5-4 The Binomial Distribution

A **binomial experiment** is a probability experiment that satisfies the following 4 requirements:

1. There must be a fixed number of trials.
2. Each trial can have only 2 outcomes. These outcomes are considered success/failure.
3. The outcomes of each trial are independent.
4. The probability of each success remains the same for each trial.

The binomial experiment leads to a special type of distribution called the binomial distribution.

The outcomes of a binomial experiment and the corresponding probabilities of these outcomes are called a **binomial distribution**.

Notation for the Binomial Distribution

1. P(S) – Probability of a success
2. P(F) – Probability of a failure
3. p – The numerical probability of a success
4. q – The numerical probability of a failure
5. *$P(S)=p$, $P(F)=1-p = q$
6. n – Number of trials
7. X – Number of successes in n trials
8. *Note $0 \leq X \leq n$, $X=0,1,2,3,\ldots,n$

Binomial Probability Formula

In a binomial experiment, the probability of exactly X successes in n trials is

$$(\quad) = \frac{!}{(\quad - \quad)!\quad !} \cdot \quad \cdot \quad ^{-}$$

Example 5-15

A coin is tossed 3 times. Find the probability of getting exactly 2 heads.

Is this Binomial?

1. There are fixed number of trials (3)
2. Two outcomes (heads or tails)
3. Outcomes are independent
4. Each time, the probability is $\frac{1}{2}$

So, n=3, X=2, p=$\frac{1}{2}$, q=$\frac{1}{2}$

Example 5-15 (cont.)

A coin is tossed 3 times. Find the probability of getting exactly 2 heads.

n=3, X=2, p=$\frac{1}{2}$, q=$\frac{1}{2}$

P(2 heads)=**P(X=2)=** $\frac{3!}{(3-2)!2!} \cdot (\frac{1}{2})^2 \cdot (\frac{1}{2})^1$

$= 3 \cdot \frac{1}{4} \cdot \frac{1}{2}$

$= \frac{3}{8} =$.

Confirm answer with sample space (what we did before) – we can get 2 heads 3 ways out of 8 total options.

Example 5-16

A survey found 1/5 of Americans visit a doctor in a given month. If 10 people are randomly selected, find probability exactly 3 go to the doctor.

Is this Binomial?

1. There are fixed number of trials (10)
2. Two outcomes (they go or they don't)
3. Outcomes are independent
4. Each time, the probability is $\frac{1}{5}$

So, n=10, X=3, p=$\frac{1}{5}$, q=$\frac{4}{5}$

Example 5-16 (cont.)

A survey found 1/5 of Americans visit a doctor in a given month. If 10 people are randomly selected, find probability exactly 3 go to the doctor.

n=10, X=3, p=$\frac{1}{5}$, q=$\frac{4}{5}$

P(3 visits)=**P(X=3)**= $\frac{10!}{(10-3)!3!} \cdot (\frac{1}{5})^3 \cdot (\frac{4}{5})^7$

$= 120 \cdot \frac{1}{125} \cdot \frac{16384}{78125}$

$= \frac{1966080}{9765625} =$.

Example 5-17

A survey found 30% of teenage consumers receive money from part-time jobs. If 5 teens are selected at random, find the probability that 3 will have part-time jobs.

Is this Binomial?
1. There are fixed number of trials (5)
2. Two outcomes (they have the job or they don't)
3. Outcomes are independent
4. Each time, the probability is 0.3

So, $n=5$, $X=3$, $p=0.3$, $q=0.7$

Example 5-17 (cont.)

A survey found 30% of teenage consumers receive money from part-time jobs. If 5 teens are selected at random, find the probability that 3 will have part-time jobs.

n=5, X=3, p=0.3, q=0.7

P(3 jobs)=P(X=3)= $\frac{5!}{(5-3)!3!} \cdot (0.3)^3 \cdot (0.7)^2$

$= 10 \cdot .027 \cdot .49$

$= 0.1323$

Analysis of Binomial Distribution

The **mean, variance, standard deviation** of a variable that has the binomial distribution can be found using the following formulas.

Mean: $= \quad \cdot$

Variance: $= \quad \cdot \quad \cdot$

Standard Deviation: $= \sqrt{\quad \cdot \quad \cdot \quad}$

Example 5-21

A coin is tossed 4 times. Find the mean, variance and standard deviation of the # of heads. Verify these new formulas work.

Use the formulas and n=4, p=$\frac{1}{2}$, q=$\frac{1}{2}$

$$= \quad \cdot \quad = 4 \cdot \frac{1}{2} =$$

$$= \quad \cdot \quad \cdot \quad = 4 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{4}{4} =$$

$$= \sqrt{\quad ^2} = \sqrt{1} =$$

Example 5-22

A die is rolled 480 times. Find the mean, variance and standard deviation of the # of 2's rolled.

This is binomial: success-get a 2, failure-don't

n=480, p=$\frac{1}{6}$, q=$\frac{5}{6}$

$$= \quad \cdot \quad = 480 \cdot \frac{1}{6} =$$

$$= \quad \cdot \quad \cdot \quad = 480 \cdot \frac{1}{6} \cdot \frac{5}{6} = \frac{80 \cdot 5}{6} \approx \quad .$$

$$= \sqrt{\quad ^2} = \sqrt{66.7} \approx \quad .$$

Example 5-22 (cont.)

A die is rolled 480 times. Find the mean, variance and standard deviation of the # of 2's rolled.

On average, there will be 80 2's. The standard deviation is 8.2.

Example 5-23

The *Statistical Bulletin* published by Metropolitan Life Insurance Co. reported that 2% of all American births result in twins. If a random sample of 8000 births is taken, find the mean, variance and standard deviation of # of births that would be twins.

This is binomial: success-have twins, failure-don't

n=8000, p=$.02$, q=$.98$

Example 5-23 (cont.)

The *Statistical Bulletin* published by Metropolitan Life Insurance Co. reported that 2% of all American births result in twins. If a random sample of 8000 births is taken, find the mean, variance and standard deviation of # of births that would be twins.

$$= \quad \cdot \quad = 8000(.02) =$$

$$= \quad \cdot \quad \cdot \quad = 8000\left(.02\right)\left(.98\right) = 160\left(.98\right) \approx \qquad .$$

$$= \sqrt{\quad ^{2}} = \sqrt{156.8} \approx \qquad .$$

Example 5-23 (cont.)

The *Statistical Bulletin* published by Metropolitan Life Insurance Co. reported that 2% of all American births result in twins. If a random sample of 8000 births is taken, find the mean, variance and standard deviation of # of births that would be twins.

For the sample, the average number of births that would result in twins is 160, the variance is around 157 and standard deviation is around 13.

Homework

- Part a - Pg. 263: 1, 3, 4ac, 7
- Part b - Pg. 263: 14-17