

CHAPTER 6

Polynomials and Polynomial Functions

Then

In Chapter 5, you graphed quadratic functions and solved quadratic equations.

Now

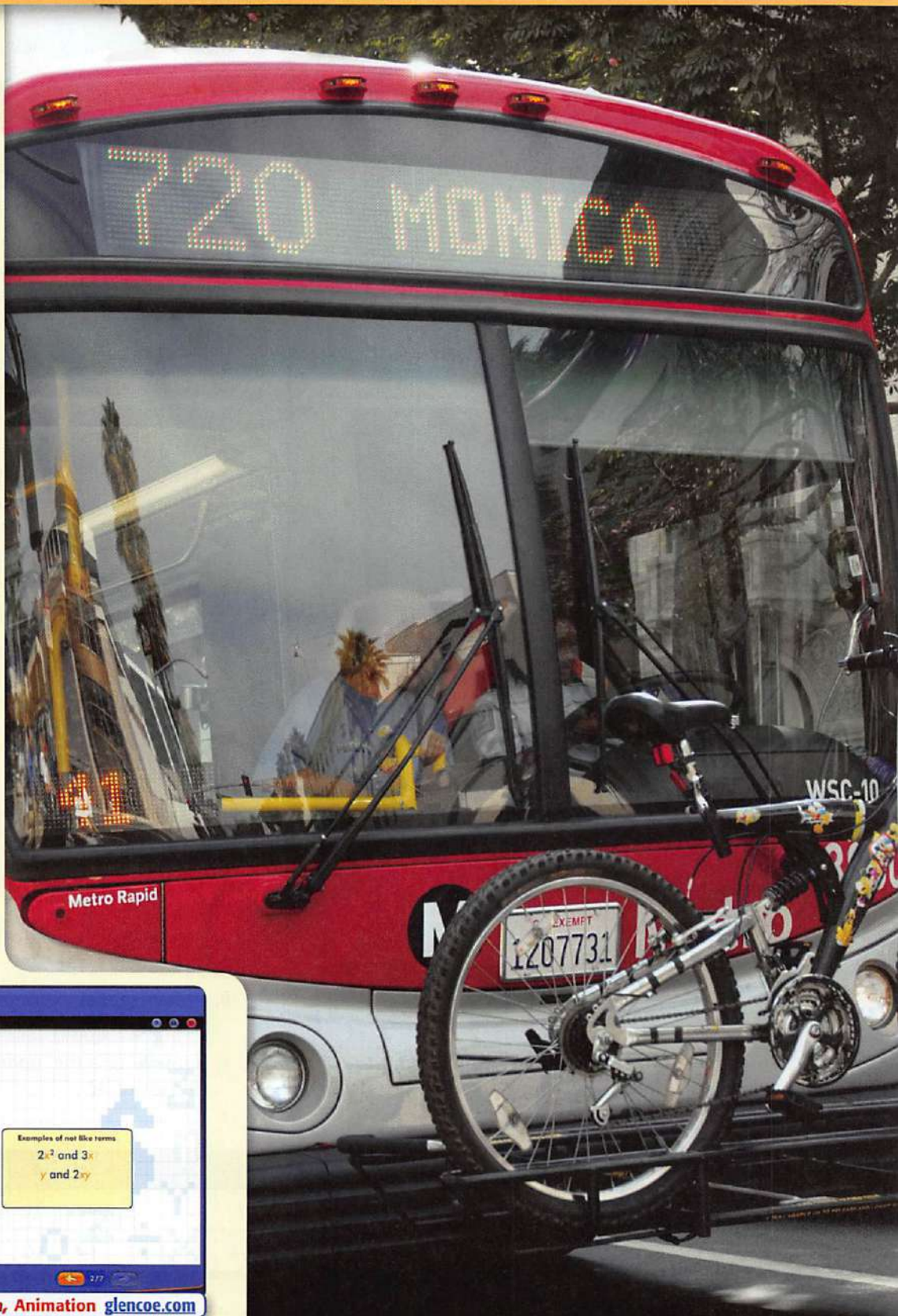
In Chapter 6, you will:

- Add, subtract, multiply, divide, and factor polynomials.
- Analyze and graph polynomial functions.
- Evaluate polynomial functions and solve polynomial equations.
- Find factors and zeros of polynomial functions.

Why?

TRANSPORTATION

Polynomial functions can be used to determine bus schedules, highway capacity, traffic patterns, average fuel costs, and the prices of new and used cars.



Polynomials and Polynomial Functions
Activity

In order to work with polynomials, you need to be able to add, subtract, multiply and divide them.

Examples of like terms	Examples of not like terms
$2x$ and $-6x$	$2x^2$ and $3x$
$15x^2$ and $2x^2$	y and $2xy$
y and $4y$	

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Get Ready for Chapter 6

Diagnose Readiness You have two options for checking Prerequisite Skills.

Text Option

Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Rewrite each difference as a sum. (Lesson 1-2)

- $-5 - 13$
- $5 - 3y$
- $5mr - 7mp$
- $3x^2y - 14xy^2$
- PARTIES** Twenty people attended a going away party for Zach. The guests left in groups of 2. By 9:00, x groups had left. Rewrite the number of guests remaining at 9:00 as a sum.

Use the Distributive Property to rewrite each expression without parentheses. (Lesson 1-2)

- $-4(a + 5)$
- $-1(3b^2 + 2b - 1)$
- $-\frac{1}{2}(2m - 5)$
- $-\frac{3}{4}(3z + 5)$
- MONEY** Mr. Chávez is buying pizza and soda for the members of the science club. A slice of pizza costs \$2.25, and a soda costs \$1.25. Write an expression to represent the amount that Mr. Chávez will spend on 15 students. Evaluate the expression by using the Distributive Property.

Solve each equation. (Lesson 5-6)

- $x^2 + 2x - 8 = 0$
- $2x^2 + 7x + 3 = 0$
- $6x^2 + 5x - 4 = 0$
- $4x^2 - 2x - 1 = 0$
- PHYSICS** If an object is dropped from a height of 50 feet above the ground, then its height after t seconds is given by $h = -16t^2 + 50$. Use the equation $0 = -16t^2 + 50$ to find how long it will take until the ball reaches the ground.

QuickReview

EXAMPLE 1

Rewrite $2xy - 3 - z$ as a sum.

$$\begin{aligned} 2xy - 3 - z & \quad \text{Original expression} \\ &= 2xy + (-3) + (-z) \quad \text{Rewrite using addition.} \end{aligned}$$

EXAMPLE 2

Use the Distributive Property to rewrite $-3(a + b - c)$.

$$\begin{aligned} -3(a + b - c) & \quad \text{Original expression} \\ &= -3(a) + (-3)(b) + (-3)(-c) \quad \text{Distributive Property} \\ &= -3a - 3b + 3c \quad \text{Simplify.} \end{aligned}$$

EXAMPLE 3

Solve $2x^2 + 8x + 1 = 0$.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Quadratic Formula} \\ &= \frac{-8 \pm \sqrt{8^2 - 4(2)(1)}}{2(2)} \quad a = 2, b = 8, c = 1 \\ &= \frac{-8 \pm \sqrt{56}}{4} \quad \text{Simplify.} \\ &= -2 \pm \frac{\sqrt{14}}{2} \quad \sqrt{56} = \sqrt{4 \cdot 14} \text{ or } 2\sqrt{14} \end{aligned}$$

The exact solutions are $-2 + \frac{\sqrt{14}}{2}$ and $-2 - \frac{\sqrt{14}}{2}$.
The approximate solutions are -0.13 and -3.87 .

Online Option

Math Online

Take a self-check Chapter Readiness Quiz at glencoe.com.

Get Started on Chapter 6

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 6. To get ready, identify important terms and organize your resources. You may wish to refer to **Chapter 0** to review prerequisite skills.

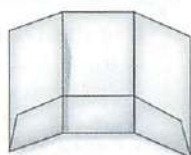
FOLDABLES® Study Organizer

Polynomials and Polynomial Functions Make this Foldable to help you organize your Chapter 6 notes about polynomials and polynomial functions. Begin with one sheet of $8\frac{1}{2}$ " by 14" paper.

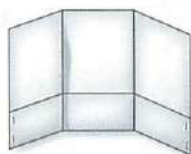
- 1** Fold a 2" tab along the bottom of a long side.



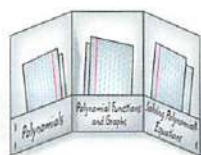
- 2** Fold along the width into thirds.



- 3** Staple the outer edges of the tab.



- 4** Label the tabs *Polynomials*, *Polynomial Functions and Graphs*, and *Solving Polynomial Equations*.



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- Study the chapter online
- Explore **Math in Motion**
- Get extra help from your own **Personal Tutor**
- Use **Extra Examples** for additional help
- Take a **Self-Check Quiz**
- Review Vocabulary** in fun ways

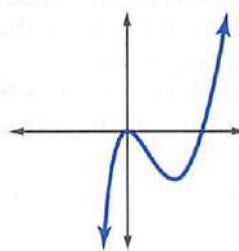
New Vocabulary

English	Español
simplify	• p. 333 • reducir
degree of a polynomial	• p. 335 • grado de un polinomio
synthetic division	• p. 342 • división sintética
polynomial in one variable	• p. 348 • polinomio de una variable
leading coefficient	• p. 348 • coeficiente líder
polynomial function	• p. 349 • función polinomial
power function	• p. 349 • función potencia
end behavior	• p. 350 • comportamiento final
relative maximum	• p. 358 • máximo relativo
relative minimum	• p. 358 • mínimo relativo
extrema	• p. 358 • extrema
turning points	• p. 358 • momentos cruciales
prime polynomials	• p. 368 • polinomios primeros
quadratic form	• p. 371 • forma de ecuación cuadrática
synthetic substitution	• p. 377 • sustitución sintética
depressed polynomial	• p. 379 • polinomio reducido

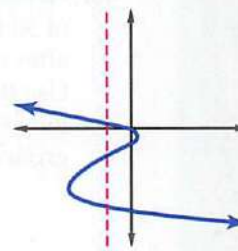
Review Vocabulary

factoring • p. 368 • **factorización** to express a polynomial as the product of monomials and polynomials

function • p. 348 • **función** a relation in which each element of the domain is paired with exactly one element in the range



Function



Not a Function

polynomial • p. 335 • **polinomio** a monomial or sum of monomials

[Multilingual eGlossary glencoe.com](http://glencoe.com)

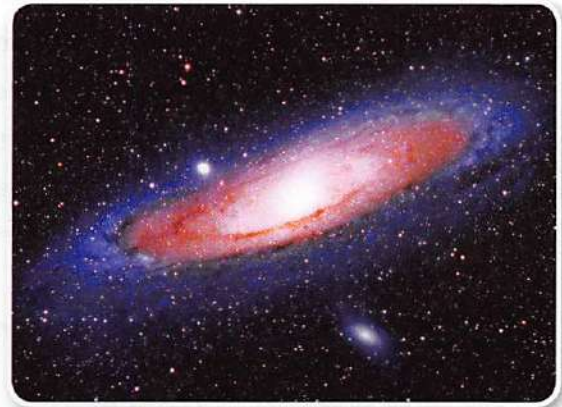
6-1

Operations with Polynomials

Why?

The light from the Sun takes approximately 8 minutes to reach Earth. So if you are outside right now you are basking in sunlight that the Sun emitted approximately 8 minutes ago.

Light travels very fast, at a speed of about 3×10^8 meters per second. How long would it take light to get here from the Andromeda galaxy, which is approximately 2.367×10^{21} meters away?



Then

You evaluated powers.
(Lesson 1-1)

Now

- Multiply, divide, and simplify monomials and expressions involving powers.
- Add, subtract, and multiply polynomials.

New Vocabulary

simplify
degree of a polynomial

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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help
- Math in Motion

Multiply and Divide Monomials To **simplify** an expression containing powers means to rewrite the expression without parentheses or negative exponents. Negative exponents are a way of expressing the multiplicative inverse of a number. The following table summarizes the properties of exponents.

Concept Summary		Properties of Exponents	For Your FOLDABLE
For any real numbers x and y , integers a and b :			
Property	Definition	Examples	
Product of Powers	$x^a \cdot x^b = x^{a+b}$	$3^2 \cdot 3^4 = 3^{2+4}$ or 3^6 $p^2 \cdot p^9 = p^{2+9}$ or p^{11}	
Quotient of Powers	If $x \neq 0$, $\frac{x^a}{x^b} = x^{a-b}$.	$\frac{9^5}{9^2} = 9^{5-2}$ or 9^3 $\frac{b^6}{b^4} = b^{6-4}$ or b^2	
Negative Exponent	$x^{-a} = \frac{1}{x^a}$ and $\frac{1}{x^{-a}} = x^a$, $x \neq 0$	$3^{-5} = \frac{1}{3^5}$ $\frac{1}{b^{-7}} = b^7$	
Power of a Power	$(x^a)^b = x^{ab}$	$(3^3)^2 = 3^{3 \cdot 2}$ or 3^6 $(d^2)^4 = d^{2 \cdot 4}$ or d^8	
Power of a Product	$(xy)^a = x^a y^a$	$(2k)^4 = 2^4 k^4$ or $16k^4$ $(ab)^3 = a^3 b^3$	
Power of a Quotient	$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$, $y \neq 0$, and $\left(\frac{x}{y}\right)^{-a} = \left(\frac{y}{x}\right)^a$ or $\frac{y^a}{x^a}$, $x \neq 0$, $y \neq 0$	$\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$ $\left(\frac{a}{b}\right)^{-5} = \frac{b^5}{a^5}$	
Zero Power	$x^0 = 1$, $x \neq 0$	$7^0 = 1$	

Recall that a *monomial* is a number, a variable, or an expression that is the product of one or more variables with nonnegative integer exponents.

When simplifying a monomial, check to be sure that it has been simplified fully.

Key Concept

Simplifying Monomials

For Your
FOLDABLE

A monomial expression is in simplified form when:

- there are no powers of powers,
- each base appears exactly once,
- all fractions are in simplest form, and
- there are no negative exponents.

Problem-Solving Tip

Check You can always check your answer using the definition of exponents.

$$\begin{aligned} \frac{q^2}{q^7} &= \frac{q \cdot q}{q \cdot q \cdot q \cdot q \cdot q \cdot q \cdot q} \\ &= \frac{1}{q^5} \end{aligned}$$

EXAMPLE 1 Simplify Expressions

Simplify each expression. Assume that no variable equals 0.

a. $(2a^{-2})(3a^3b^2)(c^{-2})$

$$(2a^{-2})(3a^3b^2)(c^{-2})$$

Original expression

$$= 2\left(\frac{1}{a^2}\right)(3a^3b^2)\left(\frac{1}{c^2}\right)$$

Definition of negative exponents

$$= \left(\frac{2}{a \cdot a}\right)(3 \cdot a \cdot a \cdot a \cdot b \cdot b)\left(\frac{1}{c \cdot c}\right)$$

Definition of exponents

$$= \left(\frac{2}{a \cdot a}\right)(3 \cdot a \cdot a \cdot a \cdot b \cdot b)\left(\frac{1}{c \cdot c}\right)$$

Divide out common factors.

$$= \frac{6ab^2}{c^2}$$

Simplify.

b. $\frac{q^2r^4}{q^7r^3}$

$$\frac{q^2r^4}{q^7r^3} = q^{2-7} \cdot r^{4-3}$$

Quotient of powers

$$= q^{-5}r$$

Subtract powers.

$$= \frac{r}{q^5}$$

Simplify.

c. $\left(\frac{-2a^4}{b^2}\right)^3$

$$\left(\frac{-2a^4}{b^2}\right)^3 = \frac{(-2a^4)^3}{(b^2)^3}$$

Power of a quotient

$$= \frac{(-2)^3(a^4)^3}{(b^2)^3}$$

Power of a product

$$= \frac{-8a^{12}}{b^6}$$

Power of a power

Check Your Progress

1A. $(2x^{-3}y^3)(-7x^5y^{-6})$

1B. $\frac{15c^5d^3}{-3c^2d^7}$

1C. $\left(\frac{a}{4}\right)^{-3}$

1D. $(-2x^3y^2)^5$

StudyTip

Power of 1 Remember that a variable with no exponent indicated can be written as a power of 1.

Operations With Polynomials The **degree of a polynomial** is the degree of the monomial with the greatest degree. For example, the degree of the polynomial $x^2 + 4x + 58$ is 2.

EXAMPLE 2 Degree of a Polynomial

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

a. $\frac{1}{4}x^4y^3 - 8x^5$

This expression is a polynomial because each term is a monomial. The degree of the first term is $4 + 3$ or 7, and the degree of the second term is 5. The degree of the polynomial is 7.

b. $\sqrt{x} + x + 4$

This expression is not a polynomial because \sqrt{x} is not a monomial.

c. $x^{-3} + 2x^{-2} + 6$

This expression is not a polynomial because x^{-3} and x^{-2} are not monomials:

$x^{-3} = \frac{1}{x^3}$ and $x^{-2} = \frac{1}{x^2}$. Monomials cannot contain variables in the denominator.

Check Your Progress

2A. $\frac{x}{y} + 3x^2$

2B. $x^5y + 9x^4y^3 - 2xy$

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You can simplify a polynomial just like you simplify a monomial. Perform the operations indicated, and combine like terms.

StudyTip

Alternative Methods Notice that Example 3a uses a horizontal method, and Example 3b uses a vertical method to simplify. Either method will yield a correct solution.

EXAMPLE 3 Simplify Polynomial Expressions

Simplify each expression.

a. $(4x^2 - 5x + 6) - (2x^2 + 3x - 1)$

Remove parentheses, and group like terms together.

$$(4x^2 - 5x + 6) - (2x^2 + 3x - 1)$$

$$= 4x^2 - 5x + 6 - 2x^2 - 3x + 1$$

Distribute the -1 .

$$= (4x^2 - 2x^2) + (-5x - 3x) + (6 + 1)$$

Group like terms.

$$= 2x^2 - 8x + 7$$

Combine like terms.

b. $(6x^2 - 7x + 8) + (-4x^2 + 9x - 5)$

Align like terms vertically and add.

$$\begin{array}{r} 6x^2 - 7x + 8 \\ (+) -4x^2 + 9x - 5 \\ \hline 2x^2 + 2x + 3 \end{array}$$

Check Your Progress

3A. $(-x^2 - 3x + 4) - (x^2 + 2x + 5)$

3B. $(3x^2 - 6) + (-x + 1)$

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You can use the Distributive Property to multiply polynomials.

EXAMPLE 4 Simplify by Using the Distributive Property

Find $3x(2x^2 - 4x + 6)$.

$$\begin{aligned} 3x(2x^2 - 4x + 6) &= 3x(2x^2) + 3x(-4x) + 3x(6) && \text{Distributive Property} \\ &= 6x^3 - 12x^2 + 18x && \text{Multiply the monomials.} \end{aligned}$$

Check Your Progress

Find each product.

4A. $\frac{4}{3}x^2(6x^2 + 9x - 12)$

4B. $-2a(-3a^2 - 11a + 20)$

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Polynomials can be used to represent real-world situations.



Real-World Career

Truck Driver

Truck drivers are considered technical professionals because they are required to obtain specialized education and professional licensure. Although state motor vehicle departments administer the Commercial Driver's License program, federal law spells out the requirements to obtain one.

Real-World EXAMPLE 5 Write a Polynomial Expression

DRIVING The U.S. Department of Transportation limits the time a truck driver can work between periods of rest to ten hours. For the first part of his shift, Tom drives at a speed of 60 miles per hour, and for the second part of the shift, he drives at a speed of 70 miles per hour. Write a polynomial to represent the distance driven.

Words 60 mph for some time, and 70 mph for the rest

Variable Let x = the number of hours he drives at 60 miles per hour.

Expression $60x + 70(10 - x)$

$$\begin{aligned} 60x + 70(10 - x) & \text{Original expression} \\ = 60x + 700 - 70x & \text{Distributive Property} \\ = 700 - 10x & \text{Combine like terms.} \end{aligned}$$

The polynomial is $700 - 10x$.

Check Your Progress

5. Paul has \$900 to invest in a savings account that has an annual interest rate of 1.8%, and a money market account that pays 4.2% per year. Write a polynomial for the interest he will earn in one year if he invests x dollars in the savings account.

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EXAMPLE 6 Multiply Polynomials

Find $(n^2 + 4n - 6)(n + 2)$.

$$\begin{aligned} (n^2 + 4n - 6)(n + 2) & \\ = n^2(n + 2) + 4n(n + 2) + (-6)(n + 2) & \text{Distributive Property} \\ = n^2 \cdot n + n^2 \cdot 2 + 4n \cdot n + 4n \cdot 2 + (-6) \cdot n + (-6) \cdot 2 & \text{Distributive Property} \\ = n^3 + 2n^2 + 4n^2 + 8n - 6n - 12 & \text{Multiply monomials.} \\ = n^3 + 6n^2 + 2n - 12 & \text{Combine like terms.} \end{aligned}$$

Check Your Progress

Find each product.

6A. $(x^2 + 4x + 16)(x - 4)$

6B. $(2x^2 - 4x + 5)(3x - 1)$

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Check Your Understanding

Example 1
p. 334

Simplify. Assume that no variable equals 0.

1. $(2a^3b^{-2})(-4a^2b^4)$ 2. $\frac{12x^4y^2}{2xy^5}$ 3. $\left(\frac{2a^2}{3b}\right)^3$ 4. $(6g^5h^{-4})^3$

Example 2
p. 335

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

5. $3x + 4y$ 6. $\frac{1}{2}x^2 - 7y$ 7. $x^2 + \sqrt{x}$ 8. $\frac{ab^3 - 1}{az^4 + 3}$

Examples 3, 4, and 6
pp. 335–336

Simplify.

9. $(x^2 - 5x + 2) - (3x^2 + x - 1)$ 10. $(3a + 4b) + (6a - 6b)$
11. $2a(4b + 5)$ 12. $3x^2(2xy - 3xy^2 + 4x^2y^3)$
13. $(n - 9)(n + 7)$ 14. $(a + 4)(a - 6)$

Example 5
p. 336

15. **EXERCISE** Tara exercises 75 minutes a day. She does cardio, which burns an average of 10 Calories a minute, and weight training, which burns an average of 7.5 Calories a minute. Write a polynomial to represent the amount of Calories Tara burns in one day if she does x minutes of weight training.

Practice and Problem Solving

 = **Step-by-Step Solutions** begin on page R20.
Extra Practice begins on page 947.

Example 1
p. 334

Simplify. Assume that no variable equals 0.

16. $(5x^3y^{-5})(4xy^3)$ 17. $(-2b^3c)(4b^2c^2)$ 18. $\frac{a^3n^7}{an^4}$ 19. $\frac{-y^3z^5}{y^2z^3}$
20. $\frac{-7x^5y^3z^4}{21x^7y^5z^2}$ 21. $\frac{9a^7b^5c^5}{18a^5b^9c^3}$ 22. $(n^5)^4$ 23. $(z^3)^6$

Example 2
p. 335

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

24. $2x^2 - 3x + 5$ 25. $a^3 - 11$ 26. $\frac{5np}{n^2} - \frac{2g}{h}$ 27. $\sqrt{m - 7}$

Examples 3, 4, and 6
pp. 335–336

Simplify.

28. $(6a^2 + 5a + 10) - (4a^2 + 6a + 12)$ 29. $(7b^2 + 6b - 7) - (4b^2 - 2)$
30. $3p(np - z)$ 31. $4x(2x^2 + y)$
32. $(x - y)(x^2 + 2xy + y^2)$ **33** $(a + b)(a^3 - 3ab - b^2)$
34. $4(a^2 + 5a - 6) - 3(2a^3 + 4a - 5)$ 35. $5c(2c^2 - 3c + 4) + 2c(7c - 8)$
36. $5xy(2x - y) + 6y^2(x^2 + 6)$ 37. $3ab(4a - 5b) + 4b^2(2a^2 + 1)$
38. $(x - y)(x + y)(2x + y)$ 39. $(a + b)(2a + 3b)(2x - y)$

Example 5
p. 336

40. **PAINTING** Connor has hired two painters to paint his house. The first painter charges \$12 an hour and the second painter charges \$11 an hour. It will take 15 hours to paint the house.

- Write a polynomial to represent the total cost of the job if Connor hires the first painter for x hours.
- Write a polynomial to represent the total cost of the job if Connor hires the second painter for y hours.



Real-World Link

The environment on Mars is very harsh. Mars has unpredictable weather patterns, as well as the deepest canyon and largest volcano in the solar system.

Source: NASA

Simplify. Assume that no variable equals 0.

41. $\left(\frac{8x^2y^3}{24x^3y^2}\right)^4$

42. $\left(\frac{12a^3b^5}{4a^6b^3}\right)^3$

43. $\left(\frac{4x^{-2}y^3}{xy^{-4}}\right)^{-2}$

44. $\left(\frac{5a^{-7}b^2}{ab^{-6}}\right)^{-3}$

45. $(a^2b^3)(ab)^{-2}$

46. $(-3x^3y)^2(4xy^2)$

47. $\frac{3c^2d(2c^3d^5)}{15c^4d^2}$

48. $\frac{-10g^6h^9(g^2h^3)}{30g^3h^3}$

49. $\frac{5x^4y^2(2x^5y^6)}{20x^3y^5}$

50. $\frac{-12n^7p^5(n^2p^4)}{36n^6p^7}$

51 ASTRONOMY Refer to the beginning of the lesson.

- How long does it take light from Andromeda to reach Earth?
- The average distance from the Sun to Mars is approximately 2.28×10^{11} meters. How long does it take light from the Sun to reach Mars?

Simplify.

52. $\frac{1}{4}g^2(8g + 12h - 16gh^2)$

53. $\frac{1}{3}n^3(6n - 9p + 18np^4)$

54. $x^{-2}(x^4 - 3x^3 + x^{-1})$

55. $a^{-3}b^2(ba^3 + b^{-1}a^2 + b^{-2}a)$

56. $(g^3 - h)(g^3 + h)$

57. $(n^2 - 7)(2n^3 + 4)$

58. $(2x - 2y)^3$

59. $(4n - 5)^3$

60. $(3z - 2)^3$

61 EDUCATION The polynomials $0.108x^2 - 0.876x + 474.1$ and $0.047x^2 + 9.694x + 361.7$ approximate the number of bachelor's degrees, in thousands, earned by males and females, respectively, where x is the number of years after 1971.

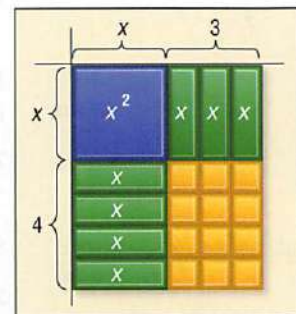
- Find the polynomial that represents the total number of bachelor's degrees (in thousands) earned by both men and women.
- Find the polynomial that represents the difference between bachelor's degrees earned by men and by women.

62. If $5^k + 7 = 5^{2k} - 3$, what is the value of k ?

63. What value of k makes $q^{41} = q^{4k} \cdot q^5$ true?

64 MULTIPLE REPRESENTATIONS Use the model at the right that represents the product of $x + 3$ and $x + 4$.

- GEOMETRIC** The area of the each rectangle is the product of its length and width. Use the model to find the product of $x + 3$ and $x + 4$.
- ALGEBRAIC** Use FOIL to find the product of $x + 3$ and $x + 4$.
- VERBAL** Explain how each term of the product is represented in the model.



H.O.T. Problems

Use Higher-Order Thinking Skills

- PROOF** Show how the property of negative exponents can be proven using the Quotient of Powers Property and the Zero Power Property.
- CHALLENGE** What happens to the quantity of x^{-y} as y increases, for $y > 0$ and $x > 1$?
- REASONING** Explain why the expression 0^{-2} is undefined.
- OPEN ENDED** Write three different expressions that are equivalent to x^{12} .
- WRITING IN MATH** Explain why properties of exponents are useful in astronomy. Include an explanation of how to find the amount of time it takes for light from a source to reach a planet.

Standardized Test Practice

70. **SHORT RESPONSE** Simplify $\frac{(2x^2)^3}{12x^4}$.

71. **STATISTICS** For the numbers a , b , and c , the average (arithmetic mean) is twice the median. If $a = 0$ and $a < b < c$, what is the value of $\frac{c}{b}$?

- A 2 C 4
B 3 D 5

72. Which is not a factor of $x^3 - x^2 - 2x$?

- F x H $x - 1$
G $x + 1$ J $x - 2$

73. **SAT/ACT** The expression $(-6 + i)^2$ is equivalent to which of the following expressions?

- A $-12i$ C $35 - 12i$
B $-12 + i$ D $37 - 12i$

Spiral Review

Solve each inequality algebraically. (Lesson 5-8)

74. $x^2 - 6x \leq 16$

75. $x^2 + 3x > 40$

76. $2x^2 - 12 \leq -5x$

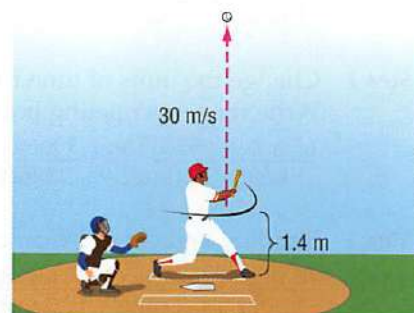
Graph each function. (Lesson 5-7)

77. $y = 3(x - 2)^2 - 4$

78. $y = -2(x + 4)^2 + 3$

79. $y = \frac{1}{3}(x + 1)^2 + 6$

80. **BASEBALL** A baseball player hits a high pop-up with an initial upward velocity of 30 meters per second, 1.4 meters above the ground. The height $h(t)$ of the ball in meters t seconds after being hit is modeled by $h(t) = -4.9t^2 + 30t + 1.4$. How long does an opposing player have to get under the ball if he catches it 1.7 meters above the ground? Does your answer seem reasonable? Explain. (Lesson 5-3)



Evaluate each determinant. (Lesson 4-5)

81. $\begin{vmatrix} 3 & 0 & -2 \\ -1 & 4 & 3 \\ 5 & -2 & -1 \end{vmatrix}$

82. $\begin{vmatrix} -2 & -4 & -6 \\ 0 & 6 & -5 \\ -1 & 3 & -1 \end{vmatrix}$

83. $\begin{vmatrix} -3 & -1 & -2 \\ -2 & 3 & 4 \\ 6 & 1 & 0 \end{vmatrix}$

84. **FINANCIAL LITERACY** A couple is planning to invest \$15,000 in certificates of deposit (CDs). For tax purposes, they want their total interest the first year to be \$800. They want to put \$1000 more in a 2-year CD than in a 1-year CD and then invest the rest in a 3-year CD. How much should they invest in each type of CD? (Lesson 3-5)

Years	1	2	3
Rate	3.4%	5.0%	6.0%

Find the slope of the line that passes through each pair of points. (Lesson 2-3)

85. $(6, -2)$ and $(-2, -9)$

86. $(-4, -1)$ and $(3, 8)$

87. $(3, 0)$ and $(-7, -5)$

88. $(\frac{1}{2}, \frac{2}{3})$ and $(\frac{1}{4}, \frac{1}{3})$

89. $(\frac{2}{5}, \frac{1}{4})$ and $(\frac{1}{10}, \frac{1}{12})$

90. $(-4.5, 2.5)$ and $(-3, -1)$

Skills Review

Factor each polynomial. (Lesson 0-3)

91. $12ax^3 + 20bx^2 + 32cx$

92. $x^2 + 2x + 6 + 3x$

93. $12y^2 + 9y + 8y + 6$

94. $2my + 7x + 7m + 2xy$

95. $8ax - 6x - 12a + 9$

96. $10x^2 - 14xy - 15x + 21y$

Real-world problems often involve units of measure. Performing operations with units is called **dimensional analysis** or **unit analysis**. You can use dimensional analysis to convert units or to perform calculations.

EXAMPLE

A car is traveling at 65 miles per hour. How fast is the car traveling in meters per second?

You want to find the speed in meters per second, so you need to change the unit of distance from miles to meters and the unit of time from hours to seconds. To make the conversion, use fractions that you can multiply.

Step 1 Change the units of length from miles to meters.
Use the relationships of miles to feet and feet to meters.

$$\frac{65 \text{ miles}}{1 \text{ hour}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} \cdot \frac{1 \text{ meter}}{3.3 \text{ feet}}$$

Step 2 Change the units of time from hours to seconds.
Write fractions relating hours to minutes and minutes to seconds.

$$\frac{65 \text{ miles}}{1 \text{ hour}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} \cdot \frac{1 \text{ meter}}{3.3 \text{ feet}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}}$$

Step 3 Simplify and check by canceling the units.

$$\begin{aligned} \frac{65 \text{ miles}}{1 \text{ hour}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} \cdot \frac{1 \text{ meter}}{3.3 \text{ feet}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} \\ = \frac{65 \cdot 5280}{3.3 \cdot 60 \cdot 60} \text{ m/s} \quad \text{Simplify.} \\ \approx 28.9 \text{ m/s} \quad \text{Use a calculator.} \end{aligned}$$

So, 65 miles per hour is about 28.9 meters per second. This answer is reasonable because the final units are m/s, not m/hr, ft/s, or mi/hr.

Exercises

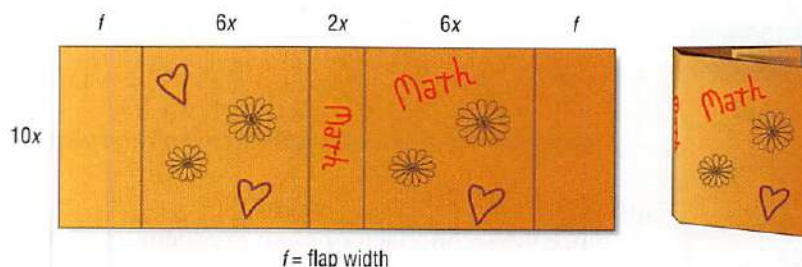
Solve each problem by using dimensional analysis. Include the appropriate units with your answer.

1. A zebra can run 40 miles per hour. How far can a zebra run in 3 minutes?
2. A cyclist traveled 43.2 miles at an average speed of 12 miles per hour. How long did the cyclist ride?
3. If you are going 50 miles per hour, how many feet per second are you traveling?
4. The equation $d = \frac{1}{2}(9.8 \text{ m/s}^2)(3.5 \text{ s})^2$ represents the distance d that a ball falls 3.5 seconds after it is dropped from a tower. Find the distance.
5. **WRITING IN MATH** Explain how dimensional analysis can be useful in checking the reasonableness of your answer.

Dividing Polynomials

Why?

Arianna needed $140x^2 + 60x$ square inches of paper to make a book jacket $10x$ inches tall. In figuring the area, she allowed for a front and back flap. If the spine is $2x$ inches wide, and the front and back are $6x$ inches wide, how wide are the front and back flaps? You can use a quotient of polynomials to help you find the answer.



f = flap width

Then

You divided monomials.
(Lesson 6-1)

Now

- Divide polynomials using long division.
- Divide polynomials using synthetic division.

New Vocabulary

synthetic division

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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Long Division In Lesson 6-1, you learned to divide monomials. You can divide a polynomial by a monomial by using those same skills.

EXAMPLE 1 Divide a Polynomial by a Monomial

Simplify $\frac{6x^4y^3 + 12x^3y^2 - 18x^2y}{3xy}$.

$$\frac{6x^4y^3 + 12x^3y^2 - 18x^2y}{3xy} = \frac{6x^4y^3}{3xy} + \frac{12x^3y^2}{3xy} - \frac{18x^2y}{3xy}$$

Sum of quotients

$$= \frac{6}{3} \cdot x^4 - 1y^3 - 1 + \frac{12}{3} \cdot x^3 - 1y^2 - 1 - \frac{18}{3} \cdot x^2 - 1y^1 - 1$$

Divide.

$$= 2x^3y^2 + 4x^2y - 6x$$

$y^1 - 1 = y^0$ or 1

Check Your Progress Simplify.

1A. $(20c^4d^2f - 16cdf^2 + 4cdf) \div (4cdf)$

1B. $(18x^2y + 27x^3y^2z)(3xy)^{-1}$

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You can use a process similar to long division to divide a polynomial by a polynomial with more than one term. The process is known as the *division algorithm*.

EXAMPLE 2 Division Algorithm

Use long division to find $(x^2 + 3x - 40) \div (x - 5)$.

$$\begin{array}{r} x + 8 \\ x - 5 \overline{) x^2 + 3x - 40} \\ \underline{(-) x^2 - 5x} \\ 8x - 40 \\ \underline{(-) 8x - 40} \\ 0 \end{array}$$

Multiply divisor by x since $\frac{x^2}{x} = x$.
Subtract. Bring down next term.
Multiply divisor by 8 since $\frac{8x}{x} = 8$.
Subtract.

The quotient is $x + 8$. The remainder is 0.

Check Your Progress Use long division to find each quotient.

2A. $(x^2 + 7x - 30) \div (x - 3)$

2B. $(x^2 - 13x + 12) \div (x - 1)$

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Just as with the division of whole numbers, the division of two polynomials may result in a quotient with a remainder. Remember that $11 \div 3 = 3 + R2$, which is often written as $3\frac{2}{3}$. The result of a division of polynomials with a remainder can be written in a similar manner.

STANDARDIZED TEST EXAMPLE 3

Which expression is equal to $(a^2 + 7a - 11)(3 - a)^{-1}$?

A $a + 10 - \frac{19}{3 - a}$

C $-a - 10 + \frac{19}{3 - a}$

B $-a + 10$

D $-a - 10 - \frac{19}{3 - a}$

Test-Taking Tip

Multiple Choice You may be able to eliminate some of the answer choices by substituting the same value for a in the original expression and the answer choices, and then evaluating.

Read the Test Item

Since the second factor has an exponent of -1 , this is a division problem.

$$(a^2 + 7a - 11)(3 - a)^{-1} = \frac{a^2 + 7a - 11}{3 - a}$$

Solve the Test Item

$$\begin{array}{r} -a - 10 \\ -a + 3 \overline{) a^2 + 7a - 11} \\ \underline{(-) a^2 - 3a} \\ 10a - 11 \\ \underline{(-) 10a - 30} \\ 19 \end{array}$$

For ease in dividing, rewrite $3 - a$ as $-a + 3$.

$$-a(-a + 3) = a^2 - 3a$$

$$7a - (-3a) = 10a$$

$$-10(-a + 3) = 10a - 30$$

$$-11 - (-30) = 19$$

The quotient is $-a - 10$, and the remainder is 19.

Therefore, $(a^2 + 7a - 11)(3 - a)^{-1} = -a - 10 + \frac{19}{3 - a}$. The answer is C.

Check Your Progress

3. Which expression is equal to $(r^2 + 5r + 7)(1 - r)^{-1}$?

F $-r - 6 + \frac{13}{1 - r}$

H $r - 6 + \frac{13}{1 - r}$

G $r + 6$

J $r + 6 - \frac{13}{1 - r}$

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Synthetic Division

Synthetic division is a simpler process for dividing a polynomial by a binomial. Suppose you want to divide $2x^3 - 13x^2 + 26x - 24$ by $x - 4$ using long division. Compare the coefficients in this division with those in Example 4.

$$\begin{array}{r} 2x^2 - 5x + 6 \\ x - 4 \overline{) 2x^3 - 13x^2 + 26x - 24} \\ \underline{(-) 2x^3 - 8x^2} \\ -5x^2 + 26x \\ \underline{(-) -5x^2 + 20x} \\ 6x - 24 \\ \underline{(-) 6x - 24} \\ 0 \end{array}$$

When the polynomial in the dividend is missing a term, a zero must be used to represent the missing term. So, with a dividend of $2x^3 - 4x^2 + 6$, a 0 will be used as a placeholder for the x -term.

$$\overline{) 2x^3 - 4x^2 + 0x + 6}$$

Key Concept

Synthetic Division

For Your

FOLDABLE

- Step 1** Write the coefficients of the dividend so that the degrees of the terms are in descending order. Write the constant r of the divisor $x - r$ in the box. Bring the first coefficient down.
- Step 2** Multiply the first coefficient by r , and write the product under the second coefficient.
- Step 3** Add the product and the second coefficient.
- Step 4** Repeat Steps 2 and 3 until you reach a sum in the last column. The numbers along the bottom row are the coefficients of the quotient. The power of the first term is one less than the degree of the dividend. The final number is the remainder.

EXAMPLE 4 Synthetic Division

Use synthetic division to find $(2x^3 - 13x^2 + 26x - 24) \div (x - 4)$.

Step 1 Write the coefficients of the dividend. Write the constant r in the box. In this case, $r = 4$. Bring the first coefficient, 2, down.

$$\begin{array}{r|rrrrr} 4 & 2 & -13 & 26 & -24 & \\ & \downarrow & & & & \\ & 2 & & & & | \end{array}$$

Step 2 Multiply the first coefficient by r : $2 \cdot 4 = 8$. Write the product under the second coefficient.

$$\begin{array}{r|rrrrr} 4 & 2 & -13 & 26 & -24 & \\ & \downarrow & 8 & & & \\ & 2 & & & & | \end{array}$$

Step 3 Add the product and the second coefficient: $-13 + 8 = -5$.

$$\begin{array}{r|rrrrr} 4 & 2 & -13 & 26 & -24 & \\ & \downarrow & 8 & & & \\ & 2 & -5 & & & | \end{array}$$

Step 4 Multiply the sum, -5 , by r : $-5 \times 4 = -20$. Write the product under the next coefficient, and add: $26 + (-20) = 6$. Multiply the sum, 6, by r : $6 \cdot 4 = 24$. Write the product under the next coefficient and add: $-24 + 24 = 0$.

$$\begin{array}{r|rrrrr} 4 & 2 & -13 & 26 & -24 & \\ & \downarrow & 8 & -20 & 24 & \\ & 2 & -5 & 6 & 0 & | \end{array}$$

CHECK Multiply the quotient by the divisor. The answer should be the dividend.

$$\begin{array}{r} 2x^2 - 5x + 6 \\ (\times) \quad x - 4 \\ \hline -8x^2 + 20x - 24 \\ \hline 2x^3 - 5x^2 + 6x \\ \hline 2x^3 - 13x^2 + 26x - 24 \end{array}$$

The quotient is $2x^2 - 5x + 6$. The remainder is 0.

Watch Out!

Synthetic Division

Remember to *add* terms when performing synthetic division.

Check Your Progress

Use synthetic division to find each quotient.

4A. $(2x^3 + 3x^2 - 4x + 15) \div (x + 3)$

4B. $(3x^3 - 8x^2 + 11x - 14) \div (x - 2)$

4C. $(4a^4 + 2a^2 - 4a + 12) \div (a + 2)$

4D. $(6b^4 - 8b^3 + 12b - 14) \div (b - 2)$

To use synthetic division, the divisor must be of the form $x - r$. If the coefficient of x in a divisor is not 1, you can rewrite the division expression so that you can use synthetic division.

Watch Out!

Divide Throughout
Remember to divide *all* terms in the numerator and denominator.

EXAMPLE 5 Divisor with First Coefficient Other than 1

Use synthetic division to find $(3x^4 - 5x^3 + x^2 + 7x) \div (3x + 1)$.

$$\begin{aligned} \frac{3x^4 - 5x^3 + x^2 + 7x}{3x + 1} &= \frac{(3x^4 - 5x^3 + x^2 + 7x) \div 3}{(3x + 1) \div 3} \\ &= \frac{x^4 - \frac{5}{3}x^3 + \frac{1}{3}x^2 + \frac{7}{3}x}{x + \frac{1}{3}} \end{aligned}$$

Rewrite the divisor with a leading coefficient of 1. Then divide the numerator and denominator by 3.

Simplify the numerator and the denominator.

Since the numerator does not have a constant term, use a coefficient of 0 for the constant term.

$$\begin{array}{r|rrrrr} x - r = x + \frac{1}{3}, \text{ so } r = -\frac{1}{3} & \rightarrow & -\frac{1}{3} & 1 & -\frac{5}{3} & \frac{1}{3} & \frac{7}{3} & 0 \\ & & & & -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} \\ \hline & & & 1 & -2 & 1 & 2 & -\frac{2}{3} \end{array}$$

The result is $x^3 - 2x^2 + x + 2 - \frac{\frac{2}{3}}{x + \frac{1}{3}}$. Now simplify the fraction.

$$\begin{aligned} \frac{\frac{2}{3}}{x + \frac{1}{3}} &= \frac{2}{3} \div \left(x + \frac{1}{3}\right) && \text{Rewrite as a division expression.} \\ &= \frac{2}{3} \div \frac{3x + 1}{3} && x + \frac{1}{3} = \frac{3x}{3} + \frac{1}{3} = \frac{3x + 1}{3} \\ &= \frac{2}{3} \cdot \frac{3}{3x + 1} && \text{Multiply by the reciprocal.} \\ &= \frac{2}{3x + 1} && \text{Simplify.} \end{aligned}$$

The solution is $x^3 - 2x^2 + x + 2 - \frac{2}{3x + 1}$.

CHECK Divide using long division.

$$\begin{array}{r} x^3 - 2x^2 + x + 2 \\ 3x + 1 \overline{) 3x^4 - 5x^3 + x^2 + 7x} \\ \underline{(-) 3x^4 + x^3} \\ -6x^3 + x^2 \\ \underline{(-) -6x^3 - 2x^2} \\ 3x^2 + 7x \\ \underline{(-) 3x^2 + x} \\ 6x + 0 \\ \underline{(-) 6x + 2} \\ -2 \end{array}$$

The result is $x^3 - 2x^2 + x + 2 - \frac{2}{3x + 1}$. ✓

Check Your Progress

Use synthetic division to find each quotient.

5A. $(8x^4 - 4x^2 + x + 4) \div (2x + 1)$

5B. $(8y^5 - 2y^4 - 16y^2 + 4) \div (4y - 1)$

5C. $(15b^3 + 8b^2 - 21b + 6) \div (5b - 4)$

5D. $(6c^3 - 17c^2 + 6c + 8) \div (3c - 4)$

Check Your Understanding

Examples 1, 2, and 4

pp. 341–343

Simplify.

- $\frac{4xy^2 - 2xy + 2x^2y}{xy}$
- $(3a^2b - 6ab + 5ab^2)(ab)^{-1}$
- $(x^2 - 6x - 20) \div (x + 2)$
- $(2a^2 - 4a - 8) \div (a + 1)$
- $(3z^4 - 6z^3 - 9z^2 + 3z - 6) \div (z + 3)$
- $(y^5 - 3y^2 - 20) \div (y - 2)$

Example 3

p. 342

7. **MULTIPLE CHOICE** Which expression is equal to $(x^2 + 3x - 9)(4 - x)^{-1}$?

- A $-x - 7 + \frac{19}{4 - x}$ B $-x - 7$ C $x + 7 - \frac{19}{4 - x}$ D $-x - 7 - \frac{19}{4 - x}$

Example 5

p. 344

Simplify.

- $(10x^2 + 15x + 20) \div (5x + 5)$
- $(18a^2 + 6a + 9) \div (3a - 2)$
- $\frac{12b^2 + 23b + 15}{3b + 8}$
- $\frac{27y^2 + 27y - 30}{9y - 6}$

Practice and Problem Solving

 = **Step-by-Step Solutions** begin on page R20.
Extra Practice begins on page 947.

Example 1

p. 341

Simplify.

- $\frac{24a^3b^2 - 16a^2b^3}{8ab}$
- $\frac{5x^2y - 10xy + 15xy^2}{5xy}$
- $\frac{4a^3b - 6ab + 2ab^2}{2ab}$
- $\frac{16c^4d^4 - 24c^2d^2}{4c^2d^2}$
- $\frac{7g^3h^2 + 3g^2h - 2gh^3}{gh}$
- $\frac{9n^3p^3 - 18n^2p^2 + 21n^2p^3}{3n^2p^2}$

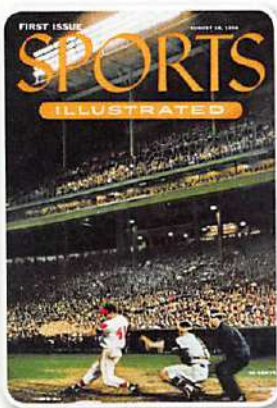
- ENERGY** Compact fluorescent light (CFL) bulbs reduce energy waste. The amount of energy waste that is reduced each day in a certain community can be estimated by $-b^2 + 8b$, where b is the number of bulbs. Divide by b to find the average amount of energy saved per CFL bulb.
- BAKING** The number of cookies produced in a factory each day can be estimated by $-w^2 + 16w + 1000$, where w is the number of workers. Divide by w to find the average number of cookies produced per worker.

Examples 2, 4, and 5

pp. 341–344

Simplify.

- $(a^2 - 8a - 26) \div (a + 2)$
- $(b^3 - 4b^2 + b - 2) \div (b + 1)$
- $(z^4 - 3z^3 + 2z^2 - 4z + 4)(z - 1)^{-1}$
- $(x^5 - 4x^3 + 4x^2) \div (x - 4)$
- $\frac{y^3 + 11y^2 - 10y + 6}{y + 2}$
- $(g^4 - 3g^2 - 18) \div (g - 2)$
- $(6a^2 - 3a + 9) \div (3a - 2)$
- $\frac{6x^5 + 5x^4 + x^3 - 3x^2 + x}{3x + 1}$
- $\frac{4g^4 - 6g^3 + 3g^2 - g + 12}{4g - 4}$
- $(2b^3 - 6b^2 + 8b) \div (2b + 2)$
- $(6z^6 + 3z^4 - 9z^2)(3z - 6)^{-1}$
- $(10y^6 + 5y^5 + 10y^3 - 20y - 15)(5y + 5)^{-1}$
- GEOMETRY** A rectangular box for a new product is designed in such a way that the three dimensions always have a particular relationship defined by the variable x . The volume of the box can be written as $6x^3 + 31x^2 + 53x + 30$, and the height is always $x + 2$. What are the width and length of the box?
- PHYSICS** The voltage V is related to current I and power P by the equation $V = \frac{P}{I}$. The power of a generator is modeled by $P(t) = t^3 + 9t^2 + 26t + 24$. If the current of the generator is $I = t + 4$, write an expression that represents the voltage.



Real-World Link

Created in 1954, *Sports Illustrated* is read by over 23 million adults each week.

Source: *Sports Illustrated*

34. **ENTERTAINMENT** A magician gives these instructions to a volunteer.

- Choose a number and multiply it by 4.
- Then add the sum of your number and 15 to the product you found.
- Now divide by the sum of your number and 3.
 - a. What number will the volunteer always have at the end?
 - b. Explain the process you used to discover the answer.

35. **BUSINESS** The number of magazine subscriptions sold can be estimated by $n = \frac{3500a^2}{a^2 + 100}$,

where a is the amount of money the company spent on advertising in hundreds of dollars and n is the number of subscriptions sold.

- a. Perform the division indicated by $\frac{3500a^2}{a^2 + 100}$.
- b. About how many subscriptions will be sold if \$1500 is spent on advertising?

Simplify.

36. $(x^4 - y^4) \div (x - y)$ 37. $(28c^3d^2 - 21cd^2) \div (14cd)$ 38. $(a^3b^2 - a^2b + 2b)(-ab)^{-1}$
39. $\frac{n^3 + 3n^2 - 5n - 4}{n + 4}$ 40. $\frac{p^3 + 2p^2 - 7p - 21}{p + 3}$ 41. $\frac{3z^5 + 5z^4 + z + 5}{z + 2}$

42. **MULTIPLE REPRESENTATIONS** Consider a rectangle with area $2x^2 + 7x + 3$ and length $2x + 1$.

- a. **CONCRETE** Use algebra tiles to represent this situation. Use the model to find the width.
- b. **SYMBOLIC** Write an expression to represent the model.
- c. **NUMERICAL** Solve this problem algebraically using synthetic or long division. Does your concrete model check with your algebraic model?

H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

43. **FIND THE ERROR** Sharon and Jamal are dividing $2x^3 - 4x^2 + 3x - 1$ by $x - 3$. Sharon claims that the remainder is 26. Jamal argues that the remainder is -100 . Is either of them correct? Explain your reasoning.
44. **CHALLENGE** If a polynomial is divided by a binomial and the remainder is 0, what does this tell you about the relationship between the binomial and the polynomial?
45. **REASONING** Review any of the division problems in this lesson. What is the relationship between the degrees of the dividend, the divisor, and the quotient?
46. **OPEN ENDED** Write a quotient of two polynomials for which the remainder is 3.
47. **WHICH ONE DOESN'T BELONG?** Identify the expression that does not belong with the other three. Explain your reasoning.

$$3xy + 6x^2$$

$$\frac{5}{x^2}$$

$$x + 5$$

$$5b + 11c - 9ad^2$$

48. **WRITING IN MATH** Use the information at the beginning of the lesson to write assembly instructions using the division of polynomials to make a paper cover for your textbook.

Standardized Test Practice

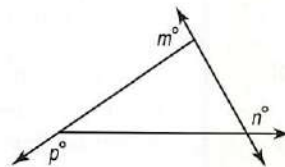
49. An office employs x women and 3 men. What is the ratio of the total number of employees to the number of women?

- A $\frac{x+3}{x}$ C $\frac{3}{x}$
 B $\frac{x}{x+3}$ D $\frac{x}{3}$

50. $(-4x^2 + 2x + 3) - 3(2x^2 - 5x + 1) =$

- F $2x^2$ H $-10x^2 + 17x$
 G $-10x^2$ J $2x^2 + 17x$

51. **GRIDDED RESPONSE** In the figure below, $m + n + p = ?$



52. **SAT/ACT** Which polynomial has degree 3?

- A $x^3 + x^2 - 2x^4$ C $x^2 + x + 12^3$
 B $-2x^2 - 3x + 4$ D $1 + x + x^3$

Spiral Review

Simplify. (Lesson 6-1)

53. $(5x^3 + 2x^2 - 3x + 4) - (2x^3 - 4x)$ 54. $(2y^3 - 3y + 8) + (3y^2 - 6y)$ 55. $4a(2a - 3) + 3a(5a - 4)$
 56. $(c + d)(c - d)(2c - 3d)$ 57. $(xy)^2(2xy^2z)^3$ 58. $(3ab^2)^{-2}(2a^2b)^2$
 59. **LANDSCAPING** Amado wants to plant a garden and surround it with decorative stones. He has enough stones to enclose a rectangular garden with a perimeter of 68 feet, but he wants the garden to cover no more than 240 square feet. What could the width of his garden be? (Lesson 5-8)

Solve each equation by completing the square. (Lesson 5-5)

60. $x^2 + 6x + 2 = 0$ 61. $x^2 - 8x - 3 = 0$ 62. $2x^2 + 6x + 5 = 0$

State the consecutive integers between which the zeros of each quadratic function are located. (Lesson 5-2)

63.

x	-7	-6	-5	-4	-3	-2	-1	0
$f(x)$	4	1	-3	-8	-1	2	8	16

64.

x	-2	-1	0	1	2	3	4	5
$f(x)$	-16	-7	-4	3	3	-4	-7	-16

65.

x	-2	-1	0	1	2	3	4	5
$f(x)$	6	1	-3	-5	-3	1	6	14

66. **BUSINESS** A landscaper can mow a lawn in 30 minutes and perform a small landscape job in 90 minutes. He works at most 10 hours per day, 5 days per week. He earns \$35 per lawn and \$125 per landscape job. He cannot do more than 3 landscape jobs per day. Find the combination of lawns mowed and completed landscape jobs per week that will maximize income. Then find the maximum income. (Lesson 3-4)

Skills Review

Find each value if $f(x) = 4x + 3$, $g(x) = -x^2$, and $h(x) = -2x^2 - 2x + 4$. (Lesson 2-1)

67. $f(-6)$ 68. $g(-8)$ 69. $h(3)$
 70. $f(c)$ 71. $g(3d)$ 72. $h(2b + 1)$

Polynomial Functions

Why?

The volume of air in the lungs during a 5-second respiratory cycle can be modeled by $v(t) = -0.037t^3 + 0.152t^2 + 0.173t$, where v is the volume in liters and t is the time in seconds. This model is an example of a polynomial function.



Then

You analyzed graphs of quadratic functions. (Lesson 5-1)

Now

- Evaluate polynomial functions.
- Identify general shapes of graphs of polynomial functions.

New Vocabulary

polynomial in one variable
leading coefficient
polynomial function
power function
end behavior
quartic function
quintic function

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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help
- Math in Motion

Polynomial Functions A **polynomial in one variable** is an expression of the form $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$, where $a_n \neq 0$, a_{n-1} , a_2 , a_1 , and a_0 are real numbers, and n is a nonnegative integer.

The polynomial is written in standard form when the values of the exponents are in descending order. The degree of the polynomial is the value of the greatest exponent. The coefficient of the first term of a polynomial in standard form is called the **leading coefficient**.

Polynomial	Expression	Degree	Leading Coefficient
Constant	12	0	12
Linear	$4x - 9$	1	4
Quadratic	$5x^2 - 6x - 9$	2	5
Cubic	$8x^3 + 12x^2 - 3x + 1$	3	8
General	$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$	n	a_n

EXAMPLE 1 Degrees and Leading Coefficients

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

a. $8x^5 - 4x^3 + 2x^2 - x - 3$

This is a polynomial in one variable. The greatest exponent is 5, so the degree is 5 and the leading coefficient is 8.

b. $12x^2 - 3xy + 8x$

This is not a polynomial in one variable. There are two variables, x and y .

c. $3x^4 + 6x^3 - 4x^8 + 2x$

This is a polynomial in one variable. The greatest exponent is 8, so the degree is 8 and the leading coefficient is -4 .

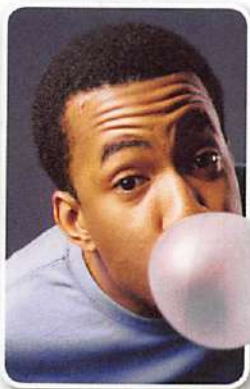
Check Your Progress

1A. $5x^3 - 4x^2 - 8x + \frac{4}{x}$

1B. $5x^6 - 3x^4 + 12x^3 - 14$

1C. $8x^4 - 2x^3 - x^6 + 3$

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A **polynomial function** is a continuous function that can be described by a polynomial equation in one variable. For example, $f(x) = 3x^3 - 4x + 6$ is a cubic polynomial function. The simplest polynomial functions of the form $f(x) = ax^b$ where a and b are real numbers are called **power functions**.

If you know an element in the domain of any polynomial function, you can find the corresponding value in the range.

Real-World Link

Total lung capacity is approximately 6 liters in a healthy young adult.

Source: Family Practice Notebook

Real-World EXAMPLE 2 Evaluate a Polynomial Function

RESPIRATION Refer to the beginning of the lesson. Find the volume of air in the lungs 2 seconds into the respiratory cycle.

By substituting 2 into the function we can find $v(2)$, the volume of air in the lungs 2 seconds into the respiration cycle.

$$v(t) = -0.037t^3 + 0.152t^2 + 0.173t \quad \text{Original function}$$

$$v(2) = -0.037(2)^3 + 0.152(2)^2 + 0.173(2) \quad \text{Replace } t \text{ with } 2.$$

$$= -0.296 + 0.608 + 0.346 \quad \text{Simplify.}$$

$$= 0.658 \text{ L} \quad \text{Add.}$$

Check Your Progress

2. Find the volume of air in the lungs 4 seconds into the respiratory cycle.

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You can also evaluate functions for variables and algebraic expressions.

EXAMPLE 3 Function Values of Variables

Find $f(3c - 4) - 5f(c)$ if $f(x) = x^2 + 2x - 3$.

To evaluate $f(3c - 4)$, replace the x in $f(x)$ with $3c - 4$.

$$f(x) = x^2 + 2x - 3 \quad \text{Original function}$$

$$f(3c - 4) = (3c - 4)^2 + 2(3c - 4) - 3 \quad \text{Replace } x \text{ with } 3c - 4.$$

$$= 9c^2 - 24c + 16 + 6c - 8 - 3 \quad \text{Multiply.}$$

$$= 9c^2 - 18c + 5 \quad \text{Simplify.}$$

To evaluate $5f(c)$, replace x with c in $f(x)$, then multiply by 5.

$$f(x) = x^2 + 2x - 3 \quad \text{Original function}$$

$$5f(c) = 5(c^2 + 2c - 3) \quad \text{Replace } x \text{ with } c.$$

$$= 5c^2 + 10c - 15 \quad \text{Distributive Property}$$

Now evaluate $f(3c - 4) - 5f(c)$.

$$f(3c - 4) - 5f(c) = (9c^2 - 18c + 5) - (5c^2 + 10c - 15)$$

$$= 9c^2 - 18c + 5 - 5c^2 - 10c + 15 \quad \text{Distribute.}$$

$$= 4c^2 - 28c + 20 \quad \text{Simplify.}$$

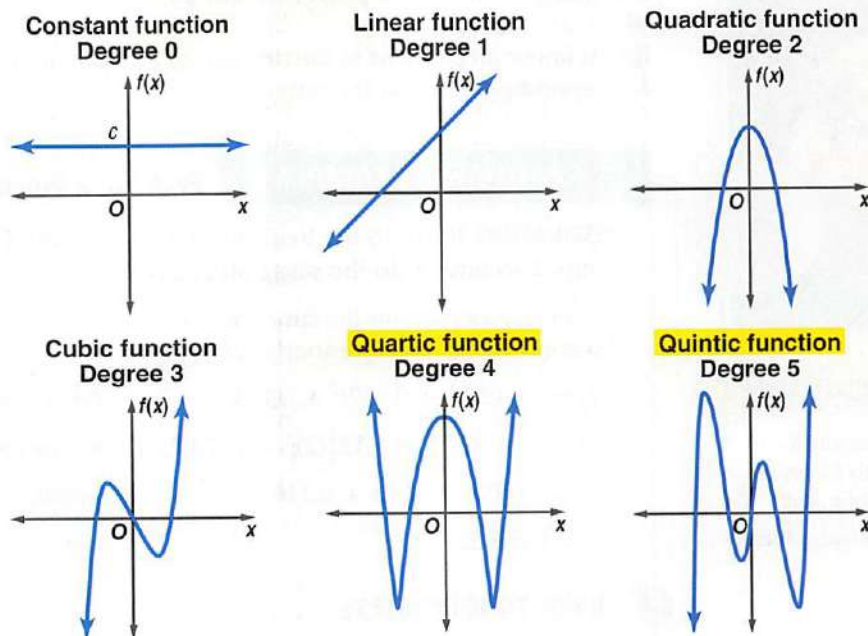
Check Your Progress

3A. Find $g(5a - 2) + 3g(2a)$ if $g(x) = x^2 - 5x + 8$.

3B. Find $h(-4d + 3) - 0.5h(d)$ if $h(x) = 2x^2 + 5x + 3$.

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Graphs of Polynomial Functions The general shapes of the graphs of several polynomial functions show the *maximum* number of times the graph of each function may intersect the x -axis. This is the same number as the degree of the polynomial.



The domain of any polynomial function is all real numbers. The **end behavior** is the behavior of the graph of $f(x)$ as x approaches positive infinity ($x \rightarrow +\infty$) or negative infinity ($x \rightarrow -\infty$). The degree and leading coefficient of a polynomial function determine the end behavior of the graph and the range of the function.

StudyTip

End behavior The leading coefficient and degree are the sole determining factors for the end behavior of a polynomial. With very large or very small numbers, the rest of the polynomial is insignificant in the appearance of the graph.

Review Vocabulary

infinity endless or boundless (Explore Lesson 1-6)

Key Concept

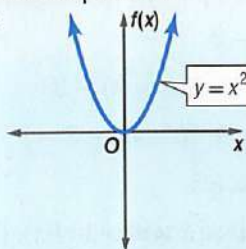
End Behavior of a Polynomial Function

For Your **FOLDABLE**

Degree: even
Leading Coefficient: positive
End Behavior:

$$f(x) \rightarrow +\infty \text{ as } x \rightarrow -\infty$$

$$f(x) \rightarrow +\infty \text{ as } x \rightarrow +\infty$$

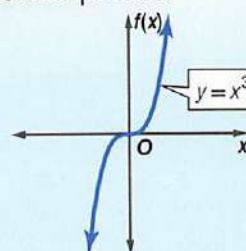


Domain: all reals
Range: all reals \geq minimum

Degree: odd
Leading Coefficient: positive
End Behavior:

$$f(x) \rightarrow -\infty \text{ as } x \rightarrow -\infty$$

$$f(x) \rightarrow +\infty \text{ as } x \rightarrow +\infty$$

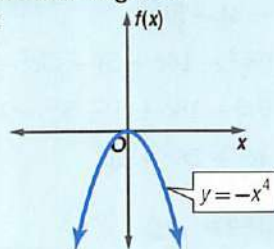


Domain: all reals
Range: all reals

Degree: even
Leading Coefficient: negative
End Behavior:

$$f(x) \rightarrow -\infty \text{ as } x \rightarrow -\infty$$

$$f(x) \rightarrow -\infty \text{ as } x \rightarrow +\infty$$

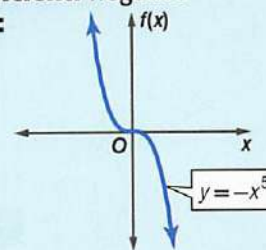


Domain: all reals
Range: all reals \leq maximum

Degree: odd
Leading Coefficient: negative
End Behavior:

$$f(x) \rightarrow +\infty \text{ as } x \rightarrow -\infty$$

$$f(x) \rightarrow -\infty \text{ as } x \rightarrow +\infty$$



Domain: all reals
Range: all reals

The number of real zeros of a polynomial equation can be determined from the graph of its related polynomial function. Recall that real zeros occur at x -intercepts, so the number of times a graph crosses the x -axis equals the number of real zeros.

StudyTip

Double roots When a graph is tangent to the x -axis, there is a *double root*, which represents two of the same root.

Review Vocabulary

zero the x -coordinate of the point at which a graph intersects the x -axis (Lesson 5-2)

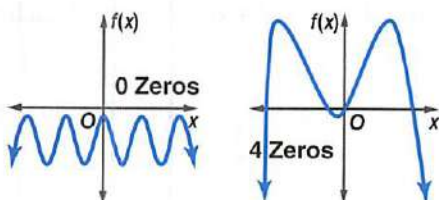
Key Concept

Zeros of Even- and Odd-Degree Functions

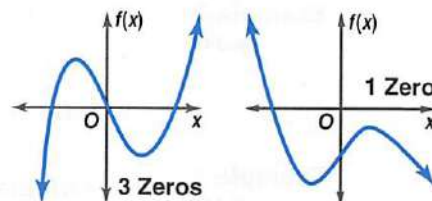
For Your
FOLDABLE

Odd-degree functions will always have an odd number of real zeros.
Even-degree functions will always have an even number of real zeros or no real zeros at all.

Even-Degree Polynomials



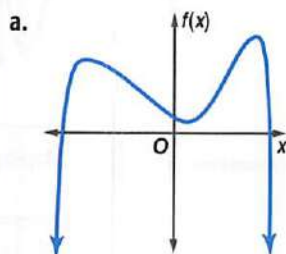
Odd-Degree Polynomials



EXAMPLE 4 Graphs of Polynomial Functions

For each graph,

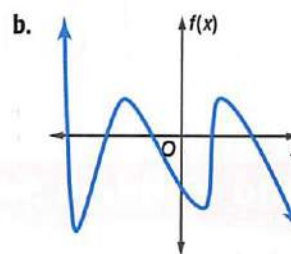
- describe the end behavior,
- determine whether it represents an odd-degree or an even-degree polynomial function, and
- state the number of real zeros.



$$f(x) \rightarrow -\infty \text{ as } x \rightarrow -\infty.$$

$$f(x) \rightarrow -\infty \text{ as } x \rightarrow +\infty.$$

Since the end behavior is in the same direction, it is an even-degree function. The graph intersects the x -axis at two points, so there are two real zeros.

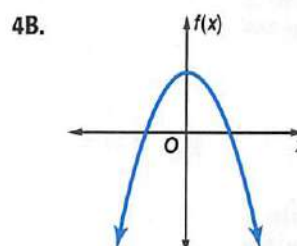
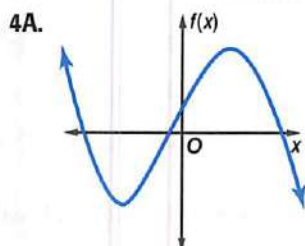


$$f(x) \rightarrow +\infty \text{ as } x \rightarrow -\infty.$$

$$f(x) \rightarrow -\infty \text{ as } x \rightarrow +\infty.$$

Since the end behavior is in opposite directions, it is an odd-degree function. The graph intersects the x -axis at five points, so there are five real zeros.

Check Your Progress



Check Your Understanding

Example 1 p. 348

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1. $11x^6 - 5x^5 + 4x^2$

2. $-10x^7 - 5x^3 + 4x - 22$

3. $14x^4 - 9x^3 + 3x - 4y$

4. $8x^5 - 3x^2 + 4xy - 5$

Example 2 p. 349

Find $w(5)$ and $w(-4)$ for each function.

5. $w(x) = -2x^3 + 3x - 12$

6. $w(x) = 2x^4 - 5x^3 + 3x^2 - 2x + 8$

Example 3 p. 349

If $c(x) = 4x^3 - 5x^2 + 2$ and $d(x) = 3x^2 + 6x - 10$, find each value.

7. $c(y^3)$

8. $-4[d(3z)]$

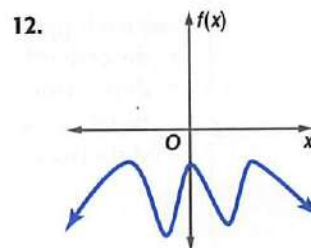
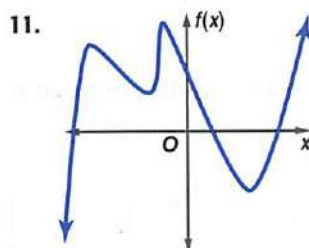
9. $6c(4a) + 2d(3a - 5)$

10. $-3c(2b) + 6d(4b - 3)$

Example 4 p. 351

For each graph,

- describe the end behavior,
- determine whether it represents an odd-degree or an even-degree function, and
- state the number of real zeros.



Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R20.
Extra Practice begins on page 947.

Example 1 p. 348

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

13. $-6x^6 - 4x^5 + 13xy$

14. $3a^7 - 4a^4 + \frac{3}{a}$

15. $8x^5 - 12x^6 + 14x^3 - 9$

16. $-12 - 8x^2 + 5x - 21x^7$

17. $15x - 4x^3 + 3x^2 - 5x^4$

18. $13b^3 - 9b + 3b^5 - 18$

19. $(d + 5)(3d - 4)$

20. $(5 - 2y)(4 + 3y)$

21. $6x^5 - 5x^4 + 2x^9 - 3x^2$

22. $7x^4 + 3x^7 - 2x^8 + 7$

Example 2 p. 349

Find $p(-6)$ and $p(3)$ for each function.

23. $p(x) = x^4 - 2x^2 + 3$

24. $p(x) = -3x^3 - 2x^2 + 4x - 6$

25. $p(x) = 2x^3 + 6x^2 - 10x$

26. $p(x) = x^4 - 4x^3 + 3x^2 - 5x + 24$

27. $p(x) = -x^3 + 3x^2 - 5$

28. $p(x) = 2x^4 + x^3 - 4x^2$

Example 3 p. 349

If $c(x) = 2x^2 - 4x + 3$ and $d(x) = -x^3 + x + 1$, find each value.

29. $c(3a)$

30. $5d(2a)$

31. $c(b^2)$

32. $d(4a^2)$

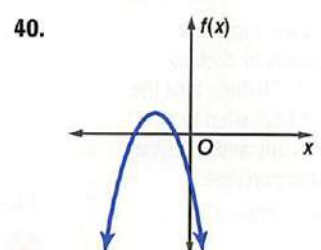
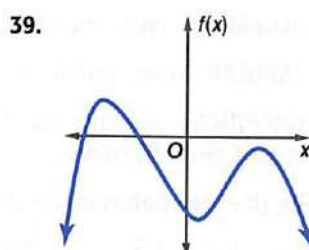
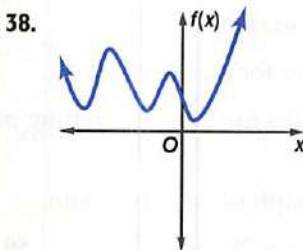
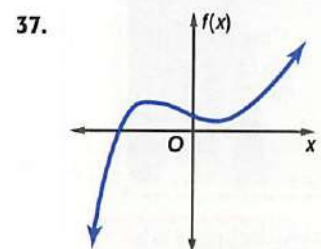
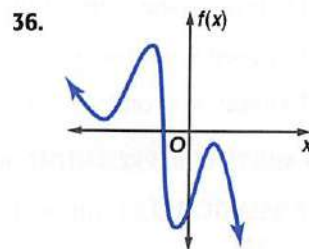
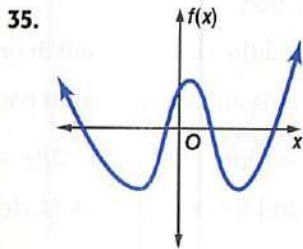
33. $d(4y - 3)$

34. $c(y^2 - 1)$

Example 4
p. 351

For each graph,

- describe the end behavior,
- determine whether it represents an odd-degree or an even-degree function, and
- state the number of real zeros.



Real-World Link

In 1947, the first commercial microwave oven hit the market. These early units were gigantic and enormously expensive, standing $5\frac{1}{2}$ feet tall, weighing over 750 pounds, and costing about \$5000 each.

Source: Microtech

41. **PHYSICS** For a moving object with mass m in kilograms, the kinetic energy KE in joules is given by the function $KE(v) = 0.5mv^2$, where v represents the speed of the object in meters per second. Find the kinetic energy of an all-terrain vehicle with a mass of 171 kilograms moving at a speed of 11 meters/second.
42. **BUSINESS** A microwave manufacturing firm has determined that their profit function is $P(x) = -0.0014x^3 + 0.3x^2 + 6x - 355$, where x is the number of microwaves sold annually.
- Graph the profit function using a calculator.
 - Determine a reasonable viewing window for the function.
 - Approximate all of the zeros of the function using the CALC menu.
 - What must be the range of microwaves sold in order for the firm to have a profit?

Find $p(-2)$ and $p(8)$ for each function.

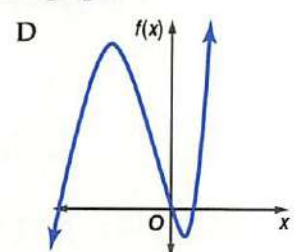
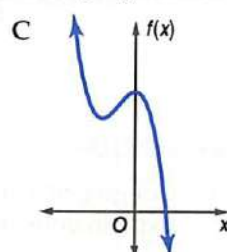
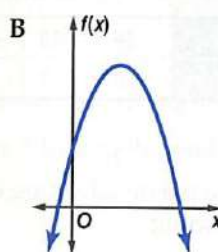
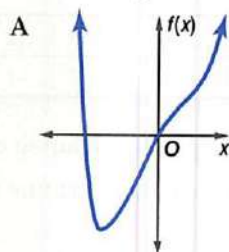
43. $p(x) = \frac{1}{4}x^4 + \frac{1}{2}x^3 - 4x^2$

44. $p(x) = \frac{1}{8}x^4 - \frac{3}{2}x^3 + 12x - 18$

45. $p(x) = \frac{3}{4}x^4 - \frac{1}{8}x^2 + 6x$

46. $p(x) = \frac{5}{8}x^3 - \frac{1}{2}x^2 + \frac{3}{4}x + 10$

Use the degree and end behavior to match each polynomial to its graph.



47. $f(x) = x^3 + 3x^2 - 4x$

48. $f(x) = -2x^2 + 8x + 5$

49. $f(x) = x^4 - 3x^2 + 6x$

50. $f(x) = -4x^3 - 4x^2 + 8$

If $c(x) = x^3 - 2x$ and $d(x) = 4x^2 - 6x + 8$, find each value.

51. $3c(a - 4) + 3d(a + 5)$

52. $-2d(2a + 3) - 4c(a^2 + 1)$

53. $5c(a^2) - 8d(6 - 3a)$

54. $-7d(a^3) + 6c(a^4 + 1)$



Real-World Link

Teens are important customers to clothing makers. Clothing tops the lists of both what teens plan to buy and what they actually purchase.

Source: NOPWorld

55. **BUSINESS** A clothing manufacturer's profitability can be modeled by $p(x) = -x^4 + 40x^2 - 144$, where x is the number of items sold in thousands and $p(x)$ is the company's profit in thousands of dollars.
- Use a table of values to sketch the function.
 - Determine the zeros of the function.
 - Between what two values should the company sell in order to be profitable?
 - Explain why only two of the zeros are considered in part c.

56. **MULTIPLE REPRESENTATIONS** Consider $g(x) = (x - 2)(x + 1)(x - 3)(x + 4)$.
- ANALYTICAL** Determine the x - and y -intercepts, roots, degree, and end behavior of $g(x)$.
 - ALGEBRAIC** Write the function in standard form.
 - TABULAR** Make a table of values for the function.
 - GRAPHICAL** Sketch a graph of the function by plotting points and connecting them with a smooth curve.

Describe the end behavior of the graph of each function.

57. $f(x) = -5x^4 + 3x^2$ 58. $g(x) = 2x^5 + 6x^4$ 59. $h(x) = -4x^7 + 8x^6 - 4x$
 60. $f(x) = 6x - 7x^2$ 61. $g(x) = 8x^4 + 5x^5$ 62. $h(x) = 9x^6 - 5x^7 + 3x^2$

H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

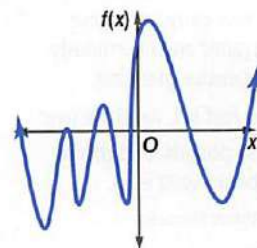
63. **FIND THE ERROR** Shenequa and Virginia are determining the number of zeros of the graph at the right. Is either of them correct? Explain your reasoning.

Shenequa

There are 7 zeros because the graph crosses the x -axis 7 times.

Virginia

There are 8 zeros because the graph crosses the x -axis 7 times, and there is a double root.



64. **CHALLENGE** Use the table to determine the minimum number of real roots and the minimum degree of the polynomial function $f(x)$.

x	-24	-18	-12	-6	0	6	12	18	24
$f(x)$	-8	-1	3	-2	4	7	-1	-8	5

65. **CHALLENGE** If $f(x)$ has a degree of 5 and a positive leading coefficient and $g(x)$ has a degree of 3 and a positive leading coefficient, determine the end behavior of $\frac{f(x)}{g(x)}$. Explain your reasoning.
66. **OPEN ENDED** Sketch the graph of an even-degree polynomial with 8 real roots, one of them a double root.
67. **REASONING** Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain.
A polynomial function that has four real roots is a fourth-degree polynomial.
68. **WRITING IN MATH** Describe what the end behavior of a polynomial function is and how to determine it.

Standardized Test Practice

69. **SHORT RESPONSE** Four students solved the same math problem. Each student's work is shown below. Who is correct?

Student A

$$x^2x^{-5} = \frac{x^2}{x^5}$$

$$= \frac{1}{x^3}, x \neq 0$$

Student C

$$x^2x^{-5} = \frac{x^2}{x^{-5}}$$

$$= x^7, x \neq 0$$

Student B

$$x^2x^{-5} = \frac{x^2}{x^{-5}}$$

$$= x^{-7}, x \neq 0$$

Student D

$$x^2x^{-5} = \frac{x^2}{x^5}$$

$$= x^3, x \neq 0$$

70. **SAT/ACT** What is the remainder when $x^3 - 7x + 5$ is divided by $x + 3$?

- A -11
B -1
C 1
D 11

71. **EXTENDED RESPONSE** A company manufactures tables and chairs. It costs \$40 to make each table and \$25 to make each chair. There is \$1440 available to spend on manufacturing each week. Let t = the number of tables produced and c = the number of chairs produced.

- The manufacturing equation is $40t + 25c = 1500$. Construct a graph of this equation.
- The company always produces two chairs with each table. Write and graph an equation to represent this situation on the same graph as the one in part a.
- Determine the number of tables and chairs that the company can produce each week.
- Explain how to determine this answer using the graph.

72. If $i = \sqrt{-1}$, then $5i(7i) =$

- F 70
G 35
H -35
J -70

Spiral Review

Simplify. (Lesson 6-2)

73. $\frac{16x^4y^3 + 32x^6y^5z^2}{8x^2y}$

74. $\frac{18ab^4c^5 - 30a^4b^3c^2 + 12a^5bc^3}{6abc^2}$

75. $\frac{18c^5d^2 - 3c^2d^2 + 12a^5c^3d^4}{3c^2d^2}$

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial. (Lesson 6-1)

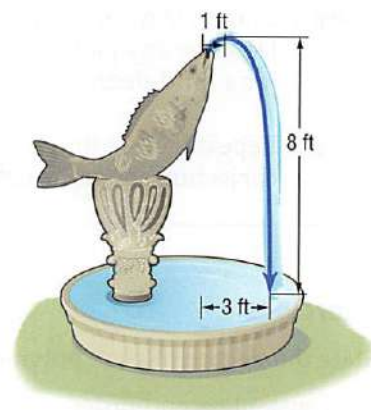
76. $8x^2 + 5xy^3 - 6x + 4$

77. $9x^4 + 12x^6 - 16$

78. $3x^4 + 2x^2 - x^{-1}$

79. **FOUNTAINS** The height of a fountain's water stream can be modeled by a quadratic function. Suppose the water from a jet reaches a maximum height of 8 feet at a distance 1 foot away from the jet. (Lesson 5-7)

- If the water lands 3 feet away from the jet, find a quadratic function that models the height $h(d)$ of the water at any given distance d feet from the jet. Then compare the graph of the function to the parent function.
- Suppose a worker increases the water pressure so that the stream reaches a maximum height of 12.5 feet at a distance of 15 inches from the jet. The water now lands 3.75 feet from the jet. Write a new quadratic function for $h(d)$. How do the changes in h and k affect the shape of the graph?



Solve each inequality. (Lesson 1-6)

80. $|2x + 4| \leq 8$

81. $|-3x + 2| \geq 4$

82. $|2x - 8| - 4 \leq -6$

Skills Review

Determine whether each function has a maximum or minimum value, and find that value. (Lesson 5-1)

83. $f(x) = 3x^2 - 8x + 4$

84. $f(x) = -4x^2 + 2x - 10$

85. $f(x) = -0.25x^2 + 4x - 5$

Algebra Lab

6-3 Polynomial Functions and Rate of Change

In Chapter 2, you learned that linear functions have a constant first-order difference. Then in Chapter 5, you learned that quadratic functions have a constant second-order difference. Now, you will examine the differences for polynomial functions with degree greater than 2.

ACTIVITY

Consider $f(x) = -2x^3$.

Step 1 Copy the table and complete row 2 for x -values from -4 through 4.

x	-4	-3	-2	-1	...	4
y						
First-order Differences						
Second-order Differences						
Third-order Differences						

Step 2 Graph the ordered pairs (x, y) and connect the points with a smooth curve.

Step 3 Find the first-order differences and complete row 3. Describe any patterns in the differences.

Step 4 Complete rows 4 and 5 by finding the second- and third-order differences. Describe any patterns in the differences. Make a conjecture about the differences for a third-degree polynomial function.

Step 5 Repeat Steps 1 through 4 using a fourth-degree polynomial function. Make a conjecture about the differences for an n th-degree polynomial function.

Exercises

State the degree of each polynomial function described.

- constant second-order difference of -3
- constant fifth-order difference of 12
- constant first-order difference of -1.25
- CHALLENGE** Write an equation for a polynomial function with real-number coefficients for the ordered pairs and differences in the table. Make sure your equation is satisfied for all of the ordered pairs (x, y) .

x	-3	-2	-1	0	1	2	3
y	-25	?	?	?	?	?	?
First-order Differences		19	7	1	1	7	19
Second-order Differences			-12	-6	0	6	12

6-4

Analyzing Graphs of Polynomial Functions

Then

You used maxima and minima and graphs of polynomials. (Lessons 5-1 and 6-3)

Now

- Graph polynomial functions and locate their zeros.
- Find the relative maxima and minima of polynomial functions.

New Vocabulary

relative maximum
relative minimum
extrema
turning points

Math Online

glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help
- Math in Motion

Why?

Annual attendance at the movies has fluctuated since the first movie theater, the Nickelodeon, opened in Pittsburgh in 1906. Overall attendance peaked during the 1920s, and it was at its lowest during the 1970s. A graph of the annual attendance to the movies can be represented by a polynomial function.

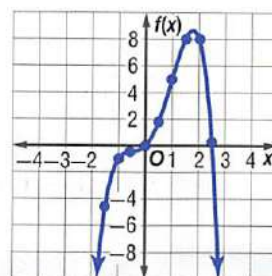


Graphs of Polynomial Functions To graph a polynomial function, make a table of values to find several points and then connect them to make a smooth continuous curve. Knowing the end behavior of the graph will assist you in completing the graph.

EXAMPLE 1 Graph of a Polynomial Function

Graph $f(x) = -x^4 + x^3 + 3x^2 + 2x$ by making a table of values.

x	$f(x)$	x	$f(x)$
-2.5	≈ -41	0.5	≈ 1.8
-2.0	-16	1.0	5.0
-1.5	≈ -4.7	1.5	≈ 8.1
-1.0	-1.0	2.0	8.0
-0.5	≈ -0.4	2.5	≈ 0.3
0.0	0.0	3.0	-21



This is an even-degree polynomial with a negative leading coefficient, so $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$ and $f(x) \rightarrow -\infty$ as $x \rightarrow +\infty$. Notice that the graph intersects the x -axis at two points, indicating there are two zeros for this function.

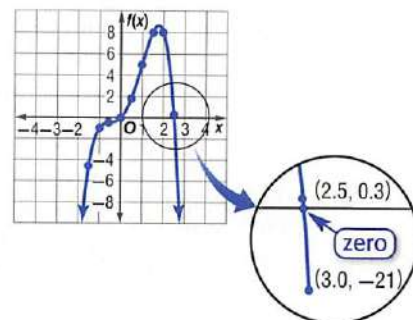
Check Your Progress

- Graph $f(x) = x^4 - x^3 - 2x^2 + 4x - 6$ by making a table of values.

 [Personal Tutor glencoe.com](http://glencoe.com)

In Example 1, one of the zeros occurred at $x = 0$. Another zero occurred between $x = 2.5$ and $x = 3.0$. Because $f(x)$ is positive for $x = 2.5$ and negative for $x = 3.0$ and all polynomial functions are continuous, we know there is a zero between these two values.

So, if the value of $f(x)$ changed signs from one value of x to the next, then there is a zero between those two x -values. This idea is called the **Location Principle**.



StudyTip

Degree Recall that the degree of the function is also the maximum number of zeros the function can have.



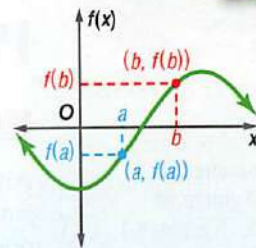
Key Concept

Location Principle

For Your
FOLDABLE

Words Suppose $y = f(x)$ represents a polynomial function and a and b are two real numbers such that $f(a) < 0$ and $f(b) > 0$. Then the function has at least one real zero between a and b .

Model



Math in Motion, Animation glencoe.com

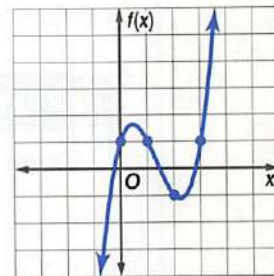
EXAMPLE 2 Locate Zeros of a Function

Determine consecutive integer values of x between which each real zero of $f(x) = x^3 - 4x^2 + 3x + 1$ is located. Then draw the graph.

Make a table of values. Since $f(x)$ is a third-degree polynomial function, it will have either 3 or 1 real zeros. Look at the values of $f(x)$ to locate the zeros. Then use the points to sketch a graph of the function.

x	$f(x)$
-2	-29
-1	-7
0	1
1	1
2	-1
3	1
4	13

← change in sign
← change in sign
← change in sign



The changes in sign indicate that there are zeros between $x = -1$ and $x = 0$, between $x = 1$ and $x = 2$, and between $x = 2$ and $x = 3$.

StudyTip

Odd Functions Some odd functions, like $f(x) = x^3$, have no turning points.

Check Your Progress

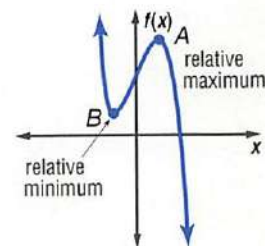
- Determine consecutive integer values of x between which each real zero of the function $f(x) = x^4 - 3x^3 - 2x^2 + x + 1$ is located. Then draw the graph.

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Maximum and Minimum Points The graph below shows the general shape of a third-degree polynomial function.

Point A on the graph is a **relative maximum** of the function since no other nearby points have a greater y -coordinate. The graph is increasing as it approaches A and decreasing as it moves from A .

Likewise, point B is a **relative minimum** since no other nearby points have a lesser y -coordinate. The graph is decreasing as it approaches B and increasing as it moves from B . The maximum and minimum values of a function are called the **extrema**.



These points are often referred to as **turning points**. The graph of a polynomial function of degree n has at most $n - 1$ turning points.

StudyTip

Maximum and Minimum A polynomial with a degree greater than 3 may have more than one relative maximum or relative minimum.

StudyTip

Rational Values Zeros and turning points will not always occur at integral values of x .

EXAMPLE 3 Maximum and Minimum Points

Graph $f(x) = x^3 - 4x^2 - 2x + 3$. Estimate the x -coordinates at which the relative maxima and relative minima occur.

Make a table of values and graph the function.

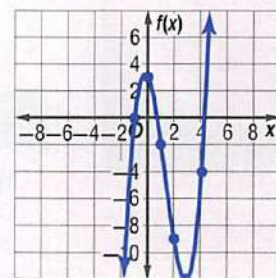
x	$f(x)$
-2	-17
-1	0
0	3
1	-2
2	-9
3	-12
4	-4
5	18

zero

indicates a relative maximum

indicates a relative minimum

zero between 4 and 5



Look at the table of values and the graph.

The value of $f(x)$ changes signs between $x = 4$ and $x = 5$, indicating a zero of the function.

The value of $f(x)$ at $x = 0$ is greater than the surrounding points, so there must be a relative maximum near $x = 0$.

The value of $f(x)$ at $x = 3$ is less than the surrounding points, so there must be a relative minimum near $x = 3$.

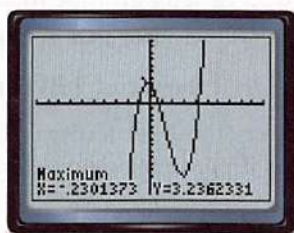
CHECK You can use a graphing calculator to find the relative maximum and relative minimum of a function and confirm your estimates.

Enter $y = x^3 - 4x^2 - 2x + 3$ in the $Y=$ list and graph the function.

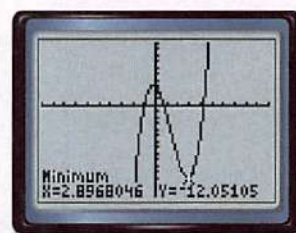
Use the **CALC** menu to find each maximum and minimum.

When selecting the left bound, move the cursor to the left of the maximum or minimum. When selecting the right bound, move the cursor to the right of the maximum or minimum.

Press **ENTER** twice.



$[-10, 10]$ scl: 1 by $[-15, 10]$ scl: 1



$[-10, 10]$ scl: 1 by $[-15, 10]$ scl: 1

The estimates for a relative maximum near $x = 0$ and a relative minimum near $x = 3$ are accurate.

Check Your Progress

3. Graph $f(x) = 2x^3 + x^2 - 4x - 2$. Estimate the x -coordinates at which the relative maxima and relative minima occur.

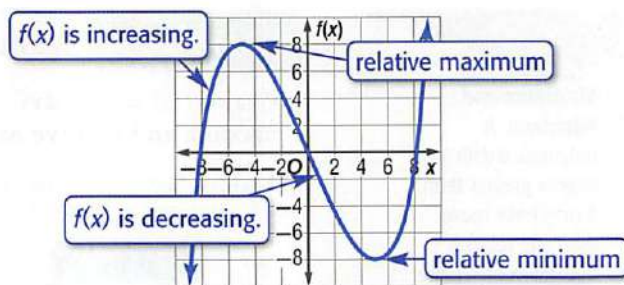


Real-World Link

Over 1.4 billion movie tickets were sold in the United States in 2006.

Source: CNN

The graph of a polynomial function can reveal trends in real-world data. It is often helpful to note when the graph is increasing or decreasing.



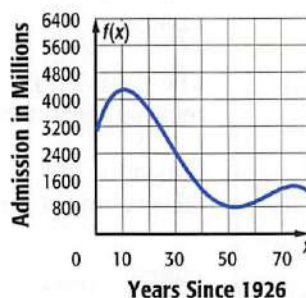
Real-World EXAMPLE 4 Graph a Polynomial Model

MOVIES Refer to the beginning of the lesson. Annual admissions to movies in the United States can be modeled by the function $f(x) = -0.0017x^4 + 0.31x^3 - 17.66x^2 + 277x + 3005$, where x is the number of years since 1926 and $f(x)$ is the annual admissions in millions.

a. Graph the function.

Make a table of values for the years 1926–2006. Plot the points and connect with a smooth curve. Finding and plotting the points for every tenth year gives a good approximation of the graph.

x	$f(x)$
0	3005
10	4302
20	3689
30	2414
40	1317
50	830
60	977
70	1374
80	1229



b. Describe the turning points of the graph and its end behavior.

There are relative maxima near 1936 and 2000 and a relative minimum between 1976 and 1981. $f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$ and $f(x) \rightarrow -\infty$ as $x \rightarrow \infty$.

c. What trends in movie admissions does the graph suggest? Is it reasonable that the trend will continue indefinitely?

Movie attendance peaked around 1936 and declined until about 1978. It then increased until 2000 and began a decline.

d. Is it reasonable that the trend will continue indefinitely?

This trend may continue for a couple of years, but the graph will soon become unreasonable as it predicts negative attendance for the future.

Check Your Progress

4. **FAX MACHINES** The annual sales of fax machines for home use can be modeled by $f(x) = -0.17x^4 + 6.29x^3 - 77.65x^2 + 251x + 1100$, where x is the number of years after 1990 and $f(x)$ is the annual sales in millions of dollars.

- Graph the function.
- Describe the turning points of the graph and its end behavior.
- What trends in fax machine sales does the graph suggest?
- Is it reasonable that the trend will continue indefinitely?

Check Your Understanding

Example 1 p. 357

Graph each polynomial equation by making a table of values.

1. $f(x) = 2x^4 - 5x^3 + x^2 - 2x + 4$

2. $f(x) = -2x^4 + 4x^3 + 2x^2 + x - 3$

3. $f(x) = 3x^4 - 4x^3 - 2x^2 + x - 4$

4. $f(x) = -4x^4 + 5x^3 + 2x^2 + 3x + 1$

Example 2 p. 358

Determine the consecutive integer values of x between which each real zero of each function is located. Then draw the graph.

5. $f(x) = x^3 - 2x^2 + 5$

6. $f(x) = -x^4 + x^3 + 2x^2 + x + 1$

7. $f(x) = -3x^4 + 5x^3 + 4x^2 + 4x - 8$

8. $f(x) = 2x^4 - x^3 - 3x^2 + 2x - 4$

Example 3 p. 359

Graph each polynomial function. Estimate the x -coordinates at which the relative maxima and relative minima occur. State the domain and range for each function.

9. $f(x) = x^3 + x^2 - 6x - 3$

10. $f(x) = 3x^3 - 6x^2 - 2x + 2$

11. $f(x) = -x^3 + 4x^2 - 2x - 1$

12. $f(x) = -x^3 + 2x^2 - 3x + 4$

Example 4 p. 360

13. **MUSIC SALES** Annual compact disc sales can be modeled by the quartic function $f(x) = 0.48x^4 - 9.6x^3 + 53x^2 - 49x + 599$, where x is the number of years after 1995 and $f(x)$ is annual sales in millions.

- Graph the function for $0 \leq x \leq 10$.
- Describe the turning points of the graph and its end behavior.
- Continue the graph for $x = 11$ and $x = 12$. What trends in compact disc sales does the graph suggest?
- Is it reasonable that the trend will continue indefinitely? Explain.

Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R20.
Extra Practice begins on page 947.

Examples 1–3 pp. 357–359

Complete each of the following.

- Graph each function by making a table of values.
- Determine the consecutive integer values of x between which each real zero is located.
- Estimate the x -coordinates at which the relative maxima and minima occur.

14. $f(x) = x^3 + 3x^2$

15. $f(x) = -x^3 + 2x^2 - 4$

16. $f(x) = x^3 + 4x^2 - 5x$

17. $f(x) = x^3 - 5x^2 + 3x + 1$

18. $f(x) = -2x^3 + 12x^2 - 8x$

19. $f(x) = 2x^3 - 4x^2 - 3x + 4$

20. $f(x) = x^4 + 2x - 1$

21. $f(x) = x^4 + 8x^2 - 12$

Example 4 p. 360

22. **FINANCIAL LITERACY** The average annual price of gasoline can be modeled by the cubic function $f(x) = 0.0007x^3 - 0.014x^2 + 0.08x + 0.96$, where x is the number of years after 1987 and $f(x)$ is the price in dollars.

- Graph the function for $0 \leq x \leq 30$.
- Describe the turning points of the graph and its end behavior.
- What trends in gasoline prices does the graph suggest?
- Is it reasonable that the trend will continue indefinitely? Explain.

Use a graphing calculator to estimate the x -coordinates at which the maxima and minima of each function occur. Round to the nearest hundredth.

23. $f(x) = x^3 + 3x^2 - 6x - 6$

24. $f(x) = -2x^3 + 4x^2 - 5x + 8$

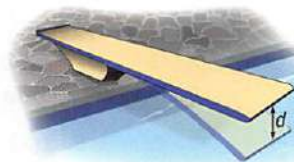
25. $f(x) = -2x^4 + 5x^3 - 4x^2 + 3x - 7$

26. $f(x) = x^5 - 4x^3 + 3x^2 - 8x - 6$

Sketch the graph of polynomial functions with the following characteristics.

27. an odd function with zeros at $-5, -3, 0, 2$ and 4
28. an even function with zeros at $-2, 1, 3,$ and 5
29. a 4-degree function with a zero at $-3,$ maximum at $x = 2,$ and minimum at $x = -1$
30. a 5-degree function with zeros at $-4, -1,$ and $3,$ maximum at $x = -2$
31. an odd function with zeros at $-1, 2,$ and 5 and a negative leading coefficient
32. an even function with a minimum at $x = 3$ and a positive leading coefficient

- 33. DIVING** The deflection d of a 10-foot-long diving board can be calculated using the function $d(x) = 0.015x^2 - 0.0005x^3$, where x is the distance between the diver and the stationary end of the board in feet.



- a. Make a table of values of the function for $0 \leq x \leq 10.$
- b. Graph the function.
- c. What does the end behavior of the graph suggest as x increases?
- d. Will this trend continue indefinitely? Explain your reasoning.



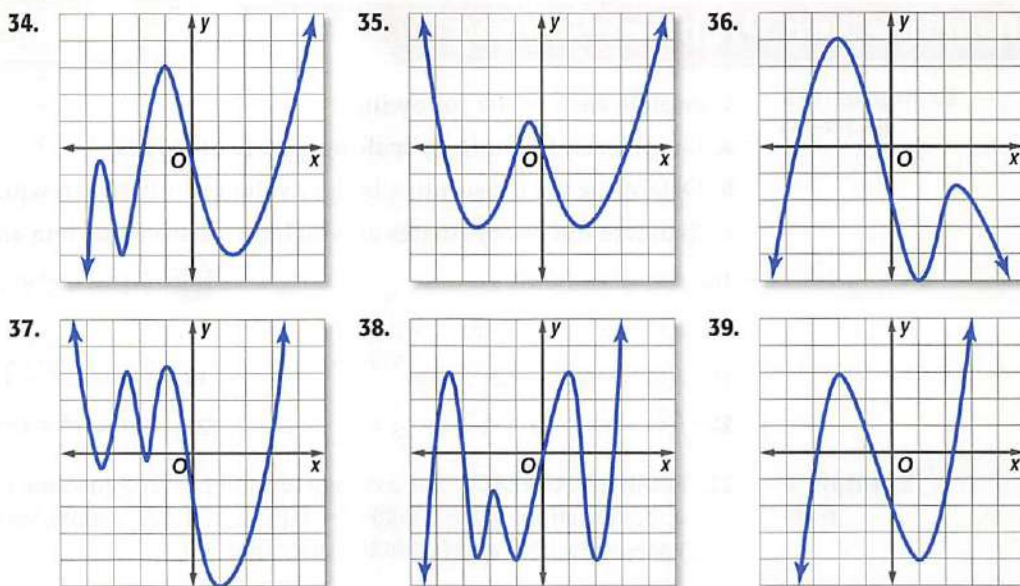
Complete each of the following.

- a. Estimate the x -coordinate of every turning point and determine if those coordinates are relative maxima or relative minima.
- b. Estimate the x -coordinate of every zero.
- c. Determine the smallest possible degree of the function.
- d. Determine the domain and range of the function.

Real-World Link

There are about 250,000 swimmers and divers on high school teams nationwide. The sport ranks 8th in popularity for girls and 10th for boys.

Source: National Federation of State High School Associations



- 40. PAGERS** The number of subscribers using pagers in the United States can be modeled by $f(x) = 0.015x^4 - 0.44x^3 + 3.46x^2 - 2.7x + 9.68$, where x is the number of years after 1990 and $f(x)$ is the number of subscribers in millions.

- a. Graph the function.
- b. Describe the end behavior of the graph.
- c. What does the end behavior suggest about the number of pager subscribers?
- d. Will this trend continue indefinitely? Explain your reasoning.



Real-World Link

Recent innovations in vending machines include telephone numbers that allow customers to call a machine and have the cost of the snack billed directly to their mobile phones.

Source: Reuters

41. **PRICING** Jin's vending machines currently sell an average of 3500 beverages per week at a rate of \$0.75 per can. She is considering increasing the price. Her weekly earnings can be represented by $f(x) = -5x^2 + 100x + 2625$, where x is the number of \$0.05 increases. Graph the function and determine the most profitable price for Jin.

For each function,

- determine the zeros, x - and y -intercepts, and turning points,
- determine the axis of symmetry, and
- determine the intervals for which it is increasing, decreasing, or constant.

42. $y = x^4 - 8x^2 + 16$

43. $y = x^5 - 3x^3 + 2x - 4$

44. $y = -2x^4 + 4x^3 - 5x$

45. $y = \begin{cases} x^2 & \text{if } x \leq -4 \\ 5 & \text{if } -4 < x \leq 0 \\ x^3 & \text{if } x > 0 \end{cases}$

46. **MULTIPLE REPRESENTATIONS** Consider the following function.

$$f(x) = x^4 - 8.65x^3 + 27.34x^2 - 37.2285x + 18.27$$

- ANALYTICAL** What are the degree, leading coefficient, and end behavior?
- TABULAR** Make a table of integer values $f(x)$ if $-4 \leq x \leq 4$. How many zeros does the function appear to have from the table?
- GRAPHICAL** Graph the function by using a graphing calculator.
- GRAPHICAL** Change the viewing window to $[0, 4]$ scl: 1 by $[-0.4, 0.4]$ scl: 0.2. What conclusions can you make from this new view of the graph?

H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

47. **REASONING** Explain why the leading coefficient and the degree are the only determining factors in the end behavior of a polynomial function.
48. **REASONING** The table below shows the values of $g(x)$, a cubic function. Could there be a zero between $x = 2$ and $x = 3$? Explain your reasoning.

x	-2	-1	0	1	2	3
$g(x)$	4	-2	-1	1	-2	-2

49. **OPEN ENDED** Sketch the graph of an odd polynomial function with 6 turning points and 2 double roots.
50. **REASONING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

For any continuous polynomial function, the y -coordinate of a turning point is also either a relative maximum or relative minimum.

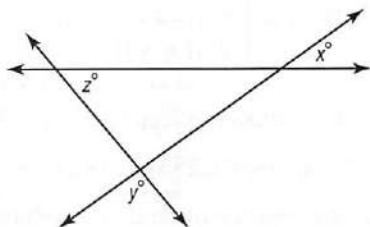
51. **REASONING** A function is said to be even if for every x in the domain of f , $f(x) = f(-x)$. Is every even-degree polynomial function also an even function? Explain.
52. **REASONING** A function is said to be *odd* if for every x in the domain, $-f(x) = f(-x)$. Is every odd-degree polynomial function also an odd function? Explain.
53. **WRITING IN MATH** Describe the process of sketching the graph of a polynomial function using its degree, leading coefficient, zeros, and turning points.

Standardized Test Practice

54. Which of the following is the factorization of $2x - 15 + x^2$?

- A $(x - 3)(x - 5)$
- B $(x - 3)(x + 5)$
- C $(x + 3)(x - 5)$
- D $(x + 3)(x + 5)$

55. **SHORT RESPONSE** In the figure below, if $x = 35$ and $z = 50$, what is the value of y ?

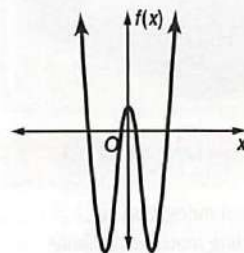


56. Which polynomial represents $(4x^2 + 5x - 3)(2x - 7)$?

- F $8x^3 - 18x^2 - 41x - 21$
- G $8x^3 + 18x^2 + 29x - 21$
- H $8x^3 - 18x^2 - 41x + 21$
- J $8x^3 + 18x^2 - 29x + 21$

57. **SAT/ACT** The figure at the right shows the graph of a polynomial function $f(x)$. Which of the following could be the degree of $f(x)$?

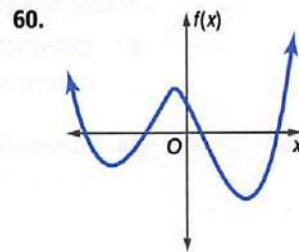
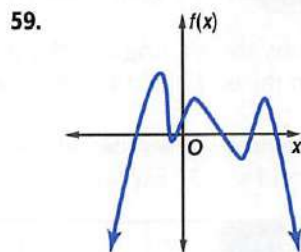
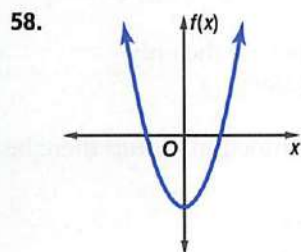
- A 2
- B 3
- C 4
- D 5



Spiral Review

For each graph,

- a. describe the end behavior,
- b. determine whether it represents an odd-degree or an even-degree function, and
- c. state the number of real zeros. (Lesson 6-3)



Simplify. (Lesson 6-2)

61. $(x^3 + 2x^2 - 5x - 6) \div (x + 1)$

62. $(4y^3 + 18y^2 + 5y - 12) \div (y + 4)$

63. $(2a^3 - a^2 - 4a) \div (a - 1)$

64. **CHEMISTRY** Tanisha needs 200 milliliters of a 48% concentration acid solution. She has 60% and 40% concentration solutions in her lab. How many milliliters of 40% acid solution should be mixed with 60% acid solution to make the required amount of 48% acid solution? (Lesson 4-6)

Skills Review

Factor. (Lesson 5-3)

65. $x^2 + 6x + 3x + 18$

66. $y^2 - 5y - 8y + 40$

67. $a^2 + 6a - 16$

68. $b^2 - 4b - 21$

69. $6x^2 - 5x - 4$

70. $4x^2 - 7x - 15$

Graphing Technology Lab

Modeling Data Using Polynomial Functions

You can use a TI-83/84 Plus graphing calculator to model data points when a curve of best fit is a polynomial function.

EXAMPLE

The table shows the distance a seismic wave produced by an earthquake travels from the epicenter. Draw a scatter plot and a curve of best fit to show how the distance is related to time. Then determine approximately how far away from the epicenter a seismic wave will be felt 8.5 minutes after an earthquake occurs.

Travel Time (min)	1	2	5	7	10	12	13
Distance (km)	400	800	2500	3900	6250	8400	10,000

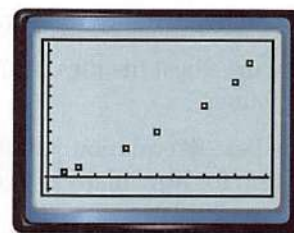
Source: University of Arizona

Step 1 Enter time in L1 and distance in L2.

KEYSTROKES: **STAT** 1 **1** **ENTER** 2 **ENTER** 5 **ENTER** 7 **ENTER** 10 **ENTER**
 12 **ENTER** 13 **ENTER** **▶** 400 **ENTER** 800 **ENTER** 2500 **ENTER** 3900 **ENTER**
 6250 **ENTER** 8400 **ENTER** 10000 **ENTER**

Step 2 Graph the scatter plot.

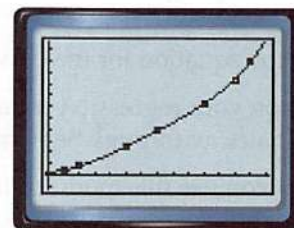
KEYSTROKES: **2nd** **[STAT PLOT]** 1 **ENTER** **▼** **ENTER** **ZOOM** 9



$[-0.2, 14.2]$ scl: 1
 by $[-1232, 11632]$ scl: 1000

Step 3 Determine and graph the equation for a curve of best fit. Use a quartic regression for the data.

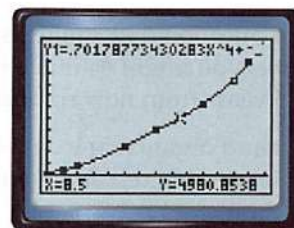
KEYSTROKES: **STAT** **▶** 7 **ENTER** **Y=** **VARS** 5 **▶** **▶** 1 **GRAPH**



Step 4 Use the **[CALC]** feature to find the value of the function for $x = 8.5$.

KEYSTROKES: **2nd** **[CALC]** 1 8.5 **ENTER**

After 8.5 minutes, the wave could be expected to be felt approximately 4980 kilometers from the epicenter.



(continued on the next page)

Exercises

The table shows how many minutes out of each eight-hour work day are used to pay one day's worth of taxes.

1. Draw a scatter plot of the data. Then graph several curves of best fit that relate the number of minutes to the number of years. Try LinReg, QuadReg, and CubicReg.
2. Write the equation for the curve that best fits the data.
3. Based on this equation, how many minutes should you expect to work each day in the year 2020 to pay one day's taxes?

Year	Minutes
1930	56
1940	86
1950	119
1960	134
1970	144
1980	147
1990	148
2000	163
2005	151

Source: Tax Foundation

The table shows the estimated number of alternative-fueled vehicles in use in the United States per year.

4. Draw a scatter plot of the data. Then graph several curves of best fit that relate the distance to the month.
5. Which curve best fits the data? Is that curve best for predicting future values?
6. Use the best-fit equation you think will give the most accurate prediction for how many alternative-fuel vehicles will be in use in the year 2012.

Year	Number of Vehicles	Year	Number of Vehicles
1995	246,855	2000	394,664
1996	265,006	2001	425,457
1997	280,205	2002	471,098
1998	295,030	2003	510,805
1999	322,302	2004	547,904

Source: U.S. Department of Energy

The table shows the average distance from the Sun to Earth during each month of the year.

7. Draw a scatter plot of the data. Then graph several curves of best fit that relate the distance to the month.
8. Write the equation for the curve that best fits the data.
9. Based on your regression equation, what is the distance from the Sun to Earth halfway through September?
10. Would you use this model to find the distance from the Sun to Earth in subsequent years? Explain your reasoning.

Month	Distance (astronomical units)
January	0.9840
February	0.9888
March	0.9962
April	1.0050
May	1.0122
June	1.0163
July	1.0161
August	1.0116
September	1.0039
October	0.9954
November	0.9878
December	0.9837

Source: The Astronomy Cafe

Extension

11. Write a question that could be answered by examining data. For example, you might estimate the number of people living in your town 5 years from now or predict the future cost of a car.
12. Collect and organize the data you need to answer the question you wrote. You may need to research your topic on the Internet or conduct a survey to collect the data you need.
13. Make a scatter plot and find a regression equation for your data. Then use the regression equation to answer the question.

Simplify. Assume that no variable equals 0. (Lesson 6-1)

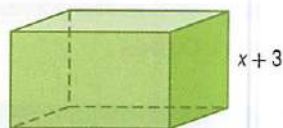
1. $(3x^2y^{-3})(-2x^3y^5)$ 2. $4t(3rt - r)$

3. $\frac{3a^4b^3c}{6a^2b^5c^3}$ 4. $\left(\frac{p^2r^3}{pr^4}\right)^2$

5. $(4m^2 - 6m + 5) - (6m^2 + 3m - 1)$

6. $(x + y)(x^2 + 2xy - y^2)$

7. **MULTIPLE CHOICE** The volume of the rectangular prism is $6x^3 + 19x^2 + 2x - 3$. Which polynomial expression represents the area of the base? (Lesson 6-1)



A $6x^4 + 37x^3 + 59x^2 + 3x - 9$

B $6x^2 + x + 1$

C $6x^2 + x - 1$

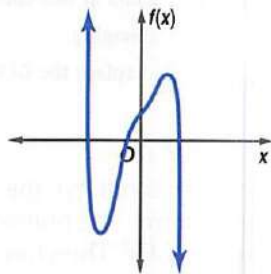
D $6x + 1$

Simplify. (Lesson 6-2)

8. $(4r^3 - 8r^2 - 13r + 20) \div (2r - 5)$

9. $\frac{3x^3 - 16x^2 + 9x - 24}{x - 5}$

10. Describe the end behavior of the graph. Then determine whether it represents an odd-degree or an even-degree polynomial function and state the number of real zeros. (Lesson 6-3)



11. **MULTIPLE CHOICE** Find $p(-3)$ if $p(x) = \frac{2}{3}x^3 + \frac{1}{3}x^2 - 5x$. (Lesson 6-3)

F 0

H 30

G 11

J 36

12. **PENDULUMS** The formula $L(t) = \frac{8t^2}{\pi^2}$ can be used to find the length of a pendulum in feet when it swings back and forth in t seconds. Find the length of a pendulum that makes one complete swing in 4 seconds. (Lesson 6-3)

13. **MULTIPLE CHOICE** Find $3f(a - 4) - 2h(a)$ if $f(x) = x^2 + 3x$ and $h(x) = 2x^2 - 3x + 5$. (Lesson 6-3)

A $-a^2 + 15a - 74$

B $-a^2 - 2a - 1$

C $a^2 + 9a - 2$

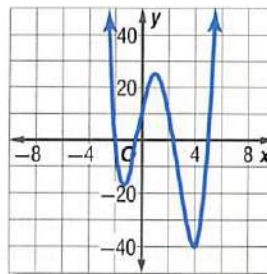
D $-a^2 - 9a + 2$

14. **ENERGY** The power generated by a windmill is a function of the speed of the wind. The approximate power is given by the function $P(s) = \frac{s^3}{1000}$, where s represents the speed of the wind in kilometers per hour. Find the units of power $P(s)$ generated by a windmill when the wind speed is 18 kilometers per hour. (Lesson 6-3)

Use $f(x) = x^3 - 2x^2 - 3x$ for Exercises 15–17. (Lesson 6-4)

15. Graph the function.
16. Estimate the x -coordinates at which the relative maxima and relative minima occur.
17. State the domain and range of the function.
18. Determine the consecutive integer values of x between which each real zero is located for $f(x) = 3x^2 - 3x - 1$. (Lesson 6-4)

Refer to the graph. (Lesson 6-4)



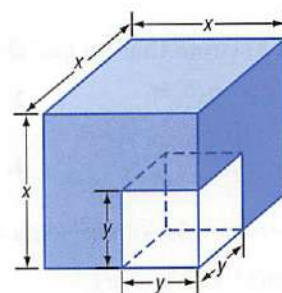
19. Estimate the x -coordinate of every turning point, and determine if those coordinates are relative maxima or relative minima.
20. Estimate the x -coordinate of every zero.
21. What is the least possible degree of the function?

Solving Polynomial Equations

Why?

A small cube is cut out of a larger cube. The volume of the remaining figure is given and the dimensions of each cube need to be determined.

This can be accomplished by factoring the cubic polynomial $x^3 - y^3$.

**Then**

You solved quadratic functions by factoring. (Lesson 5-3)

Now

- Factor polynomials.
- Solve polynomial equations by factoring.

New Vocabulary

prime polynomials
quadratic form

Math Online

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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Factor Polynomials In Lesson 5-3, you learned that quadratics can be factored just like whole numbers. Their factors, however, are other polynomials. Like quadratics, some cubic polynomials can also be factored with special rules.

Key Concept**Sum and Difference of Cubes**

For Your
FOLDABLE

Factoring Technique	General Case
Sum of Two Cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Difference of Two Cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Polynomials that cannot be factored are called **prime polynomials**.

EXAMPLE 1 Sum and Difference of Cubes

Factor each polynomial. If the polynomial cannot be factored, write *prime*.

a. $16x^4 + 54xy^3$

$$16x^4 + 54xy^3 = 2x(8x^3 + 27y^3)$$

Factor out the GCF.

$8x^3$ and $27y^3$ are both perfect cubes, so we can factor the sum of two cubes.

$$\begin{aligned} 8x^3 + 27y^3 &= (2x)^3 + (3y)^3 \\ &= (2x + 3y)[(2x)^2 - (2x)(3y) + (3y)^2] \\ &= (2x + 3y)(4x^2 - 6xy + 9y^2) \end{aligned}$$

$$(2x)^3 = 8x^3; (3y)^3 = 27y^3$$

Sum of two cubes

Simplify.

$$16x^4 + 54xy^3 = 2x(2x + 3y)(4x^2 - 6xy + 9y^2)$$

Replace the GCF.

b. $9y^3 + 5x^3$

The first term is a perfect cube, but the second term is not. So, the polynomial cannot be factored using the sum of two cubes pattern. The polynomial also cannot be factored using quadratic methods or the GCF. Therefore, it is a prime polynomial.

Check Your Progress

1A. $5y^4 - 320yz^3$

1B. $-54w^4 - 250wz^3$

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Math History Link

Sophie Germain
(1776–1831)

Sophie Germain taught herself mathematics with books from her father's library during the French Revolution, when she was confined for safety. Germain discovered the identity $x^4 + 4y^4 = (x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy)$, which is named for her.

The table below summarizes the most common factoring techniques used with polynomials. Whenever you factor a polynomial, always look for a common factor first. Then determine whether the resulting polynomial factors can be factored again using one or more of the methods below.

Concept Summary		Factoring Techniques	For Your FOLDABLE
Number of Terms	Factoring Technique	General Case	
any number	Greatest Common Factor (GCF)	$4a^3b^2 - 8ab = 4ab(a^2b - 2)$	
two	Difference of Two Squares Sum of Two Cubes Difference of Two Cubes	$a^2 - b^2 = (a + b)(a - b)$ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	
three	Perfect Square Trinomials	$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$	
	General Trinomials	$acx^2 + (ad + bc)x + bd$ $= (ax + b)(cx + d)$	
four or more	Grouping	$ax + bx + ay + by$ $= x(a + b) + y(a + b)$ $= (a + b)(x + y)$	

EXAMPLE 2 Factoring by Grouping

Factor each polynomial. If the polynomial cannot be factored, write *prime*.

a. $8ax + 4bx + 4cx + 6ay + 3by + 3cy$

$$\begin{aligned} 8ax + 4bx + 4cx + 6ay + 3by + 3cy \\ &= (8ax + 4bx + 4cx) + (6ay + 3by + 3cy) \\ &= 4x(2a + b + c) + 3y(2a + b + c) \\ &= (4x + 3y)(2a + b + c) \end{aligned}$$

Original expression
Group to find a GCF.
Factor the GCF.
Distributive Property

b. $20fy - 16fz + 15gy + 8hz - 10hy - 12gz$

$$\begin{aligned} 20fy - 16fz + 15gy + 8hz - 10hy - 12gz \\ &= (20fy + 15gy - 10hy) + (-16fz - 12gz + 8hz) \\ &= 5y(4f + 3g - 2h) - 4z(4f + 3g - 2h) \\ &= (5y - 4z)(4f + 3g - 2h) \end{aligned}$$

Original expression
Group to find a GCF.
Factor the GCF.
Distributive Property

StudyTip

Answer Checks

Multiply the factors to check your answer.

Check Your Progress

2A. $30ax - 24bx + 6cx - 5ay^2 + 4by^2 - cy^2$

2B. $13ax + 18bz - 15by - 14az - 32bx + 9ay$

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Factoring by grouping is the only method that can be used to factor polynomials with four or more terms. For polynomials with two or three terms, it may be possible to factor according to one of the patterns listed above.

When factoring two terms in which the exponents are 6 or greater, look to factor perfect squares before factoring perfect cubes.

EXAMPLE 3 Combine Cubes and Squares

Factor each polynomial. If the polynomial cannot be factored, write *prime*.

a. $x^6 - y^6$

This polynomial could be considered the difference of two squares or the difference of two cubes. The difference of two squares should always be done before the difference of two cubes for easier factoring.

$$\begin{aligned} x^6 - y^6 &= (x^3 + y^3)(x^3 - y^3) \\ &= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2) \end{aligned}$$

Difference of two squares
Sum and difference of two cubes

b. $a^3x^2 - 6a^3x + 9a^3 - b^3x^2 + 6b^3x - 9b^3$

With six terms, factor by grouping first.

$$\begin{aligned} a^3x^2 - 6a^3x + 9a^3 - b^3x^2 + 6b^3x - 9b^3 &= (a^3x^2 - 6a^3x + 9a^3) + (-b^3x^2 + 6b^3x - 9b^3) \\ &= a^3(x^2 - 6x + 9) - b^3(x^2 - 6x + 9) \\ &= (a^3 - b^3)(x^2 - 6x + 9) \\ &= (a - b)(a^2 + ab + b^2)(x^2 - 6x + 9) \\ &= (a - b)(a^2 + ab + b^2)(x - 3)^2 \end{aligned}$$

Group to find a GCF.
Factor the GCF.
Distributive Property
Difference of cubes
Perfect squares

StudyTip

Grouping 6 or more terms Group the terms that have the most common values.

Check Your Progress

3A. $a^6 + b^6$

3B. $x^5 + 4x^4 + 4x^3 + x^2y^3 + 4xy^3 + 4y^3$

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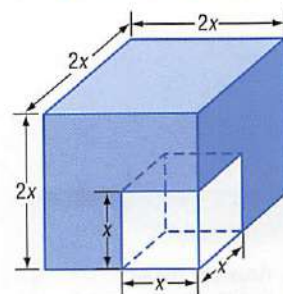
Solve Polynomial Equations In Chapter 5, you learned to solve quadratic equations by factoring and using the Zero Product Property. You can extend these techniques to solve higher-degree polynomial equations.

Real-World EXAMPLE 4 Solve Polynomial Functions by Factoring

GEOMETRY Refer to the beginning of the lesson.

If the small cube is half the length of the larger cube and the figure is 7000 cubic centimeters, what should be the dimensions of the cubes?

Since the length of the smaller cube is half the length of the larger cube, then their lengths can be represented by x and $2x$, respectively. The volume of the object equals the volume of the larger cube minus the volume of the smaller cube.



$$(2x)^3 - x^3 = 7000$$

$$8x^3 - x^3 = 7000$$

$$7x^3 = 7000$$

$$x^3 = 1000$$

$$x^3 - 1000 = 0$$

$$(x - 10)(x^2 + 10x + 100) = 0$$

$$x - 10 = 0 \quad \text{or} \quad x^2 + 10x + 100 = 0$$

$$x = 10$$

$$x = -5 \pm 5i\sqrt{3}$$

Volume of object

$$(2x)^3 = 8x^3$$

Subtract.

Divide.

Subtract 1000 from each side.

Difference of cubes

Zero Product Property

Since 10 is the only real solution, the lengths of the cubes are 10 cm and 20 cm.

Check Your Progress

4. Determine the dimensions of the cubes if the length of the smaller cube is one third of the length of the larger cube, and the volume of the object is 3250 cubic centimeters.

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In some cases, you can rewrite a polynomial in x in the form $au^2 + bu + c$. For example, by letting $u = x^2$, the expression $x^4 + 12x^2 + 32$ can be written as $(x^2)^2 + 12(x^2) + 32$ or $u^2 + 12u + 32$. This new, but equivalent, expression is said to be in **quadratic form**.



Key Concept

Quadratic Form

For Your
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Words An expression that is in quadratic form can be written as $au^2 + bu + c$ for any numbers a , b , and c , $a \neq 0$, where u is some expression in x . The expression $au^2 + bu + c$ is called the quadratic form of the original expression.

Example $12x^6 + 8x^6 + 1 = 3(2x^3)^2 + 2(2x^3)^2 + 1$

StudyTip

Quadratic Form When writing a polynomial in quadratic form, choose the expression equal to u by examining the terms with variables. Pay special attention to the exponents in those terms. Not every polynomial can be written in quadratic form.

EXAMPLE 5 Quadratic Form

Write each expression in quadratic form, if possible.

a. $150n^8 + 40n^4 - 15$

$$150n^8 + 40n^4 - 15 = 6(5n^4)^2 + 8(5n^4) - 15 \quad (5n^4)^2 = 25n^8$$

b. $y^8 + 12y^3 + 8$

This cannot be written in quadratic form since $y^8 \neq (y^3)^2$.

Check Your Progress

5A. $x^4 + 5x + 6$

5B. $8x^4 + 12x^2 + 18$

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You can use quadratic form to solve equations with larger degrees.

EXAMPLE 6 Solve Equations in Quadratic Form

Solve $18x^4 - 21x^2 + 3 = 0$.

$$18x^4 - 21x^2 + 3 = 0$$

Original equation

$$2(3x^2)^2 - 7(3x^2) + 3 = 0$$

$$2(3x^2)^2 = 18x^4$$

$$2u^2 - 7u + 3 = 0$$

Let $u = 3x^2$.

$$(2u - 1)(u - 3) = 0$$

Factor.

$$u = \frac{1}{2} \quad \text{or} \quad u = 3$$

Zero Product Property

$$3x^2 = \frac{1}{2} \quad 3x^2 = 3$$

Replace u with $3x^2$.

$$x^2 = \frac{1}{6} \quad x^2 = 1$$

Divide by 3.

$$x = \pm \frac{\sqrt{6}}{6} \quad x = \pm 1$$

Take the square root.

The solutions of the equation are 1 , -1 , $\frac{\sqrt{6}}{6}$, and $-\frac{\sqrt{6}}{6}$.

Check Your Progress

6A. $4x^4 - 8x^2 + 3 = 0$

6B. $8x^4 + 10x^2 - 12 = 0$

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Check Your Understanding

Examples 1–3 pp. 368–370

Factor completely. If the polynomial is not factorable, write *prime*.

- $3ax + 2ay - az + 3bx + 2by - bz$
- $2kx + 4mx - 2nx - 3ky - 6my + 3ny$
- $2x^3 + 5y^3$
- $16g^3 + 2h^3$
- $12qw^3 - 12q^4$
- $3w^2 + 5x^2 - 6y^2 + 2z^2 + 7a^2 - 9b^2$
- $a^6x^2 - b^6x^2$
- $x^3y^2 - 8x^3y + 16x^3 + y^5 - 8y^4 + 16y^3$
- $8c^3 - 125d^3$
- $6bx + 12cx + 18dx - by - 2cy - 3dy$

Example 4 p. 370

Solve each equation.

- $x^4 - 19x^2 + 48 = 0$
- $x^3 - 64 = 0$
- $x^3 + 27 = 0$
- $x^4 - 33x^2 + 200 = 0$

15. **LANDSCAPING** A boardwalk that is x feet wide is built around a rectangular pond. The pond is 30 feet wide and 40 feet long. The combined area of the pond and the boardwalk is 2000 square feet. What is the width of the boardwalk?



Example 5 p. 371

Write each expression in quadratic form, if possible.

- $4x^6 - 2x^3 + 8$
- $25y^6 - 5y^2 + 20$

Example 6 p. 371

Solve each equation.

- $x^4 - 6x^2 + 8 = 0$
- $y^4 - 18y^2 + 72 = 0$

Practice and Problem Solving

Step-by-Step Solutions begin on page R20.
Extra Practice begins on page 947.

Examples 1–3 pp. 368–370

Factor completely. If the polynomial is not factorable, write *prime*.

- $8c^3 - 27d^3$
- $64x^4 + xy^3$
- $a^8 - a^2b^6$
- $x^6y^3 + y^9$
- $18x^6 + 5y^6$
- $w^3 - 2y^3$
- $gx^2 - 3hx^2 - 6fy^2 - gy^2 + 6fx^2 + 3hy^2$
- $12ax^2 - 20cy^2 - 18bx^2 - 10ay^2 + 15by^2 + 24cx^2$
- $a^3x^2 - 16a^3x + 64a^3 - b^3x^2 + 16b^3x - 64b^3$
- $8x^5 - 25y^3 + 80x^4 - x^2y^3 + 200x^3 - 10xy^3$

Example 4 p. 370

Solve each equation.

- $x^4 + x^2 - 90 = 0$
- $x^4 - 16x^2 - 720 = 0$
- $x^4 - 7x^2 - 44 = 0$
- $x^4 + 6x^2 - 91 = 0$
- $x^3 + 216 = 0$
- $64x^3 + 1 = 0$

Example 5 p. 371

Write each expression in quadratic form, if possible.

- $x^4 + 12x^2 - 8$
- $-15x^4 + 18x^2 - 4$
- $8x^6 + 6x^3 + 7$
- $5x^6 - 2x^2 + 8$
- $9x^8 - 21x^4 + 12$
- $16x^{10} + 2x^5 + 6$

Example 6 p. 371

Solve each equation.

- $x^4 + 6x^2 + 5 = 0$
- $x^4 - 3x^2 - 10 = 0$
- $4x^4 - 14x^2 + 12 = 0$
- $9x^4 - 27x^2 + 20 = 0$
- $4x^4 - 5x^2 - 6 = 0$
- $24x^4 + 14x^2 - 3 = 0$



Real-World Link

Brought by Spanish explorers in the 16th century, the modern horse is one of the most important species ever introduced to North America. Closely related horse ancestors had become extinct on the continent some 10,000 years before.

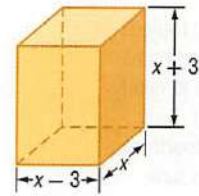
Source: Buffalo Bill Historical Center

48. **ZOOLOGY** A species of animal is introduced to a small island. Suppose the population of the species is represented by $P(t) = -t^4 + 9t^2 + 400$, where t is the time in years. Determine when the population becomes zero.

Factor completely. If the polynomial is not factorable, write *prime*.

49. $x^4 - 625$ 50. $x^6 - 64$ 51. $x^5 - 16x$ 52. $8x^5y^2 - 27x^2y^5$
53. $15ax - 10bx + 5cx + 12ay - 8by + 4cy + 15az - 10bz + 5cz$
54. $6a^2x^2 - 24b^2x^2 + 18c^2x^2 - 5a^2y^3 + 20b^2y^3 - 15c^2y^3 + 2a^2z^2 - 8b^2z^2 + 6c^2z^2$
55. $6x^5 - 11x^4 - 10x^3 - 54x^3 + 99x^2 + 90x$
56. $20x^6 - 7x^5 - 6x^4 - 500x^4 + 175x^3 + 150x^2$

57. **GEOMETRY** The volume of the figure at the right is 440 cubic centimeters. Find the value of x and the length, height, and width.

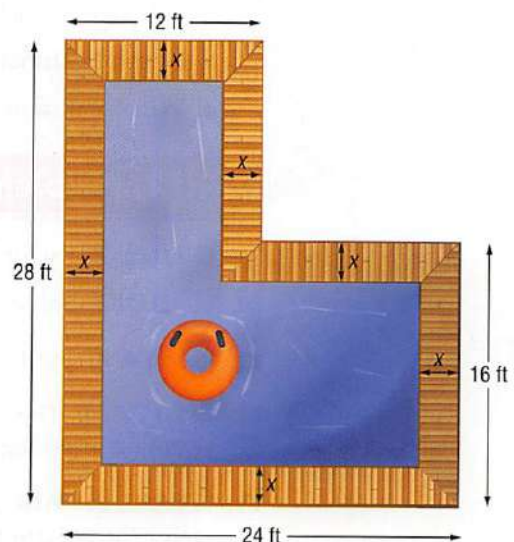


Solve each equation.

58. $8x^4 + 10x^2 - 3 = 0$ 59. $6x^4 - 5x^2 - 4 = 0$
60. $20x^4 - 53x^2 + 18 = 0$ 61. $18x^4 + 43x^2 - 5 = 0$
62. $8x^4 - 18x^2 + 4 = 0$ 63. $3x^4 - 22x^2 - 45 = 0$
64. $x^6 + 7x^3 - 8 = 0$ 65. $x^6 - 26x^3 - 27 = 0$
66. $8x^6 + 999x^3 = 125$ 67. $4x^4 - 4x^2 - x^2 + 1 = 0$
68. $x^6 - 9x^4 - x^2 + 9 = 0$ 69. $x^4 + 8x^2 + 15 = 0$

70. **GEOMETRY** A rectangular prism with dimensions $x - 2$, $x - 4$, and $x - 6$ has a volume equal to $40x$ cubic units.
- Write out a polynomial equation using the formula for volume.
 - Use factoring to solve for x .
 - Are any values for x unreasonable? Explain.
 - What are the dimensions of the prism?

71. **POOL DESIGN** Andrea wants to build a pool following the diagram at the right. The pool will be surrounded by a sidewalk of a constant width.
- If the total area of the pool itself is to be 336 ft^2 , what is x ?
 - If the value of x were doubled, what would be the new area of the pool?
 - If the value of x were halved, what would be the new area of the pool?





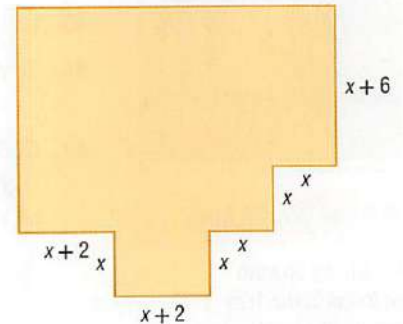
Real-World Link

Americans have tripled their spending on home theater systems to nearly \$1 billion a year. An estimated one fourth of U.S. households have some kind of home theater.

Source: USA TODAY

72. **BIOLOGY** During an experiment, the number of cells of a virus can be modeled by $P(t) = -0.012t^3 - 0.24t^2 + 6.3t + 8000$, where t is the time in hours and P is the number of cells. Jack wants to determine the times at which there are 8000 cells.
- Solve for t by factoring.
 - What method did you use to factor?
 - Which values for t are reasonable and which are unreasonable? Explain.
 - Graph the function for $0 \leq t \leq 20$ using your calculator.

73. **HOME BUILDING** Alicia's parents want their basement home theater designed according to the diagram at the right.



- Write a function in terms of x for the area of the basement.
- If the basement is to be 1366 square feet, what is x ?

74. **BIOLOGY** A population of parasites in an experiment can be modeled by $f(t) = t^3 + 5t^2 - 4t - 20$, where t is the time in days.

- Use factoring by grouping to determine the values of t for which $f(t) = 0$.
- At what times does the population reach zero?
- Are any of the values of t unreasonable? Explain.

Factor completely. If the polynomial is not factorable, write *prime*.

- $x^6 - 4x^4 - 8x^4 + 32x^2 + 16x^2 - 64$
- $y^9 - y^6 - 2y^6 + 2y^3 + y^3 - 1$
- $x^6 - 3x^4y^2 + 3x^2y^4 - y^6$

78. **CORRALS** Fredo's corral, an enclosure for livestock, is currently 32 feet by 40 feet. He wants to enlarge the area to 4.5 times its current area by increasing the length and width by the same amount.
- Draw a diagram to represent the situation.
 - Write a polynomial equation for the area of the new corral. Then solve the equation by factoring.
 - Graph the function.
 - Which solution is irrelevant? Explain.

H.O.T. Problems

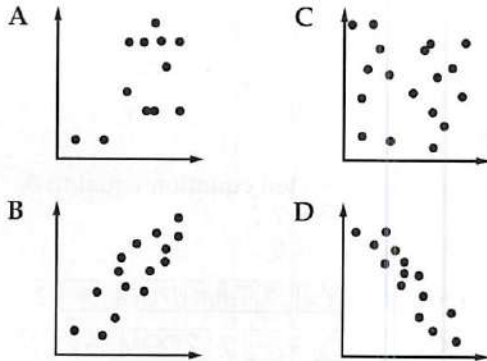
Use **H**igher-**O**rders **T**hinking Skills

- CHALLENGE** Factor $36x^{2n} + 12x^n + 1$.
- CHALLENGE** Solve $6x - 11\sqrt{3x} + 12 = 0$.
- REASONING** Find a counterexample to the statement $a^2 + b^2 = (a + b)^2$.
- OPEN ENDED** The cubic form of an equation is $ax^3 + bx^2 + cx + d = 0$. Write an equation with degree 6 that can be written in *cubic* form.
- WRITING IN MATH** Explain how the graph of a polynomial function can help you factor the polynomial.

Standardized Test Practice

84. SHORT RESPONSE Tiles numbered from 1 to 6 are placed in a bag and are drawn to determine which of six tasks will be assigned to six people. What is the probability that the tiles numbered 5 and 6 are the last two drawn?

85. STATISTICS Which of the following represents a negative correlation?



86. Which of the following most accurately describes the translation of the graph $y = (x + 4)^2 - 3$ to the graph of $y = (x - 1)^2 + 3$?

- F down 1 and to the right 3
- G down 6 and to the left 5
- H up 1 and to the left 3
- J up 6 and to the right 5

87. SAT/ACT The positive difference between k and $\frac{1}{12}$ is the same as the positive difference between $\frac{1}{3}$ and $\frac{1}{5}$. Which of the following is the value of k ?

- A $\frac{1}{60}$
- B $\frac{1}{20}$
- C $\frac{1}{15}$
- D $\frac{13}{60}$

Spiral Review

Graph each polynomial function. Estimate the x -coordinates at which the relative maxima and relative minima occur. (Lesson 6-4)

88. $f(x) = 2x^3 - 4x^2 + x + 8$

89. $f(x) = -3x^3 + 6x^2 + 2x - 1$

90. $f(x) = -x^3 + 3x^2 + 4x - 6$

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why. (Lesson 6-3)

91. $f(x) = 4x^3 - 6x^2 + 5x^4 - 8x$

92. $f(x) = -2x^5 + 5x^4 + 3x^2 + 9$

93. $f(x) = -x^4 - 3x^3 + 2x^6 - x^7$

94. ELECTRICITY The impedance in one part of a series circuit is $3 + 4j$ ohms, and the impedance in another part of the circuit is $2 - 6j$ ohms. Add these complex numbers to find the total impedance of the circuit. (Lesson 5-4)

95. SKIING All 28 members of a ski club went on a trip. The club paid a total of \$478 for the equipment. How many skis and snowboards did they rent? (Lesson 3-2)

96. GEOMETRY The sides of an angle are parts of two lines whose equations are $2y + 3x = -7$ and $3y - 2x = 9$. The angle's vertex is the point where the two sides meet. Find the coordinates of the vertex of the angle. (Lesson 3-1)

Ski Rental
\$16/day

or

Snowboard Rental
\$19/day

Skills Review

Divide. (Lesson 6-2)

97. $(x^2 + 6x - 2) \div (x + 4)$

98. $(2x^2 + 8x - 10) \div (2x + 1)$

99. $(8x^3 + 4x^2 + 6) \div (x + 2)$

Graphing Technology Lab

Solving Polynomial Inequalities

You can use a TI-83/84 Plus graphing calculator to solve polynomial inequalities.

ACTIVITY

Solve $x^4 + 2x^3 \leq 7$.

Method 1

Step 1 Graph each side of the related equation separately.

KEYSTROKES: $Y=$ X,T,θ,n \wedge 4 $+$ 2
 X,T,θ,n \wedge 3 ENTER 7 ZOOM 6

Step 2 Find the points of intersection.

KEYSTROKES: 2nd $[\text{CALC}]$ 5

Use \leftarrow or \rightarrow to position the cursor on Y1, near the first point of intersection.

Press ENTER ENTER ENTER .

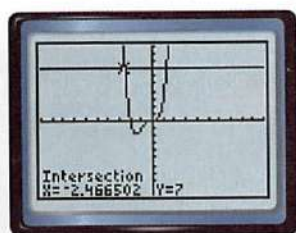
Then use \rightarrow to position the cursor near the second intersection point.

Press ENTER ENTER ENTER .

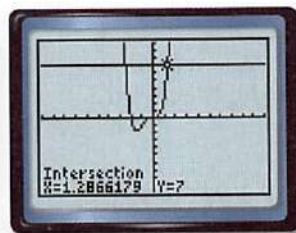
Step 3 Examine the graphs.

Determine where the graph of $y = x^4 + 2x^3$ intersects and is below $y = 7$.

The solution is approximately $-2.47 \leq x \leq 1.29$.



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

Method 2

Step 1 Graph the related equation equal to 0.

$$x^4 + 2x^3 = 7$$

$$x^4 + 2x^3 - 7 = 0$$

KEYSTROKES: $Y=$ X,T,θ,n \wedge 4 $+$ 2
 X,T,θ,n \wedge 3 $-$ 7 ZOOM 6

Step 2 Find the x -intercepts.

KEYSTROKES: 2nd $[\text{CALC}]$ 2

Use \leftarrow or \rightarrow to position the cursor to the left of the first x -intercept. Press ENTER .

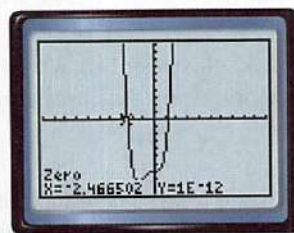
Then use \rightarrow to position the cursor to the right of the first x -intercept. Press ENTER

ENTER to display the x -intercept. Then, repeat the procedure for any remaining x -intercepts.

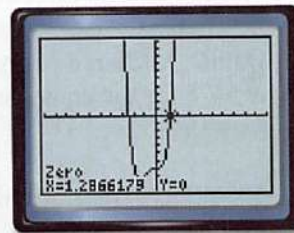
Step 3 Examine the graphs.

Determine where the graph of $y = x^4 + 2x^3 - 7$ is below the x -axis.

The solution is approximately $-2.47 \leq x \leq 1.29$.



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

Exercises

Solve each inequality. Round to the nearest hundredth.

1. $\frac{2}{3}x^3 + x^2 - 5x \geq -9$

2. $x^3 - 9x^2 + 27x \leq 20$

3. $x^3 + 1 \geq 4x^2$

4. $x^6 - 15 \leq 5x^4 - x^2$

5. $\frac{1}{2}x^5 \geq \frac{1}{5}x^2 - 2$

6. $x^8 < -x^7 + 3$

7. $x^4 - 15x^2 > -24$

8. $x^3 - 6x^2 + 4x < -6$

9. $x^4 - 15x^2 + x + 65 > 0$

6-6

The Remainder and Factor Theorems

Why?

The number of college students from the United States who study abroad can be modeled by the function $S(x) = 0.02x^4 - 0.52x^3 + 4.03x^2 + 0.09x + 77.54$, where x is the number of years since 1993 and $S(x)$ is the number of students in thousands.

You can use this function to estimate the number of U.S. college students studying abroad in 2013 by evaluating the function for $x = 20$. Another method you can use is *synthetic substitution*.



Then

You used the Distributive Property and factoring to simplify algebraic expressions. (Lesson 5-3)

Now

- Evaluate functions by using synthetic substitution.
- Determine whether a binomial is a factor of a polynomial by using synthetic substitution.

New Vocabulary

synthetic substitution
depressed polynomial

Math Online

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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Synthetic Substitution Synthetic division can be used to find the value of a function. Consider the polynomial function $f(x) = -3x^2 + 5x + 4$. Divide the polynomial by $x - 3$.

Method 1 Long Division

$$\begin{array}{r} -3x - 4 \\ x - 3 \overline{) -3x^2 + 5x + 4} \\ \underline{-3x^2 + 9x} \\ -4x + 4 \\ \underline{-4x + 12} \\ -8 \end{array}$$

Method 2 Synthetic Division

$$\begin{array}{r|rrr} 3 & -3 & 5 & 4 \\ & & -9 & -12 \\ \hline & -3 & -4 & -8 \end{array}$$

Compare the remainder of -8 to $f(3)$.

$$\begin{aligned} f(3) &= -3(3)^2 + 5(3) + 4 && \text{Replace } x \text{ with } 3. \\ &= -27 + 15 + 4 && \text{Multiply.} \\ &= -8 && \text{Simplify.} \end{aligned}$$

Notice that the value of $f(3)$ is the same as the remainder when the polynomial is divided by $x - 3$. This illustrates the **Remainder Theorem**.

Key Concept

Remainder Theorem

For Your
FOLDABLE

Words If a polynomial $P(x)$ is divided by $x - r$, the remainder is a constant $P(r)$, and

Dividend equals quotient times divisor plus remainder.

$$P(x) = Q(x) \cdot (x - r) + P(r),$$

where $Q(x)$ is a polynomial with degree one less than $P(x)$.

Example $x^2 + 6x + 2 = (x - 4) \cdot (x + 10) + 42$

Applying the Remainder Theorem using synthetic division to evaluate a function is called **synthetic substitution**. It is a convenient way to find the value of a function, especially when the degree of the polynomial is greater than 2.

EXAMPLE 1 Synthetic Substitution

If $f(x) = 3x^4 - 2x^3 + 5x + 2$, find $f(4)$.

Method 1 Synthetic Substitution

By the Remainder Theorem, $f(4)$ should be the remainder when the polynomial is divided by $x - 4$.

$$\begin{array}{r|rrrrrr}
 4 & 3 & -2 & 0 & 5 & 2 \\
 & & 12 & 40 & 160 & 660 \\
 \hline
 & 3 & 10 & 40 & 165 & 662
 \end{array}$$

Because there is no x^2 term, a zero is placed in this position as a placeholder.

The remainder is 662. Therefore, by using synthetic substitution, $f(4) = 662$.

Method 2 Direct Substitution

Replace x with 4.

$f(x) = 3x^4 - 2x^3 + 5x + 2$ Original function

$f(4) = 3(4)^4 - 2(4)^3 + 5(4) + 2$ Replace x with 4.

$= 768 - 128 + 20 + 2$ or 662 Simplify.

By using direct substitution, $f(4) = 662$. Both methods give the same result.

Check Your Progress

1A. If $f(x) = 3x^3 - 6x^2 + x - 11$, find $f(3)$.

1B. If $g(x) = 4x^5 + 2x^3 + x^2 - 1$, find $f(-1)$.

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Real-World Link

Some benefits of studying abroad include learning a new language, becoming more independent, and improving communication skills. Studying abroad also gives students a chance to experience different cultures and customs as well as try new foods.

Source: StudyAbroad

Synthetic substitution can be used in situations in which direct substitution would involve cumbersome calculations.

Real-World EXAMPLE 2 Find Function Values

COLLEGE Refer to the beginning of the lesson. How many U.S. college students will study abroad in 2013?

Use synthetic substitution to divide $0.02x^4 - 0.52x^3 + 4.03x^2 + 0.09x + 77.54$ by $x - 20$.

$$\begin{array}{r|rrrrrr}
 20 & 0.02 & -0.52 & 4.03 & 0.09 & 77.54 \\
 & & 0.4 & -2.4 & 32.6 & 653.8 \\
 \hline
 & 0.02 & -0.12 & 1.63 & 32.69 & 731.34
 \end{array}$$

In 2013, there will be about 731,340 U.S. college students studying abroad.

Check Your Progress

2. **COLLEGE** The function $C(x) = 2.46x^3 - 22.37x^2 + 53.81x + 548.24$ can be used to approximate the number, in thousands, of international college students studying in the United States x years since 2000. How many international college students can be expected to study in the U.S. in 2012?

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Factors of Polynomials The synthetic division below shows that the quotient of $2x^3 - 3x^2 - 17x + 30$ and $x + 3$ is $2x^2 - 9x + 10$.

$$\begin{array}{r|rrrr} -3 & 2 & -3 & -17 & 30 \\ & & -6 & 27 & -30 \\ \hline & 2 & -9 & 10 & 0 \end{array}$$

When you divide a polynomial by one of its binomial factors, the quotient is called a depressed polynomial. A **depressed polynomial** has a degree that is one less than the original polynomial. From the results of the division, and by using the Remainder Theorem, we can make the following statement.

$$\begin{array}{ccccccc} \text{Dividend} & & \text{equals} & & \text{quotient} & \text{times} & \text{divisor} & \text{plus} & \text{remainder.} \\ 2x^3 - 3x^2 - 17x + 30 & = & & (2x^2 - 9x + 10) & \cdot & & (x + 3) & + & 0 \end{array}$$

Since the remainder is 0, $f(-3) = 0$. This means that $x + 3$ is a factor of $2x^3 - 3x^2 - 17x + 30$. This illustrates the **Factor Theorem**, which is a special case of the Remainder Theorem.

Key Concept

Factor Theorem

For Your

FOLDABLE

The binomial $x - r$ is a factor of the polynomial $P(x)$ if and only if $P(r) = 0$.

The Factor Theorem can be used to determine whether a binomial is a factor of a polynomial. It can also be used to determine all of the factors of a polynomial.

EXAMPLE 3 Use the Factor Theorem

Determine whether $x - 5$ is a factor of $x^3 - 7x^2 + 7x + 15$. Then find the remaining factors of the polynomial.

The binomial $x - 5$ is a factor of the polynomial if 5 is a zero of the related polynomial function. Use the Factor Theorem and synthetic division.

$$\begin{array}{r|rrrr} 5 & 1 & -7 & 7 & 15 \\ & & 5 & -10 & -15 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

Because the remainder is 0, $x - 5$ is a factor of the polynomial. The polynomial $x^3 - 7x^2 + 7x + 15$ can be factored as $(x - 5)(x^2 - 2x - 3)$. The polynomial $x^2 - 2x - 3$ is the depressed polynomial. Check to see if this polynomial can be factored.

$$x^2 - 2x - 3 = (x + 1)(x - 3) \quad \text{Factor the trinomial.}$$

$$\text{So, } x^3 - 7x^2 + 7x + 15 = (x - 5)(x + 1)(x - 3).$$

You can check your answer by multiplying out the factors and seeing if you come up with the initial polynomial.

Check Your Progress

- Show that $x - 2$ is a factor of $x^3 - 7x^2 + 4x + 12$. Then find the remaining factors of the polynomial.

StudyTip

Factoring The factors of a polynomial do not have to be binomials. For example, the factors of $x^3 + x^2 - x + 15$ are $x + 3$ and $x^2 - 2x + 5$.

Check Your Understanding

Example 1 p. 378

Use synthetic substitution to find $f(4)$ and $f(-2)$ for each function.

1. $f(x) = 2x^3 - 5x^2 - x + 14$ 2. $f(x) = x^4 + 8x^3 + x^2 - 4x - 10$

Example 2 p. 378

3. **NATURE** The approximate number of bald eagle nesting pairs in the United States can be modeled by the function $P(x) = -0.16x^3 + 15.83x^2 - 154.15x + 1147.97$, where x is the number of years since 1970. About how many nesting pairs of bald eagles can be expected in 2018?

Example 3 p. 379

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

4. $x^3 - 6x^2 + 11x - 6; x - 1$ 5. $x^3 + x^2 - 16x - 16; x + 1$
6. $3x^3 + 10x^2 - x - 12; x - 1$ 7. $2x^3 - 5x^2 - 28x + 15; x + 3$

Step-by-Step Solutions begin on page R20.
Extra Practice begins on page 947.

Practice and Problem Solving

Example 1 p. 378

Use synthetic substitution to find $f(-5)$ and $f(2)$ for each function.

8. $f(x) = x^3 + 2x^2 - 3x + 1$ 9. $f(x) = x^2 - 8x + 6$
10. $f(x) = 3x^4 + x^3 - 2x^2 + x + 12$ 11. $f(x) = 2x^3 - 8x^2 - 2x + 5$
12. $f(x) = x^3 - 5x + 2$ 13. $f(x) = x^5 + 8x^3 + 2x - 15$
14. $f(x) = x^6 - 4x^4 + 3x^2 - 10$ 15. $f(x) = x^4 - 6x - 8$

Example 2 p. 378

16. **FINANCIAL LITERACY** A specific car's fuel economy in miles per gallon can be approximated by $f(x) = 0.00000056x^4 - 0.000018x^3 - 0.016x^2 + 1.38x - 0.38$, where x represents the car's speed in miles per hour. Determine the fuel economy when the car is traveling 40, 50 and 60 miles per hour.

Example 3 p. 379

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

17. $x^3 - 3x + 2; x + 2$ 18. $x^4 + 2x^3 - 8x - 16; x + 2$
19. $x^3 - x^2 - 10x - 8; x + 2$ 20. $x^3 - x^2 - 5x - 3; x - 3$
21. $2x^3 + 17x^2 + 23x - 42; x - 1$ 22. $2x^3 + 7x^2 - 53x - 28; x - 4$
23. $x^4 + 2x^3 + 2x^2 - 2x - 3; x - 1$ 24. $x^3 + 2x^2 - x - 2; x + 2$
25. $6x^3 - 25x^2 + 2x + 8; 2x + 1$ 26. $16x^5 - 32x^4 - 81x + 162; 2x - 3$

27. **BOATING** A motor boat traveling against waves accelerates from a resting position. Suppose the speed of the boat in feet per second is given by the function $f(t) = -0.04t^4 + 0.8t^3 + 0.5t^2 - t$, where t is the time in seconds.

- a. Find the speed of the boat at 1, 2, and 3 seconds.
b. It takes 6 seconds for the boat to travel between two buoys while it is accelerating. Use synthetic substitution to find $f(6)$ and explain what this means.

28. **SALES** A company's sales, in millions of dollars, of consumer electronics can be modeled by $S(x) = -1.7x^3 + 18x^2 + 26.4x + 678$, where x is the number of years since 2005.

- a. Use synthetic substitution to estimate the sales for 2010 and 2015.
b. Do you think this model is useful in estimating future sales? Explain.

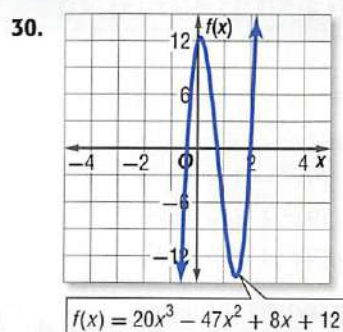
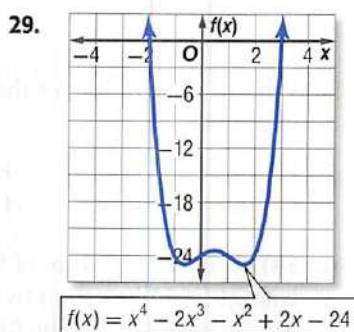



Real-World Link

The average American spends about \$8500 annually on consumer electronics.

Source: IT Facts

Use the graph to find all of the factors for each polynomial function.



31.  **MULTIPLE REPRESENTATIONS** In this problem, you will consider the function $f(x) = -9x^5 + 104x^4 - 249x^3 - 456x^2 + 828x + 432$.
- ALGEBRAIC** If $x - 6$ is a factor of the function, find the depressed polynomial.
 - TABULAR** Make a table of values for $-5 \leq x \leq 6$ for the depressed polynomial.
 - ANALYTICAL** What conclusions can you make about the locations of the other zeros based on the table? Explain your reasoning.
 - GRAPHICAL** Graph the original function to confirm your conclusions.

Watch Out!

Synthetic Substitution

Remember that synthetic substitution is used to divide a polynomial by $(x - a)$. If the binomial is $(x - a)$, use a . If the binomial is $(x + a)$, use $-a$.

Find values of k so that each remainder is 3.

32. $(x^2 - x + k) \div (x - 1)$ 33. $(x^2 + kx - 17) \div (x - 2)$
 34. $(x^2 + 5x + 7) \div (x - k)$ 35. $(x^3 + 4x^2 + x + k) \div (x + 2)$

H.O.T. Problems

Use **H**igher-**O**rders **T**hinking **S**kills

36. **OPEN ENDED** Write a polynomial function that has a double root of 1 and a double root of -5 . Graph the function.

CHALLENGE Find the solutions of each polynomial function.

37. $(x^2 - 4)^2 - (x^2 - 4) - 2 = 0$ 38. $(x^2 + 3)^2 - 7(x^2 + 3) + 12 = 0$

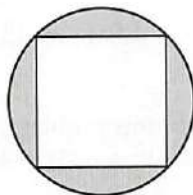
39. **REASONING** Polynomial $f(x)$ is divided by $x - c$. What can you conclude if:
- the remainder is 0?
 - the remainder is 1?
 - the quotient is 1, and the remainder is 0?
40. **CHALLENGE** Review the definition for the Factor Theorem. Provide a proof of the theorem.
41. **OPEN ENDED** Write a cubic function that has a remainder of 8 for $f(2)$ and a remainder of -5 for $f(3)$.
42. **CHALLENGE** Show that the quartic function $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ will always have a rational zero when the numbers 1, -2 , 3, 4, and -6 are randomly assigned to replace a through e , and all of the numbers are used.
43. **WRITING IN MATH** Explain how the zeros of a function can be located by using the Remainder Theorem and making a table of values for different input values and then comparing the remainders.

Standardized Test Practice

44. $27x^3 + y^3 =$

- A $(3x + y)(3x + y)(3x + y)$
- B $(3x + y)(9x^2 - 3xy + y^2)$
- C $(3x - y)(9x^2 + 3xy + y^2)$
- D $(3x - y)(9x^2 + 9xy + y^2)$

45. **GRIDDED RESPONSE** In the figure, a square with side length $2\sqrt{2}$ is inscribed in a circle. The area of the circle is $k\pi$. What is the exact value of k ?



46. What is the product of the complex numbers $(4 + i)(4 - i)$?

- F 15
- G $16 - i$
- H 17
- J $17 - 8i$

47. **SAT/ACT** The measure of the largest angle of a triangle is 14 less than twice the measure of the smallest angle. The third angle measure is 2 more than the measure of the smallest angle. What is the measure of the smallest angle?

- A 46
- B 48
- C 50
- D 82

Spiral Review

Solve each equation. (Lesson 6-5)

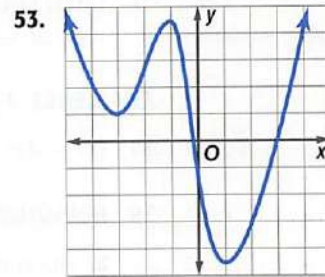
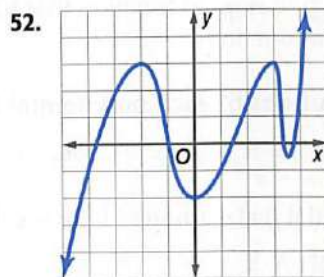
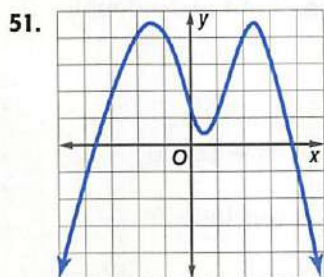
48. $x^4 - 4x^2 - 21 = 0$

49. $x^4 - 6x^2 = 27$

50. $4x^4 - 8x^2 - 96 = 0$

Complete each of the following. (Lesson 6-4)

- a. Estimate the x -coordinate of every turning point and determine if those coordinates are relative maxima or relative minima.
- b. Estimate the x -coordinate of every zero.
- c. Determine the smallest possible degree of the function.
- d. Determine the domain and range of the function.



54. **HIGHWAY SAFETY** Engineers can use the formula $d = 0.05v^2 + 1.1v$ to estimate the minimum stopping distance d in feet for a vehicle traveling v miles per hour. If a car is able to stop after 125 feet, what is the fastest it could have been traveling when the driver first applied the brakes? (Lesson 5-6)

Solve by graphing. (Lesson 3-1)

55. $y = 3x - 1$
 $y = -2x + 4$

56. $3x + 2y = 8$
 $-4x + 6y = 11$

57. $5x - 2y = 6$
 $3x - 2y = 2$

Skills Review

If $c(x) = x^2 - 2x$ and $d(x) = 3x^2 - 6x + 4$, find each value. (Lesson 6-3)

58. $c(a + 2) - d(a - 4)$

59. $c(a - 3) + d(a + 1)$

60. $c(-3a) + d(a + 4)$

61. $3d(3a) - 2c(-a)$

62. $c(a) + 5d(2a)$

63. $-2d(2a + 3) - 4c(a^2 + 1)$

Roots and Zeros

Then

You used complex numbers to describe solutions of quadratic equations. (Lesson 5-4)

Now

- Determine the number and type of roots for a polynomial equation.
- Find the zeros of a polynomial function.

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- Math in Motion

Why?

The function $g(x) = 1.384x^4 - 0.003x^3 + 0.28x^2 - 0.078x + 1.365$ can be used to model the average price of a gallon of gasoline in a given year if x is the number of years since 1990. To find the average price of gasoline in a specific year, you can use the roots of the related polynomial equation.



Synthetic Types of Roots Previously, you learned that a zero of a function $f(x)$ is any value c such that $f(c) = 0$. When the function is graphed, the real zeros of the function are the x -intercepts of the graph.

Concept Summary

For Your

FOLDABLE

Zeros, Factors, Roots, and Intercepts

Words Let $P(x) = a_nx^n + \dots + a_1x + a_0$ be a polynomial function. Then the following statements are equivalent.

- c is a zero of $P(x)$.
- c is a root or solution of $P(x) = 0$.
- $x - c$ is a factor of $a_nx^n + \dots + a_1x + a_0$.
- If c is a real number, then $(c, 0)$ is an x -intercept of the graph of $P(x)$.

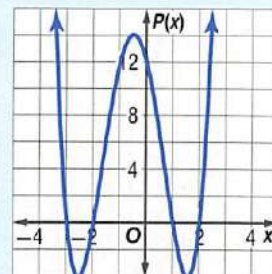
Example Consider the polynomial function $P(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$.

The zeros of $P(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$ are $-3, -2, 1,$ and 2 .

The roots of $x^4 + 2x^3 - 7x^2 - 8x + 12 = 0$ are $-3, -2, 1,$ and 2 .

The factors of $x^4 + 2x^3 - 7x^2 - 8x + 12$ are $(x + 3), (x + 2), (x - 1),$ and $(x - 2)$.

The x -intercepts of the graph of $P(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$ are $(-3, 0), (-2, 0), (1, 0),$ and $(2, 0)$.



When solving a polynomial equation with degree greater than zero, there may be one or more real roots or no real roots (the roots are imaginary numbers). Since real numbers and imaginary numbers both belong to the set of complex numbers, all polynomial equations with degree greater than zero will have at least one root in the set of complex numbers. This is the **Fundamental Theorem of Algebra**.

Key Concept

Fundamental Theorem of Algebra

For Your

FOLDABLE

Every polynomial equation with degree greater than zero has at least one root in the set of complex numbers.

ReadingMath

Repeated Roots

Polynomial equations can have double roots, triple roots, quadruple roots, and so on. In general, these are referred to as *multiple roots*.

EXAMPLE 1 Determine Number and Type of Roots

Solve each equation. State the number and type of roots.

a. $x^2 + 6x + 9 = 0$

$$x^2 + 6x + 9 = 0$$

$$(x + 3)^2 = 0$$

$$x + 3 = 0$$

$$x = -3$$

Original equation

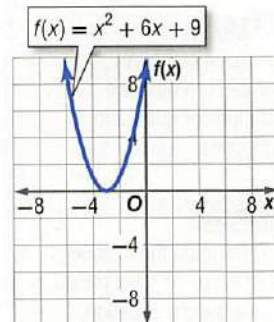
Factor.

Take the root of each side.

Solve for x .

Because $(x + 3)$ is twice a factor of $x^2 + 6x + 9$, -3 is a double root. Thus, the equation has one real repeated root, -3 .

CHECK The graph of the equation touches the x -axis at $x = -3$. Since -3 is a double root, the graph does not cross the axis. ✓



b. $x^3 + 25x = 0$

$$x^3 + 25x = 0$$

$$x(x^2 + 25) = 0$$

$$x = 0 \text{ or } x^2 + 25 = 0$$

$$x^2 = -25$$

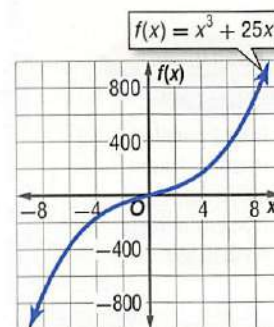
$$x = \pm\sqrt{-25} \text{ or } \pm 5i$$

Original equation

Factor.

This equation has one real root, 0 , and two imaginary roots, $5i$ and $-5i$.

CHECK The graph of this equation crosses the x -axis at only one place, $x = 0$. ✓



Check Your Progress

1A. $x^3 + 2x = 0$

1B. $x^4 - 16 = 0$

1C. $x^3 + 4x^2 - 7x - 10 = 0$

1D. $3x^3 - x^2 + 9x - 3 = 0$

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Examine the solutions for each equation in Example 1. Notice that the number of solutions for each equation is the same as the degree of each polynomial. The following corollary to the Fundamental Theorem of Algebra describes this relationship between the degree and the number of roots of a polynomial equation.

Key Concept

For Your
FOLDABLE

Corollary to the Fundamental Theorem of Algebra

Words A polynomial equation of degree n has exactly n roots in the set of complex numbers, including repeated roots.

Example $x^3 + 2x^2 + 6$
3 roots

$4x^4 - 3x^3 + 5x - 6$
4 roots

$-2x^5 - 3x^2 + 8$
5 roots

Similarly, an n th degree polynomial function has exactly n zeros.

Additionally, French mathematician René Descartes discovered a relationship between the signs of the coefficients of a polynomial function and the number of positive and negative real zeros.

StudyTip

Zero at the Origin If a zero of a function is at the origin, the sum of the number of positive real zeros, negative real zeros, and imaginary zeros is reduced by how many times 0 is a zero of the function.

Key Concept**Descartes' Rule of Signs**For Your
FOLDABLE

Let $P(x) = a_n x^n + \dots + a_1 x + a_0$ be a polynomial function with real coefficients. Then

- the number of positive real zeros of $P(x)$ is the same as the number of changes in sign of the coefficients of the terms, or is less than this by an even number, and
- the number of negative real zeros of $P(x)$ is the same as the number of changes in sign of the coefficients of the terms of $P(-x)$, or is less than this by an even number.

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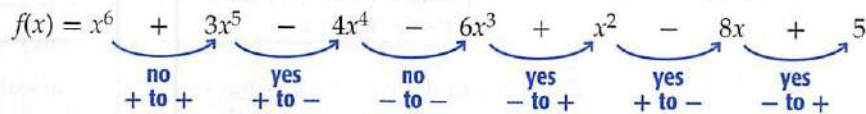
EXAMPLE 2 Find Numbers of Positive and Negative Zeros

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of $f(x) = x^6 + 3x^5 - 4x^4 - 6x^3 + x^2 - 8x + 5$.

Because $f(x)$ has degree 6, it has six zeros, either real or imaginary. Use Descartes' Rule of Signs to determine the possible number and type of *real* zeros.

Count the number of changes in sign for the coefficients of $f(x)$.

$$f(x) = x^6 \quad + \quad 3x^5 \quad - \quad 4x^4 \quad - \quad 6x^3 \quad + \quad x^2 \quad - \quad 8x \quad + \quad 5$$

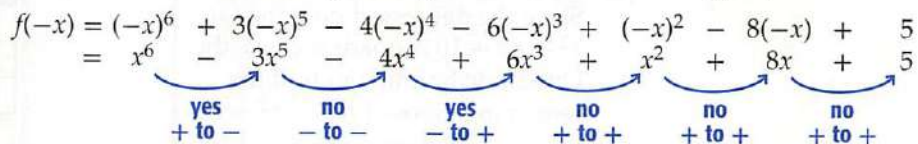


There are 4 sign changes, so there are 4, 2, or 0 positive real zeros.

Count the number of changes in sign for the coefficients of $f(-x)$.

$$f(-x) = (-x)^6 + 3(-x)^5 - 4(-x)^4 - 6(-x)^3 + (-x)^2 - 8(-x) + 5$$

$$= x^6 - 3x^5 - 4x^4 + 6x^3 + x^2 + 8x + 5$$



There are 2 sign changes, so there are 2, or 0 negative real zeros.

Make a chart of the possible combinations of real and imaginary zeros.

Number of Positive Real Zeros	Number of Negative Real Zeros	Number of Imaginary Zeros	Total Number of Zeros
4	2	0	$4 + 2 + 0 = 6$
4	0	2	$4 + 0 + 2 = 6$
2	2	2	$2 + 2 + 2 = 6$
2	0	4	$2 + 0 + 4 = 6$
0	2	4	$0 + 2 + 4 = 6$
0	0	6	$0 + 0 + 6 = 6$

Check Your Progress

2. State the possible number of positive real zeros, negative real zeros, and imaginary zeros of $h(x) = 2x^5 + x^4 + 3x^3 - 4x^2 - x + 9$.

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Find Zeros You can use the various strategies and theorems you have learned to find all of the zeros of a function.

EXAMPLE 3 Use Synthetic Substitution to Find Zeros

Find all of the zeros of $f(x) = x^4 - 18x^2 + 12x + 80$.

Step 1 Determine the total number of zeros.
Since $f(x)$ has degree 4, the function has 4 zeros.

Step 2 Determine the type of zeros.
Examine the number of sign changes for $f(x)$ and $f(-x)$.

$$f(x) = x^4 - 18x^2 + 12x + 80$$

yes
yes
no

$$f(-x) = x^4 - 18x^2 - 12x + 80$$

yes
no
yes

Because there are 2 sign changes for the coefficients of $f(x)$, the function has 2 or 0 positive real zeros. Because there are 2 sign changes for the coefficients of $f(-x)$, $f(x)$ has 2 or 0 negative real zeros. Thus, $f(x)$ has 4 real zeros, 2 real zeros and 2 imaginary zeros, or 4 imaginary zeros.

Step 3 Determine the real zeros.
List some possible values, and then use synthetic substitution to evaluate $f(x)$ for real values of x .

x	1	0	-18	12	80
-3	1	-3	-9	39	-37
-2	1	-2	-14	40	0
-1	1	-1	-17	29	51
0	1	0	-18	12	80
1	1	1	-17	-5	75
2	1	2	-14	-2	76

Each row shows the coefficients of the depressed polynomial and the remainder.

From the table, we can see that one zero occurs at $x = -2$. Since there are 2 negative real zeros, use synthetic substitution with the depressed polynomial function $f(x) = x^3 - 2x^2 - 14x + 40$ to find a second negative zero.

A second negative zero is at $x = -4$.
Since the depressed polynomial $x^2 - 6x + 10$ is quadratic, use the Quadratic Formula to find the remaining zeros of $f(x) = x^2 - 6x + 10$.

x	1	-2	-14	40
-4	1	-6	10	0
-5	1	-7	21	-65
-6	1	-8	34	-164

$$\begin{aligned}
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)} \\
 &= 3 \pm i
 \end{aligned}$$

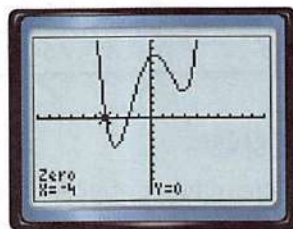
Quadratic Formula

Replace a with 1, b with -6 , and c with 10.

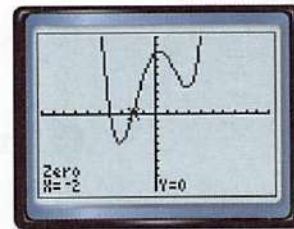
Simplify.

The function has zeros at -4 , -2 , $3 + i$, and $3 - i$.

CHECK Graph the function on a graphing calculator. The graph crosses the x -axis two times, so there are two real zeros. Use the **zero** function under the **CALC** menu to locate each zero. The two real zeros are -4 and -2 .



$[-10, 10]$ scl: 1 by $[-100, 100]$ scl: 10



$[-10, 10]$ scl: 1 by $[-100, 100]$ scl: 10

StudyTip

Testing for Zeros If a value is not a zero for a polynomial, then it will not be a zero for the depressed polynomial either, so it does not need to be checked again.

StudyTip

Locating Zeros Refer to Lesson 5-2 on how to use the **CALC** menu to locate a zero on your calculator.



Check Your Progress

3. Find all of the zeros of $h(x) = x^3 + 2x^2 + 9x + 18$.

Review Vocabulary

complex conjugates
two complex numbers of the form $a + bi$ and $a - bi$ (Lesson 5-4)

In Chapter 5, you learned that the product of complex conjugates is always a real number and that complex roots always come in conjugate pairs. For example, if one root of $x^2 - 8x + 52 = 0$ is $4 + 6i$, then the other root is $4 - 6i$.

This applies to the zeros of polynomial functions as well. For any polynomial function with real coefficients, if an imaginary number is a zero of that function, its conjugate is also a zero. This is called the **Complex Conjugates Theorem**.



Key Concept

Complex Conjugates Theorem

For Your

FOLDABLE

Words Let a and b be real numbers, and $b \neq 0$. If $a + bi$ is a zero of a polynomial function with real coefficients, then $a - bi$ is also a zero of the function.

Example If $3 + 4i$ is a zero of $f(x) = x^3 - 4x^2 + 13x + 50$, then $3 - 4i$ is also a zero of the function.

When you are given all of the zeros of a polynomial function and are asked to determine the function, convert the zeros to factors and then multiply all of the factors together. The result is the polynomial function.

EXAMPLE 4 Use Zeros to Write a Polynomial Function

Write a polynomial function of least degree with integral coefficients, the zeros of which include -1 and $5 - i$.

Understand If $5 - i$ is a zero, then $5 + i$ is also a zero according to the Complex Conjugates Theorem. So, $x + 1$, $x - (5 - i)$, and $x - (5 + i)$ are factors of the polynomial.

Plan Write the polynomial function as a product of its factors.

$$P(x) = (x + 1)[x - (5 - i)][x - (5 + i)]$$

Solve Multiply the factors to find the polynomial function.

$$\begin{aligned} P(x) &= (x + 1)[x - (5 - i)][x - (5 + i)] && \text{Write the equation.} \\ &= (x + 1)[(x - 5) + i][(x - 5) - i] && \text{Regroup terms.} \\ &= (x + 1)[(x - 5)^2 - i^2] && \text{Difference of squares} \\ &= (x + 1)[(x^2 - 10x + 25 - (-1))] && \text{Square terms.} \\ &= (x + 1)(x^2 - 10x + 26) && \text{Simplify.} \\ &= x^3 - 10x^2 + 26x + x^2 - 10x + 26 && \text{Multiply.} \\ &= x^3 - 9x^2 + 16x + 26 && \text{Combine like terms.} \end{aligned}$$

Check Because there are 3 zeros, the degree of the polynomial function must be 3, so $P(x) = x^3 - 9x^2 + 16x + 26$ is a polynomial function of least degree with integral coefficients and zeros of -1 , $5 - i$, and $5 + i$.



Check Your Progress

4. Write a polynomial function of least degree with integral coefficients having zeros that include -1 and $1 + 2i$.



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Check Your Understanding

Example 1 p. 384

Solve each equation. State the number and type of roots.

1. $x^2 - 3x - 10 = 0$

2. $x^3 + 12x^2 + 32x = 0$

3. $16x^4 - 81 = 0$

4. $0 = x^3 - 8$

Example 2 p. 385

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

5. $f(x) = x^3 - 2x^2 + 2x - 6$

6. $f(x) = 6x^4 + 4x^3 - x^2 - 5x - 7$

7. $f(x) = 3x^5 - 8x^3 + 2x - 4$

8. $f(x) = -2x^4 - 3x^3 - 2x - 5$

Example 3 p. 386

Find all of the zeros of each function.

9. $f(x) = x^3 + 9x^2 + 6x - 16$

10. $f(x) = x^3 + 7x^2 + 4x + 28$

11. $f(x) = x^4 - 2x^3 - 8x^2 - 32x - 384$

12. $f(x) = x^4 - 6x^3 + 9x^2 + 6x - 10$

Example 4 p. 387

Write a polynomial function of least degree with integral coefficients that have the given zeros.

13. 4, -1, 6

14. 3, -1, 1, 2

15. -2, 5, -3i

16. -4, 4 + i

Practice and Problem Solving

 = Step-by-Step Solutions begin on page R20.
Extra Practice begins on page 947.

Example 1 p. 384

Solve each equation. State the number and type of roots.

17. $2x^2 + x - 6 = 0$

18. $4x^2 + 1 = 0$

19. $x^3 + 1 = 0$

20. $2x^2 - 5x + 14 = 0$

21. $-3x^2 - 5x + 8 = 0$

22. $8x^3 - 27 = 0$

23. $16x^4 - 625 = 0$

24. $x^3 - 6x^2 + 7x = 0$

25. $x^5 - 8x^3 + 16x = 0$

26. $x^5 + 2x^3 + x = 0$

Example 2 p. 385

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

27. $f(x) = x^4 - 5x^3 + 2x^2 + 5x + 7$

28. $f(x) = 2x^3 - 7x^2 - 2x + 12$

29. $f(x) = -3x^5 + 5x^4 + 4x^2 - 8$

30. $f(x) = x^4 - 2x^2 - 5x + 19$

31. $f(x) = 4x^6 - 5x^4 - x^2 + 24$

32. $f(x) = -x^5 + 14x^3 + 18x - 36$

Example 3 p. 386

Find all of the zeros of each function.

33. $f(x) = x^3 + 7x^2 + 4x - 12$

34. $f(x) = x^3 + x^2 - 17x + 15$

35. $f(x) = x^4 - 3x^3 - 3x^2 - 75x - 700$

36. $f(x) = x^4 + 6x^3 + 73x^2 + 384x + 576$

37. $f(x) = x^4 - 8x^3 + 20x^2 - 32x + 64$

38. $f(x) = x^5 - 8x^3 - 9x$

39. $f(x) = x^3 - 5x^2 + 17x - 85$

40. $f(x) = x^3 + 2x$

41. $f(x) = 4x^4 + 15x^2 - 4$

42. $f(x) = 9x^4 + 9x^3 + 4x^2 + 4x$

Example 4 p. 387

Write a polynomial function of least degree with integral coefficients that have the given zeros.

43. 5, -2, -1

44. -4, -3, 5

45. -1, -1, 2i

46. -3, 1, -3i

47. 0, -5, 3 + i

48. -2, -3, 4 - 3i



Real-World Link

On average, 3500 people attend a concert in the U.S. each night.

Source: *Billboard Magazine*

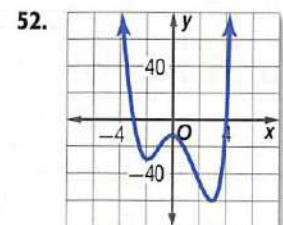
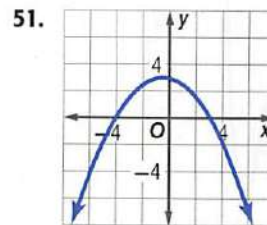
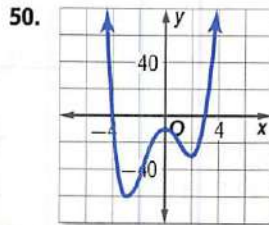
- 49. BUSINESS** A computer manufacturer determines that for each employee, the profit for producing x computers per day is $P(x) = -0.006x^4 + 0.15x^3 - 0.05x^2 - 1.8x$.
- How many positive real zeros, negative real zeros, and imaginary zeros exist?
 - What is the meaning of the zeros in this situation?

Match each graph to the given zeros.

a. $-3, 4, i, -i$

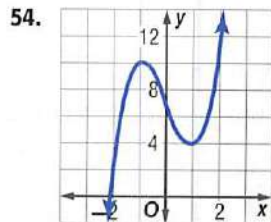
b. $-4, 3$

c. $-4, 3, i, -i$

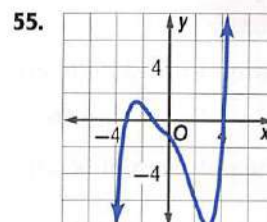


- 53. CONCERTS** The amount of money Hoshi's Music Hall took in from 2003 to 2010 can be modeled by $M(x) = -2.03x^3 + 50.1x^2 - 214x + 4020$, where x is the years since 2003.
- How many positive real zeros, negative real zeros, and imaginary zeros exist?
 - Graph the function using your calculator.
 - Approximate all real zeros to the nearest tenth. What is the significance of each zero in the context of the situation?

Determine the number of positive real zeros, negative real zeros, and imaginary zeros for each function. Explain your reasoning.



degree: 3



degree: 5

H.O.T. Problems

Use **Higher-Order Thinking Skills**

- 56. OPEN ENDED** Sketch the graph of a polynomial function with:
- 3 real, 2 imaginary zeros
 - 4 real zeros
 - 2 imaginary zeros
- 57. CHALLENGE** Write an equation in factored form of a polynomial function of degree 5 with 2 imaginary zeros, 1 nonintegral zero, and 2 irrational zeros. Explain.
- 58. WHICH ONE DOESN'T BELONG** Determine which equation is not like the others. Explain.

$$r^4 + 1 = 0$$

$$r^3 + 1 = 0$$

$$r^2 - 1 = 0$$

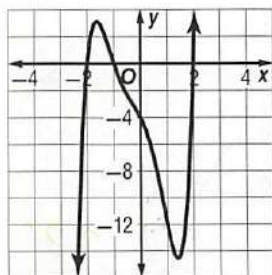
$$r^3 - 8 = 0$$

- 59. REASONING** Provide a counterexample for each statement.
- All polynomial functions of degree greater than 2 have at least 1 negative real root.
 - All polynomial functions of degree greater than 2 have at least 1 positive real root.
- 60. WRITING IN MATH** Explain to a friend how you would use Descartes' Rule of Signs to determine the number of possible positive real roots and the number of possible negative roots of the polynomial function $f(x) = x^4 - 2x^3 + 6x^2 + 5x - 12$.

Standardized Test Practice

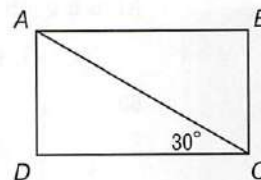
61. **SAT/ACT** Use the graph of the polynomial function below. Which is not a factor of the polynomial $x^5 + x^4 - 3x^3 - 3x^2 - 4x - 4$?

- A $x - 2$
 B $x + 2$
 C $x - 1$
 D $x + 1$



62. **SHORT RESPONSE** A window is in the shape of an equilateral triangle. Each side of the triangle is 8 feet long. The window is divided in half by a support from one vertex to the midpoint of the side of the triangle opposite the vertex. Approximately how long is the support?

63. **GEOMETRY** In rectangle $ABCD$, \overline{AD} is 8 units long. What is the length of \overline{AB} ?



- F 4 units
 G 8 units
 H $8\sqrt{3}$ units
 J 16 units
64. The total area of a rectangle is $25a^4 - 16b^2$ square units. Which factors could represent the length and width?
- A $(5a^2 + 4b)$ units and $(5a^2 + 4b)$ units
 B $(5a^2 + 4b)$ units and $(5a^2 - 4b)$ units
 C $(5a - 4b)$ units and $(5a - 4b)$ units
 D $(5a + 4b)$ units and $(5a - 4b)$ units

Spiral Review

Use synthetic substitution to find $f(-8)$ and $f(4)$ for each function. (Lesson 6-6)

65. $f(x) = 4x^3 + 6x^2 - 3x + 2$

66. $f(x) = 5x^4 - 2x^3 + 4x^2 - 6x$

67. $f(x) = 2x^5 - 3x^3 + x^2 - 4$

Factor completely. If the polynomial is not factorable, write *prime*. (Lesson 6-5)

68. $x^6 - y^6$

69. $a^6 + b^6$

70. $4x^2y + 8xy + 16y - 3x^2z - 6xz - 12z$

71. $5a^3 - 30a^2 + 40a + 2a^2b - 12ab + 16b$

72. **BUSINESS** A mall owner has determined that the relationship between monthly rent charged for store space r (in dollars per square foot) and monthly profit $P(r)$ (in thousands of dollars) can be approximated by $P(r) = -8.1r^2 + 46.9r - 38.2$. Solve each quadratic equation or inequality. Explain what each answer tells about the relationship between monthly rent and profit for this mall. (Lesson 5-8)

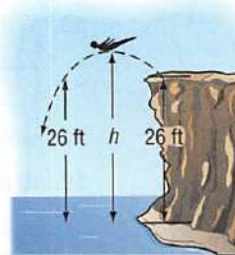
a. $-8.1r^2 + 46.9r - 38.2 = 0$

b. $-8.1r^2 + 46.9r - 38.2 > 0$

c. $-8.1r^2 + 46.9r - 38.2 > 10$

d. $-8.1r^2 + 46.9r - 38.2 < 10$

73. **DIVING** To avoid hitting any rocks below, a cliff diver jumps up and out. The equation $h = -16t^2 + 4t + 26$ describes her height h in feet t seconds after jumping. Find the time at which she returns to a height of 26 feet. (Lesson 5-3)



Skills Review

Find all of the possible values of $\pm \frac{b}{a}$ for each replacement set.

74. $a = \{1, 2, 4\}; b = \{1, 2, 3, 6\}$

75. $a = \{1, 5\}; b = \{1, 2, 4, 8\}$

76. $a = \{1, 2, 3, 6\}; b = \{1, 7\}$

Rational Zero Theorem

**Then**

You found zeros of quadratic functions of the form $f(x) = ax^2 + bx + c$. (Lesson 5-3)

Now

- Identify possible rational zeros of a polynomial function.
- Find all of the rational zeros of a polynomial function.

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- Self-Check Quiz
- Homework Help

Why?

Annual sales of recorded music in the United States can be approximated by $d(t) = 30x^3 - 478x^2 + 1758x + 12,392$, where $d(t)$ is the total sales in millions of dollars and t is the number of years since 1997. You can use this function to estimate when music sales will be \$20 billion.

Identify Rational Zeros Usually it is not practical to test all possible zeros of a polynomial function using synthetic substitution. The **Rational Zero Theorem** can help you choose some possible zeros to test. **If the leading coefficient is 1, the corollary applies.**

**Key Concept****Rational Zero Theorem**

For Your

FOLDABLE

Words If $P(x)$ is a polynomial function with integral coefficients, then every rational zero of $P(x) = 0$ is of the form $\frac{p}{q}$, a rational number in simplest form, where p is a factor of the constant term and q is a factor of the leading coefficient

Example Let $f(x) = 6x^4 + 22x^3 + 11x^2 - 80x - 40$. If $\frac{4}{3}$ is a zero of $f(x)$, then 4 is a factor of -40 , and 3 is a factor of 6.

Corollary to the Rational Zero Theorem

If $P(x)$ is a polynomial function with integral coefficients, a leading coefficient of 1, and a nonzero constant term, then any rational zeros of $P(x)$ must be factors of the constant term.

EXAMPLE 1 Identify Possible Zeros

List all of the possible rational zeros of each function.

a. $f(x) = 4x^5 + x^4 - 2x^3 - 5x^2 + 8x + 16$

If $\frac{p}{q}$ is a rational zero, then p is a factor of 16 and q is a factor of 4.

$p: \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$ $q: \pm 1, \pm 2, \pm 4$

Write the possible values of $\frac{p}{q}$ in simplest form.

$\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm \frac{1}{2}, \pm \frac{1}{4}$

b. $f(x) = x^3 - 2x^2 + 5x + 12$

If $\frac{p}{q}$ is a rational zero, then p is a factor of 12 and q is a factor of 1.

$p: \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$ $q: \pm 1$

So, $\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \text{ and } \pm 12$

**Check Your Progress**

1A. $g(x) = 3x^3 - 4x + 10$

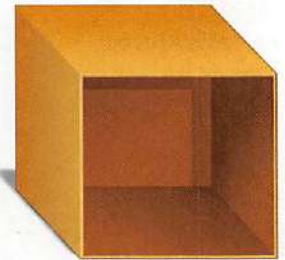
1B. $h(x) = x^3 + 11x^2 + 24$

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Find Rational Zeros Once you have written the possible rational zeros, you can test each number using synthetic substitution and use the other tools you have learned to determine the zeros of a function.

Real-World EXAMPLE 2 Find Rational Zeros

WOODWORKING Adam is building a computer desk with a separate compartment for the computer. The compartment for the computer is a rectangular prism and will be 8019 cubic inches. Find the dimensions of the computer compartment.



Let x = width, $x + 24$ = length, and $x + 18$ = height.

Write an equation for the volume.

$$\ell wh = V \quad \text{Formula for volume}$$

$$(x + 24)(x)(x + 18) = 8019 \quad \text{Substitute.}$$

$$x^3 + 42x^2 + 432x = 8019 \quad \text{Multiply.}$$

$$x^3 + 42x^2 + 432x - 8019 = 0 \quad \text{Subtract 8019 from each side.}$$

The leading coefficient is 1, so the possible rational zeros are factors of 8019.

$\pm 1, \pm 3, \pm 9, \pm 11, \pm 27, \pm 33, \pm 81, \pm 99, \pm 243, \pm 297, \pm 729, \pm 891, \pm 2673, \text{ and } \pm 8019$

Since length can only be positive, we only need to check positive values.

There is one change of sign of the coefficients, so by Descartes' Rule of Signs, there is only one positive real zero. Make a table for synthetic division and test possible values.

p	1	42	432	-8019
1	1	43	475	-7544
3	1	45	567	-6318
9	1	51	891	0

One zero is 9. Since there is only one positive real zero, we do not have to test the other numbers. The other dimensions are $9 + 24$ or 33 inches, and $9 + 18$ or 27 inches.

CHECK Multiply the dimensions and see if they equal the volume of 8019 cubic inches.

$$9 \times 33 \times 27 = 8019 \quad \checkmark$$

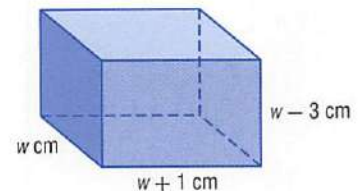
StudyTip

Descartes' Rule of Signs

Examine the signs of the coefficients of the equation. In this case, there is only one change of sign, so there is only one positive real zero.

Check Your Progress

2. **GEOMETRY** The volume of a rectangular prism is 1056 cubic centimeters. The length is 1 centimeter more than the width, and the height is 3 centimeters less than the width. Find the dimensions of the prism.



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You usually do not need to test all of the possible zeros. Once you find a zero, you can try to factor the depressed polynomial to find any other zeros.

EXAMPLE 3 Find All Zeros

Find all of the zeros of $f(x) = 5x^4 - 8x^3 + 41x^2 - 72x - 36$.

From the corollary to the Fundamental Theorem of Algebra, there are exactly 4 complex zeros. According to Descartes' Rule of Signs, there are 3 or 1 positive real zeros and exactly 1 negative real zero. The possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 36, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{3}{5}, \pm \frac{4}{5}, \pm \frac{6}{5}, \pm \frac{9}{5}, \pm \frac{12}{5}, \pm \frac{18}{5}$, and $\pm \frac{36}{5}$.

Make a table and test some possible rational zeros.

$\frac{p}{q}$	5	-8	41	-72	-36
-1	5	-13	54	-126	90
1	5	-3	38	-34	-70
2	5	2	45	18	0

Because $f(2) = 0$, there is a zero at $x = 2$.

Factor the depressed polynomial $5x^3 + 2x^2 + 45x + 18$.

$$5x^3 + 2x^2 + 45x + 18 = 0 \quad \text{Write the depressed polynomial.}$$

$$(5x^3 + 2x^2) + (45x + 18) = 0 \quad \text{Group terms.}$$

$$x^2(5x + 2) + 9(5x + 2) = 0 \quad \text{Factor.}$$

$$(x^2 + 9)(5x + 2) = 0 \quad \text{Distributive Property}$$

$$x^2 + 9 = 0 \quad \text{or} \quad 5x + 2 = 0 \quad \text{Zero Product Property}$$

$$x^2 = -9 \qquad 5x = -2$$

$$x = \pm 3i \qquad x = -\frac{2}{5}$$

There is another real zero at $x = -\frac{2}{5}$ and two imaginary zeros at $x = 3i$ and $x = -3i$.

The zeros of the function are $-\frac{2}{5}, 2, 3i$, and $-3i$.

Check Your Progress

Find all of the zeros of each function.

3A. $h(x) = 9x^4 + 5x^2 - 4$

3B. $k(x) = 2x^4 - 5x^3 + 20x^2 - 45x + 18$

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Check Your Understanding**Example 1**

p. 391

List all of the possible rational zeros of each function.

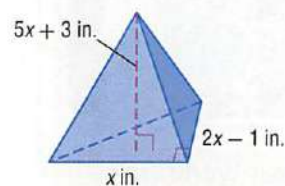
1. $f(x) = x^3 - 6x^2 - 8x + 24$

2. $f(x) = 2x^4 + 3x^2 - x + 15$

Example 2

p. 392

3. **GEOMETRY** The volume of the triangular pyramid is 210 cubic inches. Find the dimensions of the solid.



Find all of the rational zeros of each function.

4. $f(x) = x^3 - 6x^2 - 13x + 42$

5. $f(x) = 2x^4 + 11x^3 + 26x^2 + 29x + 12$

Example 3

p. 393

Find all of the zeros of each function.

6. $f(x) = 3x^3 - 2x^2 - 8x + 5$

7. $f(x) = 8x^3 + 14x^2 + 11x + 3$

8. $f(x) = 4x^4 + 13x^3 - 8x^2 + 13x - 12$

9. $f(x) = 4x^4 - 12x^3 + 25x^2 - 14x - 15$

Practice and Problem Solving

Step-by-Step Solutions begin on page R20.
Extra Practice begins on page 947.

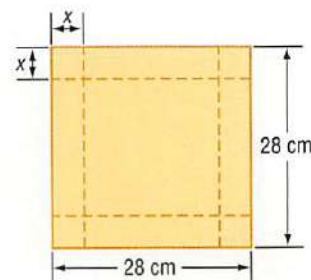
Example 1
 p. 391

List all of the possible rational zeros of each function.

- | | |
|-------------------------------------------------|------------------------------------------|
| 10. $f(x) = x^4 + 8x - 32$ | 11. $f(x) = x^3 + x^2 - x - 56$ |
| 12. $f(x) = 2x^3 + 5x^2 - 8x - 10$ | 13. $f(x) = 3x^6 - 4x^4 - x^2 - 35$ |
| 14. $f(x) = 6x^5 - x^4 + 2x^3 - 3x^2 + 2x - 18$ | 15. $f(x) = 8x^4 - 4x^3 - 4x^2 + x + 42$ |
| 16. $f(x) = 15x^3 + 6x^2 + x + 90$ | 17. $f(x) = 16x^4 - 5x^2 + 128$ |

Example 2
 p. 392

18. MANUFACTURING A box is to be constructed by cutting out equal squares from the corners of a square piece of cardboard and turning up the sides.



- Write a function $V(x)$ for the volume of the box.
- For what value of x will the volume of the box equal 1152 cubic centimeters?
- What will be the volume of the box if $x = 6$ centimeters?

Find all of the rational zeros of each function.

- | | |
|--------------------------------------|---------------------------------------------|
| 19. $f(x) = x^3 + 10x^2 + 31x + 30$ | 20. $f(x) = x^3 - 2x^2 - 56x + 192$ |
| 21. $f(x) = 4x^3 - 3x^2 - 100x + 75$ | 22. $f(x) = 4x^4 + 12x^3 - 5x^2 - 21x + 10$ |
| 23. $f(x) = x^4 + x^3 - 8x - 8$ | 24. $f(x) = 2x^4 - 3x^3 - 24x^2 + 4x + 48$ |
| 25. $f(x) = 4x^3 + x^2 + 16x + 4$ | 26. $f(x) = 81x^4 - 256$ |

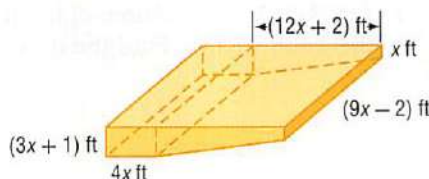
Example 3
 p. 393



Find all of the zeros of each function.

- | | |
|----------------------------------------------|----------------------------------------------------|
| 27. $f(x) = x^3 + 3x^2 - 25x + 21$ | 28. $f(x) = 6x^3 + 5x^2 - 9x + 2$ |
| 29. $f(x) = x^4 - x^3 - x^2 - x - 2$ | 30. $f(x) = 10x^3 - 17x^2 - 7x + 2$ |
| 31. $f(x) = x^4 - 3x^3 + x^2 - 3x$ | 32. $f(x) = 6x^3 + 11x^2 - 3x - 2$ |
| 33. $f(x) = 6x^4 + 22x^3 + 11x^2 - 38x - 40$ | 34. $f(x) = 2x^3 - 7x^2 - 8x + 28$ |
| 35. $f(x) = 9x^5 - 94x^3 + 27x^2 + 40x - 12$ | 36. $f(x) = x^5 - 2x^4 - 12x^3 - 12x^2 - 13x - 10$ |
| 37. $f(x) = 48x^4 - 52x^3 + 13x - 3$ | 38. $f(x) = 5x^4 - 29x^3 + 55x^2 - 28x$ |

39. SWIMMING POOLS A diagram of the swimming pool at the Midtown Community Center is shown below. The pool can hold 9175 cubic feet of water.



- Write a polynomial function that represents the volume of the swimming pool.
 - What are the possible values of x ? Which of these values are reasonable?
- 40. ROLLER COASTERS** A portion of the path of a certain roller coaster can be modeled by $f(t) = t^4 - 31t^3 + 308t^2 - 1100t + 1200$ where t represents the time in seconds and $f(t)$ represents the height of the roller coaster. Use the Rational Zero Theorem to determine the four times at which the roller coaster is at ground level.

Real-World Link

The world's largest swimming pool is along the coastline in San Alfonso del Mar, Chile. It is 1 kilometer long, about 6000 times as large as an average pool.

Source: OhGizmo!



Real-World Link

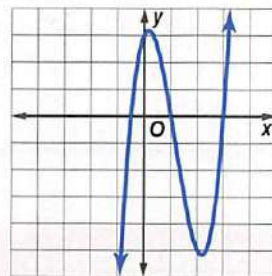
Thomas Jefferson is credited with introducing macaroni to the United States after serving as the U.S. Ambassador to France.

Source: National Pasta Association

- 41. FOOD** A restaurant orders spaghetti sauce in cylindrical metal cans. The volume of each can is about 160π cubic inches, and the height of the can is 6 inches more than the radius.
- Write a polynomial equation that represents the volume of a can. Use the formula for the volume of a cylinder, $V = \pi r^2 h$.
 - What are the possible values of r ? Which of these values are reasonable for this situation?
 - Find the dimensions of the can.

42. Refer to the graph at the right.

- Find all of the zeros of $f(x) = 2x^3 + 7x^2 + 2x - 3$ and $g(x) = 2x^3 - 7x^2 + 2x + 3$.
- Determine which function, f or g , is shown in the graph at the right.



43. MUSIC SALES Refer to the beginning of the lesson.

- Write a polynomial equation that could be used to determine the year in which music sales would be about \$20,000,000,000.
- List the possible whole number solutions for your equation in part a.
- Determine the approximate year in which music sales will reach \$20,000,000,000.
- Does the model represent a realistic estimate for all future music sales? Explain your reasoning.

Find all of the zeros of each function.

- $f(x) = x^5 + 3x^4 - 19x^3 - 43x^2 + 18x + 40$
- $f(x) = x^5 - x^4 - 23x^3 + 33x^2 + 126x - 216$

H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

- 46. FIND THE ERROR** Doug and Mika are listing all of the possible rational zeros for $f(x) = 4x^4 + 8x^5 + 10x^2 + 3x + 16$. Is either of them correct? Explain your reasoning.

Doug

$$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm \frac{1}{2}, \pm \frac{1}{4}$$

Mika

$$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{8}$$

- 47. CHALLENGE** Give a polynomial function that has zeros at $1 + \sqrt{3}$ and $5 + 2i$.
- 48. REASONING** Determine if the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.
- If all of the possible zeros of a polynomial function are integers, then the leading coefficient of the function is 1 or -1.*
- 49. OPEN ENDED** Write a function that has possible zeros of $\pm 18, \pm 9, \pm 6, \pm 3, \pm 2, \pm 1, \pm \frac{9}{2}, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm \frac{1}{2}$, and $\pm \frac{1}{4}$.
- 50. CHALLENGE** The roots of $x^2 + bx + c = 0$ are M and N . If $|M - N| = 1$, express c in terms of b .
- 51. WRITING IN MATH** Explain the process of using the Rational Zero Theorem to determine the number of possible rational zeros of a function.

Standardized Test Practice

52. **ALGEBRA** Which of the following is a zero of the function $f(x) = 12x^5 - 5x^3 + 2x - 9$?

A -6

C $\frac{3}{8}$

B $-\frac{2}{3}$

D 1

53. **SAT/ACT** How many negative real zeros does $f(x) = x^5 - 2x^4 - 4x^3 + 4x^2 - 5x + 6$ have?

F 3

H 1

G 2

J 0

54. **ALGEBRA** For all nonnegative numbers n ,

let \boxed{n} be defined by $\boxed{n} = \frac{\sqrt{n}}{2}$. If $\boxed{n} = 4$, what is the value of n ?

A 2

C 16

B 4

D 64

55. **GRIDDED RESPONSE** What is the y -intercept of a line that contains the point $(-1, 4)$ and has the same x -intercept as $x + 2y = -3$?

Spiral Review

Write a polynomial function of least degree with integral coefficients that has the given zeros. (Lesson 6-7)

56. $6, -3, \sqrt{2}$

57. $5, -1, 4i$

58. $-4, -2, i\sqrt{2}$

Given a polynomial and one of its factors, find the remaining factors of the polynomial. (Lesson 6-6)

59. $x^4 + 5x^3 + 5x^2 - 5x - 6; x + 3$

60. $a^4 - 2a^3 - 17a^2 + 18a + 72; a - 3$

61. $x^4 + x^3 - 11x^2 + x - 12; x + i$

62. **BRIDGES** The supporting cables of the Golden Gate Bridge approximate the shape of a parabola. The parabola can be modeled by the quadratic function $y = 0.00012x^2 + 6$, where x represents the distance from the axis of symmetry and y represents the height of the cables. The related quadratic equation is $0.00012x^2 + 6 = 0$. (Lesson 5-6)

- Calculate the value of the discriminant.
- What does the discriminant tell you about the supporting cables of the Golden Gate Bridge?

63. **RIDES** An amusement park ride carries riders to the top of a 225-foot tower. The riders then free-fall in their seats until they reach 30 feet above the ground. (Lesson 5-2)

- Use the formula $h(t) = -16t^2 + h_0$, where the time t is in seconds and the initial height h_0 is in feet, to find how long the riders are in free-fall.
- Suppose the designer of the ride wants the riders to experience free-fall for 5 seconds before stopping 30 feet above the ground. What should be the height of the tower?

Skills Review

Simplify. (Lesson 6-1)

64. $(x - 4)(x + 3)$

65. $3x(x^2 + 4)$

66. $x^2(x - 2)(x + 1)$

Find each value if $f(x) = 6x + 2$ and $g(x) = -4x^2$. (Lesson 2-1)

67. $f(5)$

68. $g(-3)$

69. $f(3c)$

Chapter Summary

Key Concepts

Operations with Polynomials (Lessons 6-1 and 6-2)

- To add or subtract: Combine like terms.
- To multiply: Use the Distributive Property.
- To divide: Use long division or synthetic division.

Polynomial Functions and Graphs (Lessons 6-3 and 6-4)

- Turning points of a function are called *relative maxima* and *relative minima*.

Solving Polynomial Equations (Lesson 6-5)

- You can factor polynomials by using the GCF, grouping, or quadratic techniques.

The Remainder and Factor Theorems (Lesson 6-6)

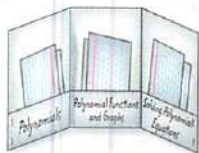
- Factor Theorem: The binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$.

Roots, Zeros, and the Rational Zero Theorem (Lessons 6-7 and 6-8)

- Complex Conjugates Theorem: If $a + bi$ is a zero of a function, then $a - bi$ is also a zero.
- Integral Zero Theorem: If the coefficients of a polynomial function are integers such that $a_0 = 1$ and $a_n = 0$, any rational zeros of the function must be factors of a_n .
- Rational Zero Theorem: If $P(x)$ is a polynomial function with integral coefficients, then every rational zero of $P(x) = 0$ is of the form $\frac{p}{q}$, a rational number in simplest form, where p is a factor of the constant term and q is a factor of the leading coefficient.

FOLDABLES® Study Organizer

Be sure the Key Concepts are noted in your Foldable.



Key Vocabulary

- | | |
|--------------------------------------------|----------------------------------------|
| degree of a polynomial (p. 335) | power function (p. 349) |
| depressed polynomial (p. 379) | prime polynomials (p. 368) |
| end behavior (p. 350) | quadratic form (p. 371) |
| extrema (p. 358) | relative maximum (p. 358) |
| leading coefficient (p. 348) | relative minimum (p. 358) |
| location principle (p. 357) | simplify (p. 333) |
| polynomial function (p. 349) | synthetic division (p. 342) |
| polynomial in one variable (p. 348) | synthetic substitution (p. 377) |
| | turning points (p. 358) |

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. The coefficient of the first term of a polynomial in standard form is called the leading coefficient.
2. Polynomials that cannot be factored are called polynomials in one variable.
3. A prime polynomial has a degree that is one less than the original polynomial.
4. A point on the graph of a function where no other nearby point has a greater y -coordinate is called a relative maximum.
5. A polynomial function is a continuous function that can be described by a polynomial equation in one variable.
6. To simplify an expression containing powers means to rewrite the expression without parentheses or negative exponents.
7. Synthetic division is a shortcut method for dividing a polynomial by a binomial.
8. The relative maximum and relative minimum of a function are often referred to as end behavior.
9. When a polynomial is divided by one of its binomial factors, the quotient is called a depressed polynomial.
10. $(x^3)^2 + 3x^3 - 8 = 0$ is a power function.

Lesson-by-Lesson Review

6-1 Operations with Polynomials (pp. 333–339)

Simplify. Assume that no variable equals 0.

11. $\frac{14x^4y}{2x^3y^5}$
12. $3t(tn - 5)$
13. $(4r^2 + 3r - 1) - (3r^2 - 5r + 4)$
14. $(x^4)^3$
15. $(m + p)(m^2 - 2mp + p^2)$
16. $3b(2b - 1) + 2b(b + 3)$

EXAMPLE 1

Simplify each expression.

a. $(-4a^3b^5)(5ab^3)$

$$\begin{aligned} (-4a^3b^5)(5ab^3) &= (-4)(5)a^{3+1}b^{5+3} \\ &= -20a^4b^8 \end{aligned}$$

Product of Powers
Simplify.

b. $(2x^2 + 3x - 8) + (3x^2 - 5x - 7)$

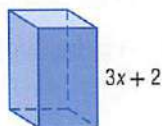
$$\begin{aligned} (2x^2 + 3x - 8) + (3x^2 - 5x - 7) \\ &= (2x^2 + 3x^2) + (3x - 5x) + [-8 + (-7)] \\ &= 5x^2 - 2x - 15 \end{aligned}$$

6-2 Dividing Polynomials (pp. 341–347)

Simplify.

17. $\frac{12x^4y^5 + 8x^3y^7 - 16x^2y^6}{4xy^5}$
18. $(6y^3 + 13y^2 - 10y - 24) \div (y + 2)$
19. $(a^4 + 5a^3 + 2a^2 - 6a + 4)(a + 2)^{-1}$
20. $(4a^6 - 5a^4 + 3a^2 - a) \div (2a + 1)$

21. **GEOMETRY** The volume of the rectangular prism is $3x^3 + 11x^2 - 114x - 80$ cubic units. What is the area of the base?



EXAMPLE 2

Simplify $(6x^3 - 31x^2 - 34x + 22) \div (2x - 1)$.

$$\begin{array}{r} 3x^2 - 14x - 24 \\ 2x - 1 \overline{) 6x^3 - 31x^2 - 34x + 22} \\ \underline{(-) 6x^3 - 3x^2} \\ -28x^2 - 34x \\ \underline{(-) -28x^2 + 14x} \\ -48x + 22 \\ \underline{(-) -48x + 24} \\ -2 \end{array}$$

The result is $3x^2 - 14x - 24 - \frac{2}{2x - 1}$.

6-3 Polynomial Functions (pp. 348–355)

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

22. $5x^6 - 3x^4 + x^3 - 9x^2 + 1$
23. $6xy^2 - xy + y^2$
24. $12x^3 - 5x^4 + 6x^8 - 3x - 3$

Find $p(-2)$ and $p(x + h)$ for each function.

25. $p(x) = x^2 + 2x - 3$
26. $p(x) = 3x^2 - x$
27. $p(x) = 3 - 5x^2 + x^3$

EXAMPLE 3

What are the degree and leading coefficient of $4x^3 + 3x^2 - 7x^7 + 4x - 1$?The greatest exponent is 7, so the degree is 7. The leading coefficient is -7 .

EXAMPLE 4

Find $p(a - 2)$ if $p(x) = 3x + 2x^2 - x^3$.

$$\begin{aligned} p(a - 2) &= 3(a - 2) + 2(a - 2)^2 - (a - 2)^3 \\ &= 3a - 6 + 2a^2 - 8a + 8 - (a^3 - 6a^2 + 12a - 8) \\ &= -a^3 + 8a^2 - 17a + 10 \end{aligned}$$

6-4 Analyzing Graphs of Polynomial Functions (pp. 357–364)

Complete each of the following.

- Graph each function by making a table of values.
- Determine the consecutive integer values of x between which each real zero is located.
- Estimate the x -coordinates at which the relative maxima and minima occur.

28. $h(x) = x^3 - 4x^2 - 7x + 10$

29. $g(x) = 4x^4 - 21x^2 + 5$

30. $f(x) = x^3 - 3x^2 - 4x + 12$

31. $h(x) = 4x^3 - 6x^2 + 1$

32. $p(x) = x^5 - x^4 + 1$

33. **BUSINESS** Milo tracked the monthly profits for his sports store business for the first six months of the year. They can be modeled by using the following six points: (1, 675), (2, 950), (3, 550), (4, 250), (5, 600), and (6, 400). How many turning points would the graph of a polynomial function through these points have? Describe them.

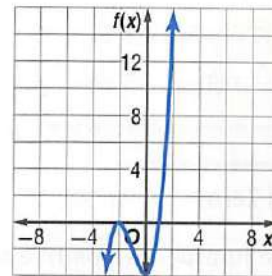
EXAMPLE 5

Graph $f(x) = x^3 + 3x^2 - 4$ by making a table of values.

Make a table of values for several values of x .

x	-3	-2	-1	0	1	2
$f(x)$	-4	0	-2	-4	0	16

Plot the points and connect the points with a smooth curve.



6-5 Solving Polynomial Equations by Factoring (pp. 368–375)

Factor completely. If the polynomial is not factorable, write *prime*.

34. $a^4 - 16$

35. $x^3 + 6y^3$

36. $54x^3y - 16y^4$

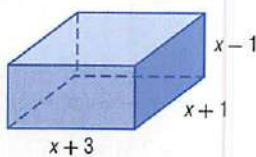
37. $6ay + 4by - 2cy + 3az + 2bz - cz$

Solve each equation.

38. $x^3 + 2x^2 - 35x = 0$

39. $8x^4 - 10x^2 + 3 = 0$

40. **GEOMETRY** The volume of the prism is 315 cubic inches. Find the value of x and the length, height, and width.



EXAMPLE 6

Factor $r^7 + 64r$.

$$\begin{aligned} r^7 + 64r &= r(r^6 + 64) && \text{Factor by GCF.} \\ &= r[(r^2)^3 + 4^3] && \text{Write as cubes.} \\ &= r(r^2 + 4)(r^4 - 4r^2 + 16) \end{aligned}$$

EXAMPLE 7

Solve $4x^4 - 25x^2 + 36 = 0$.

$$\begin{aligned} (x^2 - 4)(4x^2 - 9) &= 0 \\ x^2 - 4 = 0 &\quad \text{or} \quad 4x^2 - 9 = 0 \\ x^2 = 4 &\quad \quad \quad x^2 = \frac{9}{4} \\ x = \pm 2 &\quad \quad \quad x = \pm \frac{3}{2} \end{aligned}$$

The solutions are $-2, 2, -\frac{3}{2}$, and $\frac{3}{2}$.

6-6 The Remainder and Factor Theorems (pp. 377-382)

Use synthetic substitution to find $f(-2)$ and $f(4)$ for each function.

41. $f(x) = x^2 - 3$
 42. $f(x) = x^2 - 5x + 4$
 43. $f(x) = x^3 + 4x^2 - 3x + 2$
 44. $f(x) = 2x^4 - 3x^3 + 1$

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

45. $3x^3 + 20x^2 + 23x - 10$; $x + 5$
 46. $2x^3 + 11x^2 + 17x + 5$; $2x + 5$
 47. $x^3 + 2x^2 - 23x - 60$; $x - 5$

EXAMPLE 8

Determine whether $x - 6$ is a factor of $x^3 - 2x^2 - 21x - 18$.

$$\begin{array}{r|rrrr} 6 & 1 & -2 & -21 & -18 \\ & & 6 & 24 & 18 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

$x - 6$ is a factor because $r = 0$.

$$x^3 - 2x^2 - 21x - 18 = (x - 6)(x^2 + 4x + 3)$$

6-7 Roots and Zeros (pp. 383-390)

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

48. $f(x) = -2x^3 + 11x^2 - 3x + 2$
 49. $f(x) = -4x^4 - 2x^3 - 12x^2 - x - 23$
 50. $f(x) = x^6 - 5x^3 + x^2 + x - 6$
 51. $f(x) = -2x^5 + 4x^4 + x^2 - 3$
 52. $f(x) = -2x^6 + 4x^4 + x^2 - 3x - 3$

EXAMPLE 9

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of $f(x) = 3x^4 + 2x^3 - 2x^2 - 26x - 48$.

$f(x)$ has one sign change, so there is 1 positive real zero.

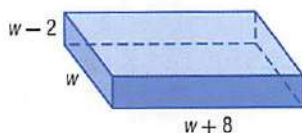
$f(-x)$ has 3 sign changes, so there are 3 or 1 negative real zeros.

There are 0 or 2 imaginary zeros.

6-8 Rational Zero Theorem (pp. 391-396)

Find all of the zeros of each function.

53. $f(x) = x^3 + 4x^2 + 3x - 2$
 54. $f(x) = 4x^3 + 4x^2 - x - 1$
 55. $f(x) = x^3 + 2x^2 + 4x + 8$
 56. **STORAGE** Melissa is building a storage box that is shaped like a rectangular prism. It will have a volume of 96 cubic feet. Using the diagram below, find the dimensions of the box.

**EXAMPLE 10**

Find all of the zeros of $f(x) = x^3 + 4x^2 - 11x - 30$.

There are exactly 3 zeros.

There are 1 positive real zero and 2 negative real zeros. The possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$.

$$\begin{array}{r|rrrr} 3 & 1 & 4 & -11 & -30 \\ & & 3 & 21 & 30 \\ \hline & 1 & 7 & 10 & 0 \end{array}$$

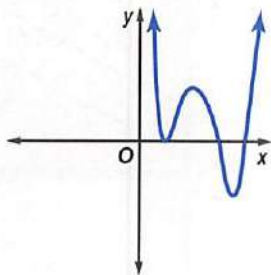
$$\begin{aligned} x^3 + 4x^2 - 11x - 30 &= (x - 3)(x^2 + 7x + 10) \\ &= (x - 3)(x + 2)(x + 5) \end{aligned}$$

Thus, the zeros are 3, -2, and -5.

Simplify.

- $(3a)^2(7b)^4$
- $(7x - 2)(2x + 5)$
- $(2x^2 + 3x - 4) - (4x^2 - 7x + 1)$
- $(4x^3 - x^2 + 5x - 4) + (5x - 10)$
- $(x^4 + 5x^3 + 3x^2 - 8x + 3) \div (x + 3)$
- $(3x^3 - 5x^2 - 23x + 24) \div (x - 3)$

7. **MULTIPLE CHOICE** How many unique real zeros does the graph have?



- A 0
B 2
C 3
D 5

8. If $c(x) = 3x^3 + 5x^2 - 4$, what is the value of $4c(3b)$?

Complete each of the following.

- Graph each function by making a table of values.
- Determine consecutive integer values of x between which each real zero is located.
- Estimate the x -coordinates at which the relative maxima and relative minima occur.

- $g(x) = x^3 + 4x^2 - 3x + 1$
- $h(x) = x^4 - 4x^3 - 3x^2 + 6x + 2$

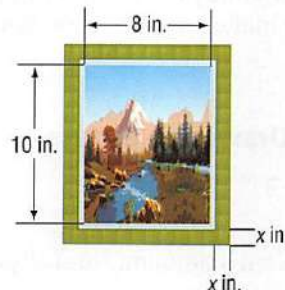
Factor completely. If the polynomial is not factorable, write *prime*.

- $8y^4 + x^3y$
- $2x^2 + 2x + 1$
- $a^2x + 3ax + 2x - a^2y - 3ay - 2y$

Solve each equation.

- $8x^3 + 1 = 0$
- $x^4 - 11x^2 + 28 = 0$

16. **FRAMING** The area of the picture and frame shown below is 168 square inches. What is the width of the frame?



17. **MULTIPLE CHOICE** Let $f(x) = x^4 - 3x^3 + 5x - 3$. Use synthetic substitution to find $f(-2)$.

- F 37
G 27
H -33
J -21

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

- $2x^3 + 15x^2 + 22x - 15; x + 5$
- $x^3 - 4x^2 + 10x - 12; x - 2$

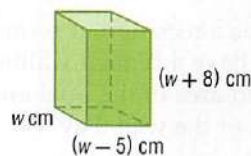
State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.

- $p(x) = x^3 - x^2 - x - 3$
- $p(x) = 2x^6 + 5x^4 - x^3 - 5x - 1$

Find all zeros of each function.

- $p(x) = x^3 - 4x^2 + x + 6$
- $p(x) = x^3 + 2x^2 + 4x + 8$

24. **GEOMETRY** The volume of the rectangular prism shown is 612 cubic centimeters. Find the dimensions of the prism.



25. List all possible rational zeros of $f(x) = 2x^4 + 3x^2 - 12x + 8$.

Draw a Picture

Drawing a picture can be a helpful way for you to visualize how to solve a problem. Sketch your picture on scrap paper or in your test booklet (if allowed). Do not make any marks on your answer sheet other than your answers.

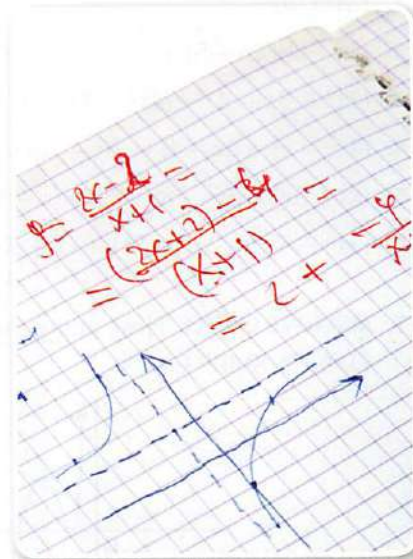
Strategies for Drawing a Picture

Step 1

Read the problem statement carefully.

Ask yourself:

- What am I being asked to solve?
- What information is given in the problem?
- What are the unknowns that I need to model and solve for?



Step 2

Sketch and label your picture.

- Draw your picture as clearly and accurately as possible.
- Label the picture carefully. Be sure to include all of the information given in the problem statement.

Step 3

Solve the problem.

- Use your picture to help you model the problem situation with an equation. Solve the equation.
- Check to be sure your answer makes sense.

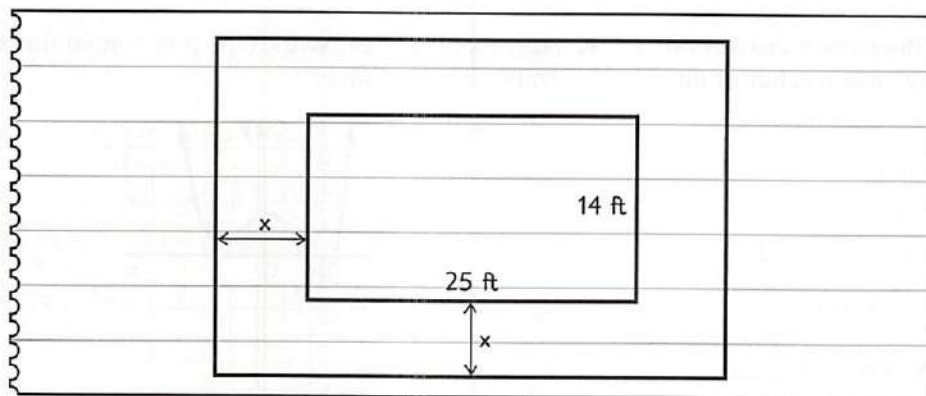
EXAMPLE

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

Mr. Nolan has a rectangular swimming pool that measures 25 feet by 14 feet. He wants to have a cement walkway installed around the perimeter of the pool. The combined area of the pool and walkway will be 672 square feet. What will be the width of the walkway?

- | | |
|-----------|-----------|
| A 2.75 ft | C 3.25 ft |
| B 3 ft | D 3.5 ft |

Draw a picture to help you visualize the problem situation. Let x represent the unknown width of the cement walkway.



The width of the pool and walkway is $14 + 2x$, and the length is $25 + 2x$. Multiply these polynomial expressions and set the result equal to the combined area, 672 square feet. Then solve for x .

$$\begin{aligned}(14 + 2x)(25 + 2x) &= 672 \\ 350 + 78x + 4x^2 &= 672 \\ 4x^2 + 78x - 322 &= 0 \\ x &= -23 \text{ or } 3.5\end{aligned}$$

Since the width cannot be negative, the width of the walkway will be 3.5 feet. The correct answer is D.

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve.

- A farmer has 240 feet of fencing that he wants to use to enclose a rectangular area for his chickens. He plans to build the enclosure using the wall of his barn as one of the walls. What is the maximum amount of area he can enclose?
A 7200 ft^2
B 4960 ft^2
C 3600 ft^2
D 3280 ft^2
- Metal washers are made by cutting a hole in a circular piece of metal. Suppose a washer is made by removing the center of a piece of metal with a 1.8-inch diameter. What is the radius of the hole if the washer has an area of 0.65π square inches?
F 0.35 in.
G 0.38 in.
H 0.40 in.
J 0.42 in.

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

- Simplify the following expression.
 $(5n^2 + 11n - 6) - (2n^2 - 5)$
 A $3n^2 + 11n - 11$ C $7n^2 + 11n - 11$
 B $3n^2 + 11n - 1$ D $7n^2 + 11n - 1$
- What is the effect on the graph of the equation $y = x^2 + 4$ when it is changed to $y = x^2 - 3$?
 F The slope of the graph changes.
 G The graph widens.
 H The graph is the same shape, and the vertex of the graph is moved down.
 J The graph is the same shape, and the vertex of the graph is shifted to the left.
- Let p represent the price that Ella charges for a necklace. Let $f(x)$ represent the total amount of money that Ella makes for selling x necklaces. The function $f(x)$ is best represented by
 A $f(x) = x + p$ C $f(x) = px$
 B $f(x) = xp^2$ D $f(x) = x^2 + p$
- Which of the following is *not* a solution to the cubic equation below?

$$x^3 - 37x - 84 = 0$$

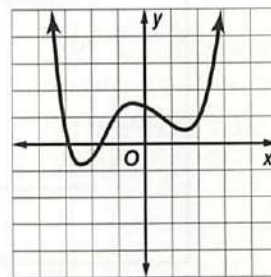
 F -4 H 6
 G -3 J 7

Test-Taking Tip

Question 5 You can use substitution to check each possible solution and identify the one that does not result in a true number sentence.

- What is the solution set for the equation $3(2x + 1)^2 = 27$?
 A $\{-5, 4\}$ C $\{2, -1\}$
 B $\{-2, 1\}$ D $\{-3, 3\}$

- How many real zeros does the polynomial function graphed below have?

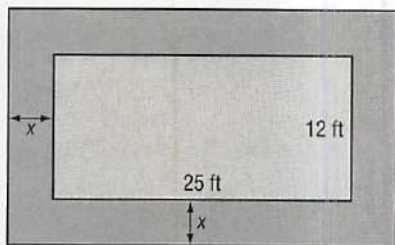


- F 2 H 4
 G 3 J 5
- For Marla's vacation, it will cost \$100 to drive her car, plus between \$0.50 and \$0.75 per mile. If she will drive her car 400 miles, what is a reasonable conclusion about c , the total cost to drive her car on the vacation?
 A $300 < c < 400$
 B $200 < c \leq 400$
 C $100 < c < 400$
 D $200 \leq c \leq 300$
 - The function $P(x) = -0.000047x^2 + 0.027x + 3$ can be used to approximate the population of Ling's home country between 1960 and 2000. The domain of the function x represents the number of years since 1960, and P is given in millions of people. Evaluate $P(20)$ to estimate the population of the country in 1980.
 F about 2 million people
 G about 2.5 million people
 H about 3 million people
 J about 3.5 million people
 - Solve $4x - 5 = 2x + 5 - 3x$ for x .
 A -2
 B -1
 C 1
 D 2

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. A stone path that is x feet wide is built around a rectangular flower garden. The garden is 12 feet wide and 25 feet long as shown below.

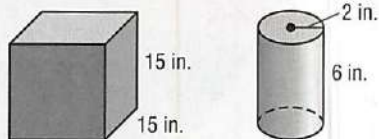


If the combined area of the garden and the stone path is 558 square feet, what is the width of the walkway? Express your answer in feet.

11. Factor $64a^4 + ab^3$ completely. Show your work.
12. Simplify $\frac{3x^3 - 4x^2 - 28x - 16}{x + 2}$. Give your answer in factored form. Show your work.
13. **GRIDDED RESPONSE** What is the value of a in the matrix equation below?

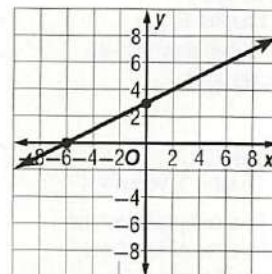
$$\begin{bmatrix} 4 & 3 \\ 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 21 \\ 9 \end{bmatrix}$$

14. **GRIDDED RESPONSE** Matt has a cubic aquarium as shown below. He plans to fill it by emptying cans of water with the dimensions as shown.



About how many cylindrical cans will it take to fill the aquarium?

15. What are the slope and y -intercept of the equation of the line graphed at the right?



Extended Response

Record your answers on a sheet of paper. Show your work.

16. The volume of a rectangular prism is 864 cubic centimeters. The length is 1 centimeter less than the height, and the width is 3 centimeters more than the height.
- Write a polynomial equation that can be used to solve for the height of the prism h .
 - How many possible roots are there for h in the polynomial equation you wrote? Explain.
 - Solve the equation from part a for all real roots h . What are the dimensions of the prism?
17. Scott launches a model rocket from ground level. The rocket's height h in meters is given by the equation $h = -4.9t^2 + 56t$, where t is the time in seconds after the launch.
- What is the maximum height the rocket will reach? Round to the nearest tenth of a meter. Show each step and explain your method.
 - How long after it is launched will the rocket reach its maximum height? Round to the nearest tenth of a second.

Need Extra Help?

If you missed Question...

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17

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