

CHAPTER 3

Systems of Equations and Inequalities

Then

In Chapter 2, you graphed equations of lines, transformed functions, and solved equations.

Now

In Chapter 3, you will:

- Solve systems of linear equations graphically and algebraically.
- Solve systems of linear inequalities graphically.
- Solve problems by using linear programming.

Why?

BUSINESS Most of the time, being successful in business means that you have to have good math skills. In this chapter, you will learn how to maximize your profits and minimize your costs. By doing this you will earn the most money possible.



Systems of Equations and Inequalities

Activity

You decide to make candy covered chocolates and candy bars for your new business. You need to know how many candies you need to make each month to maximize your profits.

It takes 0.7 hours to chocolate coat the candies and 1.2 hours to package them. For the candy bar, it takes 1.1 hours to chocolate coat and 0.8 hours to package.

With this information, develop two linear inequalities if the maximum labor hours for the chocolate coating and packaging departments are 500 and 456 hours respectively.

| | Chocolate Candies | Chocolate Bars | Total Hours |
|-----------|-------------------|----------------|-------------|
| Coating | 0.7 | 1.1 | 500 |
| Packaging | 1.2 | 0.8 | 456 |

Math in Motion, Animation glencoe.com

Get Ready for Chapter 3

Diagnose Readiness You have two options for checking Prerequisite Skills.

Text Option

Take the Quick Check below. Refer to the Quick Review for help.

QuickCheck

Graph each equation. (Lesson 2-2)

1. $x = 4y$
2. $y = \frac{1}{3}x + 5$
3. $x + 2y = 4$
4. $y = -x + 6$
5. $3x + 5y = 15$
6. $3y - 2x = -12$
7. **BUSINESS** A museum charges \$8.50 for adult tickets and \$5.25 for children's tickets. On Friday they made \$650. (Lesson 2-4)
 - a. Write an equation that can be used to model the ticket sales.
 - b. Graph the equation.

QuickReview

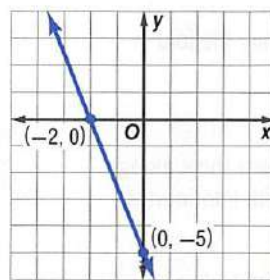
EXAMPLE 1

Graph $2y + 5x = -10$.

Find the x - and y -intercepts.

$$\begin{array}{rcl} 2(0) + 5x = -10 & & 2y + 5(0) = -10 \\ 5x = -10 & & 2y = -10 \\ x = -2 & & y = -5 \end{array}$$

The graph crosses the x -axis at $(-2, 0)$ and the y -axis at $(0, -5)$. Use these ordered pairs to graph the equation.



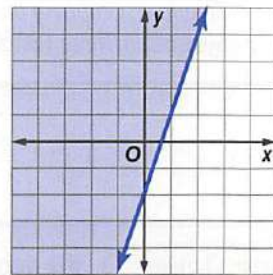
Graph each inequality. (Lesson 2-8)

8. $y < 3$
9. $x + y \geq 1$
10. $3x - y > 6$
11. $x + 2y \leq 5$
12. $y > 4x - 1$
13. $5x - 4y < 12$
14. **FUNDRAISER** The student council is selling T-shirts for \$15 and sweatshirts for \$25. They must make \$2500 to cover expenses. Write and graph an inequality to show the number of T-shirts and sweatshirts that they must sell.

EXAMPLE 2

Graph $y \geq 3x - 2$.

The boundary is the graph of $y = 3x - 2$. Since the inequality symbol is \geq , the boundary will be solid.



Test the point $(0, 0)$.

$$\begin{array}{l} 0 \geq 3(0) - 2 \quad (x, y) = (0, 0) \\ 0 \geq -2 \quad \checkmark \end{array}$$

Shade the region that includes $(0, 0)$.

Online Option

Math Online

Take a self-check Chapter Readiness Quiz at glencoe.com.

Get Started on Chapter 3

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 3. To get ready, identify important terms and organize your resources. You may wish to refer to **Chapter 0** to review prerequisite skills.

FOLDABLES®

Study Organizer

Systems of Equations and Inequalities Make this Foldable to help you organize your Chapter 3 notes about systems of equations and inequalities. Begin with a sheet of $8\frac{1}{2}$ " by 11" paper.

1 **Fold** in half along the height.



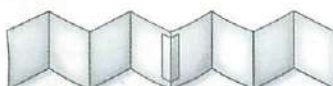
2 **Cut** along the fold.



3 **Fold** each sheet along the width into fourths.



4 **Tape** the ends of two sheets together.



5 **Label** the tabs with *Solve by Graphing*, *Substitution*



Method, Elimination Method, No Solution, Infinite Solutions, Graphing Inequalities, Optimization, and Systems in Three Variables.

Math Online glencoe.com

- Study the chapter online
- Explore **Math in Motion**
- Get extra help from your own **Personal Tutor**
- Use **Extra Examples** for additional help
- Take a **Self-Check Quiz**
- Review Vocabulary** in fun ways

New Vocabulary

English

Español

| | | |
|-------------------------------|------------|---------------------------------|
| system of equations | • p. 135 • | sistema de ecuaciones |
| break-even point | • p. 136 • | punto de equilibrio |
| consistent | • p. 137 • | consistente |
| inconsistent | • p. 137 • | inconsistente |
| independent | • p. 137 • | independiente |
| dependent | • p. 137 • | dependiente |
| substitution method | • p. 143 • | método de sustitución |
| elimination method | • p. 144 • | método de eliminación |
| system of inequalities | • p. 151 • | sistema de desigualdades |
| constraints | • p. 160 • | restricciones |
| linear programming | • p. 160 • | programación lineal |
| feasible region | • p. 160 • | región viable |
| bounded | • p. 160 • | acotada |
| unbounded | • p. 160 • | no acotado |
| optimize | • p. 162 • | optimizar |
| ordered triple | • p. 167 • | triple ordenado |

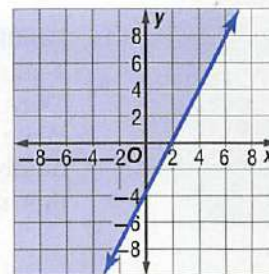
Review Vocabulary

equation • p. 135 • **ecuación** a mathematical sentence stating that two mathematical expressions are equal

inequality • p. 151 • **desigualdad** an open sentence that contains the symbol $<$, \leq , $>$, or \geq

linear equation • p. 135 • **ecuación lineal** an equation that has no operations other than addition, subtraction, and multiplication of a variable by a constant

solution • p. 135 • **solución** a replacement for the variable in an open sentence that results in a true sentence



Solving Systems of Equations by Graphing

Then

You graphed and solved linear equations.

(Lesson 2-2)

Now

- Solve systems of linear equations by using tables and graphs.
- Determine whether a system of linear equations is inconsistent, consistent and dependent, or consistent and independent.

New Vocabulary

system of equations
break-even point
consistent
inconsistent
independent
dependent

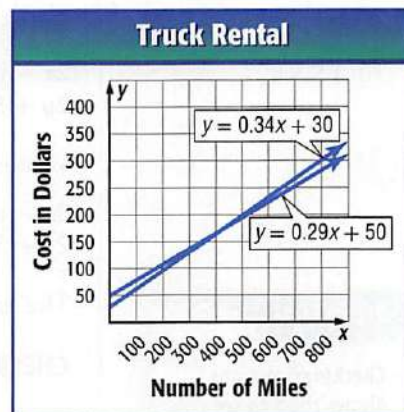
Math Online

glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Why?

Miguel is moving and needs to rent a moving truck. Rico's Truck Rental charges a flat fee of \$50 plus \$0.29 per mile. Home Movers charges a flat fee of \$30 plus \$0.34 per mile. If Miguel expects to put 500 miles on the rental truck, from which company should he rent the truck? This situation can be modeled by a system of linear equations.



Solve Systems Using Tables and Graphs A **system of equations** is two or more equations with the same variables. To solve a system of equations with two variables, find the ordered pair that satisfies all of the equations.

To solve a system of equations by using a table, first write each equation in slope-intercept form. Then substitute different values for x and solve for the corresponding y -values. For ease of use, choose 0 and 1 as your first x -values.

$$y_1 = -2x + 8$$

$$y_2 = 4x - 7$$

| x | y_1 | y_2 | Difference |
|-----|-------|-------|------------|
| 0 | 8 | -7 | 15 |
| 1 | 6 | -3 | 9 |
| 2 | 4 | 1 | 3 |
| 3 | 2 | 5 | -3 |

Because the difference between the y -values is closer to 0 for $x = 1$ than $x = 0$, a value greater than 1 should be tried next.

Because the difference between the y -values changed signs from $x = 2$ to $x = 3$, a value between these should be tried next.

The solution is between 2 and 3.

EXAMPLE 1 Solve by Using a Table

Solve the system of equations.

$$3x + 2y = -2$$

$$-4x + 5y = -28$$

Write each equation in slope-intercept form.

$$3x + 2y = -2 \rightarrow y = -1.5x - 1$$

$$-4x + 5y = -28 \rightarrow y = 0.8x - 5.6$$

Use a table to find the solution that satisfies both equations.

The solution of the system is $(2, -4)$.

| x | y_1 | y_2 | (x, y_1) | (x, y_2) |
|-----|-------|-------|------------|------------|
| 0 | -1 | -5.6 | (0, -1) | (0, -5.6) |
| 1 | -2.5 | -4.8 | (1, -2.5) | (1, -4.8) |
| 2 | -4 | -4 | (2, -4) | (2, -4) |

Check Your Progress

1A. $2x - 5y = 11$
 $-3x + 4y = -13$

1B. $4x + 3y = -17$
 $-7x - 2y = 20$

Another method for solving a system of equations is to graph the equations on the same coordinate plane. The point of intersection represents the solution.

EXAMPLE 2 Solve by Graphing

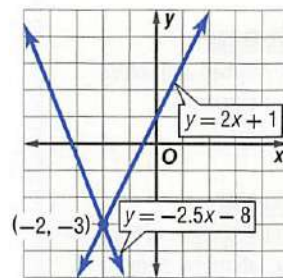
Solve the system of equations by graphing.

$$\begin{aligned} 2x - y &= -1 \\ 2y + 5x &= -16 \end{aligned}$$

Write each equation in slope-intercept form.

$$\begin{aligned} 2x - y &= -1 &\rightarrow y &= 2x + 1 \\ 2y + 5x &= -16 &\rightarrow y &= -2.5x - 8 \end{aligned}$$

The graphs of the lines appear to intersect at $(-2, -3)$.



CHECK Substitute the coordinates into each original equation.

| | | |
|-----------------------------------|-------------------------------------|---------------------------|
| $2x - y = -1$ | $2y + 5x = -16$ | Original equations |
| $2(-2) - (-3) \stackrel{?}{=} -1$ | $2(-3) + 5(-2) \stackrel{?}{=} -16$ | $x = -2$ and $y = -3$ |
| $-1 = -1$ ✓ | $-16 = -16$ ✓ | Simplify. |

The solution of the system is $(-2, -3)$.

StudyTip

Checking Solutions

Always check to see if the values work for *both* of the original equations.

Check Your Progress

2A. $4x + 3y = 12$
 $-6x + 4y = -1$

2B. $-3y + 8x = 36$
 $6x + y = -21$

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Systems of equations are used by businesses to determine the break-even point. The **break-even point** is the point at which the income equals the cost.



Real-World Link

The Internet provides countless business opportunities for teenage entrepreneurs. Roughly 1.6 million U.S. teens make some money using the Internet.

Source: *Business Week*

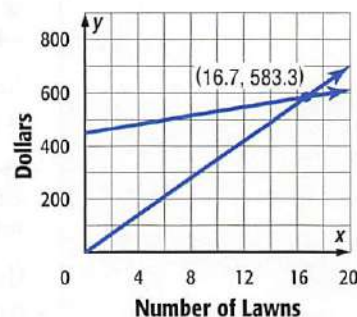
Real-World EXAMPLE 3 Break-Even Point Analysis

BUSINESS Libby borrowed \$450 to start a lawn-mowing business. She charges \$35 per lawn and incurs \$8 in operating costs per lawn. How many lawns must she mow to make a profit?

Let x = the number of lawns Libby mows, and let y = the number of dollars.

| | |
|---------------------|-------------------|
| Total Income | Total Cost |
| $y = 35x$ | $y = 8x + 450$ |

The graphs intersect at $(16.7, 583.3)$. This is the break-even point. She can only mow a whole number of lawns. If she mows 17 lawns, she will make a profit of $\$595 - \586 or $\$9$. If she mows fewer than 17 lawns, she will lose money.



Check Your Progress

- 3.** Southeast Publishing paid \$96,000 to an author for the rights to publish her book. If one book costs \$12 to produce and they plan to sell them for \$24 each, how many books must Southeast Publishing sell to break even? Show your results by graphing.

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Classify Systems of Equations Systems of equations can be classified by the number of solutions. A system of equations is **consistent** if it has at least one solution and **inconsistent** if it has no solutions. If it has exactly one solution, it is **independent**, and if it has an infinite number of solutions, it is **dependent**.

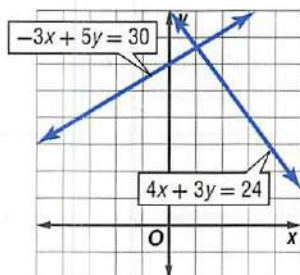
EXAMPLE 4 Classify Systems

Graph each system of equations and describe them as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

a. $4x + 3y = 24$
 $-3x + 5y = 30$

$$y = -\frac{4}{3}x + 8$$

$$y = \frac{3}{5}x + 6$$

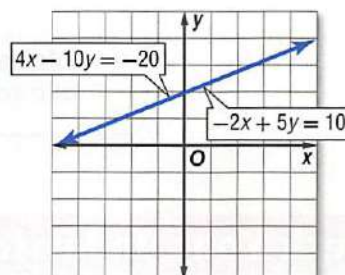


The graphs of the lines intersect at one point, so there is one solution. The system is consistent and independent.

b. $-2x + 5y = 10$
 $4x - 10y = -20$

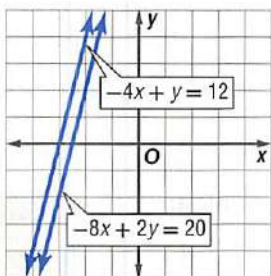
$$y = \frac{2}{5}x + 2$$

$$y = \frac{2}{5}x + 2$$



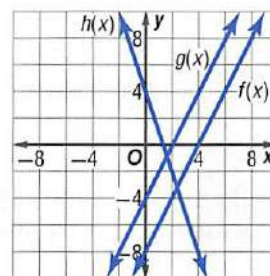
Because the equations are equivalent, their graphs are the same line. The system is consistent and dependent.

c. $-8x + 2y = 20$
 $-4x + y = 12$



The graphs of the lines do not intersect, so the graphs are parallel and there is no solution. The system is inconsistent.

d. $f(x) = 2x - 8$
 $g(x) = 2x - 4$
 $h(x) = -3x + 4$



$f(x)$ and $g(x)$ are inconsistent. $f(x)$ and $h(x)$ are consistent and independent. $g(x)$ and $h(x)$ are consistent and independent.

StudyTip

Slope and Classifying Systems If the equations have different slopes, then the system is consistent and independent.

StudyTip

Graphing Calculator Most graphing calculators have a function for determining a point of intersection of two lines.

Check Your Progress

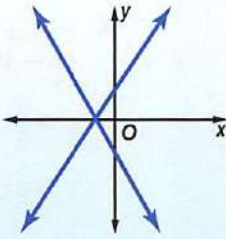
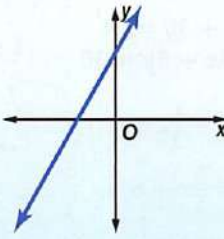
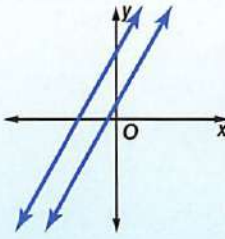
4A. $6x - 4y = 15$
 $-6x + 4y = 18$

4C. $10x - 12y = 40$
 $-5x + 6y = -20$

4B. $-4x + 5y = -17$
 $-4x - 2y = 15$

4D. $6x + y = -13$
 $6x - y = 13$

The relationship between the graph of a system of equations and the number of solutions is summarized below.

| <div> <div>Concept Summary</div> <div>Characteristics of Linear Systems</div> <div>For Your FOLDABLE</div> </div> | | |
|---|--|---|
| Consistent and Independent | Consistent and Dependent | Inconsistent |
|  |  |  |
| intersecting lines; one solution | same line; infinitely many solutions | parallel lines; no solution |

Check Your Understanding

Example 1 p. 135

Solve each system of equations by using a table.

- $y = 3x - 4$
 $y = -2x + 11$
- $4x - y = 1$
 $5x + 2y = 24$

Example 2 p. 136

Solve each system of equations by graphing.

- $y = -3x + 6$
 $2y = 10x - 36$
- $y = -x - 9$
 $3y = 5x + 5$
- $y = 0.5x + 4$
 $3y = 4x - 3$
- $-3y = 4x + 11$
 $2x + 3y = -7$
- $4x + 5y = -41$
 $3y - 5x = 5$
- $8x - y = 50$
 $x + 4y = -2$

Example 3 p. 136

- DIGITAL PHOTOS** Refer to the graphic at the right.
 - Write equations that represent the cost of printing digital photos at each lab.
 - Under what conditions is the cost to print digital photos the same at both stores?
 - When is it best to use EZ Online Digital Photos and when is it best to use the local pharmacy?

Developing Digital Photos

EZ Online Digital Photos charges

\$0.15

per digital photo

\$2.70

for shipping

Local Pharmacy charges



\$0.25

per digital photo

Example 4 p. 137

Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

- $y + 4x = 12$
 $3y = 8 - 12x$
- $-2x - 3y = 9$
 $4x + 6y = -18$
- $9x - 2y = 11$
 $5x + 4y = 13$

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Practice and Problem Solving

 **Step-by-Step Solutions** begin on page R20.
Extra Practice begins on page 947.

Example 1 p. 135

Solve each system of equations by using a table.

13. $y = 5x + 3$
 $y = x - 9$

14. $3x - 4y = 16$
 $-6x + 5y = -29$

15. $2x - 5 = y$
 $-3x + 4y = 0$

16. $x + y = 6$
 $-y + 8x = -15$

Example 2 p. 136

Solve each system of equations by graphing.

17. $4x + 3y = -24$
 $8x - 2y = -16$

18. $-3x + 2y = -6$
 $-5x + 10y = 30$

19. $-3x - 8y = 12$
 $12x + 32y = -48$

20. $6x - 5y = 17$
 $6x + 2y = 31$

21. $-10x + 4y = 7$
 $2x - 5y = 7$

22. $y - 3x = -29$
 $9x - 6y = 102$

Example 3 p. 136

23. **SHOPPING** Jerilyn has a \$10 coupon and a 15% discount coupon for her favorite store. The store has a policy that only one coupon may be used per purchase. When is it best for Jerilyn to use the \$10 coupon, and when is it best for her to use the 15% discount coupon?

24. **HABITATS** A zoo is building a new habitat for the wolves. The boundaries for the habitat are $y = 8$, $x = 4$, $y = 2x + 2$, and $y = 0.5x - 1$.

- Graph the system of equations that models the area of the new wolf enclosure.
- Find the coordinates of the vertices of the quadrilateral that represents the wolves' new habitat.
- What is the approximate area of the wolves' new home?

25. **WATER SKIING** A water ski jump is in the shape of a triangle. The graphs of $y - 2x = 1$, $4x + y = 7$, and $2y - x = -4$ contain the sides of the triangle. Find the coordinates of the vertices of the triangle.

Example 4 p. 137

Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

26. $y = 2x - 1$
 $y = 2x + 6$

27. $y = 3x - 4$
 $y = 6x - 8$

28. $x - 6y = 12$
 $3x + 18y = 14$

29. $2x + 5y = 10$
 $-4x - 10y = 20$

30. $8y - 3x = 15$
 $-16y + 6x = -30$

31. $-5x - 6y = 13$
 $12y + 10x = -26$

32. $3.2x - 2.4y = 28$
 $0.6y = -0.8x + 7$

33. $4.9x - 7.7y = -14$
 $-1.4x + 2.2y = 4$

34. $3.5x - 1.2y = 8.2$
 $-5.25x + 1.8y = 12.3$

Solve each system of equations by graphing.

35. $y - \frac{3}{4}x = -\frac{5}{2}$
 $\frac{3}{2}x + 6y = 9$

36. $5x + 2y = -\frac{5}{2}$
 $\frac{3}{2}y + \frac{1}{2}x = 3$

37. $\frac{5}{6}x + \frac{4}{3}y = -1$
 $\frac{1}{4}x + \frac{5}{3}y = \frac{7}{2}$

Use a graphing calculator to solve each system of equations. Round the coordinates of the intersection to the nearest hundredth.

38. $12y = 5x - 15$
 $4.2y + 6.1x = 11$

39. $5.8x - 6.3y = 18$
 $-4.3x + 8.8y = 32$

40. $-3.8x + 2.9y = 19$
 $6.6x - 5.4y = -23$



Real-World Link

Track and Field runners Paavo Nurmi (Finland) and Carl Lewis (U.S.) along with gymnast Larisa Latynina (former Soviet Union) currently hold the record for the most Olympic gold medals with 9 each. Latynina also holds the record for the most Olympic medals with 18.

Source: NBC Sports

- 41 OLYMPICS** The table shows the winning times in seconds for the 100-meter dash at the Olympics between 1964 and 2004.

| Years Since 1964, x | Men's Gold Medal Time | Women's Gold Medal Time |
|-----------------------|-----------------------|-------------------------|
| 0 | 10.0 | 11.4 |
| 4 | 9.90 | 11.0 |
| 8 | 10.14 | 11.07 |
| 12 | 10.06 | 11.08 |
| 16 | 10.25 | 11.06 |
| 20 | 9.99 | 10.97 |
| 24 | 9.92 | 10.54 |
| 28 | 9.96 | 10.82 |
| 32 | 9.84 | 10.94 |
| 36 | 9.87 | 10.75 |
| 40 | 9.85 | 10.93 |

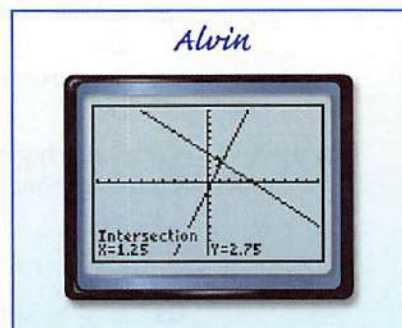
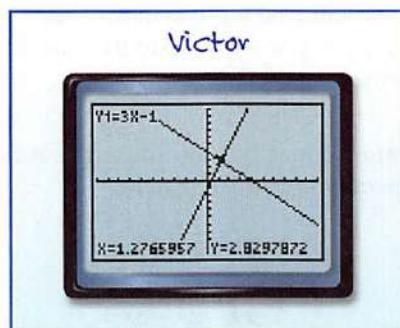


- Write equations that represent the winning times for men and women since 1964. Assume that both times continue along the same trend.
- Graph both equations. Estimate when the women's performance will catch up to the men's performance.
- Do you think that your prediction is reasonable? Explain.

H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

- REASONING** If a is consistent and dependent with b , b is inconsistent with c , and c is consistent and independent with d , then a will *sometimes*, *always*, or *never* be consistent and independent with d . Explain your reasoning.
- FIND THE ERROR** Victor and Alvin are using their calculators to solve the system $y = 3x - 1$ and $x + y = 4$. Is either of them correct? Explain your reasoning.



- CHALLENGE** Consider the system of equations $kx + 3y = 6$ and $8x + 5y = k$. Find a value of k , if it exists, for each condition.
 - The system is inconsistent.
 - The system is consistent and independent.
 - The system is consistent and dependent.
- OPEN ENDED** Write a system to illustrate each of the following.
 - an inconsistent system
 - a consistent and dependent system
 - a consistent and independent system
- WRITING IN MATH** Explain how you can determine the consistency and dependence of a system without graphing the system.

Standardized Test Practice

47. **SHORT RESPONSE** Simplify $3y(4x + 6y - 5)$.

48. **ACT/SAT** Which of the following best describes the graph of the equations?

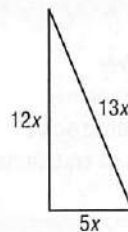
$$4y = 3x + 8$$

$$-6x = -8y + 24$$

- A The lines are parallel.
- B The lines are perpendicular.
- C The lines have the same x -intercept.
- D The lines have the same y -intercept.

49. **GEOMETRY** Which set of dimensions corresponds to a triangle similar to the one shown at the right?

- F 1 unit, 2 units, 3 units
- G 7 units, 11 units, 12 units
- H 10 units, 23 units, 24 units
- J 20 units, 48 units, 52 units



50. Move-A-Lot Rentals will rent a moving truck for \$100 plus \$0.10 for every mile it is driven. Which equation can be used to find C , the cost of renting a moving truck and driving it for m miles?

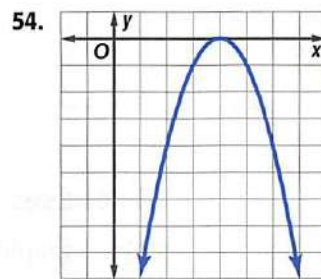
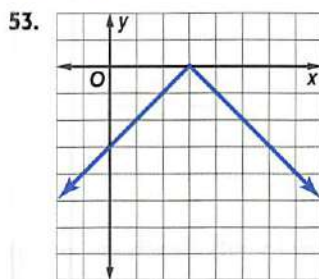
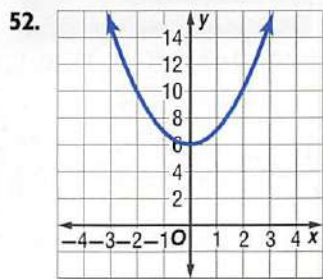
- A $C = 0.1(100 + m)$
- B $C = 100 + 0.1m$
- C $C = 100m + 0.1$
- D $C = 100(m + 0.1)$

Spiral Review

51. **CRAFTS** Priscilla sells stuffed animals at a local craft show. She charges \$10 for the small ones and \$15 for the large. To cover her expenses, she needs to sell at least \$350 worth of animals. (Lesson 2-8)

- a. Write an inequality for this situation.
- b. Graph the inequality.
- c. If she sells 10 small and 15 large animals, will she cover her expenses?

Write an equation for each function. (Lesson 2-7)



Solve each equation. Check your solution. (Lesson 1-3)

55. $2p = 14$

56. $-14 + n = -6$

57. $7a - 3a + 2a - a = 16$

58. $x + 9x - 6x + 4x = 20$

59. $27 = -9(y + 5) + 6(y + 8)$

60. $-7(p + 7) + 3(p - 4) = -17$

Skills Review

Simplify each expression. (Lesson 1-2)

61. $(2x + 4) - (3x + 2)$

62. $(y - 10) + (5y + 2)$

63. $(4x - 2y) + (-6x + 5y)$

64. $3(6x + 2y - 1)$

65. $4(5x + 2y - 2x + 8)$

66. $6(x + 4y) - 5(x + 9y)$

Graphing Technology Lab

Systems of Equations

Objective

Use a graphing calculator to solve systems of equations.

You can use a TI-83/84 Plus graphing calculator to solve systems of equations. You can use the $Y=$ menu to graph each equation on the same set of axes.

EXAMPLE Intersection of Two Graphs

Graph the system of equations in the standard viewing window.

$$3x + y = 9$$

$$x - y = -1$$

Step 1 Write each equation in the form $y = mx + b$.

$$3x + y = 9$$

$$x - y = -1$$

$$y = -3x + 9$$

$$-y = -x - 1$$

$$y = x + 1$$

Step 2 Enter $y = -3x + 9$ as Y1 and $y = x + 1$ as Y2. Then graph the lines.

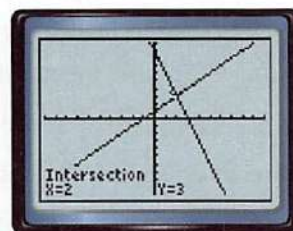
KEYSTROKES: $Y=$ $(-)$ 3 X,T,θ,n $+$ 9 ENTER

X,T,θ,n $+$ 1 ENTER ZOOM 6

Step 3 Find the intersection of the lines.

KEYSTROKES: 2nd $[\text{CALC}]$ 5 ENTER ENTER ENTER

The solution is (2, 3).



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

Exercises

Use a graphing calculator to solve each system of equations.

1. $2x + 4y = 36$
 $10y - 5x = 0$

3. $4x - 2y = 16$
 $7x + 3y = 15$

5. $5x + y = 13$
 $3x = 15 - 3y$

7. $\frac{1}{4}x + y = \frac{11}{4}$
 $x - \frac{1}{2}y = 2$

9. $3x - 6y = 6$
 $2x - 4y = 4$

2. $2y - 3x = 7$
 $5x = 4y - 12$

4. $2x + 4y = 4$
 $x + 3y = 13$

6. $4y - 5 = 20 - 3x$
 $4x - 7y + 16 = 0$

8. $3x + 2y = -3$
 $x + \frac{1}{3}y = -4$

10. $6x + 8y = -16$
 $3x + 4y = 12$

3-2

Solving Systems of Equations Algebraically

Then

You solved systems of linear equations by using tables and graphs. (Lesson 3-1)

Now

- Solve systems of linear equations by using substitution.
- Solve systems of linear equations by using elimination.

New Vocabulary

substitution method
elimination method

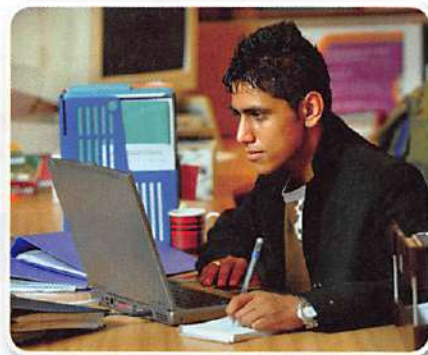
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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Why?

Alejandro has a computer support business. He estimates that the cost to run his business can be represented by $y = 48x + 500$, where x is the number of customers. He also estimates that his income can be represented by $y = 65x - 145$. A system of equations can be used to determine the number of customers he will need in order to break even.



Substitution Algebraic methods are used to find exact solutions of systems of equations. One algebraic method is called the **substitution method**.

Key Concept

Substitution Method

For Your
FOLDABLE

- Step 1** Solve one equation for one of the variables.
- Step 2** Substitute the resulting expression into the other equation to replace the variable. Then solve the equation.
- Step 3** Substitute to solve for the other variable.

Real-World EXAMPLE 1 Use the Substitution Method

BUSINESS How many customers will Alejandro need in order to break even? What will his profit be if he has 60 customers?

Understand Alejandro wants to know how many customers he needs for his income to equal his costs. He also wants to know what his profit will be if he has 60 customers.

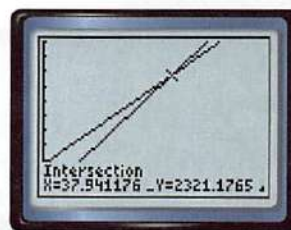
Plan Solve the system of equations. Income: $y = 65x - 145$
Cost: $y = 48x + 500$

Solve

| | |
|-------------------------|--|
| $y = 65x - 145$ | Income equation |
| $48x + 500 = 65x - 145$ | Substitute $48x + 500$ for y. |
| $500 = 17x - 145$ | Subtract $48x$ from each side. |
| $645 = 17x$ | Add 145 to each side. |
| $37.9 \approx x$ | Divide each side by 17. |

Alejandro needs 38 customers to break even. If he has 60 customers, his income will be $65(60) - 145$ or \$3755, and his costs will be $48(60) + 500$ or \$3380, so his profit will be $3755 - 3380$ or \$375.

Check You can use a graphing calculator to check this solution. The break-even point is near (37.9, 2321.2). Use the CALC function to find the cost and income with 60 customers.



[0, 65] scl: 5 by [0, 3000] scl: 300

Check Your Progress

Use substitution to solve each system of equations.

1A. $5x - 3y = 23$
 $2x + y = 7$

1B. $x - 7y = 11$
 $5x + 4y = -23$

1C. $-6x - y = 27$
 $3x + 8y = 9$

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Elimination You can use the **elimination method** to solve a system when one of the variables has the same coefficient in both equations.

Key Concept

Elimination Method

For Your
FOLDABLE

Step 1 Multiply one or both equations by a number to result in two equations that contain opposite terms.

Step 2 Add the equations, eliminating one variable. Then solve the equation.

Step 3 Substitute to solve for the other variable.

Variables can be eliminated by addition or subtraction.

EXAMPLE 2 Solve by Using Elimination

Use the elimination method to solve the system of equations.

$$\begin{aligned} 5x + 3y &= -19 \\ 8x + 3y &= -25 \end{aligned}$$

Notice that solving by substitution would involve fractions.

Step 1 Multiply one equation by -1 so the equations contain $3y$ and $-3y$.

$$8x + 3y = -25$$

Multiply by -1 .

$$-8x - 3y = 25$$

Step 2 Add the equations to eliminate one variable.

$$\begin{array}{r} 5x + 3y = -19 \\ (+) -8x - 3y = 25 \\ \hline -3x = 6 \\ x = -2 \end{array}$$

Equation 1

Equation 2 $\times (-1)$

Add the equations.

Divide each side by -3 .

Step 3 Substitute -2 for x into either original equation.

$$\begin{aligned} 8x + 3y &= -25 \\ 8(-2) + 3y &= -25 \\ -16 + 3y &= -25 \\ 3y &= -9 \\ y &= -3 \end{aligned}$$

Equation 2

$x = -2$

Multiply.

Add 16 to each side.

Divide each side by 3.

The solution is $(-2, -3)$.

StudyTip

Elimination

Remember when you add or subtract one equation from another to add or subtract every term, including the constant on the other side of the equal sign.

Review Vocabulary

Least Common Multiple

the least number that is a common multiple of two or more numbers

Check Your Progress

2A. $4x - 3y = -22$
 $2x + 3y = 16$

2B. $6x - 5y = -8$
 $4x - 5y = -12$

2C. $2x - 9y = 34$
 $-2x + 6y = -28$

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Sometimes, adding or subtracting equations will not eliminate either variable. You can use multiplication and least common multiples to find a common coefficient.

STANDARDIZED TEST EXAMPLE 3

Solve the system of equations.

$$5x + 3y = 52$$

$$9x - 4y = 56$$

A (4, 1)

B (8, 4)

C (8, 0)

D (12, 3)

Read the Test Item

You are given a system of two linear equations and are asked to find the solution.

Solve the Test Item

Neither variable has a common coefficient. The coefficients of the y -variables are 3 and 4 and their least common multiple is 12, so multiply each equation by the value that will make the y -coefficient 12.

$$5x + 3y = 52 \quad \text{Multiply by 4.} \quad 20x + 12y = 208$$

$$9x - 4y = 56 \quad \text{Multiply by 3.} \quad \begin{array}{r} (+) 27x - 12y = 168 \\ \hline 47x \qquad \qquad = 376 \\ x = 8 \end{array}$$

Add the equations.
Divide each side by 47.

Solve for y by substituting $x = 8$ into either of the original equations.

$$5x + 3y = 52$$

$$5(8) + 3y = 52$$

$$40 + 3y = 52$$

$$3y = 12$$

$$y = 4$$

Equation 1

Replace x with 8.

Multiply.

Subtract 40 from each side.

Divide each side by 3.

The correct answer is B.

Check Your Progress

3. Solve the system of equations.

$$6a - 5b = -62$$

$$8a + 7b = 54$$

F (4, 6)

G (2, -12)

H (0, -8)

J (-2, 10)

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StudyTip

Adding and Subtracting Equations

If you add or subtract two equations in a system and the result is an equation that is never true, then the system is inconsistent. When you add or subtract two equations in a system and the result is an equation that is always true, then the system is dependent.

StudyTip

Subtraction When a variable has identical coefficients in two equations, it can be eliminated by subtraction.

EXAMPLE 4 No Solution and Infinite Solutions

Use the elimination method to solve each system of equations.

a. $5x + 6y = 45$

$$-5x - 6y = 38$$

$$\begin{array}{r} 5x + 6y = 45 \\ (+) -5x - 6y = 38 \\ \hline 0 = 83 \end{array}$$

Because $0 = 83$ is not true, this system has no solutions.

b. $2x + 3y = 5$

$$6x + 9y = 15$$

$$\begin{array}{r} 3(2x + 3y = 5) = 6x + 9y = 15 \\ 6x + 9y = 15 \\ (-) 6x + 9y = 15 \\ \hline 0 = 0 \end{array} \quad \text{Multiply by 3.}$$

Because the equation $0 = 0$ is always true, there are an infinite number of solutions.

Check Your Progress

4A. $9a - 7b = 14$

$$-18a + 14b = -28$$

4B. $-6c + 12d = 81$

$$-5c + 10d = -61$$

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Math History Link

Nina Karlovna Bari
(1901–1961)

Russian mathematician Nina Karlovna Bari was considered the principal leader of mathematics at Moscow State University, shown above. She is best known for her textbooks *Higher Algebra* and *The Theory of Series*.

The following summarizes the various methods for solving systems.

Concept Summary

Solving Systems of Equations

For Your
FOLDABLE

| Method | The Best Time to Use |
|---|--|
| Table | to estimate the solution, since a table may not provide an exact solution |
| Graphing | to estimate the solution, since graphing usually does not give an exact solution |
| Substitution | if one of the variables in either equation has a coefficient of 1 or -1 |
| Elimination Using Addition | if one of the variables has opposite coefficients in the two equations |
| Elimination Using Subtraction | if one of the variables has the same coefficient in the two equations |
| Elimination Using Multiplication | if none of the coefficients are 1 or -1 and neither of the variables can be eliminated by simply adding or subtracting the equations |

Check Your Understanding

Example 1 p. 143

1. **FUNDRAISER** To raise money for new uniforms, the band boosters sell T-shirts and hats. The cost and sale price of each item is shown. The boosters spend a total of \$2000 on T-shirts and hats. They sell all of the merchandise, and make \$3375. How many T-shirts did they sell?

| Item | Cost | Sale Price |
|---------|------|------------|
| T-Shirt | \$6 | \$10 |
| Hat | \$4 | \$7 |

Solve each system of equations by using substitution.

2. $y = 2x - 10$
 $y = -4x + 8$
3. $x + 5y = 3$
 $3x - 2y = -8$
4. $a - 3b = -22$
 $4a + 2b = -4$
5. $2a + 8b = -8$
 $3a - 5b = 22$
6. $9c - 3d = -33$
 $6c + 5d = -8$
7. $6x - 7y = 23$
 $8x + 4y = 44$

Examples 2–4 pp. 144–145

Solve each system of equations by using elimination.

8. $4x - 3y = 29$
 $4x + 3y = 35$
9. $-6w - 8z = -44$
 $3w + 6z = 36$
10. $8a - 3b = -11$
 $5a + 2b = -3$
11. $3a + 5b = -27$
 $4a + 10b = -46$
12. $6x - 4y = 30$
 $12x + 5y = -18$
13. $5a + 15b = -24$
 $-2a - 6b = 28$

14. **MULTIPLE CHOICE** What is the solution of the linear system?

$$\begin{aligned} 4x + 3y &= 2 \\ 4x - 2y &= 12 \end{aligned}$$

A (8, -10)

B (2, -2)

C (-10, 14)

D no solution

Practice and Problem Solving

Step-by-Step Solutions begin on page R20.
Extra Practice begins on page 947.

Example 1

p. 143

Solve each system of equations by using substitution.

15. $-4r - 3t = -26$
 $5r + t = 27$

16. $6u + 3v = -15$
 $8u - 5v = 7$

17. $4x + 12y = 0$
 $-3x - 4y = -10$

18. $c - 5d = -16$
 $3c + 2d = -14$

19. $-5p - t = 17$
 $4p + 6t = 2$

20. $-6y + 5z = -35$
 $7y - z = 36$

21. $9y + 3x = 18$
 $-3y - x = -6$

22. $5x - 20y = 70$
 $6x + 5y = -32$

23. $-4x - 16y = -96$
 $7x + 3y = 68$

24. $-4a - 5b = 14$
 $9a + 3b = -48$

25. $-9c - 4d = 31$
 $6c + 6d = -24$

26. $8f + 3g = 12$
 $-32f - 12g = 48$

27. **TENNIS** At a park, there are 38 people playing tennis. Some are playing doubles, and some are playing singles. There are 13 matches in progress. A doubles match requires 4 players, and a singles match requires 2 players.

- Write a system of two equations that represents the number of singles and doubles matches going on.
- How many matches of each kind are in progress?

Examples 2–4

pp. 144–145



Solve each system of equations by using elimination.

28. $8x + y = 27$
 $-3x + 4y = 3$

29. $2a - 5b = -20$
 $2a + 5b = 20$

30. $6j + 4k = -46$
 $2j + 4k = -26$

31. $3x - 8y = 24$
 $-12x + 32y = 96$

32. $5a - 2b = -19$
 $8a + 5b = -55$

33. $r - 6t = 44$
 $9r + 12t = 0$

34. $6d + 5f = -32$
 $5d - 9f = 26$

35. $11u = 5v + 35$
 $8v = -6u + 62$

36. $-1.2c + 3.4d = 6$
 $6c = -30 + 17d$

37. $6g + 8h = 16$
 $-4g - 7h = -19$

38. $15j - 3k = 129$
 $-6j + 8k = -72$

39. $-4m - 9n = 30$
 $10m + 3n = 42$

Real-World Link

The Wimbledon Championship is broadcast to over 170 countries with an audience of over 1 billion people.

Source: *Tennis Magazine*

- The sum of four times a number and six times a second number is 36. The difference of five times the second number and three times the first number is 49. Find the numbers.
- Twice the sum of a number and 3 times a second number is 4. The difference of ten times the second number and five times the first is 90. Find the numbers.
- Three times the difference of four times a number and three times a second number is 117. Four times the sum of 6 times the second number and 8 times the first number is 72. Find the numbers.

43. **CYCLING** The total distance of the cycling course is 104.8 miles. Julian starts the course at 8:00 A.M. and rides at 12 miles an hour. Peter starts two hours later than Julian but decides to try to catch up with him. Peter rides at a speed of 16 miles an hour.

- Solve the system of equations to find when Peter will catch up to Julian.
- Peter wants to reduce the time it takes him to catch up to Julian by 1 hour. Explain how he could do this by changing his starting time. Explain how he could do this by changing his speed. Are your answers reasonable?

Solve each system of equations.

44. $11p + 3q = 6$
 $-0.75q - 2.75p = -1.5$

45. $8r - 5t = -60$
 $6r + 3t = -18$

46. $10t + 4v = 13$
 $-4t - 7v = 11$

47. $6w = 12 - 4x$
 $6x = -9w + 18$

48. $\frac{3}{2}y + z = 3$
 $-y - \frac{2}{3}z = -2$

49. $\frac{5}{2}a - \frac{3}{4}b = 46$
 $-\frac{7}{8}a - 3b = 10$

50. **ROWING** Allison can row a boat 1 mile upstream (against the current) in 24 minutes. She can row the same distance downstream in 13 minutes. Assume that both the rowing speed and the speed of the current are constant.

- Find the speed at which Allison is rowing and the speed of the current.
- If Allison plans to meet her friends 3 miles upstream one hour from now, will she be on time? Explain.

51. **EARTH WEEK** To reduce waste, The Green Café offers a reduced refill rate on coffee for anyone buying a Green mug. The mug costs \$2.95 and is filled with 16 ounces of coffee. The refill price is \$0.50. A 16-ounce coffee in a disposable cup costs \$0.85.

- What is the approximate break-even point for buying the mug and refills in comparison to buying coffee in disposable cups? What does this mean?
- Which offer do you think is better? Explain your reasoning.
- How would your decision change if the refillable mug offer was extended for a year?

52. **SKATING PARTY** Anita invites 21 friends to the skating rink for a birthday party. The rink rents roller skates for \$3 and inline skates for \$5. The total rental bill for all 22 students is \$96.

- Write a system of equations that represents the number of students who rented the two types of skates.
- How many students rented roller skates and how many rented inline skates?

53. **GEOMETRY** Angles A and B are supplementary and the measure of angle A is 18 degrees greater than the measure of angle B . Find the angle measures.

54. **JOBS** Levi has a job offer in which he will receive \$800 per month plus a commission of 2% of the total price of cars he sells. At his current job, he receives \$1200 per month plus a commission of 1.5% of his total sales. How much must he sell per month to make the new job a better deal?

55. **TRAVEL** A youth group went on a trip to an amusement park, travelling in two vans. The number of people in each van and the total cost of admission are shown in the table. Find the adult price and student price of admission.

| | Adults | Students | Total Cost |
|--------------|--------|----------|------------|
| Van A | 2 | 5 | \$77 |
| Van B | 2 | 7 | \$95 |



Solve each system of equations.

56. $4.1x - 3.4y = 19.97$
 $6.3x + 2.2y = 7.67$

58. $3.65x + 6.83y = -34.526$
 $-41.3x - 2.68y = 77.336$

57. $3.7x - 4.6y = 15.37$
 $-5.1x - 2.8y = 40.97$

59. $-6.79a + 3.29b = -43.792$
 $-9.14a - 6.28b = 10.658$



Real-World Link

Earth Day began in San Francisco in 1969 and is now observed by over 500 million people worldwide.

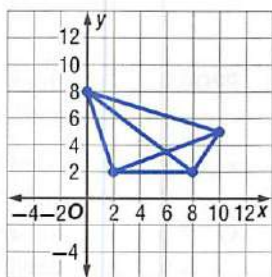
Source: Earth Week
 Charlottesville

StudyTip**Reasonable Answers**

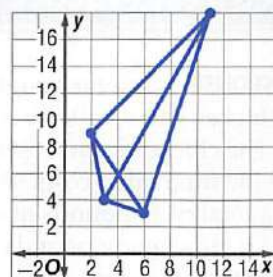
For Exercises 60 and 61, you can use the graphs to determine whether your answers are reasonable.

GEOMETRY Find the point at which the diagonals of the quadrilaterals intersect.

60.



61.



62. **ELECTIONS** In the election for student council, Candidate A received 55% of the total votes, while Candidate B received 1541 votes. If Candidate C received 40% of the votes that Candidate A received, how many total votes were cast?

63. **MULTIPLE REPRESENTATIONS** In this problem, you will explore systems of equations with three lines and two variables.

$$3y + x = 16$$

$$y - 2x = -4$$

$$y + 5x = 10$$

- TABULAR** Make a table of x - and y -values for each equation.
- ANALYTICAL** Which values from the table indicate intersections? Is there a solution that satisfies all three equations?
- GRAPHICAL** Graph the three equations on a single coordinate plane.
- VERBAL** What conditions must be met for a system of three equations with two variables to have a solution? What conditions result in no solution?

H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

64. **FIND THE ERROR** Gloria and Syreeta are solving the system $6x - 4y = 26$ and $-3x + 4y = -17$. Is either of them correct? Explain your reasoning.

Gloria

$$\begin{array}{rcl} 6x - 4y = 26 & 6(3) - 4y = 26 & \\ -3x + 4y = -17 & 18 - 4y = 26 & \\ \hline 3x = 9 & -4y = 8 & \\ x = 3 & y = -2 & \\ \text{The solution is } (3, -2). & & \end{array}$$

Syreeta

$$\begin{array}{rcl} 6x - 4y = 26 & 6(-3) - 4y = 26 & \\ -3x + 4y = -17 & -18 - 4y = 26 & \\ \hline 3x = -9 & -4y = 44 & \\ x = -3 & y = -11 & \\ \text{The solution is } (-3, -11). & & \end{array}$$

65. **CHALLENGE** Find values of a and b for which the following system has a solution of $(b - 1, b - 2)$.

$$-8ax + 4ay = -12a$$

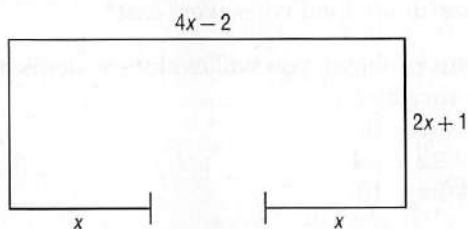
$$2bx - by = 9$$

- REASONING** Katie says that if the coefficients of each variable are identical, then she does not need to solve the system since it will always have infinite solutions. Is she correct? Explain your reasoning.
- OPEN ENDED** Write a system of equations in which one equation needs to be multiplied by 3 and the other needs to be multiplied by 4 in order to solve the system with elimination. Then solve your system.
- WRITING IN MATH** Explain why substitution is sometimes more helpful than elimination and why elimination is sometimes more helpful than substitution.

Standardized Test Practice

69. **GRIDDED RESPONSE** A caterer bought several pounds of chicken salad and several pounds of tuna salad. The chicken salad costs \$9 per pound, and the tuna salad costs \$6 per pound. He bought a total of 14 pounds of salad and paid a total of \$111. How many pounds of chicken salad did he buy?

70. A rectangular room is shown below. Which expression represents the width of the door?



- A $(4x - 2) - x - x$
 B $(2x + 1) - x - x$
 C $(4x - 2) - (2x + 1)$
 D $(4x - 2) + x + x$

71. **PROBABILITY** Which of the following is an example of dependent events?

- F rolling a 6-sided die twice and getting different numbers
 G choosing two cards from a stack of colored cards, with replacement, and both cards are red
 H flipping a coin twice and getting heads both times
 J choosing the starting line-up for a football game

72. **ACT/SAT** Peni bought a basketball and a volleyball that cost a total of \$67. If the price of the basketball b is \$4 more than twice the cost of the volleyball v , which system of linear equations could be used to determine the cost of each ball?

- | | |
|----------------|---------------|
| A $b + v = 67$ | C $b + v = 4$ |
| $b = 2v - 4$ | $b = 2v - 67$ |
| B $b + v = 67$ | D $b + v = 4$ |
| $b = 2v + 4$ | $b = 2v + 67$ |

Spiral Review

73. **EXERCISE** Refer to the graphic. (Lesson 3-1)

- For each option, write an equation that represents the cost of belonging to the gym.
- Graph the equations. Estimate the break-even point for the gym memberships.
- Explain what the break-even point means.
- If you plan to visit the gym at least once per week during the year, which option should you choose?

| OPTION 1: | OPTION 2: |
|------------------|---------------|
| \$400/yr | \$150/yr |
| Unlimited visits | \$5 per visit |

Graph each inequality. (Lesson 2-8)

74. $x + y \geq 6$

75. $4x - 3y < 10$

76. $5x + 7y \geq -20$

Write an equation of the line passing through each pair of points. (Lesson 2-4)

77. $(3, 5), (7, -3)$

78. $(8, -2), (4, 8)$

79. $(-6, -1), (-9, 11)$

80. $(-4, -4), (12, -8)$

Skills Review

Determine whether the given point satisfies each inequality. (Lesson 2-8)

81. $4x + 5y \leq 15; (2, -2)$

82. $3x + 5y \geq 8; (1, 1)$

83. $6x + 9y < -1; (0, 0)$

3-3

Solving Systems of Inequalities by Graphing

Then

You solved systems of linear equations using tables and graphs. (Lesson 3-1)

Now

- Solve systems of inequalities by graphing.
- Determine the coordinates of the vertices of a region formed by the graph of a system of inequalities.

New Vocabulary

system of inequalities

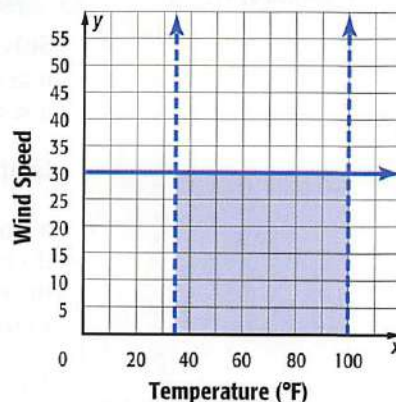
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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help
- Math in Motion

Why?

Many weather conditions need to be met before a space shuttle can launch. The temperature must be greater than 35°F and less than 100°F , and the wind speed cannot exceed 30 knots. These three conditions can be represented by the system of inequalities shown at the right.



Systems of Inequalities Solving a **system of inequalities** means finding the ordered pairs that satisfy all of the inequalities in the system.

Key Concept

Solving Systems of Inequalities

For Your

FOLDABLE

Step 1 Graph each inequality, shading the correct area.

Step 2 Identify the region that is shaded for all of the inequalities. This is the solution of the system.

Math in Motion, Animation glencoe.com

EXAMPLE 1 Intersecting Regions

Solve the system of inequalities.

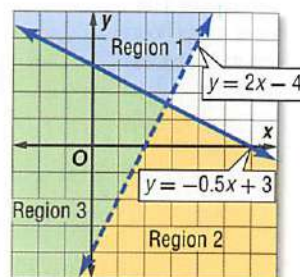
$$y > 2x - 4$$

$$y \leq -0.5x + 3$$

Solution of $y > 2x - 4 \rightarrow$ Regions 1 and 3

Solution of $y \leq -0.5x + 3 \rightarrow$ Regions 2 and 3

Region 3 is part of the solution of both inequalities, so it is the solution of the system.



CHECK

Notice that the origin is part of the solution of the system. The origin can be used as a test point. You can test the solution by substituting $(0, 0)$ for x and y in each equation.

$$y > 2x - 4$$

$$0 \stackrel{?}{>} 2(0) - 4$$

$$0 \stackrel{?}{>} 0 - 4$$

$$0 > -4 \quad \checkmark$$

$$y \leq -0.5x + 3$$

$$0 \leq -0.5(0) + 3$$

$$0 \leq 0 + 3$$

$$0 \leq 3 \quad \checkmark$$

Check Your Progress

1A. $y \leq -2x + 5$

$$y > -\frac{1}{4}x - 6$$

1B. $y \geq |x|$

$$y < \frac{4}{3}x + 5$$

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ReadingMath

Empty Set The empty set is also called the *null set*. It can be represented by \emptyset or $\{\}$.

It is possible that the regions do not intersect. When this occurs, there is no solution of the system or the solution set is the *empty set*.

EXAMPLE 2 Separate Regions

Solve the system of inequalities by graphing.

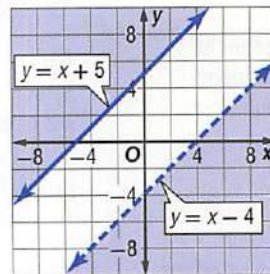
$$y \geq x + 5$$

$$y < x - 4$$

Graph both inequalities.

Since the graphs of the inequalities do not overlap, there are no points in common and there is no solution to the system.

The solution set is the empty set.



Check Your Progress

2A. $y \geq -4x + 8$
 $y < -4x + 4$

2B. $y \geq |x|$
 $y < 2x - 24$

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Real-World Link

Typical incoming freshmen will spend more than 3 times as many hours studying in college as in high school.

Source: National Survey of Student Engagement

Real-World EXAMPLE 3 Write and Use a System of Inequalities

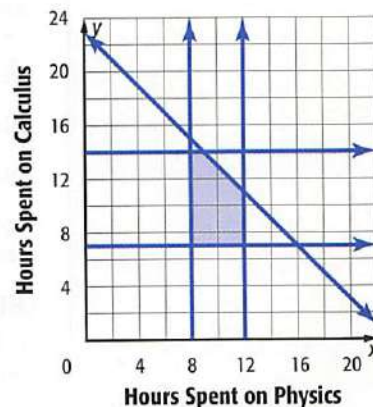
TIME MANAGEMENT Chelsea has final exams in calculus, physics, and history. She has up to 25 hours to study for the exams. She plans to study history for 2 hours. She needs to spend at least 7 hours studying for calculus, but over 14 is too much. She hopes to spend between 8 and 12 hours on physics. Write and graph a system of inequalities to represent the situation.

Calculus: at least 7 hours, but no more than 14
 $7 \leq c \leq 14$

Physics: at least 8 hours, but no more than 12
 $8 \leq p \leq 12$

Chelsea has 25 hours available, and 2 of those will be spent on history. She has up to 23 hours left for calculus and physics.
 $c + p \leq 23$

Graph all of the inequalities. Any ordered pair in the intersection is a solution of the system. One solution is 10 hours on physics and 12 hours on calculus.



Check Your Progress

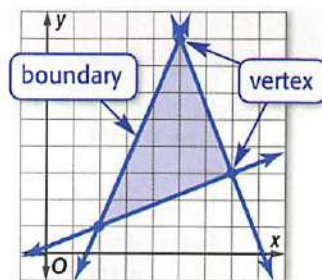
3. **TRAVEL** Mr. and Mrs. Rodriguez are driving across the country with their two children. They plan on driving a maximum of 10 hours each day. Mr. Rodriguez wants to drive at least 4 hours a day but no more than 8 hours a day. Mrs. Rodriguez can drive in between 2 and 5 hours per day. Write and graph a system of inequalities that represents this information.

Personal Tutor glencoe.com

StudyTip

Boundaries If the inequality that forms the boundary is $<$ or $>$, then the boundary is not included in the solution, and the line should be dashed.

Find Vertices of an Enclosed Region Sometimes the graph of a system of inequalities produces an enclosed region in the form of a polygon. To find the vertices of the region, determine the coordinates of the points at which the boundaries intersect.



EXAMPLE 4 Find Vertices

Find the coordinates of the vertices of the triangle formed by $y \geq 2x - 8$, $y \leq -\frac{1}{4}x + 6$, and $4y \geq -15x - 32$.

Step 1 Graph each inequality.
The coordinates $(-4, 7)$ and $(0, -8)$ can be determined from the graph. To find the coordinates of the third vertex, solve the system of equations $y = 2x - 8$ and $y = -\frac{1}{4}x + 6$.

Step 2 Substitute for y in the second equation.

$$2x - 8 = -\frac{1}{4}x + 6 \quad \text{Replace } y \text{ with } 2x - 8.$$

$$2x = -\frac{1}{4}x + 14 \quad \text{Add 8 to each side.}$$

$$\frac{9}{4}x = 14 \quad \text{Add } \frac{1}{4}x \text{ to each side.}$$

$$x = \frac{56}{9} \text{ or } 6\frac{2}{9} \quad \text{Divide each side by } \frac{9}{4}.$$

Step 3 Find y .

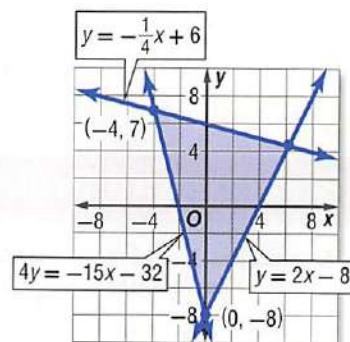
$$y = 2\left(6\frac{2}{9}\right) - 8 \quad \text{Replace } x \text{ with } 6\frac{2}{9}.$$

$$= 12\frac{4}{9} - 8 \quad \text{Distributive Property}$$

$$= 4\frac{4}{9} \quad \text{Simplify.}$$

CHECK Compare the coordinates to the coordinates on the graph. The x -coordinate of the third vertex is between 6 and 7, so $6\frac{2}{9}$ is reasonable. The y -coordinate of the third vertex is between 4 and 5, so $4\frac{4}{9}$ is reasonable.

The vertices of the triangle are at $(-4, 7)$, $(0, -8)$, and $\left(6\frac{2}{9}, 4\frac{4}{9}\right)$.



Check Your Progress

Find the coordinates of the vertices of the triangle formed by each system of inequalities.

4A. $y \geq -3x - 6$
 $2y \geq x - 16$
 $11y + 7x \leq 12$

4B. $5y \leq 2x + 9$
 $y \leq -x + 6$
 $9y \geq -2x + 5$

Check Your Understanding

Examples 1 and 2 pp. 151–152

Solve each system of inequalities by graphing.

$$\begin{aligned} 1. \quad & y \leq 6 \\ & y > -3 + x \end{aligned}$$

$$\begin{aligned} 2. \quad & y \leq -3x + 4 \\ & y \geq 2x - 1 \end{aligned}$$

$$\begin{aligned} 3. \quad & y > -2x + 4 \\ & y \leq -3x - 3 \end{aligned}$$

Example 3 p. 152

4. **COOKOUT** The most Kala can spend on hot dogs and buns for her cookout is \$35. A package of 10 hot dogs costs \$3.50. A package of buns costs \$2.50 and contains 8 buns. She needs to buy at least 40 hot dogs and 40 buns.

- Graph the region that shows how many packages of each item she can purchase.
- Give an example of three different purchases she can make.

Example 4 p. 153

Find the coordinates of the vertices of the triangle formed by each system of inequalities.

$$\begin{aligned} 5. \quad & y \geq 2x + 1 \\ & y \leq 8 \\ & 4x + 3y \geq 8 \end{aligned}$$

$$\begin{aligned} 6. \quad & y \geq -2x - 4 \\ & 6y \leq x + 28 \\ & y \geq 13x - 34 \end{aligned}$$

Practice and Problem Solving

 = **Step-by-Step Solutions** begin on page R20.
Extra Practice begins on page 947.

Examples 1 and 2 pp. 151–152

Solve each system of inequalities by graphing.

$$\begin{aligned} 7. \quad & x < 3 \\ & y \geq -4 \end{aligned}$$

$$\begin{aligned} 8. \quad & y > 3x - 5 \\ & y \leq 4 \end{aligned}$$

$$\begin{aligned} 9. \quad & y < -3x + 4 \\ & 3y + x > -6 \end{aligned}$$

$$\begin{aligned} 10. \quad & y \geq 0 \\ & y < x \end{aligned}$$

$$\begin{aligned} 11. \quad & 6x - 2y \geq 12 \\ & 3x + 4y > 12 \end{aligned}$$

$$\begin{aligned} 12. \quad & -8x > -2y - 1 \\ & -4y \geq 2x - 5 \end{aligned}$$

$$\begin{aligned} 13. \quad & 5y < 2x + 10 \\ & y - 4x > 8 \end{aligned}$$

$$\begin{aligned} 14. \quad & 3y - 2x \leq -24 \\ & y \geq \frac{2}{3}x - 1 \end{aligned}$$

$$\begin{aligned} 15. \quad & y > -\frac{2}{5}x + 2 \\ & 5y \leq -2x - 15 \end{aligned}$$

Example 3 p. 152

16. **RECORDING** Jane's band wants to spend no more than \$575 recording their first CD. The studio charges at least \$35 an hour to record. Graph a system of inequalities to represent this situation.

17. **SUMMER TRIP** Rondell has to save at least \$925 to go to Rome with his Latin class in 8 weeks. He earns \$9 an hour working at the Pizza Palace and \$12 an hour working at a car wash. By law, he cannot work more than 25 hours per week. Graph two inequalities that Rondell can use to determine the number of hours he needs to work at each job if he wants to make the trip.

Example 4 p. 153

Find the coordinates of the vertices of the triangle formed by each system of inequalities.

$$\begin{aligned} 18. \quad & x \geq 0 \\ & y \geq 0 \\ & x + 2y < 4 \end{aligned}$$

$$\begin{aligned} 19. \quad & y \geq 3x - 7 \\ & y \leq 8 \\ & x + y > 1 \end{aligned}$$

$$\begin{aligned} 20. \quad & x \leq 4 \\ & y > -3x + 12 \\ & y \leq 9 \end{aligned}$$

$$\begin{aligned} 21. \quad & -3x + 4y \leq 15 \\ & 2y + 5x > -12 \\ & 10y + 60 \geq 27x \end{aligned}$$

$$\begin{aligned} 22. \quad & 8y - 19x < 74 \\ & 38y + 26x \leq 119 \\ & 54y - 12x \geq -198 \end{aligned}$$

$$\begin{aligned} 23. \quad & 6y - 24x \geq -168 \\ & 8y + 7x > 10 \\ & 20y - 2x \leq 64 \end{aligned}$$

24. **BAKING** Rebecca wants to bake cookies and cupcakes for a bake sale. She can bake 15 cookies at a time and 12 cupcakes at a time. She needs to make at least 120 baked goods, but no more than 360, and she wants to have at least three times as many cookies as cupcakes. What combination of batches of each could Rebecca make?

- 25. CELL PHONES** Dale has a maximum of 800 minutes on his cell phone plan that he can use each month. Daytime minutes cost \$0.15, and nighttime minutes cost \$0.10. Dale plans to use at least twice as many daytime minutes as nighttime minutes. If Dale uses at least 200 nighttime minutes and does not go over his limit, what is his maximum bill? his minimum bill?



Real-World Link

The four crown classes are:

- dominant (D): trees well above the general canopy of the forest
- co-dominant (CD): part of the general canopy that receives sunlight from above and the sides
- intermediate (I): trees below the general canopy that only receive sunlight from above
- suppressed (S): trees well below the general canopy

Source: Silva Ecosystem Consultants

- 26. TREES** Trees are divided into four categories according to height and trunk circumference. In one forest, the trees are categorized by the heights and circumferences described in the table.

| Crown Class | dominant | co-dominant | intermediate | suppressed |
|---------------------------------|----------|-------------|--------------|------------|
| Height (in feet) | over 72 | 56–72 | 40–55 | under 39 |
| Trunk Circumference (in inches) | over 60 | 48–60 | 34–48 | under 33 |

Source: USDA Forest Service

- Write and graph the system of inequalities that represents the range of heights h and circumferences c for a co-dominant tree.
 - Determine the crown class of a basswood that is 48 feet tall. Find the expected trunk circumference.
- 27. CAMPING** On a camping trip, Jessica needs at least 3 pounds of food and 0.5 gallon of water per day. Marc needs at least 5 pounds of food and 0.5 gallon of water per day. Jessica's equipment weighs 10 pounds, and Marc's equipment weighs 20 pounds. A gallon of water weighs approximately 8 pounds. Each of them carries their own supplies, and Jessica is capable of carrying 35 pounds while Marc can carry 50 pounds.
- Graph the inequalities that represent how much they can carry.
 - How many days can they camp, assuming that they bring all their supplies in at once?
 - Who will run out of supplies first?

Solve each system of inequalities by graphing.

28. $y \geq |2x + 4| - 2$
 $3y + x \leq 15$

29. $y \geq |6 - x|$
 $|y| \leq 4$

30. $|y| \geq x$
 $y < 2x$

31. $y > -3x + 1$
 $4y \leq x - 8$
 $3x - 5y < 20$

32. $6y + 2x \leq 9$
 $2y + 18 \geq 5x$
 $y > -4x - 9$

33. $|x| > y$
 $y \leq 6$
 $y \geq -2$

34. $2x + 3y \geq 6$
 $y \leq |x - 6|$

35. $8x + 4y < 10$
 $y > |2x - 1|$

36. $y \geq |x - 2| + 4$
 $y \leq \lfloor x \rfloor - 3$

- 37. MUSIC** Steve is trying to decide what to put on his MP3 player. Audio books are 3 hours long and songs are 2.5 minutes long. Steve wants no more than 4 audio books on his MP3 player, but at least ten songs and one audio book. Each book costs \$15.00 and each song costs \$0.95. Steve has \$63 to spend on books and music. Graph the inequalities to show possible combinations of books and songs that Steve can have.

- 38. JOBS** Louie has two jobs and can work no more than 25 total hours per week. He wants to earn at least \$150 per week. Graph the inequalities to show possible combinations of hours worked at each job that will help him reach his goal.

| Job | Pay |
|--------|--------|
| Busboy | \$6.50 |
| Clerk | \$8.00 |



Real-World Link

Dodgeball has been played in the United States for over 100 years and is still played in schools today.

Source: Kidzworld

- 39. TIME MANAGEMENT** Ramir uses his spare time to write a novel and to exercise. He has budgeted 35 hours per week. He wants to exercise at least 7 hours a week but no more than 15. He also hopes to write between 20 and 25 hours per week. Write and graph a system of inequalities that represents this situation.

Find the coordinates of the vertices of the figure formed by each system of inequalities.

- | | | |
|-----------------------------|----------------------------|------------------------------|
| 40. $y \geq 2x - 12$ | 41. $y \geq -x - 8$ | 42. $2y - x \geq -20$ |
| $y \leq -4x + 20$ | $2y \geq 3x - 20$ | $y \geq -3x - 6$ |
| $4y - x \leq 8$ | $4y + x \leq 24$ | $y \leq -2x + 2$ |
| $y \geq -3x + 2$ | $y \leq 4x + 22$ | $y \leq 2x + 14$ |

- 43. FINANCIAL LITERACY** Mr. Hoffman is investing \$10,000 in two funds. One fund will pay 6% interest, and a riskier second fund will pay 10% interest. What is the least amount Mr. Hoffman can invest in the risky fund and still earn at least \$740 after one year?
- 44. DODGEBALL** A high school is selecting a dodgeball team to play in a fund-raising exhibition against their rival. There can be between 10 and 15 players on the team and there must be more girls than boys on the team.
- Write and graph a system of inequalities to represent the situation.
 - List all of the possible combinations of boys and girls for the team.
 - Explain why there is not an infinite number of possibilities.

H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

- 45. CHALLENGE** Find the area of the region defined by the following inequalities.

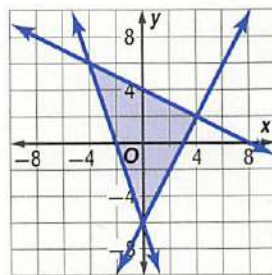
$$\begin{aligned} y &\geq -4x - 16 \\ 4y &\leq 26 - x \\ 3y + 6x &\leq 30 \\ 4y - 2x &\geq -10 \end{aligned}$$

- 46. OPEN ENDED** Write a system of two inequalities in which the solution:
- lies only in the third quadrant.
 - does not exist.
 - lies only on a line.
 - lies on exactly one point.

- 47. CHALLENGE** Write a system of inequalities to represent the solution shown at the right.

- 48. REASONING** Determine whether the following statement is *true* or *false*. If false, give a counterexample.

A system of two linear inequalities has either no points or infinitely many points in its solution.



- 49. WRITING IN MATH** Write a how-to manual for determining where to shade when graphing a system of inequalities.
- 50. WRITING IN MATH** Explain how you would test to see whether $(-4, 6)$ is a solution of a system of inequalities.

Standardized Test Practice

51. To be a member of the marching band, a student must have a grade-point average of at least 2.0 and must have attended at least five after-school practices. Choose the system of inequalities that best represents this situation.

- A $x \geq 2$
 $y \geq 5$
 B $x \leq 2$
 $y \leq 5$
 C $x < 2$
 $y < 5$
 D $x > 2$
 $y > 5$

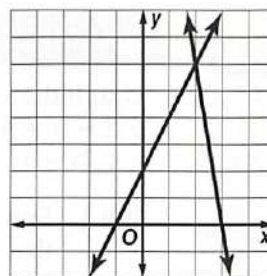
52. **ACT/SAT** The table at the right shows a relationship between x and y . Which equation represents this relationship?

- F $y = 3x - 2$
 G $y = 3x + 2$
 H $y = 4x + 1$
 J $y = 4x - 1$

| x | y |
|-----|-----|
| 1 | 5 |
| 2 | 8 |
| 3 | 11 |
| 4 | 14 |
| 5 | 17 |
| 6 | 20 |

53. **SHORT RESPONSE** If $3x = 2y$ and $5y = 6z$, what is the value of x in terms of z ?

54. **GEOMETRY** Look at the graph below. Which of these statements describes the relationship between the two lines?



- A They intersect at $(6, 2)$.
 B They intersect at $(0, 2)$.
 C They intersect at $(3.5, 0)$.
 D They intersect at $(2, 6)$.

Spiral Review

55. **GEOMETRY** Find the coordinates of the vertices of the parallelogram whose sides are contained in the lines with equations $y = 3$, $y = 7$, $y = 2x$, and $y = 2x - 13$. (Lesson 3-2)

Solve each system of equations by graphing. (Lesson 3-1)

56. $y = 6 - x$
 $y = x + 4$

57. $x + 2y = 2$
 $2x + 4y = 8$

58. $x - 2y = 8$
 $\frac{1}{2}x - y = 4$

Graph each function. Identify the domain and range. (Lesson 2-7)

59. $g(x) = \begin{cases} 0 & \text{if } x < 0 \\ -x + 2 & \text{if } x \geq 0 \end{cases}$

60. $h(x) = \begin{cases} x + 3 & \text{if } x \leq -1 \\ 2x & \text{if } x > -1 \end{cases}$

61. $h(x) = \begin{cases} -1 & \text{if } x < -2 \\ 1 & \text{if } x > 2 \end{cases}$

62. **EXTENDED RESPONSE** For each meeting of the Putnam High School book club, \$25 is taken from the activities account to buy snacks and materials. After their sixth meeting, there will be \$350 left in the activities account. (Lesson 2-4)

- If no money is put back into the account, what equation can be used to show how much money is left in the activities account after having x number of meetings?
- How much money was originally in the account?
- After how many meetings will there be no money left in the activities account?

Skills Review

Find each value if $f(x) = 2x + 5$ and $g(x) = 3x - 4$. (Lesson 2-1)

63. $f(-3)$

64. $g(-2)$

65. $f(-1)$

66. $g(-0.5)$

67. $f(-0.25)$

68. $g(-0.75)$

Graphing Technology Lab

Systems of Linear Inequalities

- Other Calculator Keystrokes
- Graphing Technology Personal Tutor

Objective

Use a graphing calculator to solve systems of inequalities.

You can graph systems of linear inequalities with a graphing calculator by using the $Y=$ menu. You can choose different graphing styles to shade above or below a line.

EXAMPLE Intersection of Two Graphs

Graph the system of inequalities in the standard viewing window.

$$y \geq -3x + 4$$

$$y \leq 2x - 1$$

Step 1 Enter $-3x + 4$ as Y_1 . Because y is greater than $-3x + 4$, shade above the line.

KEYSTROKES: $Y=$ \leftarrow \leftarrow ENTER ENTER \rightarrow \rightarrow

$(-)$ 3 $[X,T,\theta,n]$ + 4 ENTER



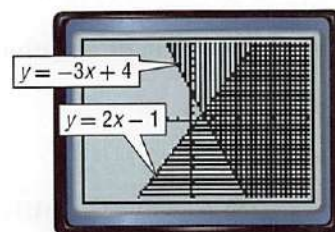
Step 2 Enter $2x - 1$ as Y_2 . Because y is less than $2x - 1$, shade below the line.

KEYSTROKES: \leftarrow \leftarrow ENTER ENTER ENTER \rightarrow \rightarrow 2

$[X,T,\theta,n]$ - 1 ENTER

Step 3 Display the graphs in the standard viewing window.

KEYSTROKES: ZOOM 6



$[-10, 10]$ scl: 1 by $[-10, 10]$ scl: 1

Notice the shading pattern above the line $y = -2x + 3$ and the shading pattern below the line $y = x + 5$. The intersection of the graphs is the region where the patterns overlap. This region includes all the points that satisfy the system $y \geq -2x + 3$ and $y \leq x + 5$.

Exercises

Use a graphing calculator to solve each system of inequalities.

1. $y \geq 3$
 $y \leq -x + 1$

4. $y \geq 2x + 1$
 $y \leq -x - 1$

7. $5y + 3x \geq 11$
 $3y - x \leq -8$

2. $y \geq -4x$
 $y \leq -5$

5. $2y \geq 3x - 1$
 $3y \leq -x + 7$

8. $10y - 7x \geq -19$
 $7y - 5x \leq 11$

3. $y \geq 2 - x$
 $y \leq x + 3$

6. $y + 5x \geq 12$
 $y - 3 \leq 10$

9. $\frac{1}{6}y - x \geq -3$
 $\frac{1}{5}y + x \leq 7$

Solve each system of equations by graphing.
(Lesson 3-1)

1. $y = 2x + 4$
 $y = -x - 2$
2. $5x + 2y = 3$
 $5x - 4y = 9$
3. $x = 2y - 4$
 $x = -3y + 1$
4. $2x - 5y = 14$
 $4x + 3y = -24$

5. **MULTIPLE CHOICE** What type of system is shown? (Lesson 3-1)

$$\begin{aligned} 2x + 4y &= 5 \\ 3x + 6y &= 11 \end{aligned}$$

- A consistent and dependent
B consistent and independent
C inconsistent
D none of the above

Solve each system of equations by using either substitution or elimination. (Lesson 3-2)

6. $y = x + 4$
 $x + y = -12$
7. $3x + 5y = -7$
 $6x - 4y = 0$
8. $\frac{1}{3}x - \frac{3}{8}y = 28$
 $\frac{1}{7}x + \frac{5}{7}y = -37$
9. $\frac{1}{3}x = y + 2$
 $x = 5y - 2$
10. $5x + 2y = 4$
 $3y - 4x = -40$
11. $8x - 3y = -13$
 $-3x + 5y = 1$
12. $6x - 5y = 92$
 $9x + 2y = 100$
13. $4y + 7x = -92$
 $5x - 6y = 14$

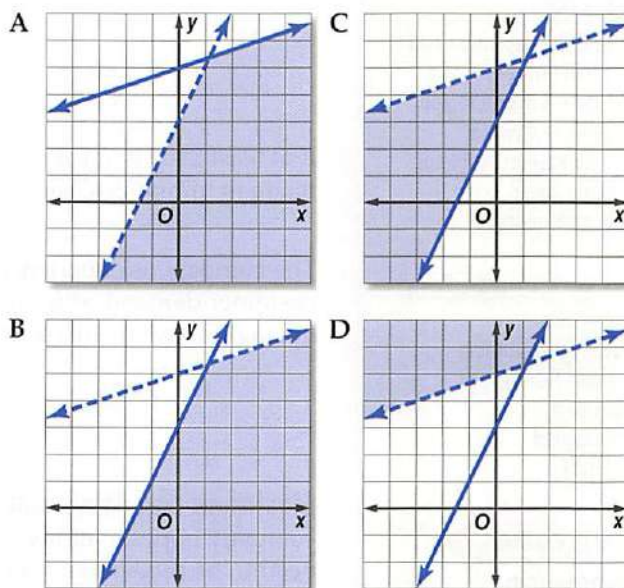
14. **SHOPPING** Main St. Media sells all DVDs for one price and all books for another price. Alex bought 4 DVDs and 6 books for \$170, while Matt bought 3 DVDs and 8 books for \$180. What is the cost of a DVD and the cost of a book? (Lesson 3-2)

15. **MULTIPLE CHOICE** On Thursday, the art museum sold 56 fewer tickets than they sold on Friday. They sold a total of 406 tickets on Thursday and Friday. Which system of equations can be used to find the number of tickets sold on each day? (Lesson 3-2)

- F $f - t = 56$
 $f + t = 406$
G $t - 56 = f$
 $f + t = 406$
H $f - t = 406$
 $f + t = 56$
J $f - t = 56$
 $f + 406 = t$

16. **MULTIPLE CHOICE** Which graph shows the solution of the system of inequalities? (Lesson 3-3)

$$\begin{aligned} y &\leq 2x + 3 \\ y &< \frac{1}{3}x + 5 \end{aligned}$$



Solve each system of inequalities by graphing.
(Lesson 3-3)

17. $x + y > 6$
 $x - y < 0$
18. $y \geq 2x - 5$
 $y \leq x + 4$
19. $3x + 4y \leq 12$
 $6x - 3y \geq 18$
20. $5y + 2x \leq 20$
 $4x + 3y > 12$

21. **MULTIPLE CHOICE** Tia spent \$42 on 2 cans of primer and 1 can of paint for her room. If the price of paint p is 150% of the price of primer r , which system of equations can be used to find the price of paint and primer? (Lesson 3-1)

- F $p = r + \frac{1}{2}r$
 $p + 2r = 42$
G $p = r + 2r$
 $p + \frac{1}{2}r = 42$
H $r = p + \frac{1}{2}p$
 $p + 2p = 42$
J $r = p + 2p$
 $p + \frac{1}{2} = 42$

22. **ART** Rai can spend no more than \$225 on the art club's supply of brushes and paint. A box of 3 brushes costs \$7.50. A box of 10 tubes of paint costs \$21.45. She needs at least 20 brushes and 56 tubes of paint. Graph the region that shows how many boxes of each item can be purchased. (Lesson 3-3)

Optimization with Linear Programming

Then

You solved systems of linear inequalities by using graphs. (Lesson 3-3)

Now

- Find the maximum and minimum values of a function over a region.
- Solve real-world optimization problems using linear programming.

New Vocabulary

constraints
linear programming
feasible region
bounded
unbounded
optimize

Math Online

glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help
- Math in Motion

Why?

An electronics company produces digital audio players and digital video players. A sign on the company bulletin board is shown.

Keeping Costs Down: We Can Do It!

Our Goal: Production per Shift

| Player | Minimum | Maximum | Cost per Unit |
|--------|---------|---------|---------------|
| audio | 600 | 1500 | \$55 |
| video | 800 | 1700 | \$95 |

If at least 2000 items must be produced per shift, how many of each type should be made to minimize costs?

The company is experiencing limitations, or **constraints**, on production caused by customer demand, shipping, and the productivity of their factory. A system of inequalities can be used to represent these constraints.

Maximum and Minimum Values Situations often occur in business in which a company hopes to either maximize profits or minimize costs and many constraints need to be considered. These issues can often be addressed by the use of systems of inequalities in linear programming.

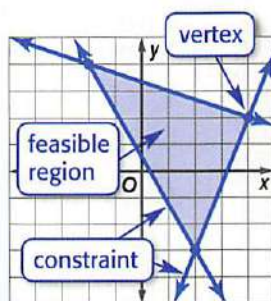
Linear programming is a method for finding maximum or minimum values of a function over a given system of inequalities with each inequality representing a constraint. After the system is graphed and the vertices of the solution set, called the **feasible region**, are substituted into the function, you can determine the maximum or minimum value.

Key Concept

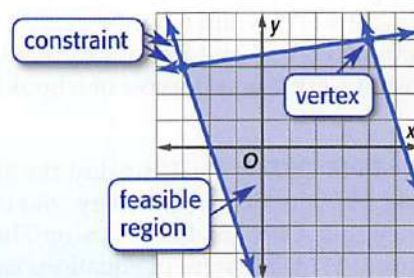
Feasible Regions

For Your

FOLDABLE



The feasible region is enclosed, or **bounded**, by the constraints. The maximum or minimum value of the related function *always* occurs at a vertex of the feasible region.



The feasible region is open and can go on forever. It is **unbounded**. Unbounded regions have either a maximum or a minimum.

ReadingMath

Function Notation

The notation $f(x, y)$ is used to represent a function with two variables, x and y . It is read f of x and y .

EXAMPLE 1 Bounded Region

Graph the system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function for this region.

$$\begin{aligned} 3 &\leq y \leq 6 \\ y &\leq 3x + 12 \\ y &\leq -2x + 6 \\ f(x, y) &= 4x - 2y \end{aligned}$$

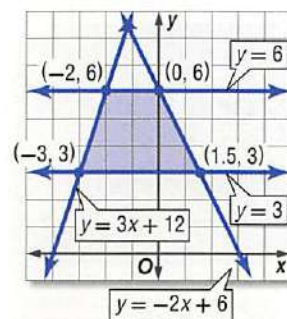
Step 1 Graph the inequalities and locate the vertices.

Step 2 Evaluate the function at each vertex.

| (x, y) | $4x - 2y$ | $f(x, y)$ |
|------------|-----------------|-----------|
| $(-3, 3)$ | $4(-3) - 2(3)$ | -18 |
| $(1.5, 3)$ | $4(1.5) - 2(3)$ | 0 |
| $(0, 6)$ | $4(0) - 2(6)$ | -12 |
| $(-2, 6)$ | $4(-2) - 2(6)$ | -20 |

← maximum

← minimum



The maximum value is 0 at $(1.5, 3)$. The minimum value is -20 at $(-2, 6)$.

Check Your Progress

1A. $-2 \leq x \leq 6$
 $1 \leq y \leq 5$
 $y \leq x + 3$
 $f(x, y) = -5x + 2y$

1B. $-6 \leq y \leq -2$
 $y \leq -x + 2$
 $y \leq 2x + 2$
 $f(x, y) = 6x + 4y$

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When a system of inequalities does not form a closed region, it is unbounded.

Watch Out!

Maximum Value Do not assume that there is no maximum if the feasible region is unbounded above the vertices. Test points are needed to determine if there is a minimum or maximum.

EXAMPLE 2 Unbounded Region

Graph the system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the function for this region.

$$\begin{aligned} 2y + 3x &\geq -12 \\ y &\leq 3x + 12 \\ y &\geq 3x - 6 \\ f(x, y) &= 9x - 6y \end{aligned}$$

Graph the inequalities and evaluate the function at each vertex.

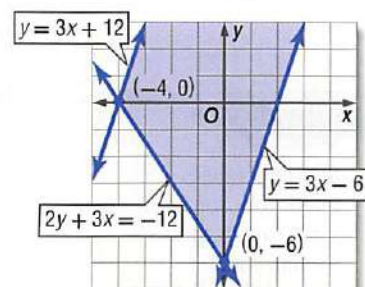
| (x, y) | $9x - 6y$ | $f(x, y)$ |
|-----------|----------------|-----------|
| $(-4, 0)$ | $9(-4) - 6(0)$ | -36 |
| $(0, -6)$ | $9(0) - 6(-6)$ | 36 |

The maximum value is 36 at $(0, -6)$. There is no minimum value. Notice that another point in the feasible region, $(0, 8)$, yields a value of -48 , which is less than -36 .

Check Your Progress

2A. $y \leq 8$
 $y \geq -x + 4$
 $y \leq -x + 10$
 $f(x, y) = -6x + 8y$

2B. $y \geq x - 9$
 $y \leq -4x + 16$
 $y \geq -4x - 4$
 $f(x, y) = 10x + 7y$



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Optimization To **optimize** means to seek the best price or amount to minimize costs or maximize profits. This is often obtained with the use of linear programming.



Real-World Career

Operations Manager

Operations management is an area of business that is concerned with the production of goods and services, and involves the responsibility of ensuring that business operations are efficient and effective. A master's degree in business and experience in operations are preferred.

StudyTip

Reasonableness

Check your solutions for reasonableness by thinking of the context of the problem.

Key Concept

Optimization with Linear Programming

For Your
FOLDABLE

- Step 1** Define the variables.
- Step 2** Write a system of inequalities.
- Step 3** Graph the system of inequalities.
- Step 4** Find the coordinates of the vertices of the feasible region.
- Step 5** Write a linear function to be maximized or minimized.
- Step 6** Substitute the coordinates of the vertices into the function.
- Step 7** Select the greatest or least result. Answer the problem.

▶ **Math in Motion, Animation** glencoe.com

Real-World EXAMPLE 3

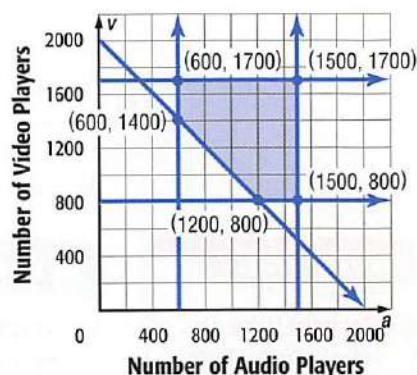
Optimization with Linear Programming

BUSINESS Refer to the application at the beginning of the lesson. Use linear programming to determine how many of each type of digital player should be made per shift.

Step 1 Let a = number of audio players produced.
Let v = number of video players produced.

Step 2 $600 \leq a \leq 1500$
 $800 \leq v \leq 1700$
 $a + v \geq 2000$

Steps 3 and 4 The system is graphed at the right. Note the vertices of the feasible region.



Step 5 The function to be minimized is $f(a, v) = 55a + 95v$.

Step 6

| (a, v) | $55a + 95v$ | $f(a, v)$ |
|--------------|-----------------------|-----------|
| (600, 1700) | $55(600) + 95(1700)$ | 194,500 |
| (600, 1400) | $55(600) + 95(1400)$ | 166,000 |
| (1500, 1700) | $55(1500) + 95(1700)$ | 244,000 |
| (1500, 800) | $55(1500) + 95(800)$ | 158,500 |
| (1200, 800) | $55(1200) + 95(800)$ | 142,000 |

← maximum

← minimum

Step 7 1200 audio players and 800 video players should be produced to minimize costs.

Check Your Progress

3. **JEWELRY** Each week, Mackenzie can make between 10 and 25 necklaces and 15 to 40 pairs of earrings. If she earns profits of \$3 on each pair of earrings and \$5 on each necklace, and she plans to sell at least 30 pieces of jewelry, how many earrings and necklaces should she make to maximize profit?

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Check Your Understanding

Examples 1 and 2 p. 161

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

1. $y \leq 5$
 $x \leq 4$
 $y \geq -x$
 $f(x, y) = 5x - 2y$

2. $y \leq -3x + 6$
 $-y \leq x$
 $y \leq 3$
 $f(x, y) = 8x + 4y$

3. $y \geq -3x + 2$
 $9x + 3y \leq 24$
 $y \geq -4$
 $f(x, y) = 2x + 14y$

4. $-2 \leq y \leq 6$
 $3y \leq 4x + 26$
 $y \leq -2x + 2$
 $f(x, y) = -3x - 6y$

5. $-3 \leq y \leq 7$
 $4y \geq 4x - 8$
 $6y + 3x \leq 24$
 $f(x, y) = -12x + 9y$

6. $y \leq 2x + 6$
 $y \geq 2x - 8$
 $y \geq -2x - 18$
 $f(x, y) = 5x - 4y$

Example 3 p. 162

7. **FINANCIAL LITERACY** The total number of workers' hours per day available for production in a skateboard factory is 85 hours. There are 40 hours available for finishing decks and quality control each day. The table shows the number of hours needed in each department for two different types of skateboards.

| Skateboard Manufacturing Time | | |
|-------------------------------|-----------------|--------------------------------|
| Board Type | Production Time | Deck Finishing/Quality Control |
| Pro Boards | 1.5 hours | 2 hours |
| Specialty Boards | 1 hour | 0.5 hour |



- Write a system of inequalities to represent the situation.
- Draw the graph showing the feasible region.
- List the coordinates of the vertices of the feasible region.
- If the profit on a pro board is \$50 and the profit on a specialty board is \$65, write a function for the total profit on the skateboards.
- Determine the number of each type of skateboard that needs to be made to have a maximum profit. What is the maximum profit?

 = **Step-by-Step Solutions** begin on page R20.
Extra Practice begins on page 947.

Practice and Problem Solving

Examples 1 and 2 p. 161

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

8. $1 \leq y \leq 4$
 $4y - 6x \geq -32$
 $2y \geq -x + 4$
 $f(x, y) = -6x + 3y$

9. $2 \geq x \geq -3$
 $y \geq -2x - 6$
 $4y \leq 2x + 32$
 $f(x, y) = -4x - 9y$

10. $-2 \leq x \leq 4$
 $5 \leq y \leq 8$
 $2x + 3y \leq 26$
 $f(x, y) = 8x - 10y$

11. $-8 \leq y \leq -2$
 $y \leq x$
 $y \leq -3x + 10$
 $f(x, y) = 5x + 14y$

12. $x + 4y \geq 2$
 $2x + 4y \leq 24$
 $2 \leq x \leq 6$
 $f(x, y) = 6x + 7y$

13. $3 \leq y \leq 7$
 $2y + x \leq 8$
 $y - 2x \leq 23$
 $f(x, y) = -3x + 5y$

Examples 1 and 2
p. 161

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

14. $-9 \leq x \leq -3$
 $-9 \leq y \leq -5$
 $3y + 12x \leq -75$
 $f(x, y) = 20x + 8y$

17. $x \geq -6$
 $y + x \leq -1$
 $2x + 3y \geq -9$
 $f(x, y) = -10x - 12y$

20. $y \leq x + 4$
 $y \geq x - 4$
 $y \leq -x + 10$
 $y \geq -x - 10$
 $f(x, y) = -10x + 9y$

15. $x \geq -8$
 $3x + 6y \leq 36$
 $2y + 12 \geq 3x$
 $f(x, y) = 10x - 6y$

18. $-5 \geq y \geq -17$
 $y \leq 3x + 19$
 $y \leq -4x + 15$
 $f(x, y) = 8x - 3y$

21. $-4 \leq x \leq 8$
 $-8 \leq y \leq 6$
 $y \geq x - 6$
 $4y + 7x \leq 31$
 $f(x, y) = 12x + 8y$

16. $y \geq |x - 2|$
 $y \leq 8$
 $8y + 5x \leq 49$
 $f(x, y) = -5x - 15y$

19. $-8 \leq x \leq 16$
 $y \geq 2x - 10$
 $2y + x \leq 80$
 $f(x, y) = 12x + 15y$

22. $y \geq |x + 1| - 2$
 $0 \leq y \leq 6$
 $-6 \leq x \leq 2$
 $x + 3y \leq 14$
 $f(x, y) = 5x + 4y$

Example 3
p. 162

23. **COOKING** Jenny's Bakery makes two types of birthday cakes: yellow cake, which sells for \$25, and strawberry cake, which sells for \$35. Both cakes are the same size, but the decorating and assembly time required for the yellow cake is 2 hours, while the time is 3 hours for the strawberry cake. There are 450 hours of labor available for production. How many of each type of cake should be made to maximize revenue?

24. **BUSINESS** The manager of a travel agency is printing brochures and fliers to advertise special discounts on vacation spots during the summer months. Each brochure costs \$0.08 to print, and each flier costs \$0.04 to print. A brochure requires 3 pages, and a flier requires 2 pages. The manager does not want to use more than 600 pages, and she needs at least 50 brochures and 150 fliers. How many of each should she print to minimize the cost?

25. **PAINTING** Sean has 20 days to paint as many play houses and sheds as he is able. The sheds can be painted at a rate of 2.5 per day, and the play houses can be painted at a rate of 2 per day. He has 45 structures that need to be painted.

- Write a system of inequalities to represent the possible ways Sean can paint the structures.
- Draw a graph showing the feasible region and list the coordinates of the vertices of the feasible region.
- If the profit is \$26 per shed and \$30 per play house, how many of each should he paint?
- What is the maximum profit?



Real-World Link

43–47% of teens see a movie in theaters at least once a month. The average American sees 5.8 movies per year in theaters.

Source: Chapel Hill

26. **MOVIES** Employees at a local movie theater work 8-hour shifts from noon to 8 P.M. or from 4 P.M. to midnight. The table below shows the number of employees needed and their corresponding pay. Find the numbers of day-shift workers and night-shift workers that should be scheduled to minimize the cost. What is the minimal cost?

| Time | noon to 4 P.M. | 4 P.M. to 8 P.M. | 8 P.M. to midnight |
|----------------------------|----------------|------------------|--------------------|
| Number of Employees Needed | at least 5 | at least 14 | 6 |
| Rate per Hour | \$5.50 | \$7.50 | \$7.50 |



Real-World Link

Plastic bottles can be recycled into rugs, swimming goggles, dog bowls, back packs, skateboard wheels, winter jackets, and more.

Source: Earth 911

- 27. BUSINESS** Each car on a freight train can hold 4200 pounds of cargo and has a capacity of 480 cubic feet. The freight service handles two types of packages: small—which weigh 25 pounds and are 3 cubic feet each, and large—which are 50 pounds and are 5 cubic feet each. The freight service charges \$5 for each small package and \$8 for each large package.

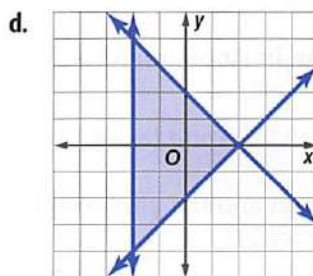
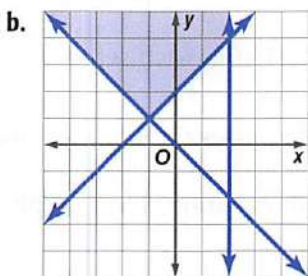
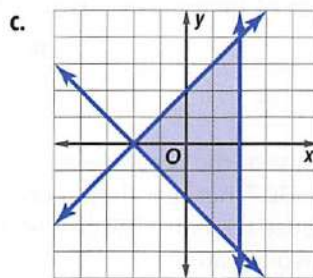
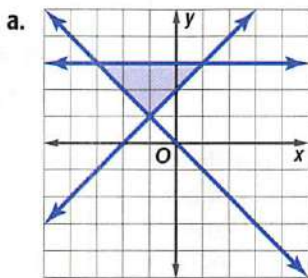
- Find the number of each type of package that should be placed on a train car to maximize revenue.
- What is the maximum revenue per train car?
- In this situation, is maximizing the revenue necessarily the best thing for the company to do? Explain.

- 28. RECYCLING** A recycling plant processes used plastic into food or drink containers. The plant processes up to 1200 tons of plastic per week. At least 300 tons must be processed for food containers, while at least 450 tons must be processed for drink containers. The profit is \$17.50 per ton for processing food containers and \$20 per ton for processing drink containers. What is the profit if the plant maximizes processing?

H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

- 29. OPEN ENDED** Create a set of inequalities that forms a bounded region with an area of 20 units² and lies only in the fourth quadrant.
- 30. CHALLENGE** Find the area of the bounded region formed by the following constraints: $y \geq |x| - 3$, $y \leq -|x| + 3$, and $x \geq |y|$.
- 31. WHICH ONE DOESN'T BELONG?** Identify the system of inequalities that is not the same as the other three. Explain your reasoning.



- 32. REASONING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

An unbounded region will not have both a maximum and minimum value.

- 33. WRITING IN MATH** Upon determining a bounded feasible region, Ayumi noticed that vertices $A(-3, 4)$ and $B(5, 2)$ yielded the same maximum value for $f(x, y) = 16y + 4x$. Kelvin confirmed that her constraints were graphed correctly and her vertices were correct. Then he said that those two points were not the only maximum values in the feasible region. Explain how this could have happened.

Standardized Test Practice

34. Kelsey worked 350 hours during the summer and earned \$2978.50. She earned \$6.85 per hour when she worked at a video store and \$11 per hour as an architectural intern. Let x represent the number of hours she worked at the video store and y represent the number of hours that she interned. Which system of equations represents this situation?

- A $x + y = 350$
 $11x + 6.85y = 2978.50$
B $x + y = 350$
 $6.85x + 11y = 2978.50$
C $x + y = 2978.50$
 $6.85x + 11y = 350$
D $x + y = 2978.50$
 $11x + 6.85y = 350$

35. **SHORT RESPONSE** A family of four went out to dinner. Their bill, including tax, was \$60. They left a 17% tip on the total cost of their bill. What is the total cost of the dinner including tip?

36. **ACT/SAT** For a game she is playing, Liz must draw a card from a deck of 26 cards, one with each letter of the alphabet on it, and roll a die. What is the probability that Liz will draw a letter in her name and roll an odd number?

- F $\frac{2}{3}$ H $\frac{1}{26}$
G $\frac{1}{13}$ J $\frac{3}{52}$

37. **GEOMETRY** Which of the following best describes the graphs of $y = 3x - 5$ and $4y = 12x + 16$?

- A The lines have the same y -intercept.
B The lines have the same x -intercept.
C The lines are perpendicular.
D The lines are parallel.

Spiral Review

Solve each system of inequalities by graphing. (Lesson 3-3)

38. $3x + 2y \geq 6$
 $4x - y \geq 2$

39. $4x - 3y < 7$
 $2y - x < -6$

40. $3y \leq 2x - 8$
 $y \geq \frac{2}{3}x - 1$

41. **BUSINESS** Last year the chess team paid \$7 per hat and \$15 per shirt for a total purchase of \$330. This year they spent \$360 to buy the same number of shirts and hats because the hats now cost \$8 and the shirts cost \$16. Write and solve a system of two equations that represents the number of hats and shirts bought each year. (Lesson 3-2)

Write an equation in slope-intercept form for the line that satisfies each set of conditions. (Lesson 2-4)

42. passes through (5, 1) and (8, -4)

43. passes through (-3, 5) and (3, 2)

Find the x -intercept and the y -intercept of the graph of each equation. Then graph the equation. (Lesson 2-2)

44. $5x + 3y = 15$

45. $2x - 6y = 12$

46. $3x - 4y - 10 = 0$

47. $2x + 5y - 10 = 0$

48. $y = x$

49. $y = 4x - 2$

Skills Review

Evaluate each expression if $x = -1$, $y = 3$, and $z = 7$. (Lesson 1-1)

50. $x + y + z$

51. $2x - y + 2z$

52. $-x + 4y - 3z$

53. $4x + 2y - z$

54. $5x - y + 4z$

55. $-3x - 3y + 3z$

Systems of Equations in Three Variables

Then

You solved linear equations with two variables. (Lesson 3-2)

Now

- Solve systems of linear equations in three variables.
- Solve real-world problems using systems of linear equations in three variables.

New Vocabulary

ordered triple

Math Online

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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help
- Math in Motion

Why?

Seats closest to an amphitheater stage cost \$30. The seats in the next section cost \$25, and lawn seats are \$20. There are twice as many seats in section B as in section A. When all 19,200 seats are sold, the amphitheater makes \$456,000.

A system of equations in three variables can be used to determine the number of seats in each section.

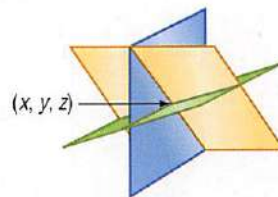


Systems in Three Variables Like systems of equations in two variables, systems in three variables can have one solution, infinite solutions, or no solution. A solution of such a system is an **ordered triple** (x, y, z) .

The graph of an equation in three variables is a three-dimensional graph in the shape of a plane. The graphs of a system of equations in three variables form a system of planes.

One Solution

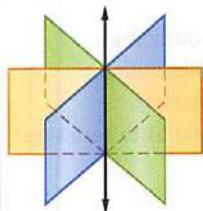
The three individual planes intersect at a specific point.



Infinitely Many Solutions

The planes intersect in a line.

Every coordinate on the line represents a solution of the system.



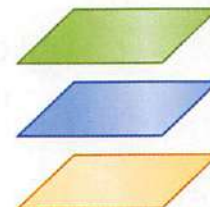
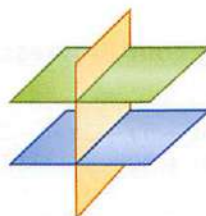
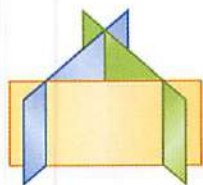
The planes intersect in the same plane.

Every equation is equivalent. Every coordinate in the plane represents a solution of the system.



No Solution

There are no points in common with all three planes.



Solving systems of equations in three variables is similar to solving systems of equations in two variables. Use the strategies of substitution and elimination to find the ordered triple that represents the solution of the system.

EXAMPLE 1 A System with One Solution

Solve the system of equations.

$$\begin{aligned} 3x - 2y + 4z &= 35 \\ -4x + y - 5z &= -36 \\ 5x - 3y + 3z &= 31 \end{aligned}$$

The coefficient of 1 in the second equation makes y a good choice for elimination.

Step 1 Eliminate one variable by using two pairs of equations.

$$\begin{array}{rcl} 3x - 2y + 4z & = & 35 \\ -4x + y - 5z & = & -36 \end{array} \quad \text{Multiply by 2.} \quad \begin{array}{rcl} 3x - 2y + 4z & = & 35 \\ (+) -8x + 2y - 10z & = & -72 \\ \hline -5x & & -6z = -37 \end{array}$$

Equation 1
Equation 2 $\times 2$

$$\begin{array}{rcl} -4x + y - 5z & = & -36 \\ 5x - 3y + 3z & = & 31 \end{array} \quad \text{Multiply by 3.} \quad \begin{array}{rcl} -4x + y - 5z & = & -36 \\ (+) 15x - 9y + 9z & = & 93 \\ \hline 11x - 8y - 4z & = & 57 \end{array}$$

Equation 2 $\times 3$
Equation 1

The y -terms in each equation have been eliminated. We now have a system of two equations and two variables, x and z .

Step 2 Solve the system of two equations.

$$\begin{array}{rcl} -5x - 6z & = & -37 \\ -7x - 12z & = & -77 \end{array} \quad \text{Multiply by } -2. \quad \begin{array}{rcl} -5x - 6z & = & -37 \\ (+) 14x + 24z & = & 154 \\ \hline 9x + 18z & = & 117 \\ \hline 3x & = & -3 \\ x & = & -1 \end{array}$$

Eliminate z .
Divide by 3.

Use substitution to solve for z .

$$\begin{array}{rcl} -5x - 6z & = & -37 \\ -5(-1) - 6z & = & -37 \\ 5 - 6z & = & -37 \\ -6z & = & -42 \\ z & = & 7 \end{array} \quad \begin{array}{l} \text{Equation with two variables} \\ \text{Substitution} \\ \text{Multiply.} \\ \text{Subtract 5 from each side.} \\ \text{Divide each side by } -6. \end{array}$$

The result is $x = -1$ and $z = 7$.

Step 3 Substitute the two values into one of the original equations to find y .

$$\begin{array}{rcl} -4x + y - 5z & = & -36 \\ -4(-1) + y - 5(7) & = & -36 \\ 4 + y - 35 & = & -36 \\ y - 31 & = & -36 \\ y & = & -5 \end{array} \quad \begin{array}{l} \text{Equation 1} \\ \text{Substitution} \\ \text{Multiply.} \\ \text{Add 31 to each side.} \end{array}$$

CHECK

$$\begin{array}{rcl} -4x + y - 5z & = & -36 \\ -4(-1) + (-5) - 5(7) & \stackrel{?}{=} & -36 \\ 4 + (-5) - 35 & \stackrel{?}{=} & -36 \\ -36 & = & -36 \quad \checkmark \end{array}$$

Equation 2
 $x = -1, y = -5, z = 7$
Simplify.

The solution is $(-1, -5, 7)$.

Check Your Progress

1A. $\begin{aligned} 2x + 4y - 5z &= 18 \\ -3x + 5y + 2z &= -27 \\ -5x + 3y - z &= -17 \end{aligned}$

1B. $\begin{aligned} 4x - 3y + 6z &= 18 \\ -x + 5y + 4z &= 48 \\ 6x - 2y + 5z &= 0 \end{aligned}$

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StudyTip

Choosing Variables

If one of the equations is missing a variable already, use the other two equations and eliminate the same variable.

Math in Motion,
Interactive Lab
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StudyTip

Checking Solutions

Always substitute your answer into all of the original equations to confirm your answer.

When solving a system of three linear equations with three variables, it is important to check your answer using all three of the original equations. This is necessary because it is possible for a solution to work for two of the equations but not the third.

EXAMPLE 2 No Solution and Infinite Solutions

Solve each system of equations.

a. $5x + 4y - 5z = -10$
 $-4x - 10y - 8z = -16$
 $6x + 15y + 12z = 24$

Eliminate x in the second two equations.

$$\begin{array}{rcl} -4x - 10y - 8z = -16 & \xrightarrow{\text{Multiply by 3.}} & -12x - 30y - 24z = -48 \\ 6x + 15y + 12z = 24 & \xrightarrow{\text{Multiply by 2.}} & (+) 12x + 30y + 24z = 48 \\ & & \hline & & 0 = 0 \end{array}$$

The equation $0 = 0$ is always true. This indicates that the last two equations represent the same plane. Check to see if this plane intersects the first plane.

$$\begin{array}{rcl} 5x + 4y - 5z = -10 & \xrightarrow{\text{Multiply by 4.}} & 20x + 16y - 20z = -40 \\ -4x - 10y - 8z = -16 & \xrightarrow{\text{Multiply by 5.}} & (+) -20x - 50y - 40z = -80 \\ & & \hline & & -34y - 60z = -120 \end{array}$$

The planes intersect in a line. So, there are an infinite number of solutions.

b. $-6a + 9b - 12c = 21$
 $-2a + 3b - 4c = 7$
 $10a - 15b + 20c = -30$

Eliminate a in the first two equations.

$$\begin{array}{rcl} -6a + 9b - 12c = 21 & & -6a + 9b - 12c = 21 \\ -2a + 3b - 4c = 7 & \xrightarrow{\text{Multiply by } -3.} & (+) 6a - 9b + 12c = -21 \\ & & \hline & & 0 = 0 \end{array}$$

The equation $0 = 0$ is always true. This indicates that the first two equations represent the same plane. Check to see if this plane intersects the last plane.

$$\begin{array}{rcl} -2a + 3b - 4c = 7 & \xrightarrow{\text{Multiply by 5.}} & -10a + 15b - 20c = 35 \\ 10a - 15b + 20c = -30 & & (+) 10a - 15b + 20c = -30 \\ & & \hline & & 0 = 5 \end{array}$$

The equation $0 = 5$ is never true. So, there is no solution of this system.

StudyTip

Infinite and No Solutions When solving a system of more than two equations, $0 = 5$ will always yield no solution, while $0 = 0$ may not always yield infinite solutions.

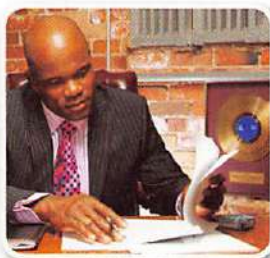
Check Your Progress

2A. $-4x - 2y - z = 15$
 $12x + 6y + 3z = 45$
 $2x + 5y + 7z = -29$

2B. $3x + 5y - 2z = 13$
 $-5x - 2y - 4z = 20$
 $-14x - 17y + 2z = -19$

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Real-World Problems When solving problems involving three variables, use the four-step plan to help organize the information. Identify the three variables and what they represent. Then use the information in the problem to form equations using the variables. Once you have three equations and all three variables are represented, you can solve the problem.



Real-World Career

Music Management

Music management involves acting as the talent manager to the artists. Other duties include negotiating with record labels, music promoters, and tour promoters. Managers usually earn a percentage of the artist's income. A bachelor's degree is usually required.

Problem-Solving Tip

Look for a Pattern

Looking for a pattern can help to identify a function and write an equation.

EXAMPLE 3 Write and Solve a System of Equations

CONCERTS Refer to the application at the beginning of the lesson. Write and solve a system of equations to determine how many seats are in each section of the amphitheater.

Understand Define the variables. x = seats in section A
 y = seats in section B
 z = lawn seats

Plan There are 19,200 seats.

$$x + y + z = 19,200$$

The total revenue is \$456,000.

$$30x + 25y + 20z = 456,000$$

Equation 1

Equation 2

There are twice as many seats in section B as in section A.

$$y = 2x$$

Equation 3

Solve Solve the system.

Step 1 Substitute $y = 2x$ in the first two equations.

$$x + y + z = 19,200$$

$$x + 2x + z = 19,200$$

$$3x + z = 19,200$$

$$30x + 25y + 20z = 456,000$$

$$30x + 25(2x) + 20z = 456,000$$

$$80x + 20z = 456,000$$

Equation 1

$y = 2x$

Add.

Equation 2

$y = 2x$

Simplify.

Step 2 Solve the system of two equations in two variables.

$$3x + z = 19,200$$

$$80x + 20z = 456,000$$

Multiply by -20 .

$$-60x - 20z = -384,000$$

$$(+)\ 80x + 20z = 456,000$$

$$20x = 72,000$$

$$x = 3600$$

Step 3 Substitute to find z .

$$3x + z = 19,200$$

$$3(3600) + z = 19,200$$

$$10,800 + z = 19,200$$

$$z = 8400$$

Remaining equation in two variables

$x = 3600$

Distribute.

Simplify.

Step 4 Substitute to find y .

$$y = 2x$$

$$y = 2(3600) \text{ or } 7200$$

Equation 3

$x = 3600$

The solution is (3600, 7200, 8400). There are 3600 seats in section A, 7200 in section B, and 8400 lawn seats.

Check Substitute the values into either of the first two equations.

Check Your Progress

- Ms. Garza invested \$50,000 in three different accounts. She invested three times as much money in an account that paid 8% interest than an account that paid 10% interest. The third account earned 12% interest. If she earned a total of \$5160 in interest in a year, how much did she invest in each account?

Check Your Understanding

Examples 1 and 2

pp. 168–169

Solve each system of equations.

1. $-3a - 4b + 2c = 28$
 $a + 3b - 4c = -31$
 $2a + 3c = 11$

2. $3y - 5z = -23$
 $4x + 2y + 3z = 7$
 $-2x - y - z = -3$

3. $3x + 6y - 2z = -6$
 $2x + y + 4z = 19$
 $-5x - 2y + 8z = 62$

4. $-4r - s + 3t = -9$
 $3r + 2s - t = 3$
 $r + 3s - 5t = 29$

5. $3x + 5y - z = 12$
 $-2x - 3y + 5z = 14$
 $4x + 7y + 3z = 38$

6. $2a - 3b + 5c = 58$
 $-5a + b - 4c = -51$
 $-6a - 8b + c = 22$

Example 3

p. 170

7. **DOWNLOADING** Heather downloaded some television shows. A sitcom uses 0.3 gigabyte of memory; a drama, 0.6 gigabyte; and a talk show, 0.6 gigabyte. She downloaded 7 programs totaling 3.6 gigabytes. There were twice as many episodes of the drama as the sitcom.

- Write a system of equations for the number of episodes of each type of show.
- How many episodes of each show did she download?

 = **Step-by-Step Solutions** begin on page R20.
Extra Practice begins on page 947.

Practice and Problem Solving

Examples 1 and 2

pp. 168–169

Solve each system of equations.

8. $-5x + y - 4z = 60$
 $2x + 4y + 3z = -12$
 $6x - 3y - 2z = -52$

9. $4a + 5b - 6c = 2$
 $-3a - 2b + 7c = -15$
 $-a + 4b + 2c = -13$

10. $-2x + 5y + 3z = -25$
 $-4x - 3y - 8z = -39$
 $6x + 8y - 5z = 14$

11. $4r + 6s - t = -18$
 $3r + 2s - 4t = -24$
 $-5r + 3s + 2t = 15$

12. $-2x + 15y + z = 44$
 $4x + 3y + 3z = 18$
 $-3x + 6y - z = 8$

13. $4x + 2y + 6z = 13$
 $-12x + 3y - 5z = 8$
 $-4x + 7y + 7z = 34$

14. $8x + 3y + 6z = 43$
 $-3x + 5y + 2z = 32$
 $5x - 2y + 5z = 24$

15. $-6x - 5y + 4z = 53$
 $5x + 3y + 2z = -11$
 $8x - 6y + 5z = 4$

16. $-9a + 3b - 2c = 61$
 $8a + 7b + 5c = -138$
 $5a - 5b + 8c = -45$

17. $2x - y + z = 1$
 $x + 2y - 4z = 3$
 $4x + 3y - 7z = -8$

18. $x + 2y = 12$
 $3y - 4z = 25$
 $x + 6y + z = 20$

19. $r - 3s + t = 4$
 $3r - 6s + 9t = 5$
 $4r - 9s + 10t = 9$

Example 3

p. 170

20. **SWIMMING** A friend e-mails you the results of a recent high school swim meet. The e-mail states that 24 individuals placed, earning a combined total of 53 points. First place earned 3 points, second place earned 2 points, and third place earned 1 point. There were as many first-place finishers as second- and third-place finishers combined.

- Write a system of three equations that represents how many people finished in each place.
- How many swimmers finished in first place, in second place, and in third place?
- Suppose the e-mail had said that the athletes scored a combined total of 47 points. Explain why this statement is false and the solution is unreasonable.

21. **AMUSEMENT PARKS** Nick goes to the amusement park to ride roller coasters, bumper cars, and water slides. The wait for the roller coasters is 1 hour, the wait for the bumper cars is 20 minutes long, and the wait for the water slides is only 15 minutes long. Nick rode 10 total rides during his visit. Because he enjoys roller coasters the most, the number of times he rode the roller coasters was the sum of the times he rode the other two rides. If Nick waited in line for a total of 6 hours and 20 minutes, how many of each ride did he go on?



Real-World Link

Most mechanics would recommend you get an oil change every 6 months and a radiator flush every other year.

Source: Popular Mechanics

22. **BUSINESS** Ramón usually gets one of the routine maintenance options at Annie's Garage. Today however, he needs a different combination of work than what is listed.



- Assume that the price of an option is the same price as purchasing each item separately. Find the prices for an oil change, a radiator flush, and a brake pad replacement.
- If Ramón wants his brake pads replaced and his radiator flushed, how much should he plan to spend?

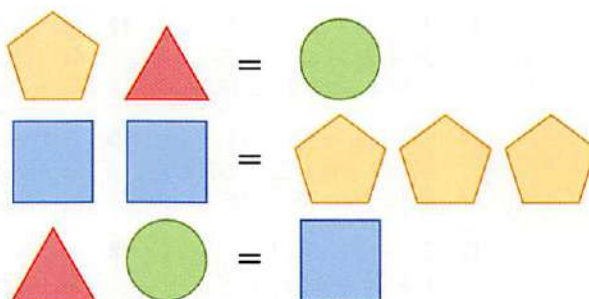
23. **FINANCIAL LITERACY** Kate invested \$100,000 in three different accounts. If she invested \$30,000 more in account A than account C and is expected to earn \$6300 in interest, how much did she invest in each account?

| Account | Expected Interest |
|---------|-------------------|
| A | 4% |
| B | 8% |
| C | 10% |

H.O.T. Problems

Use **H**igher-**O**rder **T**hinking Skills

24. **REASONING** Write a system of equations to represent the three rows of figures below. Use the system to find the number of red triangles that will balance one green circle.



25. **CHALLENGE** The general form of an equation for a parabola is $y = ax^2 + bx + c$, where (x, y) is a point on the parabola. If three points on a parabola are $(2, -10)$, $(-5, -101)$, and $(6, -90)$, determine the values of a , b , and c and write the general form of the equation.
26. **PROOF** Consider the following system and prove that if $b = c = -a$, then $ty = a$.
- $$\begin{aligned} rx + ty + vz &= a \\ rx - ty + vz &= b \\ rx + ty - vz &= c \end{aligned}$$
27. **OPEN ENDED** Write a system of three linear equations that has a solution of $(-5, -2, 6)$. Show that the ordered triple satisfies all three equations.
28. **REASONING** Use the diagrams of solutions of systems of equations on page 167 to consider a system of inequalities in three variables. Describe the solution of such a system.
29. **WRITING IN MATH** Use your knowledge of solving a system of three linear equations with three variables to explain how to solve a system of four equations with four variables.

Standardized Test Practice

30. What is the solution of the system of equations shown below?

$$\begin{cases} x - y + z = 0 \\ -5x + 3y - 2z = -1 \\ 2x - y + 4z = 11 \end{cases}$$

- A (0, 3, 3)
 B (2, 5, 3)
 C no solution
 D infinitely many solutions
31. Which of the following represents a correct procedure for solving each equation?

F $-3(x - 7) = -16$
 $-3x - 21 = -16$
 $-3x = 5$

$x = -\frac{5}{3}$

G $7 - 4x = 3x + 27$
 $7 - 7x = 27$

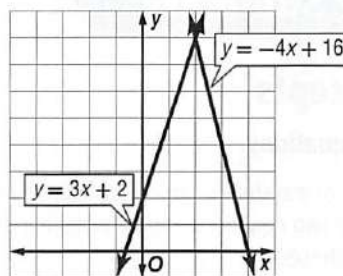
$-7x = -\frac{20}{7}$

$x = 20$

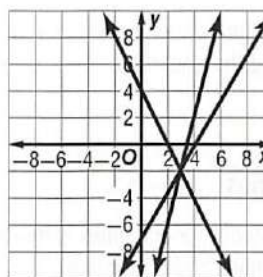
H $2(x - 4) = 20$
 $2x - 8 = 20$
 $2x = 12$
 $x = 6$

J $6(2x + 1) = 30$
 $12x + 6 = 30$
 $12x = 24$
 $x = 2$

32. **EXTENDED RESPONSE** Use the graph to find the solution of the systems of equations. Describe one way to check the solution.



33. **ACT/SAT** The graph shows which system of equations?



A $y + 14 = 4x$
 $y = 4 - 2x$

$-7 = y - \frac{5}{3}x$

B $y - 14 = 4x$
 $y = 4 + 2x$

$-7 = y + \frac{5}{3}x$

C $y + 14x = 4$
 $-2y = 4 + y$

$-7 = y - \frac{5}{3}x$

D $y - 14x = 4$
 $2x = 4 + y$

$7 = y - \frac{5}{3}x$

Spiral Review

A feasible region has vertices at $(-3, 2)$, $(1, 3)$, $(6, 1)$, and $(2, -2)$. Find the maximum and minimum values of each function. (Lesson 3-4)

34. $f(x, y) = 2x - y$

35. $f(x, y) = x + 5y$

36. $f(x, y) = y - 4x$

37. $f(x, y) = -x + 3y$

38. **SKI CLUB** The ski club's budget for the year is \$4250. They are able to find skis for \$75 per pair and boots for \$40 per pair. They know they should buy more boots than skis because the skis are adjustable to several sizes of boots. (Lesson 3-3)

- a. Give an example of three different purchases that the ski club can make.
 b. Suppose the ski club wants to spend all of its budget. What combination of skis and boots should they buy? Explain.

Skills Review

Solve each system of equations. (Lesson 3-2)

39. $x = y + 5$
 $3x + y = 19$

40. $3x - 2y = 1$
 $4x + 2y = 20$

41. $5x + 3y = 25$
 $4x + 7y = -3$

42. $y = x - 7$
 $2x - 8y = 2$

Chapter Summary

Key Concepts

Systems of Equations (Lessons 3-1 and 3-2)

- The solution of a system of equations can be found by graphing the two equations and determining at what point they intersect.
- In the substitution method, one equation is solved for a variable and substituted to find the value of another variable.
- In the elimination method, one variable is eliminated by adding or subtracting the equations.

Systems of Inequalities (Lesson 3-3)

- The solution of a system of inequalities is found by graphing the inequalities and determining the intersection of the graphs.

Linear Programming (Lesson 3-4)

- Finding maximum or minimum values of a function over a given system of inequalities with each inequality representing a constraint.
- To optimize means to seek the optimal price or amount that is desired to minimize costs or maximize profits.

Systems of Equations in Three Variables (Lesson 3-5)

- A system of equations in three variables can be solved algebraically by using the substitution method or the elimination method.

FOLDABLES® Study Organizer

Be sure the Key Concepts are noted in your Foldable.



Key Vocabulary

| | |
|------------------------------------|--|
| bounded (p. 160) | independent (p. 137) |
| break-even point (p. 136) | linear programming (p. 160) |
| consistent (p. 137) | optimize (p. 162) |
| constraints (p. 160) | ordered triple (p. 167) |
| dependent (p. 137) | substitution method (p. 143) |
| elimination method (p. 144) | system of equations (p. 135) |
| feasible region (p. 160) | system of inequalities (p. 151) |
| inconsistent (p. 137) | unbounded (p. 160) |

Vocabulary Check

Choose the term from the list above to complete each sentence.

- _____ is a method for finding maximum or minimum values of a function over a given system of inequalities with each inequality representing a constraint.
- The _____ is an algebraic method for solving systems of linear equations that eliminates a variable by adding or subtracting the equations.
- A system of equations is _____ if it has at least one solution.
- To _____ means to seek the best price or profit using linear programming.
- A feasible region that is open and can go on forever is called _____.
- The _____ is the point at which the income equals the cost.
- A system of equations is _____ if it has an infinite number of solutions.
- A(n) _____ is two or more equations with the same variables.
- A system of equations is _____ if it has no solutions.
- To solve a system of equations by the _____, solve one equation for one variable in terms of the other. Then substitute in the other equation.

Lesson-by-Lesson Review

3-1 Solving Systems of Equations by Graphing (pp. 135–141)

Solve each system of equations by graphing.

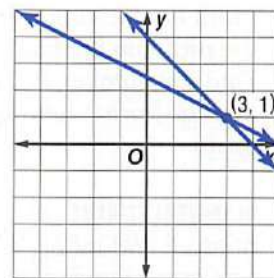
11. $3x + 4y = 8$
 $x - 3y = -6$
12. $x + \frac{8}{3}y = 12$
 $\frac{1}{2}x + \frac{4}{3}y = 6$
13. $y - 3x = 13$
 $y = \frac{1}{3}x + 5$
14. $6x - 14y = 5$
 $3x - 7y = 5$
15. **LAWN CARE** André and Paul each mow lawns. André charges a \$30 service fee and \$10 per hour. Paul charges a \$10 service fee and \$15 per hour. After how many hours will André and Paul charge the same amount?

EXAMPLE 1

Solve the system of equations by graphing.

$$x + y = 4 \quad x + 2y = 5$$

Graph both equations on the coordinate plane.



The solution of the system is (3, 1).

3-2 Solving Systems of Equations Algebraically (pp. 143–150)

Solve each system of equations by using either substitution or elimination.

16. $x + y = 6$
 $3x - 2y = -2$
17. $5x - 2y = 4$
 $-2y + x = 12$
18. $x + y = 3.5$
 $x - y = 7$
19. $3y - 5x = 0$
 $2y - 4x = -2$
20. **SCHOOL SUPPLIES** At an office supply store, Emilio bought 3 notebooks and 5 pens for \$13.75. If a notebook costs \$1.25 more than a pen, how much does a notebook cost? How much does a pen cost?

EXAMPLE 2

Solve the system of equations by using either substitution or elimination.

$$3x + 2y = 1 \quad y = -x + 1$$

Substitute $-x + 1$ for y in the first equation. Then solve for y .

$$\begin{array}{l|l} 3x + 2y = 1 & y = -x + 1 \\ 3x + 2(-x + 1) = 1 & = -(-1) + 1 \text{ or } 2 \\ 3x - 2x + 2 = 1 & \\ x + 2 = 1 & \\ x = -1 & \end{array} \quad \begin{array}{l} \text{The solution is } (-1, 2). \end{array}$$

3-3 Solving Systems of Inequalities by Graphing (pp. 151–157)

Solve each system of inequalities by graphing.

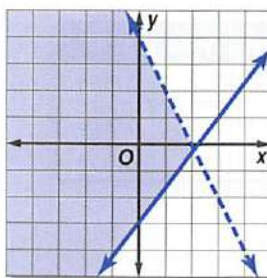
21. $y < 2x - 3$
 $y \geq 4$
22. $|y| > 2$
 $x > 3$
23. $y \geq x + 3$
 $2y \leq x - 5$
24. $y > x + 1$
 $x < -2$
25. **JEWELRY** Payton makes jewelry to sell at her mother's clothing store. She spends no more than 3 hours making jewelry on Saturdays. It takes her 15 minutes to set up her supplies and 25 minutes to make each bracelet. Draw a graph that represents this.

EXAMPLE 3

Solve the system of inequalities by graphing.

$$y \geq \frac{3}{2}x - 3$$

$$y < 4 - 2x$$



The solution of the system is the region that satisfies both inequalities. The solution of this system is the shaded region.

3-4

Linear Programming (pp. 160–166)

- 26. FLOWERS** A florist can make a grand arrangement in 18 minutes or a simple arrangement in 10 minutes. The florist makes at least twice as many of the simple arrangements as the grand arrangements. The florist can work only 40 hours per week. The profit on the simple arrangements is \$10 and the profit on the grand arrangements is \$25. Find the number and type of arrangements that the florist should produce to maximize profit.
- 27. MANUFACTURING** A shoe manufacturer makes outdoor and indoor soccer shoes. There is a two-step process for both kinds of shoes. Each pair of outdoor shoes requires 2 hours in step one and 1 hour in step two, and produces a profit of \$20. Each pair of indoor shoes requires 1 hour in step one and 3 hours in step 2, and produces a profit of \$15. The company has 40 hours of labor available per day for step one and 60 hours available for step two. What is the manufacturer's maximum profit? What is the combination of shoes for this profit?

EXAMPLE 4

A gardener is planting two types of herbs in a 5184-square-inch garden. Herb A requires 6 square inches of space, and herb B requires 24 square inches of space. The gardener will plant no more than 300 plants. If herb A can be sold for \$8 and herb B can be sold for \$12, how many of each herb should be sold to maximize income?

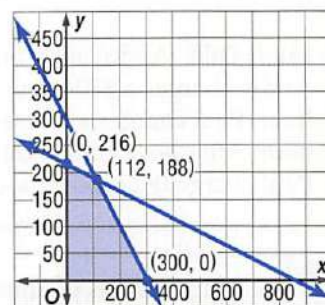
Let a = the number of herb A and
 b = the number of herb B.

$$\begin{aligned} a &\geq 0, b \geq 0, \\ 6a + 24b &\leq 5184, \\ \text{and } a + b &\leq 300 \end{aligned}$$

Graph the inequalities. The vertices of the feasible region are $(0, 0)$, $(300, 0)$, $(0, 216)$, and $(112, 188)$.

The profit function is $f(a, b) = 8a + 12b$.

The maximum value of \$3152 occurs at $(112, 188)$. So the gardener should plant 112 of herb A and 188 of herb B.



3-5

Solving Systems of Equations in Three Variables (pp. 167–173)

Solve each system of equations.

- 28.** $a - 4b + c = 3$ **29.** $2x - z = 14$
 $b - 3c = 10$ $3x - y + 5z = 0$
 $3b - 8c = 24$ $4x + 2y + 3z = -2$

- 30. AMUSEMENT PARKS** Dustin, Luis, and Marci went to an amusement park. They purchased snacks from the same vendor. Their snacks and how much they paid are listed in the table. How much did each snack cost?

| Name | Hot Dogs | Popcorn | Soda | Price |
|--------|----------|---------|------|---------|
| Dustin | 1 | 2 | 3 | \$15.25 |
| Luis | 2 | 0 | 3 | \$14.00 |
| Marci | 1 | 2 | 1 | \$10.25 |

EXAMPLE 5

Solve the system of equations.

$$\begin{aligned} x + y + 2z &= 6 \\ 2x + 5z &= 12 \\ x + 2y + 3z &= 9 \end{aligned}$$

$$\begin{array}{rcl} 2x + 2y + 4z &= & 12 \\ (-) x + 2y + 3z &= & 9 \\ \hline x + z &= & 3 \end{array}$$

Equation 1 $\times 2$
Equation 3
Subtract.

Solve the system of two equations.

$$\begin{array}{rcl} 2x + 5z &= & 12 \\ (-) 2x + 2z &= & 6 \\ \hline 3z &= & 6 \\ z &= & 2 \end{array}$$

Equation 1
 $2 \times (x + z = 3)$
Subtract.
Divide each side by 2.

Substitute 2 for z in one of the equations with two variables, and solve for y . Then, substitute 2 for z and the value you got for y into an equation from the original system to solve for x .

The solution is $(1, 1, 2)$.

Solve each system of equations.

1. $2x - 3y = 9$

$4x + 3y = 9$

3. $-x + y = 2$

$4x - 3y = -3$

2. $x + 2y = 7$

$y = 5x - 2$

4. $\frac{1}{2}x + \frac{1}{3}y = 7$

$\frac{1}{5}x - \frac{2}{3}y = -2$

Solve each system of inequalities by graphing.

5. $x + y \leq 4$

$y \geq x$

7. $x - y > 0$

$4 + y \leq 2x$

6. $2x + 3y > 12$

$3x - y < 21$

8. $2y - 5x \leq 6$

$4x + y < -4$

9. **MULTIPLE CHOICE** Which statement best describes the graphs of the two equations?

$x + 4y = 8$

$3x + 12y = 2$

- A The lines are parallel.
 B The lines are the same.
 C The lines intersect in only one point.
 D The lines intersect in more than one point, but are not the same.

Solve each system of equations.

10. $x - 2y + 3z = 1$

$4y - 4z = 12$

$8y - 14z = 0$

11. $x + y + z = 4$

$x + 3y + 3z = 10$

$2x + y - z = 3$

12. $2x - y - 2z = 5$

$10x + 8z = -4$

$3x - y = 1$

13. $2x + 3y + z = 0$

$3x + y = 1$

$x - 2y + z = 9$

14. **MULTIPLE CHOICE** Seela rented a raft from River Rafter's Inc. She paid \$100 to rent the raft and \$25 for each hour. Martin rented a raft from Oscar's Outdoor Shop. He paid \$50 to rent the raft and \$35 per hour. For what number of hours will both rafting companies charge the same amount?

F 0

G 4

H 5

J 10

15. **CARPENTRY** Cal's Carpentry makes tables and chairs. The process involves some carpentry time and some finishing time. The carpentry times and finishing times are listed in the table below.

| Product | Carpentry Time (hr) | Finishing Time (hr) |
|---------|---------------------|---------------------|
| chair | 3 | 0.5 |
| table | 2 | 1 |

Cal's Carpentry can work for a maximum of 108 carpentry hours and 20 finishing hours per day. The profit is \$35 for a table and \$25 for a chair. How many tables and chairs should be made each day to maximize profit?

- a. Using c for the number of chairs and t for the number of tables, write a system of inequalities to represent this situation.
 b. Draw the graph showing the feasible region.
 c. Determine the number of tables and chairs that need to be made to maximize profit. What is the maximum profit?

16. **DRAMA** On opening night of the drama club's play, they made \$1366. They sold a total of 199 tickets. They charged \$8.50 for each adult ticket and \$5.00 for each child's ticket. Write a system of equations that can be used to find the number of adult tickets and the number of children's tickets sold.

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and the minimum values of the given function.

17. $5 \geq y \geq -3$

$4x + y \leq 5$

$-2x + y \leq 5$

$f(x, y) = 4x - 3y$

18. $x \geq -10$

$1 \geq y \geq -6$

$3x + 4y \leq -8$

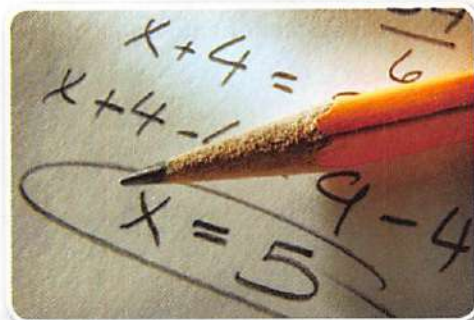
$2y \geq x - 10$

$f(x, y) = 2x + y$

19. **GEOMETRY** An isosceles trapezoid has shorter base of measure a , longer base of measure c , and congruent legs of measure b . The perimeter of the trapezoid is 58 inches. The average of the bases is 19 inches and the longer base is twice the leg plus 7.
- a. Find the lengths of the sides of the trapezoid.
 b. Find the area of the trapezoid.

Short Answer Questions

Short answer questions require you to provide a solution to the problem, along with a method, explanation, and/or justification used to arrive at the solution.



Strategies for Solving Short Answer Questions

Step 1

Short answer questions are typically graded using a **rubric**, or a scoring guide. The following is an example of a short answer question scoring rubric.

| Scoring Rubric | |
|--|-------|
| Criteria | Score |
| Full Credit: The answer is correct and a full explanation is provided that shows each step. | 2 |
| Partial Credit: <ul style="list-style-type: none"> The answer is correct but the explanation is incomplete. The answer is incorrect but the explanation is correct. | 1 |
| No Credit: Either an answer is not provided or the answer does not make sense. | 0 |

Step 2

In solving short answer questions, remember to...

- explain your reasoning or state your approach to solving the problem.
- show all of your work or steps.
- check your answer if time permits.

EXAMPLE

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

Company A charges a monthly fee of \$14.50 plus \$0.05 per minute for cell phone service. Company B charges \$20.00 per month plus \$0.04 per minute. For what number of minutes would the total monthly charge be the same with each company?

Read the problem carefully. You are given information about two different cell phone companies and their monthly charges. Since the situation involves a fixed amount and a variable rate, you can set up and solve a system of equations.

Example of a 2-point response:

Set up and solve a system of equations.

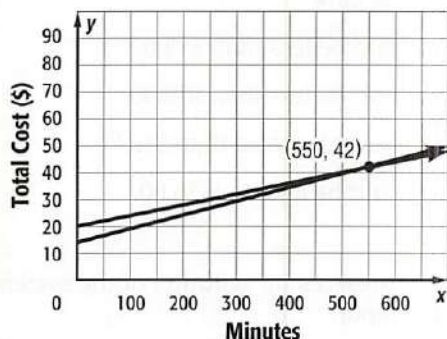
flat fee + rate \times minutes = total charges

y = total charges, x = minutes used

$$y = 14.5 + 0.05x \text{ (Company A)}$$

$$y = 20 + 0.04x \text{ (Company B)}$$

Solve the system by graphing.



The solution is (550, 42). So, with each company, if the customer uses 550 minutes the total monthly charge is \$42.

The steps, calculations, and reasoning are clearly stated. The student also arrives at the correct answer. So, this response is worth the full 2 points.

Exercises

Read each problem. Eliminate any unreasonable answers. Then use the information in the problem to solve.

1. Shawn and Jerome borrowed \$1400 to start a lawn mowing business. They charge their customers \$45 per lawn, and with each lawn that they mow, they incur \$10.50 in operating expenses. How many lawns must they mow in order to start earning a profit?
2. A circle of radius r is circumscribed about a square. What is the exact ratio of the area of the circle to the area of the square?

3. Mr. Williams can spend no more than \$50 on art supplies. Packages of paint brushes cost \$4.75 each, and boxes of colored pencils cost \$6.50 each. He wants to buy at least 2 packages of each supply. Write a system of inequalities and plot the feasible region on a coordinate grid. Give three different solutions to the system.
4. Marla sells engraved necklaces over the Internet. She purchases 50 necklaces for \$400, and it costs her an additional \$3 for each personalized engraving. If she charges \$20 for each necklace, how many will she need to sell in order to make a profit of at least \$225?

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Use the system of equations shown below.

$$\begin{aligned} 8x - 2y &= -18 \\ -5x + 3y &= 20 \end{aligned}$$

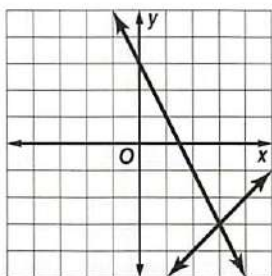
Which ordered pair is the solution of the system of linear equations?

- A $(-1, 5)$ C $(-3, -\frac{2}{3})$
B $(\frac{1}{2}, -4)$ D $(-2, 2)$

Test-Taking Tip

Question 1 If time is short, you can test each possible answer choice in the equations to find the correct answer. This might be faster than solving the system algebraically.

2. What is the solution of the system of equations graphed below?



- F $(-3, -3)$ H $(3, -3)$
G $(-3, 3)$ J $(3, 3)$

3. A bakery sells cookies, doughnuts, and bagels. Cindy bought 3 cookies, 2 doughnuts, and 1 bagel for \$4.84. Andrew bought 10 cookies, 12 doughnuts, and 6 bagels for \$25.12. Linda bought 12 doughnuts and 10 bagels for \$27.38. How much does a doughnut cost?

- A \$0.49 C \$0.69
B \$0.59 D \$0.79

4. The table at the right shows the cost of a pizza depending on the diameter of the pizza.

| Diameter (in.) | Cost (\$) |
|----------------|-----------|
| 6 | 5.00 |
| 10 | 8.10 |
| 15 | 11.70 |
| 20 | 15.00 |

Which conclusion can be made based on the information in the table?

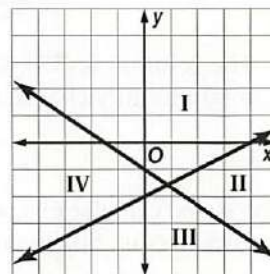
- F A 12-inch would be less than \$9.00.
G A 24-inch would be less than \$18.00.
H An 18-inch would be more than \$13.70.
J An 8-inch would be less than \$6.00.

5. Which region represents the solution of the system of inequalities below?

$$y \leq \frac{1}{2}x - 2$$

$$y \leq -\frac{2}{3}x - 1$$

- A Region I
B Region II
C Region III
D Region IV



6. Which of the following points is *not* a vertex of the feasible region for the system of linear inequalities below?

$$x \geq 0, y \geq 0$$

$$y \leq -2x + 6$$

- F $(0, 0)$ G $(0, 3)$ H $(0, 6)$ J $(3, 0)$

7. Marilyn bought a pair of jeans and a sweater at her favorite clothing store. She spent \$120, not including tax. If the price of the sweater p was \$12 less than twice the cost of the jeans j , which system of linear equations could be used to determine the price of each item?

$$\begin{aligned} j + p &= 120 \\ p &= 2j - 12 \end{aligned}$$

$$\begin{aligned} j + 120 &= p \\ p &= 2j - 12 \end{aligned}$$

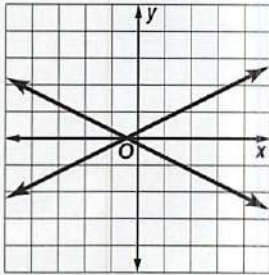
$$\begin{aligned} j + p &= 120 \\ j &= 2p - 12 \end{aligned}$$

$$\begin{aligned} j + p &= 12 \\ p &= 2j - 120 \end{aligned}$$

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. Which of the following terms does *not* describe the system of equations graphed below: consistent, dependent, independent, or intersecting?



9. **GRIDDED RESPONSE** Miranda traveled half of her trip by train. She then traveled one fourth of the rest of the distance by bus. She rented a car and drove the remaining 120 miles. How many miles away was her destination?
10. Julio is solving the system of equations $8x - 2y = 12$ and $-15x + 2y = -19$ by using elimination. His work is shown below.

$$\begin{array}{rcl} 8x - 2y & = & 12 \\ -15x + 2y & = & -19 \\ \hline -7x & = & 7 \\ x & = & -1 \end{array} \qquad \begin{array}{rcl} 8x - 2y & = & 12 \\ 8(-1) - 2y & = & 12 \\ -8 - 2y & = & 12 \\ -2y & = & 20 \\ y & = & -10 \end{array}$$

The solution is $(-1, -10)$.

- What error does Julio make?
 - What is the correct solution of the system of equations? Show your work.
11. Write the equation of the line, in slope-intercept form, that passes through the points $(0, 2)$ and $(2, 0)$.

12. **GRIDDED RESPONSE** Heather is starting a small business giving tennis lessons on the weekends and evenings. Considering equipment expenses, travel, and court rental fees, she determines that the cost of running her business will be represented by the function $C(x) = 25x + 345$. The function C represents her cost, in dollars, when she has x clients taking lessons. Heather's income from giving lessons is given by the function $I(x) = 50x - 105$. How many clients will Heather need in order to break even?

Extended Response

Record your answers on a sheet of paper. Show your work.

13. Suppose Tonya is baking cookies and muffins for a bake sale. Each tray of cookies uses 5 cups of flour and 2 cups of sugar. Each tray of muffins uses 5 cups of flour and 1 cup of sugar. She has 40 cups of flour and 15 cups of sugar available for baking. Tonya will make \$12 profit for each tray of cookies that is sold and \$8 profit for each tray of muffins sold.
- Let x represent the number of trays of cookies baked, and let y represent the number of trays of muffins baked. Write a system of inequalities to model the different number of trays Tonya can bake.
 - Graph the system of inequalities to show the feasible region. List the coordinates of the vertices of the feasible region.
 - Write a profit function for selling x trays of cookies and y trays of muffins.
 - How many trays of cookies and muffins should Tonya bake to maximize the profit? What will the total profit be?

Need Extra Help?

| If you missed Question... | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Go to Lesson or Page... | 3-2 | 3-1 | 3-5 | 1-3 | 3-3 | 3-4 | 2-4 | 3-1 | 2-4 | 3-2 | 2-3 | 3-3 | 3-4 |