Warm Up

Evaluate.

1. 5 • 4 • 3 • 2 • 1 120 **2**. 7 • 6 • 5 • 4 • 3 • 2 • 1 3. 4. 5040 4•3•2•1 7•6•5•4•3•2•1 4 210 3•2 4•3•2 5. 6. $\frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)(3 \cdot 2 \cdot 1)}$ 8 • 7 • 6 • 5 • 4 • 3 • 2 • 1 10 $\overline{(4 \cdot 3 \cdot 2 \cdot 1)(4 \cdot 3 \cdot 2 \cdot 1)}^{70}$

Fundamental Counting Principle

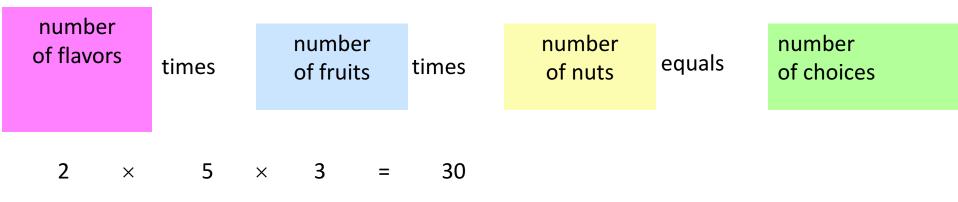
If there are *n* items and m_1 ways to choose a first item, m_2 ways to choose a second item after the first item has been chosen, and so on, then there are $m_1 \cdot m_2 \cdot \ldots \cdot m_n$ ways to choose *n* items.

Example 1A: Using the Fundamental Counting Principle

To make a yogurt parfait, you choose one flavor of yogurt, one fruit topping, and one nut topping. How many parfait choices are there?

Yogurt Parfait (choose 1 of each)				
Flavor	Fruit	Nuts		
Plain	Peaches	Almonds		
Vanilla	Strawberries	Peanuts		
	Bananas	Walnuts		
	Raspberries			
	Blueberries			

Example 1A Continued



There are 30 parfait choices.

Example 1b

A password is 4 letters followed by 1 digit. Uppercase letters (A) and lowercase letters (a) may be used and are considered different. How many passwords are possible?

Since both upper and lower case letters can be used, there are 52 possible letter choices.

letter letter letter number

 $52 \times 52 \times 52 \times 52 \times 10 = 73,116,160$

There are 73,116,160 possible passwords.

Example 2: Paint Colors

A paint manufacturer wishes to manufacture several different paints. The categories include

- Color: red, blue, white, black, green, brown, yellow
- Type: latex, oil
- Texture: flat, semi gloss, high gloss
- Use: outdoor, indoor

How many different kinds of paint can be made if you can select one color, one type, one texture, and one use?

$$\begin{pmatrix} \# \text{ of } \\ \text{colors} \end{pmatrix} \begin{pmatrix} \# \text{ of } \\ \text{types} \end{pmatrix} \begin{pmatrix} \# \text{ of } \\ \text{textures} \end{pmatrix} \begin{pmatrix} \# \text{ of } \\ \text{uses} \end{pmatrix}$$

$$7 \cdot 2 \cdot 3 \cdot 2$$

84 different kinds of paint

n Factorial

For any whole number n,

WORDS	NUMBERS	ALGEBRA
The factorial of a number is the product of the natural numbers less than or equal to the number. 0! is defined as 1.	6! = 6 • 5 • 4 • 3 • 2 • 1 = 720	$n! = n \cdot (n - 1) \cdot (n - 2) \cdot (n - 3) \cdot \dots \cdot 1$

A **permutation** is a selection of a group of objects in which order is <u>important</u>.

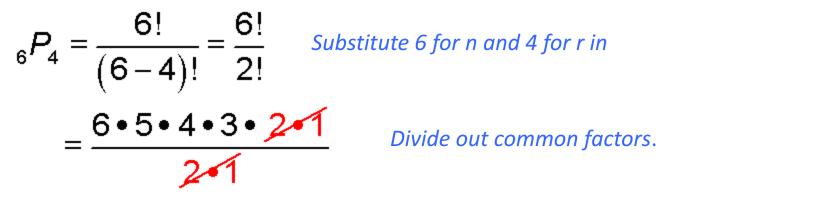
Permutations					
NUMBERS	ALGEBRA				
The number of permutations of 7 items taken 3 at a time	The number of permutations of <i>n</i> items taken <i>r</i> at a time				
is	is				
$_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!}.$	${}_{n}P_{r}=\frac{n!}{(n-r)!}.$				

Example 3A: Finding Permutations

 $\frac{n}{(n-r)!}$

How many ways can a student government select a president, vice president, secretary, and treasurer from a group of 6 people?

This is the equivalent of selecting and arranging 4 items from 6.



 $= 6 \bullet 5 \bullet 4 \bullet 3 = 360$

There are 360 ways to select the 4 people.

Example 3B: Finding Permutations

How many ways can a stylist arrange 5 of 8 vases from left to right in a store display?

$${}_{8}P_{5} = \frac{8!}{(8-5)!} = \frac{8!}{3!}$$

$$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$$
Divide out common factors.
$$= 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$$

$$= 6720$$

There are 6720 ways that the vases can be arranged.

Example 4: Business Locations

Suppose a business owner has a choice of 5 locations in which to establish her business. She decides to rank each location according to certain criteria, such as price of the store and parking facilities. How many different ways can she rank the 5 locations?

(first become third third fourth fifth choice choice choice choice to $3 \cdot 2 \cdot 1$

120 different ways to rank the locations

Using factorials, 5! = 120.

Using permutations, ${}_5P_5 = 120$.

Example 5: Television News Stories

A television news director wishes to use 3 news stories on an evening show. One story will be the lead story, one will be the second story, and the last will be a closing story. If the director has a total of 8 stories to choose from, how many possible ways can the program be set up?

Since there is a lead, second, and closing story, we know that order matters. We will use permutations.

$$_{8}P_{3} = \frac{8!}{5!} = 336$$
 or $_{8}P_{3} = \begin{cases} 8 \cdot 7 \cdot 6 = 336 \\ 3 \end{cases}$

A **<u>combination</u>** is a grouping of items in which order <u>does not matter</u>. There are generally fewer ways to select items when order does not matter.

Combinations					
	NUMBERS	ALGEBRA			
	The number of combinations	The number of combinations			
	of 7 items taken 3 at a	of <i>n</i> items taken <i>r</i> at a			
	time is	time is			
	$_{7}C_{3} = \frac{7!}{3!(7-3)!}.$	${}_{n}C_{r} = \frac{n!}{r!(n-r)!}.$			

When deciding whether to use permutations or combinations, first decide whether order is important. Use a permutation if order matters and a combination if order does not matter.

Example 6: Application

There are 12 different-colored cubes in a bag. How many ways can Randall draw a set of 4 cubes from the bag?

Step 1 Determine whether the problem represents a permutation of combination.

The order does not matter. The cubes may be drawn in any order. It is a combination.

Example 6 Continued

Step 2 Use the formula for combinations.

$${}_{12}C_4 = \frac{12!}{4!(12-4)!} = \frac{12!}{4!(8!)} \qquad n = 12 \text{ and } r = 4$$

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(4 \cdot 3 \cdot 2 \cdot 1)(8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1)} \qquad \begin{array}{l} \text{Divide out} \\ \text{common} \\ \text{factors.} \end{array}$$

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{12 \cdot 11 \cdot 10 \cdot 59}{4 \cdot 3 \cdot 2 \cdot 1} = 495$$

There are 495 ways to draw 4 cubes from 12.

Example 7: Newspaper Editing

A newspaper editor has received 8 books to review. He decides that he can use 3 reviews in his newspaper. How many different ways can these 3 reviews be selected?

The placement in the newspaper is not mentioned, so order does not matter. We will use combinations.

$$_{8}C_{3} = \frac{8!}{5!3!} = 8!/(5!3!) = 56$$

or
$${}_{8}C_{3} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2} = 56$$
 or ${}_{8}C_{3} = \frac{{}_{8}P_{3}}{3!} = 56$

Example 8: Combination Locks

A combination lock consists of the 26 letters of the alphabet. If a 3-letter combination is needed, find the probability that the combination will consist of the letters ABC in that order. The same letter can be used more than once. (*Note: A combination lock is really a* permutation lock.)

There are $26 \cdot 26 \cdot 26 = 17,576$ possible combinations. The letters ABC in order create one combination.

$$P(\mathsf{ABC}) = \frac{1}{17,576}$$

Example 9: Committee Selection

A store has 6 *TV Graphic magazines and 8 Newstime* magazines on the counter. If two customers purchased a magazine, find the probability that one of each magazine was purchased.

TV Graphic: One magazine of the 6 magazines *Newstime*: One magazine of the 8 magazines Total: Two magazines of the 14 magazines

$$\frac{{}_{6}C_{1} \cdot {}_{8}C_{1}}{{}_{14}C_{2}} = \frac{6 \cdot 8}{91} = \boxed{\frac{48}{91}}$$

Homework

Pg. 220 # 7, 11, 13, 27, 33, 35