Warm Up

Evaluate.

1. $5 \bullet 4 \bullet 3 \bullet 2 \bullet 1$

2. $7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1$

120

3.

$$\frac{4 \bullet 3 \bullet 2 \bullet 1}{3 \bullet 2}$$

4

4.

5040

$$\frac{7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1}{4 \bullet 3 \bullet 2}$$

210

5.

$$\frac{5 \bullet 4 \bullet 3 \bullet 2 \bullet 1}{(2 \bullet 1)(3 \bullet 2 \bullet 1)}$$

10

6.

$$\frac{8 \bullet 7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1}{(4 \bullet 3 \bullet 2 \bullet 1)(4 \bullet 3 \bullet 2 \bullet 1)}$$

70

Fundamental Counting Principle

If there are n items and m_1 ways to choose a first item, m_2 ways to choose a second item after the first item has been chosen, and so on, then there are $m_1 \cdot m_2 \cdot \ldots \cdot m_n$ ways to choose n items.

Example 1A: Using the Fundamental Counting Principle

To make a yogurt parfait, you choose one flavor of yogurt, one fruit topping, and one nut topping. How many parfait choices are there?

Yogurt Parfait (choose 1 of each)		
Flavor	**Fruit**	**Nuts**
Plain	Peaches	Almonds
Vanilla	Strawberries	Peanuts
	Bananas	Walnuts
	Raspberries	
	Blueberries	

Example 1A Continued

number of flavors times number of fruits times number of nuts equals number of choices

$$2 \times 5 \times 3 = 30$$

There are 30 parfait choices.

Example 1b

A password is 4 letters followed by 1 digit. Uppercase letters (A) and lowercase letters (a) may be used and are considered different. How many passwords are possible?

Since both upper and lower case letters can be used, there are 52 possible letter choices.

letter letter letter letter number

$52 \times 52 \times 52 \times 52 \times 10 = 73{,}116{,}160$

There are 73,116,160 possible passwords.

Example 2: Paint Colors

A paint manufacturer wishes to manufacture several different paints. The categories include

Color: red, blue, white, black, green, brown, yellow

Type: latex, oil

Texture: flat, semi gloss, high gloss

Use: outdoor, indoor

How many different kinds of paint can be made if you can select one color, one type, one texture, and one use?

$$\left(\begin{array}{c}\text{\# of}\\ \text{colors}\end{array}\right)\left(\begin{array}{c}\text{\# of}\\ \text{types}\end{array}\right)\left(\begin{array}{c}\text{\# of}\\ \text{textures}\end{array}\right)\left(\begin{array}{c}\text{\# of}\\ \text{uses}\end{array}\right)$$

$$7 \cdot 2 \cdot 3 \cdot 2$$

$$\boxed{84 \text{ different kinds of paint}}$$

n Factorial

For any whole number *n*,

WORDS	NUMBERS	ALGEBRA
The **factorial** of a number is the product of the natural numbers less than or equal to the number. 0! is defined as 1.	$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$	$n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \ldots \cdot 1$

A **permutation** is a selection of a group of objects in which order is important.

Permutations

NUMBERS	ALGEBRA
The number of permutations of 7 items taken 3 at a time is $_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!}$.	The number of permutations of n items taken r at a time is $_nP_r = \frac{n!}{(n-r)!}$.

Example 3A: Finding Permutations

How many ways can a student government select a president, vice president, secretary, and treasurer from a group of 6 people?

This is the equivalent of selecting and arranging 4 items from 6.

$$_6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!}$$

Substitute 6 for n and 4 for r in $\frac{n!}{(n-r)!}$

$$= \frac{6 \bullet 5 \bullet 4 \bullet 3 \bullet \cancel{2 \bullet 1}}{\cancel{2 \bullet 1}}$$

Divide out common factors.

= 6 • 5 • 4 • 3 = 360

There are 360 ways to select the 4 people.

Example 3B: Finding Permutations

How many ways can a stylist arrange 5 of 8 vases from left to right in a store display?

$$_{8}P_{5} = \frac{8!}{(8-5)!} = \frac{8!}{3!}$$

$$= \frac{8 \bullet 7 \bullet 6 \bullet 5 \bullet 4 \bullet \cancel{3 \bullet 2 \bullet 1}}{\cancel{3 \bullet 2 \bullet 1}}$$

Divide out common factors.

$= 8 \bullet 7 \bullet 6 \bullet 5 \bullet 4$

$= 6720$

There are 6720 ways that the vases can be arranged.

Example 4: Business Locations

Suppose a business owner has a choice of 5 locations in which to establish her business. She decides to rank each location according to certain criteria, such as price of the store and parking facilities. How many different ways can she rank the 5 locations?

$$\begin{pmatrix}\text{first}\\ \text{choice}\end{pmatrix}\begin{pmatrix}\text{second}\\ \text{choice}\end{pmatrix}\begin{pmatrix}\text{third}\\ \text{choice}\end{pmatrix}\begin{pmatrix}\text{fourth}\\ \text{choice}\end{pmatrix}\begin{pmatrix}\text{fifth}\\ \text{choice}\end{pmatrix}$$

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$\boxed{\text{120 different ways to rank the locations}}$$

Using factorials, $5! = 120$.

Using permutations, ${}_5P_5 = 120$.

Example 5: Television News Stories

A television news director wishes to use 3 news stories on an evening show. One story will be the lead story, one will be the second story, and the last will be a closing story. If the director has a total of 8 stories to choose from, how many possible ways can the program be set up?

Since there is a lead, second, and closing story, we know that order matters. We will use permutations.

$$ {}_8P_3 = \frac{8!}{5!} = 336 \qquad \text{or} \qquad {}_8P_3 = \underbrace{8 \cdot 7 \cdot 6}_{3} = 336 $$

A **combination** is a grouping of items in which order does not matter. There are generally fewer ways to select items when order does not matter.

Combinations

NUMBERS	ALGEBRA
The number of combinations of 7 items taken 3 at a time is $_7C_3 = \frac{7!}{3!(7-3)!}$.	The number of combinations of n items taken r at a time is $_nC_r = \frac{n!}{r!(n-r)!}$.

When deciding whether to use permutations or combinations, first decide whether order is important. Use a permutation if order matters and a combination if order does not matter.

Example 6: Application

There are 12 different-colored cubes in a bag. How many ways can Randall draw a set of 4 cubes from the bag?

Step 1 Determine whether the problem represents a permutation of combination.

The order does not matter. The cubes may be drawn in any order. It is a combination.

Example 6 Continued

Step 2 Use the formula for combinations.

$$_{12}C_4 = \frac{12!}{4!(12-4)!} = \frac{12!}{4!(8!)}$$

n = 12 and r = 4

$$= \frac{12 \bullet 11 \bullet 10 \bullet 9 \bullet \cancel{8 \bullet 7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1}}{(4 \bullet 3 \bullet 2 \bullet 1)(\cancel{8 \bullet 7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1})}$$

Divide out common factors.

$$= \frac{12 \bullet 11 \bullet 10 \bullet 9}{4 \bullet 3 \bullet 2 \bullet 1} = \frac{\cancel{12} \bullet 11 \bullet \cancel{10}^{5} \bullet 9}{\cancel{4} \bullet \cancel{3} \bullet \cancel{2} \bullet 1} = 495$$

There are 495 ways to draw 4 cubes from 12.

Example 7: Newspaper Editing

A newspaper editor has received 8 books to review. He decides that he can use 3 reviews in his newspaper. How many different ways can these 3 reviews be selected?

The placement in the newspaper is not mentioned, so order does not matter. We will use combinations.

$$ {}_8C_3 = \frac{8!}{5!3!} = 8!/\left(5!3!\right) = 56 $$

$$ \text{or} \quad {}_8C_3 = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2} = 56 \quad \text{or} \quad {}_8C_3 = \frac{{}_8P_3}{3!} = 56 $$

Example 8: Combination Locks

A combination lock consists of the 26 letters of the alphabet. If a 3-letter combination is needed, find the probability that the combination will consist of the letters ABC in that order. The same letter can be used more than once. (*Note: A combination lock is really a* permutation lock.)

There are 26·26·26 = 17,576 possible combinations.

The letters ABC in order create one combination.

$$P(\text{ABC}) = \frac{1}{17,576}$$

Example 9: Committee Selection

A store has 6 *TV Graphic magazines and 8 Newstime* magazines on the counter. If two customers purchased a magazine, find the probability that one of each magazine was purchased.

TV Graphic: One magazine of the 6 magazines

Newstime: One magazine of the 8 magazines

Total: Two magazines of the 14 magazines

$$\frac{{}_6C_1 \cdot {}_8C_1}{{}_{14}C_2} = \frac{6 \cdot 8}{91} = \boxed{\frac{48}{91}}$$

Homework

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