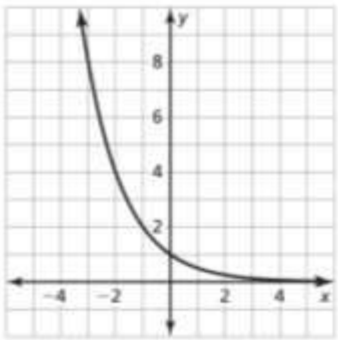
Chapter 6 Review **ANSWER KEY**

1. On page 353, do 1-3.

1. Graph must have points (0,1) and (1, ½). Other point could be (-1, 2).

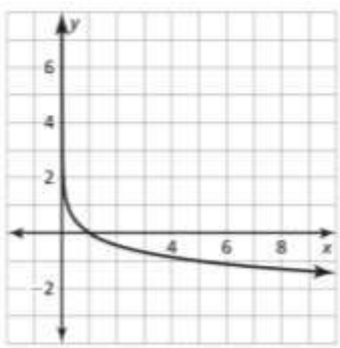
1.



The domain is all real numbers, the range is $y > 0$, and the asymptote is $y = 0$.

2. Graph must have points (1, 0) and (1/5, 1). Other point could be (5, -1).

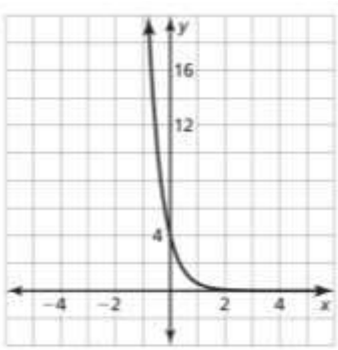
2.



The domain is $x > 0$, the range is all real numbers, and the asymptote is $x = 0$.

2. This is a transformation. The graph of y = e^x would have points (0, 1) and (1, 2.718). However, these points need to be adjusted to $(\frac{x}{-2}, 4y)$. So, the adjusted points would be at (0, 4) and (-1/2, 10.873). You can use your calculator to find other points as well.

3.



The domain is all real numbers, the range is $y > 0$, and the asymptote is $y = 0$.

2. On page 353, do 7-10.

7. $\log_3 52 = \log_3 4 + \log_3 13$

$\approx 1.262 + 2.335$

$= 3.597$

8. $\log_3 \frac{13}{9} = \log_3 13 - \log_3 9$

$= \log_3 13 - 2$

$\approx 2.335 - 2$

$= 0.335$

9. $\log_3 16 = 2 \log_3 4$

$\approx 2(1.262)$

$= 2.524$

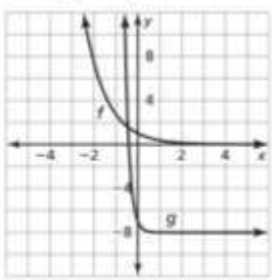
10. $\log_3 8 + \log_3 \frac{1}{2} = \log_3 2 + \log_3 4 - \log_3 2$

$= \log_3 4$

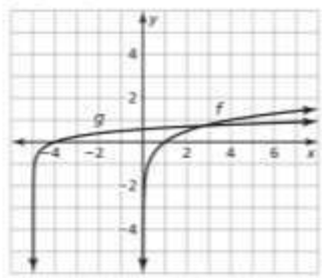
≈ 1.262

3. On page 351, do 18 and 19.

18. Notice that the function is of the form $g(x) = e^{-ax} + k$, where $a = 5$ and $k = -8$. So, the graph of g is a horizontal shrink by a factor of $\frac{1}{5}$ followed by a translation 8 units down of the graph of f.



19. Notice that the function is of the form $g(x) = a \log_4(x - h)$, where $h = -5$ and $a = \frac{1}{2}$. So, the graph of g is a vertical shrink by a factor of $\frac{1}{2}$ and a translation 5 units left of the graph of f.



4. On page 352, do 31-33.

31. $5^x = 8$

$\log_5 5^x = \log_5 8$

$x = \log_5 8$

$x \approx 1.29$

The solution is $x = \log_5 8$, or about 1.29.

32. $\log_3 (2x - 5) = 2$

$3^{\log_3(2x-5)} = 3^2$

$2x - 5 = 9$

$2x = 14$

$x = 7$

The solution is $x = 7$.

Check

$\log_3 (2 \cdot 7 - 5) \stackrel{?}{=} 2$

$\log_3 9 \stackrel{?}{=} 2$

$2 = 2$ ✓

33. $\ln x + \ln (x + 2) = 3$

$\ln [x(x + 2)] = 3$

$e^{\ln[x(x+2)]} = e^3$

$x(x + 2) = e^3$

$x^2 + 2x = e^3$

$x^2 + 2x - e^3 = 0$

$x \approx -5.59$ or $x \approx -3.59$

Check

$\ln (-5.59) + \ln (-5.59 + 2) \stackrel{?}{=} 3$

Because $\ln (-5.59)$ is not defined, -5.59 is not a solution.

$\ln 3.59 + \ln (3.59 + 2) \stackrel{?}{=} 3$

$\ln 3.59 + \ln 5.59 \stackrel{?}{=} 3$

$1.278 + 1.721 \approx 3$ ✓

The apparent solution $x \approx -5.59$ is extraneous. The only solution is $x \approx 3.59$.

5. On page 338, do 5-16 either the odds or evens.

5. $7^{3x+5} = 7^{1-x}$

$3x + 5 = 1 - x$

$4x + 5 = 1$

$4x = -4$

$x = -1$

6. $e^{2x} = e^{3x-1}$

$2x = 3x - 1$

$-x = -1$

$x = 1$

7. $5^{x-3} = 25^{x-5}$

$5^{x-3} = 5^{2(x-5)}$

$x - 3 = 2(x - 5)$

$x - 3 = 2x - 10$

$-x - 3 = -10$

$-x = -7$

$x = 7$

8. $6^{2x-6} = 36^{3x-5}$

$6^{2x-6} = 6^{2(3x-5)}$

$2x - 6 = 2(3x - 5)$

$2x - 6 = 6x - 10$

$-4x - 6 = -10$

$-4x = -4$

$x = 1$

9. $3^x = 7$

$\log_3 3^x = \log_3 7$

$x = \log_3 7$

$x \approx 1.771$

10. $5^x = 33$

$\log_5 5^x = \log_5 33$

$x = \log_5 33$

$x \approx 2.173$

11. $49^{5x+2} = \left(\frac{1}{7}\right)^{11-x}$

$7^{2(5x+2)} = 7^{-1(11-x)}$

$2(5x + 2) = -1(11 - x)$

$10x + 4 = -11 + x$

$9x + 4 = -11$

$9x = -15$

$x = -\frac{5}{3}$

12. $512^{5x-1} = \left(\frac{1}{8}\right)^{-4-x}$

$2^{9(5x-1)} = 2^{-3(-4-x)}$

$9(5x - 1) = -3(-4 - x)$

$45x - 9 = 12 + 3x$

$42x - 9 = 12$

$42x = 21$

$x = \frac{1}{2}$

13. $7^{5x} = 12$

$\log_7(7^{5x}) = \log_7 12$

$5x = \log_7 12$

$x = \frac{1}{5}\log_7 12$

$x \approx 0.255$

14. $11^{6x} = 38$

$\log_{11} 11^{6x} = \log_{11} 38$

$6x = \log_{11} 38$

$x = \frac{1}{6}\log_{11} 38$

$x \approx 0.253$

15. $3e^{4x} + 9 = 15$

$3e^{4x} = 6$

$e^{4x} = 2$

$\ln e^{4x} = \ln 2$

$4x = \ln 2$

$x = \frac{1}{4}\ln 2$

$x \approx 0.173$

16. $2e^{2x} - 7 = 5$

$2e^{2x} = 12$

$e^{2x} = 6$

$\ln e^{2x} = \ln 6$

$2x = \ln 6$

$x = \frac{1}{2}\ln 6$

$x \approx 0.896$

6. On page 339, do 33-40, odds or evens.

33. $\log_2 x + \log_2 (x-2) = 3$

$$\log_2[x(x-2)] = 3$$
$$2^{\log_2[x(x-2)]} = 2^3$$
$$x(x-2) = 8$$
$$x^2 - 2x = 8$$
$$x^2 - 2x - 8 = 0$$
$$(x-4)(x+2) = 0$$
$$x - 4 = 0 \quad or \quad x + 2 = 0$$
$$x = 4 \qquad\qquad x = -2$$

Because $\log_2(-2)$ is not defined, -2 is not a solution. The apparent solution $x = -2$ is extraneous. So, the only solution is $x = 4$.

34. $\log_6 3x + \log_6(x-1) = 3$

$$\log_6[3x(x-1)] = 3$$
$$6^{\log_6[3x(x-1)]} = 6^3$$
$$3x(x-1) = 216$$
$$3x^2 - 3x = 216$$
$$x^2 - x = 72$$
$$x^2 - x - 72 = 0$$
$$(x-9)(x+8) = 0$$
$$x - 9 = 0 \quad or \quad x + 8 = 0$$
$$x = 9 \qquad\qquad x = -8$$

Because $\log_6(-24)$ is not defined, -8 is not a solution. The apparent solution $x = -8$ is extraneous. So, the only solution is $x = 9$.

35. $\ln x + \ln(x+3) = 4$

$$\ln[x(x+3)] = 4$$
$$e^{\ln[x(x+3)]} = e^4$$
$$x(x+3) = e^4$$
$$x^2 + 3x = e^4$$
$$x^2 + 3x - e^4 = 0$$
$$x \approx 6.039 \quad or \quad x \approx -9.039$$

The approximate apparent solution $x \approx -9.039$ is extraneous. So, the only solution is $x \approx 6.039$.

36. $\ln x + \ln(x-2) = 5$

$$\ln[x(x-2)] = 5$$
$$e^{\ln[x(x-2)]} = e^5$$
$$x(x-2) = e^5$$
$$x^2 - 2x = e^5$$
$$x^2 - 2x - e^5 = 0$$
$$x \approx 13.223 \quad or \quad x \approx -11.223$$

The apparent solution $x \approx -11.223$ is extraneous. So, the only solution is $x \approx 13.223$.

37. $\log_3 3x^2 + \log_3 3 = 2$

$$\log_3 9x^2 = 2$$
$$3^{\log_3 9x^2} = 3^2$$
$$9x^2 = 9$$
$$x^2 = 1$$
$$x = \pm 1$$

The solutions are $x = -1$ and $x = 1$.

38. $\log_4(-x) + \log_4(x+10) = 2$

$$\log_4[-x(x+10)] = 2$$
$$4^{\log_4[-x(x+10)]} = 4^2$$
$$-x(x+10) = 16$$
$$-x^2 - 10x = 16$$
$$-x^2 - 10x - 16 = 0$$
$$x^2 + 10x + 16 = 0$$
$$(x+2)(x+8) = 0$$
$$x + 2 = 0 \quad or \quad x + 8 = 0$$
$$x = -2 \qquad\qquad x = -8$$

The solutions are $x = -2$ and $x = -8$.

39. $\log_3(x-9) + \log_3(x-3) = 2$

$$\log_3[(x-9)(x-3)] = 2$$
$$3^{\log_3[(x-9)(x-3)]} = 3^2$$
$$(x-9)(x-3) = 9$$
$$x^2 - 12x + 27 = 9$$
$$x^2 - 12x + 18 = 0$$
$$x \approx 10.24 \quad or \quad x \approx 1.76$$

The apparent solution $x \approx 1.76$ is extraneous. So, the only solution is $x \approx 10.24$.

40. $\log_5(x+4) + \log_5(x+1) = 2$

$$\log_5[(x+4)(x+1)] = 2$$
$$5^{\log_5[(x+4)(x+1)]} = 5^2$$
$$(x+4)(x+1) = 25$$
$$x^2 + 5x + 4 = 25$$
$$x^2 + 5x - 21 = 0$$
$$x \approx 2.72 \quad or \quad x \approx -7.72$$

The apparent solution $x \approx -7.72$ is extraneous. So, the only solution is $x \approx 2.72$.

7. On page 352, do #39.

39. Enter the data into a graphing calculator and perform a logarithmic regression. The model is $s = 3.95 + 27.48 \ln t$. There are about $s = 3.95 + 27.48 \ln 6 \approx 53$ pairs of shoes sold after week 6.

8. On page 353, do 13-15.

13. One thousand wells will produce

$y = 12.263 \ln 1000 - 45.381 \approx 39$ billion barrels.

$$y = 12.263 \ln x - 45.381$$
$$x = 12.263 \ln y - 45.381$$
$$x + 45.381 = 12.263 \ln y$$
$$\frac{x + 45.381}{12.263} = \ln y$$
$$e^{\frac{x+45.381}{12.263}} = y$$
$$e^{(x/12.263) + (45.381/12.263)} = y$$
$$\left(e^{x/12.263}\right)\left(e^{45.381/12.263}\right) = y$$
$$\left(e^{1/12.263}\right)^x \left(e^{45.381/12.263}\right) = y$$
$$40.473(1.085)^x \approx y$$

The function $y \approx 40.473(1.085)^x$ represents the number of wells needed to produce a certain number of billions of barrels of oil.

14. a. The function is $L(x) = 100e^{-0.02x}$.

b. The function in part (a) represents exponential decay since the base, $e^{-0.02} \approx 0.98$, is greater than 0 and less than 1.

c. When the depth is 40 meters, the percent of surface light is $L(40) = 100\, e^{-0.02(40)} \approx 44.9$ or about 44.9%.

15. *Sample answer:* The three ways to find the exponential model are:

1. Use two points and the model $y = ab^x$ to determine the values of a and b.
2. Convert the pairs to $(x, \ln y)$, then solve the related linear equation for y.
3. Enter the points into a graphing calculator and perform exponential regression.

The model is $y = 4200\,(0.89)^x$ and the snowmobile is worth \$2500 in about 4.5 years.

9. Starting with section 5.6 on page 281, and finishing with section 6.7 on page 346, do one word problem of your choice from each set of exercises. Write down the page and question number that you pick.

(answers are shown in the examples)

10. Write down each of the logarithm properties discussed in section 6.5. Then "translate" it into your own words.

(see page 328-329)

11. Make corrections on your 5.6-6.3 quiz. If you scored well, redo five random questions from the quiz.

(see your teacher)

On page 355, do #8 a, b, c, and d.

8a. Cannot be factored. Complete the square or use quadratic formula. Answer: $2 \pm \sqrt{14}$

8b. Square roots method is easiest. Answer: $\pm 2i\sqrt{3}$

8c. Since this one will require foiling the $(x-1)^2$ and then combining like terms, it seems easier to graph. Put the left side of equation in Y_1 and right side into Y_2. Then, solving using 2^{nd} Trace INTERSECT. Answer: x = 0.149 or x = 3.351

8d. Factoring is really easy here. Answer: x = 6 or x = -3