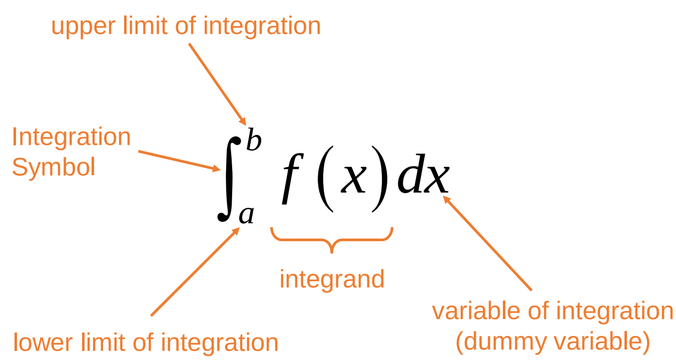
What you'll Learn About

- Terminology and Notation of Integration
- The Definite Integral
- Area under a curve using geometry
- Properties of Definite Integrals

Evaluate the definite integral using geometry



It is called a dummy variable because the answer does not depend on the variable chosen.

8) $\int_3^7 -20dx$

8A) $\int_2^7 22dx$

14) $\int_{.5}^{1.5} (-2x+4)dx$

16) $\int_{-4}^{0} \sqrt{16-x^2}\,dx$

	18) $\int_{-1}^{1}(1-\lvert x\rvert)dx$ 28) $\int_{a}^{\sqrt{3}a}(x)dx$ $Graph\ f(x)=\frac{1}{2}x^2$ using areas under the curve $\int_0^1 xdx =$ $\int_0^2 xdx =$ $\int_0^3 xdx =$ $\int_0^4 xdx =$ $\int_0^5 xdx =$

Use properties of Definite Integrals to answer the following

$\int_1^9 f(x)dx = -1 \qquad \int_7^9 f(x)dx = 5 \qquad \int_7^9 h(x)dx = 4$

$a) \int_1^9 -2f(x)dx =$

$b) \int_7^9 [f(x) + h(x)]dx =$

$c) \int_7^9 [2f(x) - 3h(x)]dx =$

$d) \int_9^1 f(x)dx =$

$e) \int_1^7 f(x)dx =$

$f) \int_9^7 [h(x) - f(x)]dx =$

$g) \int_9^9 h(x)dx =$

What you'll Learn About

- Average Value
- How to take the anti-derivative of a function
- How to evaluate the anti-derivative of a function (Part of the Fundamental Theorem of Calculus

8) $\int_3^7 -20dx =$

a) $\int_3^6 5dx =$

b) $\int_0^1 x^2 dx =$

d) $\int_0^1 x^3 dx =$

19) $\int_{\pi}^{2\pi} \sin x dx =$

22a) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc^2 x dx =$

24a) $\int_{-1}^{4} -5x^3 dx =$

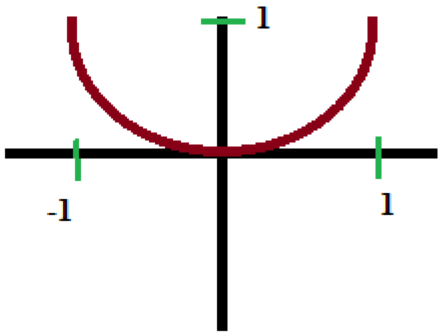
28) $\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx =$

	$30a)\int_1^2 \frac{1}{x^3}\,dx =$	$30)\int_0^5 x^{3/2}dx =$
	$34)\int_0^{\pi} (1+\cos x)dx =$	$40)\int_0^4 \frac{1-\sqrt{x}}{\sqrt{x}}\,dx =$

	$39a)\int_0^1 (1+x)^3\,dx =$ $39a)\int_0^1 (1+x)^3\,dx =$
	$39b)\int_0^1 (1+2x)^3\,dx =$ A) $\int_2^5 5^x\,dx =$

Find the average value of the function without integrating.

16. $f(x) = 1 - \sqrt{1 - x^2}$



32. $y = \dfrac{1}{x}$ [e, 2e]

	Using the calculator to compute area $A)\ \int_0^8 \frac{1}{5+3\cos(x)}$ $B)$ Find the Area of the region between the x - axis and the graph of $y = \sqrt{9 - 4x^2}$. C) For what value of x does $\int_0^x t^2 dt = 2$ D) For what value of x does $\int_0^x e^{-t^3} dt = .5695$ E) Find the area of the region in the first quadrant enclosed by the coordinate axes and the graph of $x^5 + y^5 = 1$. F) Find the average value of $\sqrt{\sin x}$ on the interval [1, 2].

What you'll Learn About

- Analyzing antiderivatives graphically
- Connecting Antiderivatives to Area
- Taking the derivative of an integral

A) *Find* $\frac{d}{dx}\int_{1}^{x}(\cos t)dt$

B) *Find* $\frac{d}{dx}\int_{1}^{x^3}(\cos t)dt$

C) *Find* $\frac{d}{dx}\int_{x^3}^{x^2}(\cos t)dt$

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$ for the given function

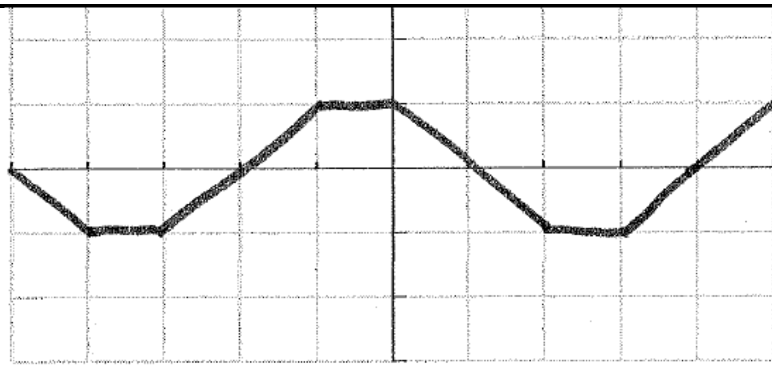
2) $\mathrm{y}=\int_{2}^{x}\left(3t+\cos t^{2}\right)dt$

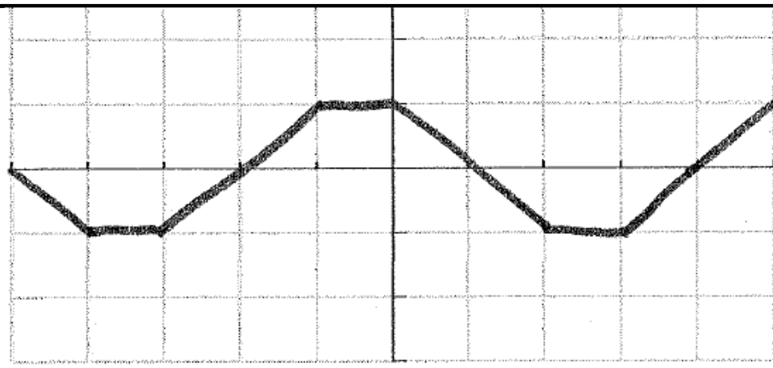
10) $\mathrm{y}=\int_{6}^{x^{2}}\left(\cot(3t)\right)dt$

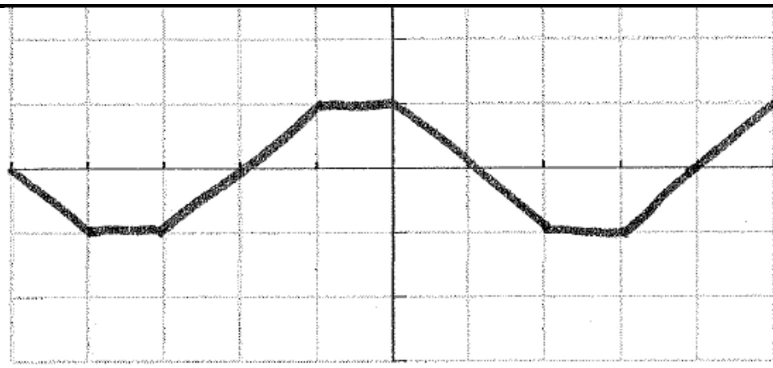
12) $\mathrm{y}=\int_{\pi}^{\pi-x}\left(\frac{1+\sin^{2}t}{1+\cos^{2}t}\right)dt$

14) $\mathrm{y}=\int_{x}^{7}\left(\sqrt{2t^{4}+t+1}\right)dt$

20) $\mathrm{y}=\int_{\sin x}^{\cos x}\left(t^{2}\right)dt$

Using the Fundamental Theorem of Calculus	 Graph of $f(t)$ Given: $g(x)=\int_{-2}^{x} f(t)dt$. Find each of the following: 1. $g(4) =$ 2. $g'(1) =$ 3. $g''(-1) =$ 4. $g''(-3) =$ 5. $g'(0) =$ 6. $g(1) =$ 7. $g(-3) =$ 8. $g(-4) =$

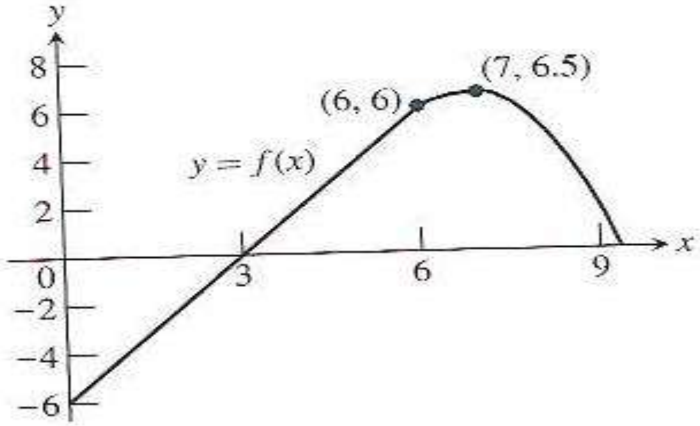
	 Graph of $f(t)$ 9. Find the equation of the tangent line to the graph of g at x = -2 10. Determine any relative/local maxima or minima on the interval (-5,2) 11. Determine the absolute maximum and minimum of g on [-5, 2].



Graph of $f(t)$

11. Let h(x) = g(x) - .5x^2 - x. Determine the critical values of h(x) on -5 < x < 5.

12. Let n(x) = $[g(x)]^2$ + f(x). Find $n'(1) =$



y = f(x) is the differentiable function whose graph is shown in the figure. The position at time t (seconds) of the particle moving along a coordinate axis is $s = \int_0^x f(t)dt$

a) What is the particle's velocity at time t = 3?

b) Is the acceleration of the particle at time t = 3 positive or negative?

c) What is the particle's position at time t = 3?

d) When does the particle pass through the origin?

e) Approximately when is the acceleration 0?

f) When is the particle moving toward the origin?

g) When is the particle moving away from the origin?

h) On which side of the origin does the particle lie at time t = 9?

What you'll Learn About

- How to find the area under the curve using rectangles and trapezoids
- What Right Riemann Sums, Left Riemann Sums, Midpoint Riemann Sums and Trapezoidal Sums are

1. Use the data below and 4 sub-intervals to approximate the area under the curve using **Right Riemann Sums**.

t	0	2	5	9	10
H(t)	66	60	52	44	43

1. Use the data below and 4 sub-intervals to approximate the area under the curve using **Left Riemann Sums**

t	0	2	5	9	10
H(t)	66	60	52	44	43

5. Use the data below to approximate the area under the curve using **Right Riemann Sums** and **Left Riemann Sums** with 5 sub-intervals.

T	0	8	20	25	32	40
P(t)	3	5	-10	-8	-4	7

1. Use the data below and 4 sub-intervals to approximate the area under the curve using the **Trapezoidal approximations**.

t	0	2	5	9	10
H(t)	66	60	52	44	43

2. Use the data below and 4 sub-intervals to approximate the area under the curve using **Trapezoidal approximations**.

t(hours)	0	2	5	7	8
E(t) (hundreds of entries)	0	4	13	21	23

4. Use the data below to approximate the area under the curve using **Midpoint Riemann Sums** with 3 sub-intervals.

T	0	2	4	6	8	10
P(t)	0	46	53	57	60	62

13. Use the data below to approximate the area under the curve using a **midpoint Riemann sum** with 3 sub-intervals

T (sec)	0	60	120	180	240	300	360
a(t) ft/sec^2	24	30	28	30	26	24	26

t(minutes)	0	4	9	15	20
W(t) degrees F	55.0	57.1	61.8	67.9	71.0

2012 #1

The temperature of water in a tub at time t is modeled by a strictly increasing, twice differentiable function, W, where W(t) is measured in degrees Fahrenheit and t is measured in minutes. At time t = 0, the temperature of the water is 55° F. The water is heated for 30 minutes, beginning at time t = 0. Values of W(t) at selected times t for the first 20 minutes are given in the table above.

c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)dt.$ Use a left Riemann sum with four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)dt.$ Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	–220	150

2015 BC3: Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differential function v. Selected values of v(t), where t is measured in minutes and v(t) is measured in meters per minute, are given in the table above.

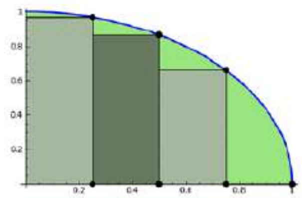
A) Use the data in the table to estimate the value of $v'(16)$

B) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\,dt$ in the context of the problem. Approximate the value of $\int_0^{40} |v(t)|\,dt$ using a right Riemann sum with four sub-intervals indicated in the table.

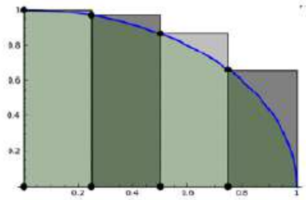
C) Bob is riding his bicycle along the same path. For $0 \le t \le 10$. Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and B(t) is measured in meters per minute. Find Bob's acceleration at time t = 5.

D) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

Left and right Riemann sums



Right Riemann Sum



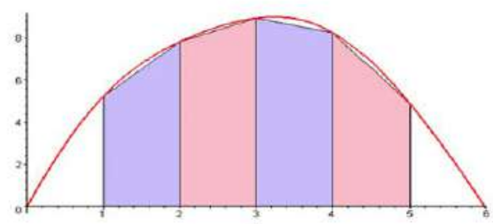
Left Riemann Sum

Correct justification for over and under approximations:

f(x)	**Left Riemann Sum**	**Right Riemann Sum**
Increasing ($f'(x) > 0$)	Under approximates the area because *f*(x) is increasing	Over approximates the area because *f*(x) is increasing
Decreasing ($f'(x) < 0$)	Over approximates the area because *f*(x) is decreasing	Under approximates the area because *f*(x) is decreasing

Incorrect Reasoning: *The left Riemann Sum is an under approximation because the rectangles are all underneath or below the graph. Stating that the rectangles are below the function is not acceptable mathematical reasoning. It merely restates that it is an under approximation but does not explain WHY.*

Trapezoidal approximations



Over/Under Approximations with Trapezoidal Approximations

f(x)	**Trapezoidal Sum**
Concave Up ($f''(x) > 0$)	Over approximates the area because $f''(x) > 0$
Concave Down ($f''(x) < 0$)	Under approximates the area because $f''(x) < 0$