Chapter 10

2. a. The large cube is made up of $3 \bullet 3 \bullet 3$ small cubes. Because each small cube contains \$3, the total amount of money in the large cube is $3 \bullet 3 \bullet 3 \bullet 3 = 3^4$.

b. $3^4 = 81$

There is \$81 in the large cube.

3. a. $10^{26} = 100{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000$

The diameter of the observable universe is 100,000,000,000,000,000,000,000,000 meters.

b. $10^{21} = 1{,}000{,}000{,}000{,}000{,}000{,}000{,}000$

The diameter of the Milky Way Galaxy is 1,000,000,000,000,000,000,000 meters.

This can be written as one sextillion meters.

c. $10^{16} = 10{,}000{,}000{,}000{,}000{,}000$

The diameter of the solar system is 10,000,000,000,000,000 meters.

This can be written as ten quadrillion meters.

d. $10^{7} = 10{,}000{,}000$

The diameter of Earth is 10,000,000 meters.

This can be written as ten million meters.

e. $10^{6} = 1{,}000{,}000$

The length of the Lake Erie shoreline is 1,000,000 meters.

This can be written as one million meters.

f. $10^{5} = 100{,}000$

The width of Lake Erie is 100,000 meters.

This can be written as one hundred thousand meters.

4. Wives: 7^1

Sacks: $7 \bullet 7 = 7^2$

Cats: $7 \bullet 7 \bullet 7 = 7^3$

Kits: $7 \bullet 7 \bullet 7 \bullet 7 = 7^4$

5. You can use exponents to write the product of repeated factors.

Sample answer: The formula for the volume of a cube, $V = s^3$, is an example of how exponents are used in real life. Exponents are also used in measuring astronomical distances.

10.1 On Your Own (pp. 412–413)

1. $\frac{1}{4} \bullet \frac{1}{4} \bullet \frac{1}{4} \bullet \frac{1}{4} \bullet \frac{1}{4}$

Because $\frac{1}{4}$ is used as a factor 5 times, its exponent is 5.

So, $\frac{1}{4} \bullet \frac{1}{4} \bullet \frac{1}{4} \bullet \frac{1}{4} \bullet \frac{1}{4} = \left(\frac{1}{4}\right)^5$.

2. $0.3 \bullet 0.3 \bullet 0.3 \bullet 0.3 \bullet x \bullet x$

Because 0.3 is used as a factor 4 times, its exponent is 4. Because x is used as a factor 2 times, its exponent is 2.

So, $0.3 \bullet 0.3 \bullet 0.3 \bullet 0.3 \bullet x \bullet x = (0.3)^4 x^2$.

3. $-5^4 = -(5 \bullet 5 \bullet 5 \bullet 5) = -625$

4. $\left(-\frac{1}{6}\right)^3 = \left(-\frac{1}{6}\right) \bullet \left(-\frac{1}{6}\right) \bullet \left(-\frac{1}{6}\right) = -\frac{1}{216}$

5. $\left|-3^3 \div 27\right| = \left|-27 \div 27\right| = \left|-1\right| = 1$

6. $9 - 2^5 \bullet 0.5 = 9 - 32 \bullet 0.5 = 9 - 16 = -7$

7. The diameter is 1.8 meters, so the radius is 0.9 meter.

Inner sphere: $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(0.9)^3$

$= \frac{4}{3}\pi(0.729)$

$= 0.972\pi$

Outer sphere: $\frac{9}{2}\pi = 4.5\pi$

The volume of the inflated space is $4.5\pi - 0.972\pi = 3.528\pi$, or about 11.08 cubic meters.

10.1 Exercises (pp. 414–415)

Vocabulary and Concept Check

1. -3^4 is the negative of 3^4, so the base is 3, the exponent is 4, and its value is -81. $(-3)^4$ has a base of -3, an exponent of 4, and a value of 81.

2. The second one does not belong because it is an incorrect statement about the expression. The power is the entire expression 5^3.

Practice and Problem Solving

3. $3 \bullet 3 \bullet 3 \bullet 3$

Because 3 is used as a factor 4 times, the exponent is 4. So, $3 \bullet 3 \bullet 3 \bullet 3 = 3^4$.

4. $(-6) \bullet (-6)$

Because -6 is used as a factor 2 times, the exponent is 2.

So, $(-6) \bullet (-6) = (-6)^2$.

Chapter 10

5. $\left(-\frac{1}{2}\right) \bullet \left(-\frac{1}{2}\right) \bullet \left(-\frac{1}{2}\right)$

Because $-\frac{1}{2}$ is used as a factor 3 times, the exponent is 3.

So, $\left(-\frac{1}{2}\right) \bullet \left(-\frac{1}{2}\right) \bullet \left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3$.

6. $\frac{1}{3} \bullet \frac{1}{3} \bullet \frac{1}{3}$

Because $\frac{1}{3}$ is used as a factor 3 times, the exponent is 3.

So, $\frac{1}{3} \bullet \frac{1}{3} \bullet \frac{1}{3} = \left(\frac{1}{3}\right)^3$.

7. $\pi \bullet \pi \bullet \pi \bullet x \bullet x \bullet x \bullet x$

Because π is used as a factor 3 times, the exponent is 3.
Because x is used as a factor 4 times, the exponent is 4.

So, $\pi \bullet \pi \bullet \pi \bullet x \bullet x \bullet x \bullet x = \pi^3 x^4$.

8. $(-4) \bullet (-4) \bullet (-4) \bullet y \bullet y$

Because -4 is used as a factor 3 times, the exponent is 3.
Because y is used as a factor 2 times, the exponent is 2.

So, $(-4) \bullet (-4) \bullet (-4) \bullet y \bullet y = (-4)^3 y^2$.

9. $6.4 \bullet 6.4 \bullet 6.4 \bullet 6.4 \bullet b \bullet b \bullet b$

Because 6.4 is used as a factor 4 times, the exponent is 4.
Because b is used as a factor 3 times, the exponent is 3.

So, $6.4 \bullet 6.4 \bullet 6.4 \bullet 6.4 \bullet b \bullet b \bullet b = (6.4)^4 b^3$.

10. $(-t) \bullet (-t) \bullet (-t) \bullet (-t) \bullet (-t)$

Because $-t$ is used as a factor 5 times, the exponent is 5.

So, $(-t) \bullet (-t) \bullet (-t) \bullet (-t) \bullet (-t) = (-t)^5$.

11. $5^2 = 5 \bullet 5 = 25$

12. $-11^3 = -(11 \bullet 11 \bullet 11) = -1331$

13. $(-1)^6 = (-1) \bullet (-1) \bullet (-1) \bullet (-1) \bullet (-1) \bullet (-1) = 1$

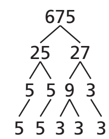
14. $\left(\frac{1}{2}\right)^6 = \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} = \frac{1}{64}$

15. $\left(-\frac{1}{12}\right)^2 = \left(-\frac{1}{12}\right) \bullet \left(-\frac{1}{12}\right) = \frac{1}{144}$

16. $-\left(\frac{1}{9}\right)^3 = -\left(\frac{1}{9} \bullet \frac{1}{9} \bullet \frac{1}{9}\right) = -\frac{1}{729}$

17. The negative sign is not part of the base;
$-6^2 = -(6 \bullet 6) = -36$.

18.



The prime factorization of 675 is $5 \bullet 5 \bullet 3 \bullet 3 \bullet 3$, or $5^2 \bullet 3^3$.

19. $-\left(\frac{1}{4} \bullet \frac{1}{4} \bullet \frac{1}{4} \bullet \frac{1}{4}\right)$

Because $\frac{1}{4}$ is used as a factor 4 times, the exponent is 4.

So, $-\left(\frac{1}{4} \bullet \frac{1}{4} \bullet \frac{1}{4} \bullet \frac{1}{4}\right) = -\left(\frac{1}{4}\right)^4$.

20. The largest doll is 12 inches and the other 3 are $\frac{7}{10}$ the height of the next larger doll. Use $\frac{7}{10}$ as a factor 3 times. So, an expression for the height of the smallest doll is

$$12 \bullet \frac{7}{10} \bullet \frac{7}{10} \bullet \frac{7}{10} = 12 \bullet \left(\frac{7}{10}\right)^3.$$

$$12 \bullet \left(\frac{7}{10}\right)^3 = 12 \bullet \frac{343}{1000} = 4.116$$

The height of the smallest doll is 4.116 inches.

21. $5 + 3 \bullet 2^3 = 5 + 3 \bullet 8 = 5 + 24 = 29$

22. $2 + 7 \bullet (-3)^2 = 2 + 7 \bullet 9 = 2 + 63 = 65$

23. $(13^2 - 12^2) \div 5 = (169 - 144) \div 5 = 25 \div 5 = 5$

24.
$$\begin{aligned} \frac{1}{2}(4^3 - 6 \bullet 3^2) &= \frac{1}{2}(64 - 6 \bullet 9) \\ &= \frac{1}{2}(64 - 54) \\ &= \frac{1}{2}(10) \\ &= 5 \end{aligned}$$

25. $\left|\frac{1}{2}(7 + 5^3)\right| = \left|\frac{1}{2}(7 + 125)\right| = \left|\frac{1}{2}(132)\right| = |66| = 66$

26. $\left|\left(-\frac{1}{2}\right)^3 \div \left(\frac{1}{4}\right)^2\right| = \left|-\frac{1}{8} \div \frac{1}{16}\right| = \left|-\frac{1}{8} \bullet \frac{16}{1}\right| = |-2| = 2$

27.

h	1	2
$2^h - 1$	$2^1 - 1 = 1$	$2^2 - 1 = 3$
2^{h-1}	$2^{1-1} = 2^0 = 1$	$2^{2-1} = 2^1 = 2$

h	3	4
$2^h - 1$	$2^3 - 1 = 7$	$2^4 - 1 = 15$
2^{h-1}	$2^{3-1} = 2^2 = 4$	$2^{4-1} = 2^3 = 8$

h	5
$2^h - 1$	$2^5 - 1 = 31$
2^{h-1}	$2^{5-1} = 2^4 = 16$

You should choose getting paid $2^h - 1$ dollars because when you work more than 1 hour, you will get paid more than the other option.

28. a. $C = 100(0.99988)^t = 100(0.99988)^4 \approx 99.95$

After 4 years, the amount of carbon-14 remaining is about 99.95 grams.

b. $$\text{percent remaining} = \frac{\text{amount remaining}}{\text{original amount}} = \frac{99.95}{100} = 99.95\%$$

After 4 years, 99.95% of the carbon-14 remains.

29. a. To travel from A-440 to A, it takes 12 notes.

b. $F = 440(1.0595)^n = 440(1.0595)^{12} \approx 880$

The frequency of A is about 880 vibrations per second.

c. *Sample answer:* For a 12-note increase, the frequency approximately doubles.

Fair Game Review

30. The statement $8 \bullet x = x \bullet 8$ represents the Commutative Property of Multiplication.

31. The statement $(2 \bullet 10)x = 2(10 \bullet x)$ represents the Associative Property of Multiplication.

32. The statement $3(x \bullet 1) = 3x$ represents the Identity Property of Multiplication.

33. B; $\dfrac{x}{18} = \dfrac{24}{27}$

$\dfrac{x}{18} = \dfrac{8}{9}$

$x = 16$

Section 10.2

10.2 Activity (pp. 416–417)

1. a.

Product	Repeated Multiplication Form	Power
$2^2 \bullet 2^4$	$2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2$	2^6
$(-3)^2 \bullet (-3)^4$	$(-3) \bullet (-3) \bullet (-3) \bullet (-3) \bullet (-3) \bullet (-3)$	$(-3)^6$
$7^3 \bullet 7^2$	$7 \bullet 7 \bullet 7 \bullet 7 \bullet 7$	7^5
$5.1^1 \bullet 5.1^6$	$5.1 \bullet 5.1 \bullet 5.1 \bullet 5.1 \bullet 5.1 \bullet 5.1 \bullet 5.1$	5.1^7
$(-4)^2 \bullet (-4)^2$	$(-4) \bullet (-4) \bullet (-4) \bullet (-4)$	$(-4)^4$
$10^3 \bullet 10^5$	$10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10 \bullet 10$	10^8
$\left(\frac{1}{2}\right)^5 \bullet \left(\frac{1}{2}\right)^5$	$\frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2}$	$\left(\frac{1}{2}\right)^{10}$

b. To find the product of two powers with the same base, add their exponents.

$a^m \bullet a^n = a^{m+n}$

c. $2^2 \bullet 2^4 = 2^{2+4} = 2^6$

$(-3)^2 \bullet (-3)^4 = (-3)^{2+4} = (-3)^6$

$7^3 \bullet 7^2 = 7^{3+2} = 7^5$

$5.1^1 \bullet 5.1^6 = 5.1^{1+6} = 5.1^7$

$(-4)^2 \bullet (-4)^2 = (-4)^{2+2} = (-4)^4$

$10^3 \bullet 10^5 = 10^{3+5} = 10^8$

$\left(\frac{1}{2}\right)^5 \bullet \left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^{5+5} = \left(\frac{1}{2}\right)^{10}$

Using the rule to simplify the products results in the values in the third column of the table in part (a).

d. $2^6 = 64$

$(-3)^6 = 729$

$7^5 = 16{,}807$

$5.1^7 \approx 89{,}741.1$

$(-4)^4 = 256$

$10^8 = 100{,}000{,}000$

$\left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$