of y = sec x



Describe the graph of the function in terms of a basic trigonometric function. Locate the vertical asymptotes and graph 2 periods of the function.

A) $y = \sec(4x)$

$y = 2\sec\frac{4x}{3}$

C) $y = -\sec(x) + 2$

D. $y = 2\sec\left(\frac{1}{2}x - 2\right)$

Describe the graph of the function in terms of a basic trigonometric function. Locate the vertical asymptotes and graph 2 periods of the function.

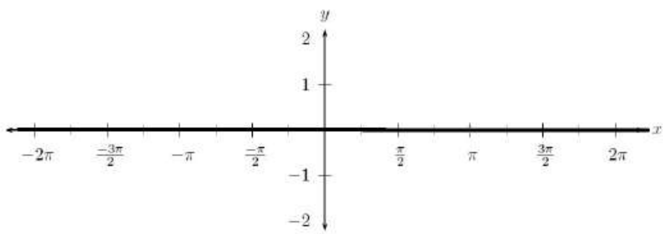
A) $y = \csc\left(\frac{x}{3}\right)$

B) $y = 4\csc 2\pi x$

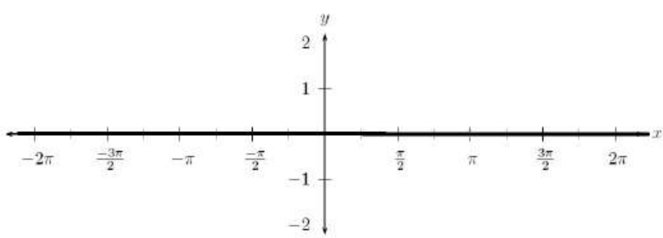
C) $y = -\csc(x) + 1$

D. $y = 2\csc\left(\frac{1}{3}x - 2\right)$

The graph of y = tan x



The graph of y = cot x



Describe the graph of the function in terms of a basic trigonometric function. Locate the vertical asymptotes and graph 2 periods of the function.

A) $y = 2\tan(3x)$

B) $y = 5\tan\left(\pi x - \frac{\pi}{3}\right)$

C) $y = -\tan\left(2\left(x - \frac{\pi}{4}\right)\right) + 2$

	Describe the transformations required to obtain the graph of the given function form a basic trigonometric graph. A) y = 2cot(3x) B) $y = 5\cot\left(\pi x - \frac{\pi}{3}\right)$ C) $y = -3\cot\frac{1}{2}(x-1)+1$