4-4

Multiplication Rules

4.4 Multiplication Rules

- Two events *A* and *B* are **independent events** if the fact that *A* occurs does not affect the probability of *B* occurring.

Multiplication Rules

$$P(A \text{ and } B) = P(A) \cdot P(B) \quad \text{Independent}$$

$$P(A \text{ and } B) = P(A) \cdot P(B|A) \quad \text{Dependent}$$

Example 4-23: Tossing a Coin

A coin is flipped and a die is rolled. Find the probability of getting a head on the coin and a 4 on the die.

Independent Events

$$P(\text{Head and } 4) = P(\text{Head}) \cdot P(4)$$

$$= \frac{1}{2} \cdot \frac{1}{6} = \boxed{\frac{1}{12}}$$

This problem could be solved using sample space.

H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6

Example 4-26: Survey on Stress

A Harris poll found that 46% of Americans say they suffer great stress at least once a week. If three people are selected at random, find the probability that all three will say that they suffer great stress at least once a week.

Independent Events

$$
\begin{aligned}
P(\text{S and S and S}) &= P(\text{S}) \cdot P(\text{S}) \cdot P(\text{S}) \\
&= (0.46)(0.46)(0.46) \\
&= \boxed{0.097}
\end{aligned}
$$

Example 4-28: University Crime

At a university in western Pennsylvania, there were 5 burglaries reported in 2003, 16 in 2004, and 32 in 2005. If a researcher wishes to select at random two burglaries to further investigate, find the probability that both will have occurred in 2004.

Dependent Events

$$P(\mathrm{C}_1 \text{ and } \mathrm{C}_2) = P(\mathrm{C}_1) \cdot P(\mathrm{C}_2 | \mathrm{C}_1)$$

$$= \frac{16}{53} \cdot \frac{15}{52} = \frac{60}{689}$$

4.4 Conditional Probability

- **Conditional probability** is the probability that the second event *B* occurs given that the first event *A* has occurred.

Conditional Probability

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Example 4-33: Parking Tickets

The probability that Sam parks in a no-parking zone *and gets a parking ticket is 0.06,* and the probability that Sam cannot find a legal parking space and has to park in the no-parking zone is 0.20. On Tuesday, Sam arrives at school and has to park in a no-parking zone. Find the probability that he will get a parking ticket.

N= parking in a no-parking zone, T= getting a ticket

$$P(\text{T}|\text{N}) = \frac{P(\text{N and T})}{P(\text{N})} = \frac{0.06}{0.20} = \boxed{0.30}$$

Example 4-34: Women in the Military

A recent survey asked 100 people if they thought women in the armed forces should be permitted to participate in combat. The results of the survey are shown.

Gender	Yes	No	Total
Male	32	18	50
Female	8	42	50
Total	40	60	100

Example 4-34: Women in the Military

a. Find the probability that the respondent answered yes (Y), given that the respondent was a female (F).

Gender	Yes	No	Total
Male	32	18	50
Female	8	42	50
Total	40	60	100

$$P\left(\mathrm{Y} \mid \mathrm{F}\right)=\frac{P\left(\mathrm{F\ and\ Y}\right)}{P\left(\mathrm{F}\right)}=\frac{\frac{8}{100}}{\frac{50}{100}}=\frac{8}{50}=\boxed{\frac{4}{25}}$$

Example 4-34: Women in the Military

b. Find the probability that the respondent was a male (M), given that the respondent answered no (N).

Gender	Yes	No	Total
Male	32	18	50
Female	8	42	50
Total	40	60	100

$$P(\text{M}|\text{N}) = \frac{P(\text{N and M})}{P(\text{N})} = \frac{\frac{18}{100}}{\frac{60}{100}} = \frac{18}{60} = \boxed{\frac{3}{10}}$$

Probabilities for “At Least” and “At Most”

- At Least
  - All probabilities larger than the given probability
  - EX: You must be at least 5 feet to ride the roller coaster means everyone 5 feet or taller
- At Most
  - All probabilities smaller than the given probability
  - EX: I have at most $20 in my purse means I have $20 or less

Example 1: Bow Ties

The Neckware Association of America reported that 3% of ties sold in the United States are bow ties (B). If 4 customers who purchased a tie are randomly selected, find the probability that at least 1 purchased a bow tie.

$$P(\text{B}) = 0.03,\ P(\overline{\text{B}}) = 1 - 0.03 = 0.97$$

$$P(\text{no bow ties}) = P(\overline{\text{B}}) \cdot P(\overline{\text{B}}) \cdot P(\overline{\text{B}}) \cdot P(\overline{\text{B}})$$

$$= (0.97)(0.97)(0.97)(0.97) = 0.885$$

$$P(\text{at least 1 bow tie}) = 1 - P(\text{no bow ties})$$

$$= 1 - 0.885 = \boxed{0.115}$$

Homework

- Pg. 209 #1, 7, 8, 32, 34