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Chapter 8: Exponential and Logarithmic Functions

Exponential Functions

EXPONENTIAL GROWTH AND DECAY

Exponential Growth	$y = ab^x$, where a ≠ 0, **b > 1**
Exponential Decay	$y = ab^x$, where a ≠ 0, **0 < b < 1**

Determine if the function represents exponential growth or decay.

1. $f(x) = 12(2.5)^x$ **2.** $f(x) = 500\left(\frac{1}{2}\right)^x$ **3.** $f(x) = -19(3)^x$ **4.** $f(x) = 5^x$

Growth Factor	**Decay Factor**
b is your growth factor. If you know the rate of increase ***r***, it can be found using the equation: $b = 1 + r$	***b*** is your decay factor. If you know the rate of decrease ***r***, it can be found using the equation: $b = 1 - r$

Find the factor for each rate exponential growth or decay.

1. 3% Growth **2.** 5% Decay **3.** 0.75% Decay **4.** 13.2% Growth

Applications

1) Matt bought a new car at a cost of $\$25,000$. The car depreciates approximately 15% of its value each year.

a) What is the decay factor for the value of this car?

b) Write an equation to model the depreciation of this car.

c) Use your equation to estimate the value of the car in 10 years.

2) A zombie like infection has spread to a local population. The infection can be traced back to one person. Each zombie must attack one other person each day to stay "alive".

a) What is the growth factor for the number of zombies?

b) Write an equation to model the growth of the number of zombies.

c) Use your equation to estimate how many people are infected after 15 days?

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Logarithmic Functions

To evaluate logs, remember : $\log_b y = x$ **means what exponent (x) is required to go from a base of "b" to reach a value of "y"?**

NOTE: A logarithm with base 10 is called a common logarithm and therefore $\log_{10} x = \log x$

Evaluate the following logarithms

1. $\log 100$ **2.** $\log_4 64$ **3.** $\log_5 \frac{1}{125}$

Change of Base Formula	$\log_h k = \frac{\log_m k}{\log_m h}$

Without using your calculator, rewrite the expressions using the change of base formula.

1. $\log_6 36$ **2.** $\log_3 18$ **3.** $\log_2 8$ **4.** $\log_4 108$

Relationships between Logs and Exponentials	$b^x = y \Rightarrow \log_b y = x$

Change to logarithmic form:

1. $4^3 = 64$ **2.** $5^4 = 625$ **3.** $2^6 = 64$ **4.** $8^{\frac{1}{3}} = 2$

Change to exponential form:

1. $\log_3 27 = 3$ **2.** $\log_5 y = b$ **3.** $\log z = 0.5$ **4.** $\log_4 256 = 4$

The Natural Log Function

"e" **Euler's Number**	• ***e* is an irrational number discovered by Leonhard Euler, a Swiss mathematician and physicist in the 1700s** $e \approx 2.71828$ • **When *e* is used as the base of a logarithm it is called a natural logarithm (denoted *ln*).** $\log_e x = \ln x$

Using a calculator, evaluate the following:

1. e^3 **2.** $e^{\sqrt{5}}$ **3.** $\ln 1$ **4.** $\ln 0.9$

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*****Remember:** $e^x = y \leftrightarrow x = \ln y$ *******

Write an equivalent equation.

1. $e^x = 20$ **2.** $e^5 \approx 148.41$ **3.** $\ln x = 5$ **4.** $\ln 4 \approx 1.39$

Properties of Logarithms

Properties of Logarithmic/Natural Log Functions:

Product Property	$\log_b xy = \log_b x + \log_b y$	$\ln uv = \ln u + \ln v$
Quotient	$\log_b \frac{x}{y} = \log_b x - \log_b y$	$\ln \frac{u}{v} = \ln u - \ln v$
Power	$\log_b x^p = p \log_b x$	$\ln u^p = p \ln u$

Expand the logarithms. Simplify if possible.

1. $\log_4 16hk$ **2.** $\log_2 8x^2$ 3. $\log_5 \frac{x^3}{25}$ 4. $\log_3 4x^4y^5$

Condense the logarithms. Simplify if possible.

1. $\log_2 7 + \log_2 x - \log_2 y$ **2.** $5\log_x m - 2\log_x n$

3. $2\log_4 8x + \log_4 2y - \log_4 2x$ **4.** $\frac{1}{4}\log_{10} x + 5\log_{10}(2x-3)$

Solving Exponential Equations

Remember all the properties we have learned!

Solve the equations by using common bases.

1. $4^x = 4^8$ **2.** $e^{2y} = e^{14}$ **3.** $3^x = 81$

4. $2^{x+4} = 8$ **5.** $125 = 5^{2x-1}$

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To solve equations where you cannot use common bases:

1. **Isolate the base with the variable on one side of the equal sign**
2. **Take the log of each side**
3. **Use the power property to change the exponent to a coefficient**
4. **solve for x**

Solve the equations using logs.

1. $6^x = 99$ **2.** $18^{6x} = 26$ **3.** $9^{x+10} + 3 = 81$

Solving Logarithmic Functions

Remember all the properties we have learned!

$b^x = y \leftrightarrow \log_b y = x$	$e^x = y \leftrightarrow \ln y = x$
$\log_b (xy) = \log_b x + \log_b y$	$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$
$\log_b (x^n) = n \log_b x$	$\log_b x = \frac{\log x}{\log b}$
$b^{\log_b x} = x$	$\log_b b^x = x$

One-to-One Property	If $b^x = b^y$, then $x = y$	If $\log_b x = \log_b y$, then $x = y$

To solve equations with one log:

1. **Isolate the log on one side of the equal sign**
2. **Change the equation to exponential form**
3. **solve for x**

Remember: $\ln$ is $\log_e$ when converting to exponential form.

Solve the logarithmic equations.

1. $x = \log_4 64$ **2.** $\log_2 x = 18$ **3.** $4\ln(x+5) = 8$

4. $\log_x 20 = 3$ **5.** $2 + 2\log x = 10$ **6.** $\ln x + 5 = 5$