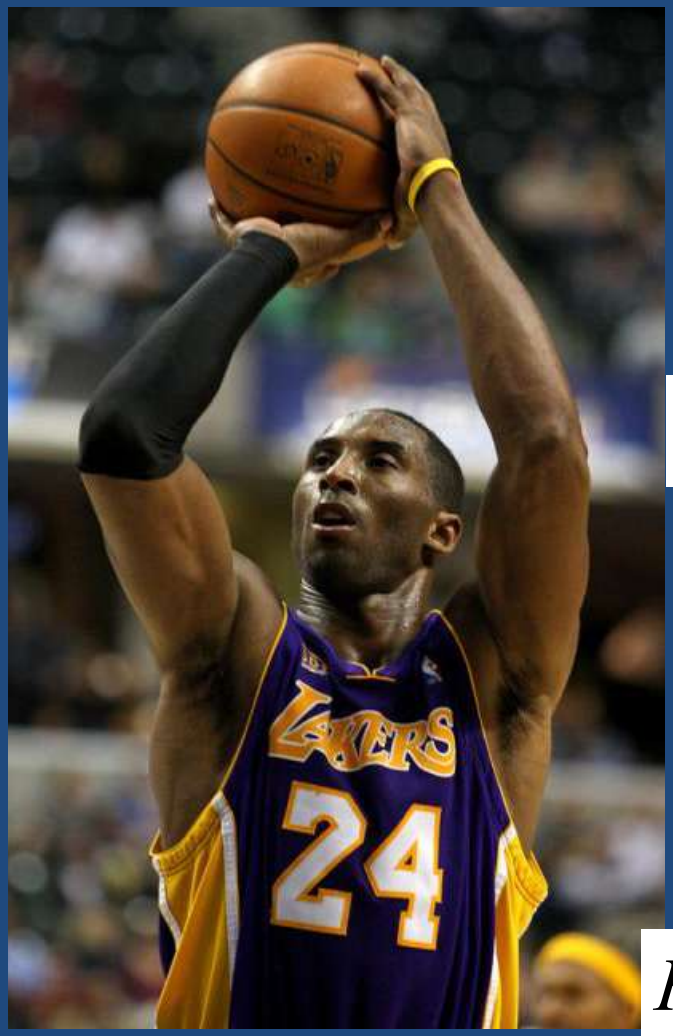
Part IV –Randomness and Probability

Ch. 17 – Probability Models
(Day 1 – The Geometric Model)

- Kobe Bryant is an 84% free throw shooter. Suppose he takes 8 shots in a game. Assuming each shot is independent, find:



–The probability that he makes all 8 shots $P(8 \text{ made}) =$

–The probability that he misses at least one shot

$P(\text{at least one miss}) =$

–The probability that he misses the first two shots

$P(miss\ 1st\ 2) =$

–The probability that the first shot he misses is the third one taken

$P(3rd\ is\ 1st\ miss) =$

Bernoulli Trials

- In this chapter, we will be looking at probability models that involve a specific type of random situation – the Bernoulli trial
- Bernoulli trials have 3 requirements:
  - There are two possible outcomes (we will call these “success” and “failure”)
  - Each trial is independent
  - The probability of success (p) remains the same for each trial

Examples of Bernoulli Trials

- Flipping a coin
- Shooting free throws (assuming shots are independent)
- Asking a “yes” or “no” question to a group of randomly selected survey respondents
- Rolling a die to try to get a 3
  - (but not rolling a die and recording how many times each number comes up)

Are these Bernoulli trials?

- You are rolling 5 dice and need to get at least two 6's to win the game
  - Two outcomes? **Yes (win or don't)**
  - Independent? **Yes (first game doesn't affect next)**
  - Probability stays the same for each trial? **Yes**
- We record the eye colors found in a group of 500 people
  - Two outcomes? **No (more than 2 eye colors)**
  - Independent?
  - Probability stays the same for each trial?

Are these Bernoulli trials?

- A manufacturer recalls a doll because about 3% have buttons that are not properly attached. Customers return 37 of these dolls to the local toy store. Is the manufacturer likely to find any dangerous buttons?
  - Two outcomes? Yes (dangerous or not)
  - Independent? Yes (If there were many dolls made)
  - Probability stays the same for each trial? Yes
- A city council of 11 Republicans and 8 Democrats picks a committee of 4 at random. What's the probability that they choose all Democrats?
  - Two outcomes? Yes (Republican or Democrat)
  - Independent? No(choose from small #, not replaced)
  - Probability stays the same for each trial? No

A note about independence…

- The last example on the previous slide was not independent because each time we removed a Democrat from the group, this changed the probability of choosing Democrats.
- But what if we had chosen our committee from the whole city of 550,000 people? Then removing one Democrat wouldn't make any noticeable difference.
- If the population is large enough, then our trials will be "close enough" to independent
- The 10% condition: Bernoulli trials should be independent, but if they aren't, we can still proceed as long as the sample size is less than 10% of the population size

The Geometric Setting

- Once we have decided that a situation involves Bernoulli trials, we then have to determine our variable of interest – in other words, what is the question asking us to find out?
- If the variable of interest (X) is the number of trials required to obtain the first success in a set of Bernoulli trials, then we are dealing with the geometric distribution

Examples of Geometric Situations

- Flip a coin until you get a tail
- Shoot free throws until you make one
- Draw cards from a deck with replacement until you draw a spade
- Roll a die until you get a 4

Calculating Geometric Probabilities

- When rolling a die, what is the probability that it will take 5 rolls to get the first three?

$P(\text{1st 3 on 5th try}) =$

- What is the probability that it will take more than 7 rolls?

$P(\text{more than 7 rolls to get a 3}) =$

Calculating Geometric Probabilities

$$P(X = n) = (1 - p)^{n-1} p$$

Where p = the probability of success for one trial

And X = the number of trials needed to obtain one success

$$P(X > n) = (1 - p)^{n}$$

- In a large population, 18% of people believe that there should be prayer in public school. If a pollster selects individuals at random, what is the probability that the 6th person he selects will be the first to support school prayer?

$$P(X = 6) =$$

- What is the probability that it will take him more than 4 people to find one who supports school prayer?

$$P(X > 4) =$$

Mean of a Geometric Random Variable

- If X is a geometric random variable with probability of success *p* on each trial, then the mean, or expected value, of X is $\mu = \frac{1}{p}$
- That is, it takes an average of 1/p trials to have the first success
- In the previous example, how many people do we expect the pollster to have to survey in order to find one who supports school prayer?

$E(X) =$

Kobe again…

- Remember that Kobe Bryant is an 84% free throw shooter. What is the average number of shots he will have to take before he makes one?

$$E(X) =$$

Calculator

- Geometpdf(p,x)
  - Find the probability of an individual outcome
- Geometcdf(p,x)
  - Find the probability of finding the first success on or before the xth trial.

Homework 17-1

- p. 401 #1, 7-12

- Use the examples from your notes!