Chapter 6 Homework Problems

1. The basketball player Shaquille O’Neal makes about half of his free throws over an entire season. We will use the calculator to simulate 100 free throws shot independently by a player who has a probability of .5 of making each shot. We let the number 1 represent the outcome “Hit” and 0 represent a “Miss”.

a. Enter the command randInt (0, 1, 100) and then Store it L1. This tells the calculator to randomly select a hit (1) or a miss (0). Press Enter.

b. What percent of the 100 shots were hits?

c. Examine the shot sequence of hits and misses. How long was the longest run of shots made? Of shots missed?

2. You read in a book on poker that the probability of being dealt three of a kind in a 5-card poker hand is 1/50. Explain in simple language what that means.

3. Probability is a measure of how likely an event is to occur. Match one of the probabilities that follow with each statement about an event.

0 .2 .4 .6 .9 1

a. The event is impossible and will never occur.

b. The event is certain. It will occur on every trial of the random phenomenon.

c. The event is very unlikely, but it will occur once in a while in a long trial of sequences.

d. The event will occur more often than not.

4. In each of the following situations, describe a sample space S for the indicated random phenomenon. In some cases, you have some freedom in your choice of S.

a. A seed is planted in the ground. It either germinates or fails to grow.

b. A patient with a usually fatal form of cancer is given a new treatment. The response variable is the length of time that the patient lives after treatment.

c. A student enrolled in a statistics course and at the end of the semester receives a letter grade.

d. A basketball player shoots 4 free throws. You record the number of baskets that she makes.

e. A basketball player shoots four free throws. You record the sequence of hits and misses.

f. A year after knee surgery, a patient is asked to rate the amount of pain in the knee. A seven-point scale is used, with 1 corresponding to no pain and 7 corresponding to extreme pain.

5. For the following, use a tree diagram to determine the outcomes in the sample space. Then, write the sample space in set notation.

a. Toss 2 coins.

b. Toss 3 coins.

c. Toss 4 coins.

6. List the sample space for tossing two, 6-sided dice.

7. All human blood can by typed as one of O, A, B, or AB, but the distribution of the types varies a bit with race. Here is the distribution of the blood type of a randomly chosen black American:

Blood Type	O	A	B	AB
Probability	.49	.27	.20	?

A. What is the probability of type AB blood? Why?

B. Maria has type B blood. She can safely receive blood transfusions from people with blood types O and B. What is the probability that a randomly chosen black American can donate blood to Maria?

8. A sociologist studying social mobility in Denmark finds that the probability that the son of a lower-class father remains in a lower class is .46. What is the probability that the son moves to one of the higher classes?

9. Government data assign a single cause for each death that occurs in the United States. The data shows that the probability is 0.45 that a randomly chosen death was due to cardiovascular disease, and 0.22 that it was due to cancer. What is the probability that a death was due either to cardiovascular disease or to cancer? What is the probability that the death was due to some other cause?

10. The New York Times (August 21, 1989) reported a poll that interviewed a random sample of 1025 women. The married women in the sample were asked whether their husbands did their fair share of household chores. Here are the results:

Outcome	Probability
Does more than his fair share	0.12
Does his fair share	0.61
Does less than his fair share	

These proportions are probabilities for the random phenomenon of choosing a married woman at random and asking her opinion.

A. What must be the probability that the women chosen says that her husband does less than his fair share? Why?

B. The event “I think my husband does at least his fair share” contains the first outcomes. What is its probability?

11. Choose an acre of land in Canada at random. The probability is 0.35 that it is a forest and 0.03 that it is pasture.

A. What is the probability that the acre chosen is not forested?

B. What is the probability that it is forested or pastured?

C. What is the probability that a randomly chosen acre in Canada is something other than forest or pasture?

12. Select a first-year college student at random and ask what his or her academic rank was in high school. Here are the probabilities, based on proportions from a large sample of first-year students:

Rank	Top 20%	2nd 20%	3rd 20%	4th 20%	Lowest 20%
Probability	.41	.23	.29	.06	.01

A. What is the sum of these probabilities? Why would you expect the sum to have this value?

B. What is the probability that a randomly chosen first-year college student was not in the top 20% of his or her high school class?

C. What is the probability that a first-year student was in the top 40% in high school?

13. A general can plan a campaign to fight one major battle or three small battles. He believes that he has a probability 0.6 of winning the large battle and probability .8 of winning each of the small battles. Victories or defeats in the small battles or all three battles are independent. The general must win wither the large battle or three small battles to win the campaign. Which strategy should he choose?

14. An automobile manufacturer buys computer chips from a supplier. The supplier sends a shipment containing 5% defective chips. Each chip chosen from this shipment has a probability of 0.05 of being defective, and each automobile uses 12 chips selected independently. What is the probability that all 12 chips in a car will work properly?

15. A string of Christmas lights contains 20 lights. The lights are wired in a series, so that if any light fails the whole string will go dark. Each light has probability 0.02 of failing during a 3-year period. The lights fail independently of each other. What is the probability that the string of lights will remain bright for 3 years?

16. An athlete suspected of having used steroids is given two tests that operate independently of each other. Test A has probability 0.9 of being positive if steroids have been used. Test B has probability

0.8 of being positive if steroids have been used. What is the probability of neither test being positive if steroids have been used?

17. In each of the following situations, state whether or not the given assignment of probabilities to individual outcomes is legitimate, that is satisfies the rules of probability. If not, give specific reasons for your answer.

A. When a coin is spun, P(H)=0.55 and P(T)=.45.

B. When two coins are tossed, P(HH)=0.4, P(HT)=0.4, P(TH)=0.4 and P(TT)=0.4.

C. When a die is rolled, the number of spots on the up-face has P(1)=1/2, P(4)=1/6, P(5)=1/6, and P(6)=1/6.

18. Choose a new car or light truck at random and note its color. Here are the probabilities of the most popular colors for vehicles made in North America in 2000:

Color:	Silver	White	Black	Dark Green	Dark Blue	Medium Red
Probability:	0.176	0.172	0.113	0.089	0.088	0.067

A. What is the probability that the vehicle you choose has any color other than the six listed?

B. What is the probability that a randomly chosen vehicle is either silver or white?

C. Choose two vehicles at random. What is the probability that both are silver or white?

19. Choose an American worker at random and classify his or her occupation into one of the following classes. These classes are used in government employment data.

A Managerial and professional
B Technical, sales, administrative support
C Service occupations
D Precision production, craft, and repair
E Operators, fabricators, and laborers
F Farming, forestry, fishing

The table below gives the probabilities that a randomly chosen worker falls into each of 12 sex-by-occupation classes:

	A	B	C	D	E	F
Male	.14	.11	.06	.11	.12	.03
Female	.09	.20	.08	.01	.04	.01

A. Verify that this is a legitimate assignment of probabilities to these outcomes.

B. What is the probability that the worker is female?

C. What is the probability that the worker is not engaged in farming, forestry, or fishing?

D. Classes D and E include most mechanical and factory jobs. What is the probability that the worker holds a job in one of these classes?

E. What is the probability that the worker does not hold a job in Classes D or E?

20. A roulette wheel has 38 slots, 0, 00, and 1 to 36. The slots 0 and 00 are colored green, 18 of the other are red, and 18 are black. The dealer spins the wheel and at the same time rolls a small ball along the wheel in the opposite direction. The wheel is carefully balanced so that the ball equally likely to land in any slot when the wheel slows. Gamblers can bet on various combinations of numbers and colors.

A. What is the probability that the ball will land in any one slot?

B. If you bet on "red", you win if the ball lands in a red slot. What is the probability of winning?

C. The slot numbers are laid out on a board which gamblers place their bets. One column if numbers on the board contains all multiples of 3, that is, 3, 6, 9, …, 36. You place a "column bet" that wins if any of these numbers comes up. What is your probability of winning?

21. A six-sided die has four green and two red faces and is balanced so that each face is equally likely to come-up. The die will be rolled several times. You must choose one of the following three sequences of colors; you will win $25 if the first rolls of the die give the sequence you have chosen.

RGRRR
RGRRRG
GRRRRR

Which sequence do you chose? Explain your choice. (In a psychological study, 63% of 260 students who had not studied probability chose the second sequence. Were they correct?)

22. The gene for albinism in humans is recessive. That is, carriers of this gene have probability ½ of passing it on to a child, and the child is albino only if both parents pass the albinism gene. Parents pass their genes independently of each other. If both parents carry the albinism gene, what is the probability that their first child is albino? If they have two children (who inherit independently of each other), what is the probability that both are albino? That neither are albino?

23. Call a household prosperous if its income exceeds $100,000. Call the household educated if the householder completed college. Select an American household at random, and let A be the event that the selected household is prosperous and B the event that it is educated. According to the Census Bureau, P(A)=0.134, P(B)=0.254, and the joint probability that a household is both prosperous and educated is P(A and B)=.0800. What is the probability P(A or B) that the household selected is either prosperous or educated?

24. Draw a Venn diagram that shows the relation between events A and B from exercise 23. Indicate each of the following events on your diagram and use the information to calculate the probability of each event. Finally, describe in words what each event is.

A. $\{A \text{ and } B\}$

C. $\{A^C \text{ and } B\}$

B. $\{A\ and\ B^C\}$ D. $\{A^C\ and\ B^C\}$

25. Consolidated Builders has bid on two large construction projects. The company president believes that the probability of winning the first contract that the probability of winning the first contract (event A) is 0.6, that the probability of winning the second (event B) is 0.5, and that the joint probability of winning both jobs (event {*A and B*}) is 0.3. What is the probability of the event {*A or B*} that Consolidated will win at least one of the jobs?

26. In the setting of the previous exercise, are events *A and B* independent? Do the calculation that proves it.

27. Draw a Venn diagram for your work in exercise 25.

A. Consolidated wins both jobs.

B. Consolidated wins the first job but not the second.

C. Consolidated does not win the first job but does win the second.

D. Consolidated does not win either job.

28. Common sources of caffeine are coffee, tea, and cola drinks.
Suppose that

55% of adults drink coffee
25% of adults drink tea
45% of adults drink cola

and also that

15% drink both coffee and tea
5% drink all three beverages
25% drink both coffee and cola
5% drink only tea

Draw a Venn diagram marked with this information. Use it along with the addition rules to answer the following questions.

A. What percent of adults drink ONLY cola?

B. What percent drink none of these beverages?

29. Musical styles other than rock and pop are becoming more popular. A survey of college students finds that 40% like country music, 30% gospel music, and 10% like both.

A. Make a Venn diagram with these results.

B. What percent of college students like country but not gospel?

C. What percent like neither country nor gospel?

30. Choose an employed person at random. Let A be the event that the person chosen is a woman, and B the event that the person holds a managerial job. Government data tells us that *P(A)*=.46 and the probability of managerial and professional jobs amongst women is *P(B|A)*=.32. Find the probability that a randomly chosen employed person is a woman holding a managerial or professional position.

31. From the table below, choose an adult American woman at random. Use the information to answer the following questions.

		AGE (in thousands)		
	18 to 24	**25 to 64**	**65 and over**	**Total**
Married	3,046	48,116	7,767	58,929
Never Married	9,289	9,252	768	19,309
Widowed	19	2,425	8,636	11,080
Divorced	260	8,916	1,091	10,267
Total	12,614	68,709	18,262	99,585

A. What is the probability that the women chosen is 65 years or older?

B. What is the conditional probability that the women chosen is married, given that she is 65 or over?

C. How many are both married and in the over-65 age group? What is the joint probability that the women we choose is married woman at least 65 years old?

D. Verify that the three probabilities you found in A, B, and C satisfy the general population rule.

E. Find the probability that the woman chosen is a widow

F. The conditional probability that the woman chosen is a widow, given that she is at least 65 years old.

G. The conditional probability that the woman chosen is a widow, given that she is between 25 and 64 years old.

H. Are the events "widow" and "at least 65 years old" independent? How do you know?

I. What is the conditional probability that the woman chosen is 18 to 24 years old, given that she is married?

J. Earlier, we found the *P(married|age 18 to 21)*=.241. Complete this sentence: 0.241 is the proportion of women who are _______ among those women who are _______.

32. Here are the counts are the counts (in thousands) of earned degrees in the United States in a recent year, classified by level and by sex of the degree recipient:

	Bachelor's	**Master's**	**Professional**	**Doctorate**	**Total**
Female	616	194	30	16	856
Male	529	171	44	26	770
Total	1145	365	74	42	1626

A. If you choose a degree recipient at random, what is the probability that the person you choose is a woman?

B. What is the conditional probability that you choose a woman, given that the person chosen received a professional degree?

C. Are the events "choose a woman" and "choose a professional degree recipient" independent? How do you know?

33. ELISA tests are used to screen donated blood for the presence of the AIDS virus. The test actually detects antibodies, substances that the body produces when the virus is present. When the antibodies are present, ELISA is positive with the probability about 0.997 and negative with probability 0.003. When the blood tested is not contaminated with AIDS antibodies, ELISA gives a positive result with the probability about 0.015 and a negative result with probability 0.985. (Because ELISA is designed to keep the AIDS virus out of blood supplies, the higher the probability 0.015 of a false positive is acceptable in exchange for the low probability 0.003 of failing to detect contaminated blood. These probabilities depend on the expertise of the particular laboratory doing the test.) Suppose 1% of a large population carries the AIDS antibody in their blood.

A. Draw a tree diagram for selecting a person from this population (outcomes: the person does or does not carry the AIDS antibody) and for testing his or her blood (outcomes: positive or negative).

B. What is the probability that the ELISA test for AIDS is positive for a randomly chosen person from this population?

C. What is the probability that a person has the antibody given that the ELISA test is positive? (This exercise illustrates a fact that is important when considering proposals for widespread testing for AIDS or illegal drugs: if the condition being tested is uncommon in the population, most positive will be false positives.)