Chapter 10: Technology and Distance Learning in the Teaching of Mathematics

Contents

Note to reader: The use of the non-binary, singular pronouns *they*, *them*, *their*, *theirs*, *themself*, and *themselves* in this framework is intentional.

Purpose of Technology in Mathematics Learning

As stated in the California Common Core State Standards for Mathematics (CA CCSSM), and as described throughout this document, the “learning of mathematics” is two-fold: the learning of grade-level content standards, and the learning of standards for mathematical practice. This chapter advocates for technology use which supports both the learning of meaningful mathematical content and the fostering of the productive habits of mind and habits of interaction embodied by the Standards for Mathematical Practice (SMPs). This first section describes the purpose of technology in the learning of mathematics, the second section introduces overarching principles meant to guide such technology use, and the third section provides general guidance for distance learning which is applicable, but not limited, to mathematics instruction.

Technology use in the teaching of mathematics has one primary purpose: To facilitate interactive experiences that enrich the learning of both content standards and SMPs. Given the increasing role that technology plays in commercial, societal, and cultural aims, the use of technology in educational settings is also situated within society’s aim of creating an informed and skilled populace. While introducing students to technology is certainly important in and of itself—and can even be a necessity (see the Distance Learning section)—this introduction can be accomplished in service to the primary purpose described above. In other words, this chapter provides guidance on how technology use can best support mathematics instructional objectives, rather than adjusting instructional objectives to support the use of technology.

Technological Pedagogical Content Knowledge Framework

The Association for Mathematics Teacher Educators (AMTE) published a framework in 2009 for research and guidance on best practice in the use of technology in mathematics education. Technological pedagogical content knowledge (TPACK), based on the work of Mishra and Koehler (2006), is a specialized type of knowledge and skills that enables an educator to draw upon content knowledge (knowledge of mathematics), technological knowledge (knowledge of, and facility with, relevant technology) and pedagogical knowledge (knowledge of teaching and learning strategies) as they create meaningful learning experiences for students. In short, this knowledge is the synthesis of three areas of expertise for educators: mathematics, teaching, and technology. Thus, the guidance in this chapter is directed toward establishing and increasing this type of knowledge. The figure here illustrates those relationships between these types of knowledge.

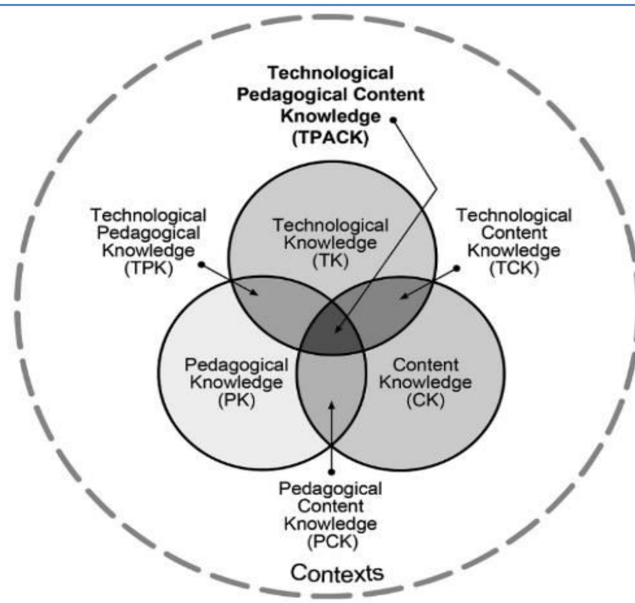


Figure 33.1 Technological pedagogical content knowledge model

Source: Koehler & Mishra (2009). Reproduced by permission of the publisher, © 2012 by tpack.org

According to the TPACK Framework, educators with robust technological pedagogical content knowledge are able to

a. Incorporate knowledge of learner characteristics, orientation, and thinking to foster learning of mathematics with technology;
b. Facilitate technology-enriched, mathematical experiences that foster creativity, develop conceptual understanding, and cultivate higher order thinking skills;
c. Promote mathematical discourse between and among instructors and learners in a technology-enriched learning community;
d. Use technology to support learner-centered strategies that address the diverse needs of all learners of mathematics; and
e. Encourage learners to become responsible for and reflect upon their own technology-enriched mathematics learning.

Principles for Technology Use in Mathematics Learning

The following principles are meant to guide effective incorporation of technology into the teaching of mathematics. This section addresses uses of technology specifically for mathematics learning; uses of technology supporting remote learning in general are discussed in a later section. While technology use varies widely across the state, and across the grade levels, these principles can serve as guideposts for districts and schools as they consider utilizing various technologies to support learning.

Principle 1: Strategic Use of Technology in a Learning Environment Can Facilitate Powerful Learning of Mathematics.

According to the National Council of Teachers of Mathematics (2015), the **strategic use of technology** in the teaching and learning of mathematics is the use of digital and physical tools by students and teachers in thoughtfully designed ways and at carefully determined times so that the capabilities of the technology enhance how students and educators learn, experience, communicate, and do mathematics. Strategic use of technology supports all students in their learning and is consistent with research on best practices in teaching and learning.

A **technology-rich environment**, when used strategically, can be a powerful tool for learning deeper mathematics. A technology-rich environment is one in which the technology serves a clearly defined pedagogical purpose (Zinger et al., 2017). In establishing a technology-rich environment for learners, three primary factors must be taken into account: access, usage, and skills (ITU, 2009). Access refers to the availability of technology for teachers and learners, usage refers to its prevalence in learning experiences, and skills refers to the knowledge level required, both for teachers and for students, to use the technology appropriately. In considering whether or not to use specific technology, each of these factors can help guide decisions. For example, if all students have **access** to a particular technology, and the teacher has the **skills** and support to enable learning which relies upon the technology, but future coursework **uses** different technology, then this difference in **usage** should be considered before adopting the technology.

By contrast, a **technocentrist** educational approach is one in which technology is considered both a means and an end (Zinger, Tate, & Warschauer, 2017). In other words, the aim of a technocentrist approach would be to train learners in using technologies with the hope that learners would use new knowledge of technologies readily outside of the classroom or in future learning situations. This approach, focused on technological learning rather than content-area learning, has been found to be ineffective in large-scale projects (Zinger et al., 2017).

Portrait of a Technology-Rich Setting

- All students have access to a particular technology intended to support specific mathematics content and practices (access)
- Teachers have knowledge about the pedagogical use of the technology—for example, through appropriate professional learning (skills)
- The lesson/task/activity relies upon the technology as an integral part of students' interactions with the content (usage)

National Council of Teachers of Mathematics Recommendations

According to the National Council of Teachers of Mathematics (NCTM), there are two types of technologies teachers can use in creating learning environments for students: **content-specific mathematics technologies** and **content-neutral technologies** (National Council of Teachers of Mathematics, 2015). **Content specific technologies** support students in exploring and identifying mathematical concepts and relationships. These include computation/visualization programs, such as Desmos or Geogebra, or virtual manipulatives or games (e.g., www.mathlearningcenter.org/resources/apps and www.mathplayground.com/), and calculation (e.g., www.wolframalpha.com/). **Content-neutral technologies**, such as spreadsheets, word processors, and drawing programs, both online and offline, help students collaborate with peers and communicate work with teachers. By using both types of these technologies, students are supported in learning mathematics content and practicing skills, as well as in developing higher-order thinking skills such as visualizing, reasoning, and problem solving.

NCTM recommends the following guidance on the strategic use of technology:

- Mathematics is the focus of instruction and drives the use of the technology. Teachers capitalize on the capabilities of technology to accomplish mathematics learning goals. As research over the years has consistently pointed out (see Reys, B. and Arbaugh, F., 2001), calculator use does not seem to hinder the learning of rich mathematics. It does hinder the learning of procedural mathematics, however, especially when that is believed to be the primary objective. In considering the use of technology, the belief that rote algorithms and procedural skills (which are easily replaced by calculators) are the most important mathematics to be learned which must be reconsidered. Students learn to negotiate the use of technology in ways that facilitate larger aims only when they are given larger aims to accomplish with the technology.
- Strategic use does not imply continuous use. Teachers should carefully consider when and how often to rely upon technology in learning experiences. Although technology mediates a major portion of each day's work in distance learning environments (see the second half of this chapter), teachers in these situations should still be mindful of over-relying on certain forms of technology when other skills are needing to be fostered. For example, students should, at times, draw lines on a coordinate grid on paper using a ruler instead of always using an online graphing system. This helps develop fine motor skills and encourages attending to precision (SMP. 6) in the same way that drawing geometry shapes by hand does. Or, in encouraging the development of number sense, teachers may wish to have students focus on mental math strategies such as "making ten" and composing/decomposing numbers. Pan balances are another example which encourage students' ability to visualize maintaining balance by "disappearing" equal quantities from both sides of a balance as a valuable precursor to solving linear equations. In simply combining terms, either using technology or not, without considering equations in their totality, students quickly lose sight of the larger aims of what they are doing much of the time. Teachers can support students by modeling deliberate use of technology only after a problem is considered thoughtfully at first.
- Teachers can meaningfully connect technology use in classroom learning to the use of technology on state and local assessments. When technology use is

interwoven with learning throughout the school year, students grow in comfort and familiarity with using technology to record and capture their thinking, and the practice time teachers provide in using the California Assessment for Student Progress and Performance (CAASPP) system becomes, in turn, more productive. In distance learning environments, assessment can take many more forms than computer-based exams. For example, students can view and compile portfolios of their work for a unit, or quarter, in their learning management system, and record a video reflecting upon their progress. Formative assessment, both in face-to-face and remote learning situations, is a powerful driver for learning, see Chapter 11: Assessment in the 21st Century and research-based Distance Learning Principle 6 on p. XX.

Sample Task: Rescue Rover (Integrated Math II/Mathematics: Investigating and Connecting II)

To teacher:

This activity promotes understanding of G-SRT-8, F-BF-1, A-REI-4, A-CED-1, 2, 3, and SMPs 1, 3, 4 and 5 as well as the NGSS Science and Engineering Practices. Students are to work in teams of four. Teachers should be mindful of students' personalities and work habits when assigning them into groups in order to achieve effective cooperation and collaboration. There may be a need to frontload some additional scientific vocabulary, such as geology or technology terms.

To students:

You are working with a team on a mission involving the remote collection of scientific data from the surface of Mars. There are two active rovers, Molly and Dixie, that are collecting soil samples, atmospheric data, and collecting evidence of past organic material.

[Incoming]: You suddenly receive a distress signal!

Dixie was moving around a rock outcropping and accidentally dislodged a boulder, pinning Dixie against the rock face. If this wasn't bad enough, Dixie's nuclear power supply was also damaged and is emitting radiation with increasing intensity. This radiation will eventually melt Dixie's internal wiring unless the emergency release panel can be removed. In Dixie's position, the rock face is blocking access to the panel. The situation is truly dire for Dixie!

Fortunately, the other rover, Molly, is across the field and could potentially reach Dixie in time to open the casing and remove the power supply, using Molly's robotic tool armature.

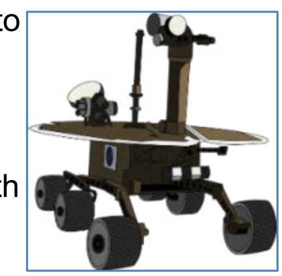
Your Team's Tasks:

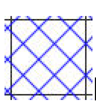
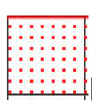
- Your team's first task is to determine exactly what the team goals are for this situation.
- Then using the map below, list what information your team will need to help achieve these goals?

(Note: Each different pattern represents a different terrain on Mars)

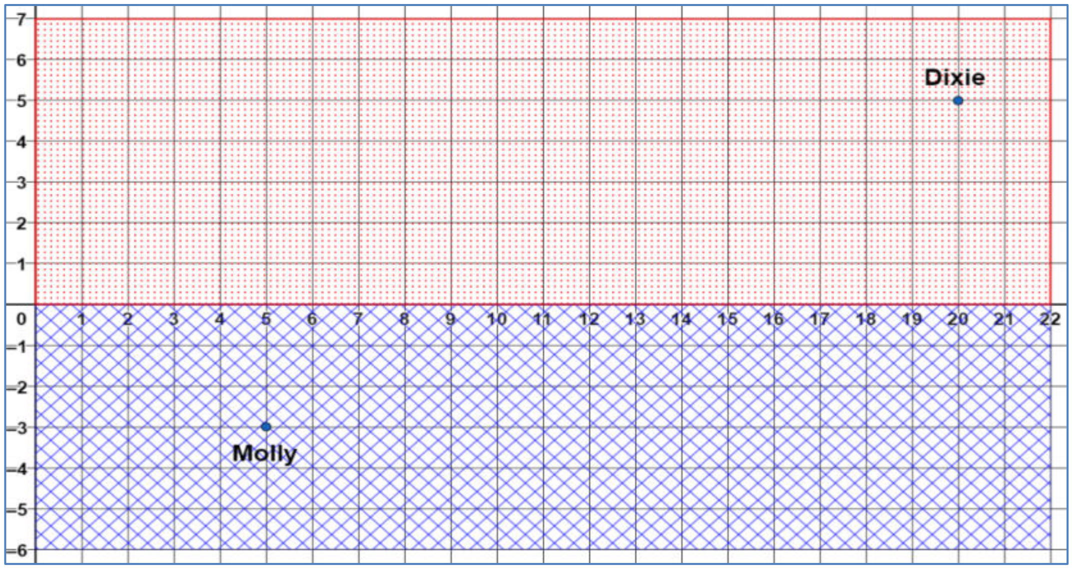
Additional specifics to the mission:

There are two main terrains that Molly will be moving through to get to Dixie. The first is fairly firm bedrock where Molly can average 20 km/hr. The second surface is rugged gravel and the rover will only average 10 km/hr. Work with your team to clearly plot out Molly's path and determine the amount of time it will take to get Molly to Dixie. Every minute counts!



1. Fairly Firm Bedrock Rugged Gravel

   Legend: Each vertical and horizontal unit represents 10 km.



Show all your work and explain your reasoning below.

a. What is the least amount of time that it takes for Molly to reach Dixie? Explain the evidence which supports the shortest time duration that your team found.
b. Is the shortest distance to the settlement a straight path between the two? Explain how you know.
c. Is the shortest distance always the fastest possible path? Explain your reasoning.
d. Your team would like you to automate this process in case one rover needs to reach another. Describe what parts of your solution process could be programmed. What parameters would there be?

Implementation:

After students spend some time grappling with the problem they can begin to understand that in picking different points on the x-axis at which Molly crosses from one terrain to the other, there will be different time durations for the entire journey. So, in order to minimize the time, they must find the point on the border between terrains which gives the least time.

Technology to the Rescue:

Because calculating distances using the distance formula and dividing by Molly's rate to find the time for that leg of the journey is tedious work by hand for each point chosen, students can divide this work up in their teams by having each team member calculate the total time for a point of their choosing, and another team member can pick a different point, and so on. But what if there were three types of terrain, or five, or 10? The case for using technology to automate these processes is easily made at this point, and is at the heart of NCTM's first recommendation: Mathematics is the focus of instruction and drives the use of the technology. Teachers capitalize on the capabilities of technology to accomplish mathematics learning goals. Because optimizing for time involves some fairly complicated calculus, the use of technology can enable students to automate their processes and find the minimum distance.

An open-source software program which enables modeling of blended algebra and geometry problems is Geogebra, which works in browsers and as an app. The teacher can encourage students to set up the diagram on Geogebra, with points at the locations of Molly and Dixie. They can then place a free point on the border and use the distance function to output the total distance. Using the algebra command lines, the time for the first "leg" (from Molly to border point) can be programmed in as the quotient of the distance divided by Molly's rate in the bedrock. Similarly, the time for the second "leg" (from border point to Dixie) can be found. Then these times can be totaled to find the total time, and this total time can be adjusted by grabbing and moving the border point along the border until the minimum time is found. Thousands of calculations in an instant! Students can discuss and explore variations on this design, such as different initial starting points for Molly and Dixie, various other terrains and different rates of travel, elevation changes, and others. In this way, students have a different understanding of the importance of the key mathematical relationships between time, distance, and speed since they must use these relationships in creating their model to answer questions.

Principle 2: Support for Teachers of Mathematics Accompanies Use of Learning Technologies.

Supporting teachers in their pedagogical development is the most critical part of effecting positive change in students' learning experiences. This chapter recommends the adoption of technology only when it is accompanied by changes to teaching practices which make the technology an integral and sustained component of the instruction, and when ongoing support can be provided to teachers.

Administrators play a pivotal role in supporting teachers as they explore, adopt, and implement new technologies in their instructional practice. Introducing technology use into students' learning experiences requires consideration of a school's mission, values, and budget; and active communication amongst school and district personnel. Below are guidelines to inform administrators and policymakers in state, county, district and school offices as they support teachers in strategic uses of technology:

- Adoption of technology occurs only when it is accompanied by changes to teaching practices which make the technology an integral and sustained component of the instruction, and when ongoing support can be provided to teachers. Technologies can have short half-lives; restraint should be exercised in adoption of popular technologies in the moment.
- Time is provided to teachers to explore particular technologies to learn, reflect upon, and integrate technology into learning experiences for students. This is critical for all technology, as it is a hard lesson to have technology fail to work, or work improperly, at the point of students' experiences with it. Delays, pauses, system updates, and the like can sabotage momentum in the flow of instruction.
- Technology support for teachers is ongoing and readily available. This support can take the form of workshops, peer collaboration, conference attendance, virtual meetings, but of critical importance is that this time be provided and incentivized. In particular, the encouragement and support of peers is of great benefit as teachers expand their knowledge of strategic technology use.
- Effective professional development focused on the use of technology in mathematics learning is differentiated, reflecting the multitude of knowledge and comfort levels that teachers have with regards to technology. A successful plan for professional development recognizes that for teachers to learn to use technology in ways that enhance and increase student learning, they must go through "a process of entry, adoption, adaptation, appropriation, and invention as they navigate through the integration of technology in their classrooms" (Zinger, 2017).
- In order to not overwhelm teachers, and in deference to the multitude of knowledge and comfort levels they have, training should focus on one tool, or aspect of one system, at a time. After teachers are given opportunities to

implement in their classes, then further tools can be introduced (Warschauer, 2014).

- Professional development includes specific criteria for teachers to rely upon as they select worthwhile applications, games, or other software that can accomplish learning objectives.

Principle 3: Learning Technologies Are Accessible for All Students.

Technology use in mathematics classrooms must contribute to making the mathematics community more equitable. Thus, administrators and teachers must give special attention to issues of access when designing instructional uses of technology. In general, a key consideration is in exacerbating the “digital divide,” the gap in knowledge and skills between populations of students which have access to technology, through wealth and privilege, and those that do not. Reducing, rather than widening, the divide should be an effect of well-designed uses of technology in schooling. For a particular technological tool, other considerations include:

- The linguistic or cultural assumptions embedded in the technological tool under consideration. Is the tool designed with a particular student profile in mind, thus disadvantaging students who don’t fit that profile? If so, another tool should be found or the existing tool modified to address these issues.
- Differences in prior exposure to related technology—perhaps necessitating different supports for different students. Appropriate and equitable supports must be provided to provide equal access for all.
- Providing the necessary classroom materials for technology use, both hardware and software.
- Providing initial and ongoing technology support that is readily available to students, even in rural and remote settings. Technology can be used as a vehicle to better understand the students, their interests, and other culturally-relevant information, as it relates to equity. For example, polling students can provide teachers with immediate information regarding their students’ interests, thus enabling teachers to vary activities which can then engage the interests of more of their students.

- Allowing for widely varying levels of internet capabilities and connection speeds among students and their families, including limited internet rural internet capabilities and potential outages/interruptions.
- Aligning technology use to create equitable learning experiences using assessment platform technology. For example, affording class time for students to become familiar with the Smarter Balanced system used in the administration of the CAASPP (https://www.cde.ca.gov/ta/tg/ca).

Vignette: Polygon Properties Puzzles (Grade 4)

Students in Ms. Thompson's grade-four class have been exploring the attributes of polygons. They have compared and contrasted physical models and illustrations of polygons, attending to features such as angle size, number of sides, and whether the figures have any parallel or perpendicular sides. Lessons have included polygons that students view as "typical" as well as atypical examples. Today, Ms. Thompson will ask her students to show their understanding by drawing polygons that meet specific criteria. She has adapted five challenges from *About Teaching Mathematics* (Burns, 2007). Students will illustrate the figures using technology, specifically Whiteboard. Some of the standards addressed in the lesson include:

- SMP.1, 3, 5, 6, 7
- Content Standards: 3.G.1; 4.G.1, 2, 4.MD.5; 5.G.3, 4
- English Language Development standards: PI.1; PI.2; PI.3; PI.4; PI.5; PI.9; PI.12

Ms. Thompson is deliberate and selective in the use of technology. She plans to use Whiteboard for this lesson as she finds it can facilitate the use of mathematical practices and increase focus on the mathematics content. Her expectation is that this use of technology will:

- reduce the challenge of drawing straight lines by using Whiteboard's "line" tool
- encourage collaboration and discourse between partners who are sharing one Chromebook, and later, among the larger group

- support students who are English learners (ELs) and students with disabilities in accessing the tasks and finding meaning in their learning
- increase engagement for the many students who are enthusiastic users of technology
- foster growth mindsets and promote the correction of errors and revision of work in progress
- enable the class to see and compare various student products in a highly visible, large scale format via Google Casting or the link sharing within Whiteboard
- use class time efficiently, allowing for full discussion and analysis
- serve as a quick way to engage in the formative assessment process as student work is instantly transmitted to the teacher’s view.

Ms. Thompson has used Google Classroom (and other learning management systems) and Whiteboard (by the Math Learning Institute: https://apps.mathlearningcenter.org/whiteboard/) often in the past for lessons. These students have worked in collaborative groups for several months, sharing and explaining their thinking digitally. They know how to share their work using links or the share code and posting them into their assignments on Google Classroom. The class has established effective collaboration protocols (e.g., stay on your own page, let everyone speak, do not delete others’ work, add to someone else’s thinking, everyone has equal access to the tool). Students are arranged in four-person table groups. They know how to partner up and then switch partners in their table group quickly. The class has a system for Chromebook management: one partner is responsible for getting two Chromebooks out before the morning meeting; the second partner returns the devices to the charging station during afternoon clean-up time.

The teacher considered language barriers and the needs of individual students as she planned partners and heterogeneous groups. Ms. Thompson has 12 ELs in this class. To support their learning, she:

- has placed one Emerging EL student with a language proficient Spanish speaking student to help with translations and collaboration

- will create and display sentence frames for this student to use during discussion and collaboration
- will provide her seven ELs who are at the Bridging stage and her four ELs who are at the Expanding level, with sentence stems to support them as they discuss and explain their thinking
- has paired a student with an **Individualized Educational Plan (IEP)** for reading with a student who can help them access the written material
- situated two students who have IEPs for math with partners who are supportive and able to share the work equitably and inclusively

In this lesson, students will use a now-familiar classroom routine, “Convince Yourself, a Friend, a Skeptic.” They will:

1. Solve each problem with a partner (convince yourself)
2. Justify their mathematical argument to the other pair in their table group, who will ask questions and encourage further explanation (convince a friend/pair)
3. Prepare to convince the class, who will challenge and probe any inconsistencies (convince a skeptic).

Ms. Thompson begins the lesson by focusing attention on an image the class explored the day before: a square that is not oriented on the horizontal. She asks partners to describe the figure using precise mathematical terms, as they did in the previous lesson.

Students offer many of the terms that emerged in the earlier lesson, which Ms. Thompson records for the class: *square, rectangle, tilted square, diamond, right angles, square corners, parallel sides, perpendicular, equal side lengths.* Several students raise their hands to challenge the term “diamond,” arguing that it is an informal term and that “*a square is still a square, even if it is tilted!*” Ms. Thompson comments that students have shown they could convince others and could take the role of skeptics; she encourages them to continue to attend to the properties of polygons in today’s lesson, too.

Ms. Thompson tells the students this time, they will share one Chromebook with their designated partner, using Whiteboard to illustrate a series of polygons with particular properties. This causes excitement among her students; almost all are enthusiastic about using Whiteboard and working with their partners.

Ms. Thompson tells the class that they will draw a series of polygons that include specific properties. As she posts each one, students will read the task aloud together, and then think quietly about how they might draw the figure. Once they have an idea, they should show a “thumb up,” to signal that they are ready to start work on the Chromebook. After partners solve each problem, they must convince the other partners at the table, and plan to explain and justify their thinking in the whole-class “skeptics” discussion.

Ms. Thompson posts Task 1: “Make a triangle with one right angle and no two sides the same length.”

The class reads the statement aloud twice, carefully and slowly. Ms. Thompson signals for quiet thinking, and watches as students begin responding with thumbs up. When she is satisfied that partners are ready to begin, she invites them to start illustrating on Whiteboard.

As anticipated, students are successful and confident on the first task, having had ample practice exploring triangles of various types. Ms. Thompson displays four responses for the class to consider, selecting examples that are oriented differently. Students express surprise about how many different ways the figure can be drawn and still meet the requirements. Ms. Thompson asks students to talk with their partners, using the sentence frames as necessary, in their role as skeptics, and be ready to question, challenge, or probe any inconsistencies they note in the triangles displayed. After a few moments, a few questions/challenges are posed:

- How can we tell if C has a right angle, when it’s “lying down” like that?
- Is B really a right angle triangle if the right angle is pointing to the left?
- Convince us about D, too! It’s pointing to the left!

Ms. Thompson invites the partners whose images are being questioned to respond. In two cases, students ask if they can measure side lengths to assure that they are all different. Ms. Thompson allows the class to reach consensus independently, agreeing that all four examples are right triangles with three sides of different lengths.

Ms. Thompson presents Task 2: “Make a triangle with exactly two congruent angles.”

The procedures from the first task are duplicated here: read aloud, pause to think, then collaborate with a partner—but this time the second partner is the lead illustrator.

Ms. Thompson circulates, stopping beside her Emerging EL student and partner to listen. To provide support for but not single out her EL student, she asks the pair to draw or use hands to demonstrate what is meant by “congruent” angles. A brief exchange assures her that the partners are working effectively; she reminds the pair to rehearse how they could defend their illustration to their table partners and the class. Several student pairs are discussing congruence as she moves through the groups; some referring to their journals or the word wall listing mathematics terms. In quick check-ins with the remaining groups comprised of EL students, Ms. Thompson notes that two of the Bridging students are letting their partners do most of the talking; she reminds students of the classroom norms for related to “equal voices,” then engages with each pair in ways that engage the quieter students. After instilling this balance, she encourages each, noting that partner time is a time for safe practice. Before leaving each group, she reminds the students that what she is heard is worth sharing when the time comes to discuss with the class, inviting her EL students to reiterate for their peers what they developed in pairs.

When Ms. Thompson posts several students’ illustrations, she includes an example with 3 congruent angles, not “exactly” two; as the task specified. This non-example promotes energetic discussion and respectful challenges from friendly skeptics.

The class continues with two more tasks:

- Task 3: “Make a four-sided polygon with no parallel sides”
- Task 4: “Make a four-sided polygon with one right angle and all sides different lengths”

As Ms. Thompson circulates, encourages, and listens intently, she acquires insights into students’ understandings and strengths, and uncovers a few misconceptions. She notes with satisfaction that students are actively using mathematical practices, in particular, SMPs 3 and 6. These observations guide her as she orchestrates the skeptics' discussion for each task.

Ms. Thompson will use students' responses to the final task, an exit ticket, as a formative assessment. She has designed two exit tickets so that each student can express and share their own understanding independently rather than with support from their partner.

She tells the class, rather than repeating the "Convince Yourself, a Friend, a Skeptic" routine, they will respond independently. Each student may choose to respond using paper and pencil, or the Whiteboard. Those who respond digitally share their work via the link sharing button and post it into their Google Classroom assignment. The paper copies are collected.

The exit ticket tasks involve concepts of parallel sides and angle measurement, which are key understandings in the grade four standards (4.MD.5, 6; 4.G.1,2).

Task 5:

a) Make a four-sided polygon with no right angles but with opposite sides parallel
b) Make a four-sided polygon with at least two angles greater than 90°.

As she reflects on the lesson, Ms. Thompson notes:

- Whiteboard's immediacy expedited the students' creation, and the teacher's selection and presentation, of work samples
- images were large, detailed, and easily viewed by all students
- with few exceptions, students were engaged throughout the lesson
- all students were able to use the technology to make their own polygons
- partners shared the use of the device smoothly
- the level of challenge was appropriate for almost all students
- three of the seven ELs who are at the Bridging stage were willing to speak with their individual partners, but remained quiet in table and whole-class discussions
- two of the four ELs at the Expanding level justified their reasoning confidently during the whole-class discussion.

During the next lesson, Ms. Thompson will create an opportunity for students to correct any misunderstandings that were revealed, as well as solidify their learning by sharing and analyzing examples of Task 5 illustrations.

[Note: The following sections, aside from the Desmos vignette, were primarily taken from the California Department of Education's Distance Learning website and will be tailored to mathematics teaching and learning in coordination with the Sacramento County Office of Education's Distance Learning work, projected to be completed in April, 2021.]

Distance Learning

Distance learning presents a unique set of challenges and opportunities for students, parents, teachers and schools. Technology plays a vital role in facilitating meaningful learning of mathematics within a distance learning format. It is important to develop structures that continue to place students at the center of learning, while also being mindful of the varied contexts of at-home supports. This section is adapted from the guidance in planning, implementation and evaluation of online instruction from the California Department of Education (CDE) Distance Learning web page at https://www.cde.ca.gov/ls/he/hn/guidanceplanning.asp.

While nothing will replace the invaluable connection developed through in-person teaching, there are best practices local educational agencies (**LEA**s) may consider in order to maximize instructional time. It is important to consider utilizing the time educators spend directly interacting with students to be focused, planned, and designed to further student learning goals. Learners will need opportunities for guided learning with an educator, as well as opportunities to work with peers, families, and community members to apply their learning and practice their skills. This guidance is not all-encompassing as instructional time can be a nuanced area. These suggestions are recommended best practices and do not constitute legal advice or a legal service.

Common Definitions

The definitions below are designed to provide a common understanding of the various models of learning and their unique distinctions and to avoid the common error of applying terms interchangeably. It is important to note that not all distance learning requirements outlined in the statute are included in this document. Readers should consider the CDE’s Frequently Asked Questions (https://www.cde.ca.gov/ci/cr/dl/distlearningfaqs.asp) and additional guidance documents as they plan for and engage in distance learning to ensure all requirements are met.

Distance Learning. Instruction in which the pupil and instructor are in different locations and pupils are under the general supervision of a certificated employee of the local educational agency. Distance learning may include but is not limited to all of the following:

- Interaction, instruction, and check-in between teachers and pupils through the use of a computer or communications technology
- Video or audio instruction in which the primary mode of communication between the pupil and certificated employee is online interaction, instructional television, video, telecourses, or other instruction that relies on computer or communications technology
- The use of print materials incorporating assignments that are the subject of written or oral feedback (*Education Code* [*EC*] 43500[a])

Blended Learning. Combination of in-person and distance instruction.

The below terms are used throughout the document as we continue to discuss ways in which LEAs offer high-quality distance learning in accordance with *EC* Section 43503.

In-Person Instruction. Instruction under the immediate physical supervision and control of a certificated employee of the local educational agency while engaged in educational activities required of the pupil.

Synchronous Learning. Synchronous learning takes place in real-time,” with delivery of instruction and/or interaction with participants such as a live whole-class, small group, or individual meeting via an online platform or in-person when possible.

Asynchronous Learning. Asynchronous learning occurs without direct, simultaneous interaction of participants such as videos featuring direct instruction of new content students watch on their own time.

Time Value. Instructional time for distance learning is calculated based on the time value of synchronous and/or asynchronous assignments made by and certified by a certificated employee of the LEA. Time value for distance learning is different than time value used previously in independent study programs which include an evaluation of the time value of work product.

Distance Learning assignments can include assigned instruction or activities delivered through synchronous or asynchronous means. Synchronous opportunities may include full group instruction, peer interaction, and collaboration, two-way communication, small group breakouts, or individual office hours. The delivery method should match the purpose of the current learning outcome, corresponding task, and program placement (i.e., Language Acquisition Program). At times it may be appropriate for new content to be delivered asynchronously utilizing synchronous time for peer interaction, small group breakouts, or individual office hours. Inversely, at times content may require synchronous opportunities to include direct instruction on new content. All modes should provide students a means of checking for understanding and progressing based on that understanding. For English learners, this means of checking for understanding should include opportunities to have oral conversations to elaborate on the language necessary to articulate what is understood and ask questions for clarifying what is not fully comprehended. For students with disabilities, instructional time may be determined by the IEP team, as instructional delivery should be appropriately adapted to the unique needs of the student. Additionally, instruction and activities should be aligned to learning objectives and goals specified in the IEP.

There is an opportunity for staff to develop integrated lessons to maximize instructional time. An example might include integrating science and math standards in a performance task. Educators will need to support the development of independent learning skills through modeling, scaffolding, student conferences, feedback, and reflection. Although family support is important, applied learning experiences should take into account that many families will not be able to provide extensive support. During time allotted for applied learning, it is important for an adult to be available for questions. Educators will need to be especially in tune with language needs and provide sufficient language scaffolds to ensure the student can participate fully in the development of content and the development of the necessary linguistic structures to meet the language demands of the lesson. Integrated ELD is critical for English learners’ access to the material and should be an integral part of the lesson planning and delivery process in all subjects. Structures may need to be in place to provide support for students that may not have an English-speaking adult or family member to support applied learning. Consider using expanded learning staff or other staff from community-based organizations to support individual students or learning pods of students. Collaboration with families can be especially important when developing learning opportunities for students with disabilities, particularly students with extensive support needs. Gauging the needs of the family in supporting the student will be key to ensuring successful student outcomes. LEAs are encouraged to engage service providers and paraprofessionals in collaboratively supporting the students and family to ensure meaningful access to learning opportunities.

The ratio of synchronous and asynchronous and the sequence of these chunks of instructional time will depend on the course structure, instructional methods, access to technology tools, student needs, and whether learning is taking place entirely online or if the class is using a blended model. As such, it is important to emphasize that these two types of instructional time do not need to be chunked or sequenced in any particular way. For example, a teacher may choose to have students spend an estimated 30 minutes independently reading a text to prepare for a 20-minute, teacher-facilitated, synchronous discussion, followed by a 30-minute group research task, and then another -minute check-in discussion. Some English learners may need materials in the primary language to support their independent learning. Parents may need guidance as to how to support their child to enhance and support the student using these materials.

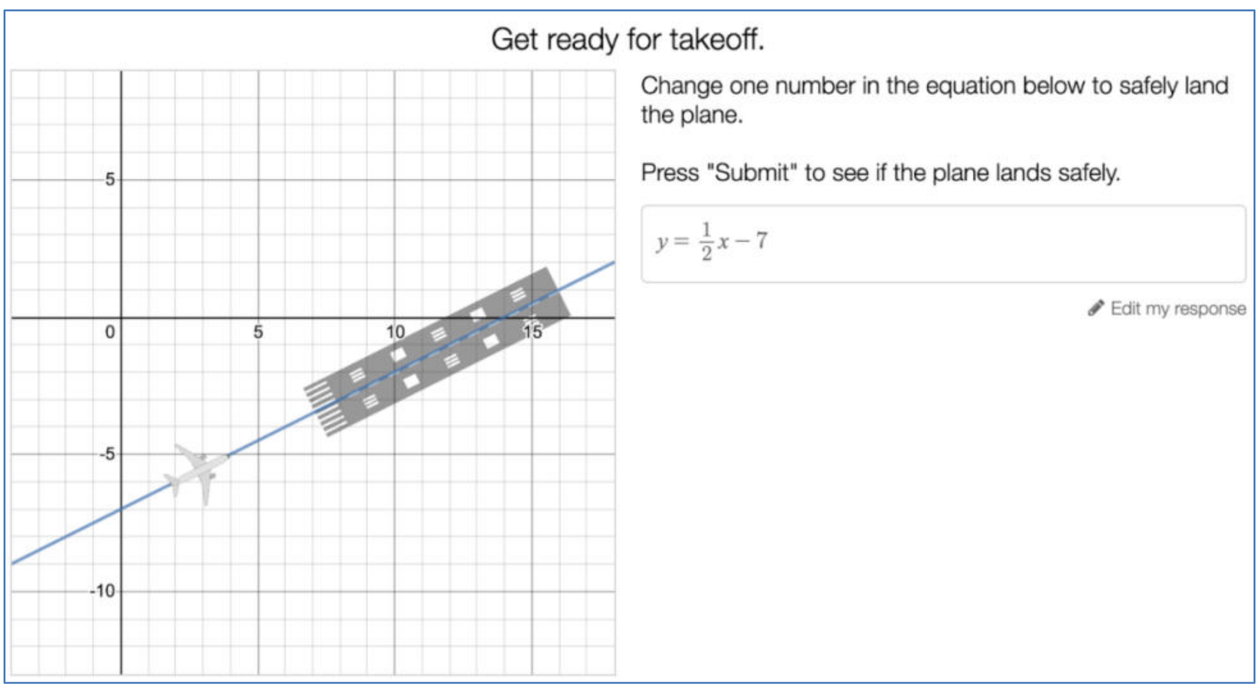
In the context of a multilingual program, instructional minutes in each language should be aligned to the percentage of minutes dedicated to that language based on the program design. For example, if 80 percent of the instructional minutes in a dual-language immersion program are dedicated to Spanish, then 80 percent of the 230 instructional minutes in a third-grade classroom should be dedicated to Spanish instruction and interaction.

Snapshot: Landing the Plane (Grade Eight)

This snapshot illustrates a use of technology to provide access for all students to sense-making mathematics, in remote or in-person settings.

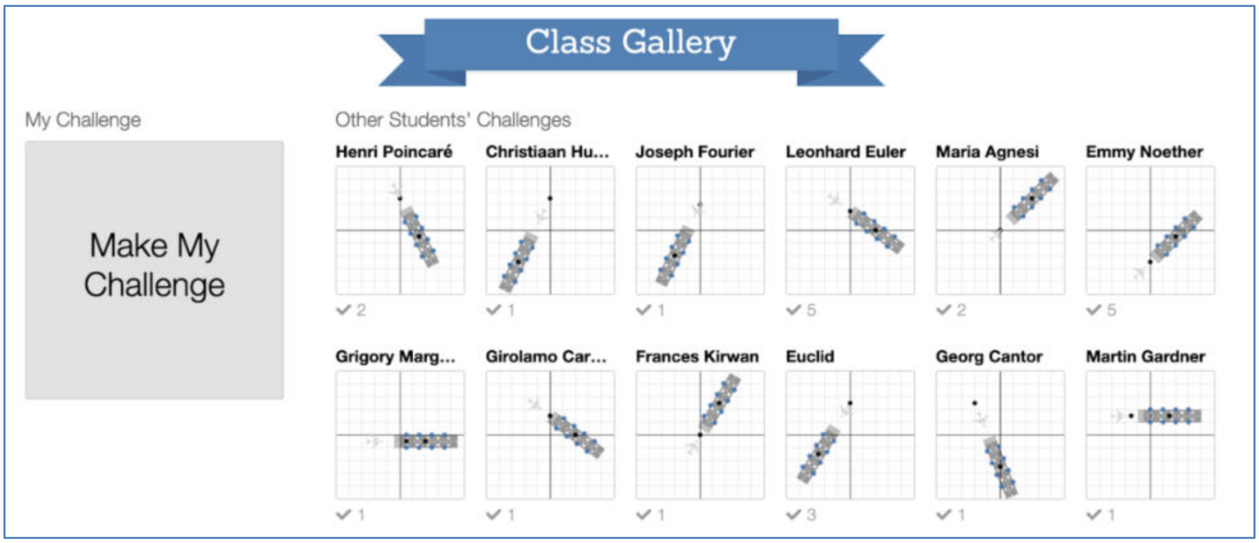
During distance learning, Ms. Trejo and her grade-eight students have less than half the synchronous learning time they had last year. She is planning strategically, trying to understand how best to use the time they're together and the time they are apart. She also values math that is conceptual and math that is learned through social interactions between students. The physical distance of school closure, and the tendency of computer-based mathematics to isolate students behind a monitor, puts both of those goals at great risk. Ms. Trejo decides that a Desmos activity called *Land the Plane*, should work as well in her current remote-instruction setting as it has in her in-person past instruction.

Using her classroom learning management system, she invites students to work asynchronously Land the Plane (https://teacher.desmos.com/activitybuilder/custom/582b81f4bf3030840aacf265). The activity asks students to plot the linear equation of a plane so that it lands on a runway. Students can work on much of the activity by themselves because the activity gives them *interpretive* rather than *evaluative* feedback. Instead of seeing "right" or "wrong" as their feedback, students see the plane travel along the graph of whatever linear equation they plot. They learn from that feedback and try again.



During their limited synchronous time, Ms. Trejo focuses her and her class's energy on some questions that computers *can't* interpret or evaluate. In one case, students are presented with two hypothetical students' linear equations and are asked to argue in favor of one. Ms. Trejo uses the Desmos Snapshot tool to select and (anonymously) present unique student answers, and then invites students to discuss the strengths and weaknesses of those answers.

During this synchronous time, she also asks students to participate in the "Challenge Creator," an activity where students create runway challenges for their classmates to solve.



She thinks this activity is best when students can discuss and debate each other's challenges, and sure enough, students debate in the chat which challenges were too easy, which seem impossible, and how many tries they took to solve the hardest ones, all while learning that they themselves can be authors of rich math questions, not just teachers and textbooks.

Distance learning has put many of Ms. Trejo's pedagogical and mathematical goals at risk, but she has found digital tools that enhance, rather than undermine, her students' mathematical connections and creativity.

Research-Based Distance Learning Principles

Research on effective distance and blended instruction can provide helpful principles for educators. It is useful to know that well-designed online or blended instruction can be as or more effective than in-classroom learning alone. While many worry that distance learning is necessarily less effective than in-person learning, many studies show that well-designed distance learning that has the features described below is often more effective than traditional in-classroom learning alone (US Department of Education, 2010; see also Policy Analysis for California Education, 2020). Key elements include:

1. A strategic combination of synchronous and asynchronous instruction: Combining synchronous activities where students meet regularly online (or in-person) with their classmates and teachers, with asynchronous activities where students think deeply and engage with the subject matter and other students independently are more effective than fully synchronous on-line courses.

   Synchronous time should be set for reasonable amounts of time, punctuated with other activities to avoid attention fatigue. It can be used for short mini-lectures and for many kinds of student to student and student to teacher interaction as described below. Many students also benefit from synchronous individual or small-group support in addition to whole-group distance instruction.

   Asynchronous time can provide an opportunity for students to gain exposure to concepts prior to engaging in synchronous time or as a follow up to dive more deeply into concepts that have been introduced through independent activities such as reading articles, watching videos or software-based presentations with voiceover, or completing modules online. Teachers can also use these asynchronous modules to provide targeted scaffolding or essential background information for those students in need of extra support in a particular area.

2. Student control over how they engage with asynchronous instruction: Research shows that students do better when they can go at their own pace, on their own time, when they have some choice over the learning materials to use and the learning strategies that work best for them, and when materials are set-up to enable them to engage deeply and critically with course content by managing how they use videos or print materials. As one successful on-line teacher explains:

   > Rather than assigning only worksheets or reading questions that can often lead to frustration and disengagement, offer students approaches that are universally designed so they can build and apply knowledge based on their interests and readiness levels. For example, provide a recorded lecture, two or three videos, and two readings about the topic. The students can listen or watch the lecture and then choose to complete a combination of

the remaining content options. Provide links to reading assignments at different reading levels so that all students find a path to comprehension, with tools like Newsela, Rewordify, News in Levels, and more. Give two or three choices for completing a task, such as writing, recording a video, building a slide deck, using a game-based education platform to demonstrate math concepts, or historical and literary events, through building. Allow students to upload their work onto the classroom learning platform to share with peers.

3. Frequent, direct, and meaningful interaction. The more interaction students have with other students, with their teachers, and with interactive content, the stronger the learning gains. In online learning environments where there is little student-student, student-instructor, and student-content interaction, students often become disengaged. Activities such as experiments, debates, data analysis, and groups solving challenging applications together can serve to synthesize and extend student knowledge. Students can interact with peers and the teacher in multiple formats – whole group and small group discussion in synchronous instruction (for example in zoom breakout rooms), chat rooms and discussion boards that may be synchronous or asynchronous, quick polls and votes followed by debate and discussion are all means to improve engagement, and create positive effects on learning gains, as do interactive materials.

4. Collaborative learning opportunities. Opportunities for students to engage in interdependent cooperative learning are important and improve achievement. Teachers can structure learning opportunities that encourage collaboration by accommodating flexible grouping options for completing work and by setting class norms for collaborative activities. This includes group engagement in shared projects and presentations as well as smaller daily activities. Small groups can work on tasks together during synchronous time in breakout rooms and then return to share their ideas. Asynchronous tasks can also be structured to offer opportunities for students to collaborate and build learning together, for example through discussion boards and by providing peer feedback. Students can pursue

projects in asynchronous time by being taught to set up their own collaboration in on-line platforms.

5. Interactive materials. High-quality distance learning incorporates the use of interactive multimedia materials, typically during asynchronous learning. For example, researchers found that 8th-grade students whose teachers integrated the use of the Pathways to Freedom Electronic Field Trips—an online collection of interactive activities designed by Maryland Public Television—in their teaching about slavery and the Underground Railroad, outperformed those who had the same unit without these materials. Fifth-grade science students who used a virtual web-based science lab, allowing them to conduct virtual experiments while teachers observed student work and gave feedback online, outperformed those who did an in-person science lab. Elementary special education students across 5 urban schools who used a web-based program supporting writing in action by prompting attention to the topical organization and structure of ideas during the planning and composing phases of writing outperformed those who had the same materials in hardcopy in the classroom (US Department of Education, 2010).

6. Assessment through formative feedback, reflection, and revision. Formative assessment is very important in online and blended learning, and it promotes stronger learning when it provides feedback that allows students to reflect on and revise their work. For example, researchers found that students performed better when they used a formative online self-assessment strategy that gave them resources to explore when they answered an item incorrectly. Similarly, students who received quizzes that allowed them the opportunity for additional practice on item types that had been answered incorrectly did better over time than those who received quizzes identifying only right and wrong answers. Studies have found positive effects of a variety of reflection tools during on-line learning, ranging from questions asking students to reflect on their problem-solving activities to prompts for students to provide explanations regarding their work, student reflection exercises during and after online learning activities, and learning guidance systems which ask questions as students design studies or conduct other

activities that support students' thinking processes without offering direct answers (US Department of Education, 2010).

7. Explicit teaching of self-management strategies. Students who receive instruction in self-regulation learning strategies perform better in online learning. Teachers can help students with tools that help them schedule their time, set goals, and evaluate their own work. They can also provide checklists that are readily available to students and parents that break out the steps for task completion to help them understand the scope of the work and the milestones they'll accomplish along the way.

Ensuring Support for Distance Learning

As districts prepare for the start or restart of a distance learning program, it is important they consider: ways to engage and support families and staff, the utilization of common tools, and the identification of success criteria. For students with disabilities, LEAs should plan for how IEPs can be executed in a distance learning environment. It is also important to establish the ELD program expectations, schedules, and guidance as to how to make sure both designated and integrated ELD is provided consistently throughout all subjects.

Preparing Families and Staff for Distance Learning

In order to ensure parents and staff (including community partners where applicable) feel comfortable and prepared to engage in distance learning, it is important to solicit feedback, understanding their experience in distance learning, if any, as well as offering multiple opportunities to discuss expectations and engage with technology in a low-stakes setting. It is important to engage with parents in the language which is spoken in the home.

Focus	Considerations
Understanding context	• Students Perspectives on Distance Learning [Survey (in multiple languages) or focus groups]

	○ What did you like best about distance learning? ○ What part of distance learning was the most challenging? ○ If you could do one thing to improve Distance Learning what would it be? • Teacher Perspectives (Survey or focus groups) ○ What worked well in distance learning over the spring? ○ What were some of the biggest challenges? ○ What do you need to be successful in distance learning in the fall? ○ Were you able to support various types of student needs including English learners, students with disabilities, foster youth, socioeconomically disadvantaged youth, etc.? ○ What strategies did you use to provide integrated and designated English language development (ELD)? Where do you need additional assistance? • Parents (Survey or focus group with appropriate translations) ○ What worked well with distance learning? ○ What was the most difficult? ○ How would you improve distance learning to better support your child? ○ What support would you like/need as Distance Learning continues? ○ Was the information provided in a language and manner accessible to you and your family?
During the initial opening/ reopening	• Considerations for Staff ○ Offer professional development on a common digital platform by site (See Common Tools Below).

	- Support a common use of platforms. Example: If using google classroom, are all teachers logging homework in the same place? - Ensure all staff are informed of students with disabilities (SWDs) current IEP and 504 accommodations and if concerned who to contact to discuss supports needed in Distance Learning - Provide professional learning on integrated and designated ELD in the distance learning context and ensure that all staff are aware of the requirement that both integrated and designated ELD are provided to English learners. - Provide professional learning on dual language instruction in the distance learning context and ensure that all staff are aware of the instructional minute requirements and plan for language use schedules to ensure language models continue as designed. - Plan for a schedule of agreed-upon times of IEP meetings to ensure all team members are available to be present. - Collaborate with the IEP team to schedule services for students within the agreed-upon instructional minutes schedule. • Considerations for Parents/Students - Over the course of a week consider offering opportunities for 1:1 meetings or meetings in groups with parents and students. It may be helpful to offer evening options for parents that work full-time. Ensure that interpreters are available for parents who speak languages other than English to the extent possible.

	○ Review the digital platform with the student and parent. ○ Discuss the rhythm of learning that will be established: Where and/when is work posted? How do they submit assignments? ○ Ask parents about the best form of communication and feedback loop.
On-going	• Considerations for Staff ○ Offer tiered (ranging from beginning to mastering) professional development opportunities for staff to continue to build their capacity in areas to support distance learning such as learning platforms, engaging strategies, or tools and resources. ○ Utilize staff meeting time to review success criteria (see below), address emerging needs, celebrate successes and identify areas for ongoing professional development. ○ Establish a regular time for grade-level teams to collaborate in developing shared resources, review student work, and co-create lessons. ○ Provide ongoing professional learning on integrated and designated ELD in the distance learning context and time for teachers to collaborate on addressing the needs of English learners (successes and next steps). ○ Provide regular time for grade-level teams to collaborate with special education teams (SAI;SLP;OT;APE..) to discuss supports/challenges in DL model and GE curriculum • Considerations for Parents/Students ○ Establish a regular time for parents to receive support with technology. Explore platforms that are available

	in languages other than English to ensure that parents and students have access and that the home language is seen as an asset. o Establish regular office hours for students to connect with their teachers and peers. o Ensure that communications with parents are translated to the extent possible and that translators are available for teachers to contact parents who speak languages other than English.

Tips for Success

- Consider other means of communication other than email. Survey data that shows 1 in 3 families of English Learners do not have an email address.
- Google and several text messaging apps provide alternative phone numbers that link to your personal phone number so that it is kept private. Calling the alternative phone number will connect to you directly. These services are typically free of charge. Several text messaging apps provide translation services for two-way translation (from English to the parent's preferred language and also translating their response back to English).
- Consider creating videos regarding how to access the digital platform for future viewing.
- Consider providing guidance in multiple languages including video, written material, digital material, and technology platforms, apps, and others.

Use of Common Tools

Consistency across grade-levels will support the success of students and families as they prepare to engage in distance learning. Consistency also provides opportunities for teachers to marshal resources. As an example, if teachers are all using the same high-quality curriculum, they might develop or curate videos for asynchronous learning and share with colleagues. Consistent use of platforms allows parents with multiple children to learn and offer support in a focused area. Similarly, students with multiple teachers will have space to focus on content as opposed to navigating multiple digital platforms for learning.

Focus	Considerations
Common district-wide digital (learning management system) platform	• Select one common digital platform for appropriate grade-spans, i.e., kindergarten and first grade may utilize a different platform than second-grade and above. • Ensure support is provided to teachers on how to use the platform in a consistent manner. • Ensure support is provided to parents on how to use the system and that this support is available in multiple languages.
Use of common high-quality instructional materials and resources	• Identify the district adopted materials for each subject area. • Ensure every teacher has access to the required curriculum, including ELD and Special Education. • As a staff, use the categories of investigation identified in this Framework (see chapters six, seven, and eight) for focus and planning in distance learning. • As a staff, use the CA CCSSM to identify the new content introduced in each grade-level for focus and planning. • As a staff, use the *ELA/ELD Framework* and the ELD standards to ensure that instructional materials include both integrated and designated ELD for English learners. Integrated ELD should be provided in all subject areas. • As a staff, discuss multilingual program needs. • Identify necessary supports to build staff, parent, and student capacity around the curriculum that will be used.

	(see Preparing Families and Staff for Distance Learning above)
Use of common diagnostic, formative, and summative assessments	• Administer a common grade-appropriate diagnostic assessment at the beginning of the year to establish a baseline for student learning • Plan for the administration of common assessments to use for grade-level collaboration, including assessments in other languages for multilingual programs and the English language proficiency for ELD progress. • Provide timely, personalized feedback to students on formative and summative assessments including acknowledgment of the receipt of their work and a way for students to track their grades. • Communicate to parents and students progress in learning regularly ensuring translations when appropriate.

Success Criteria

It is important for districts to review the past and current local data in order to identify metrics for success in the distance learning setting. As an example, if an LEA previously saw high rates of chronic absenteeism with their students with disabilities, a clear improvement outcome should be established with a plan to monitor participation rates for that student group. Success criteria will clearly communicate the vision of the LEA regarding student performance and allow staff to monitor progress, to celebrate success, and identify needs early.

Focus	Considerations

Identify metrics to monitor progress in DL over time	• Identify anticipated student needs based on previous data and on formative assessments within the first month of school. • Develop clear, consistent ways to solicit feedback from students, parents, and staff • Identify and develop common assessments at each grade level. • Identify local data to review regularly, including specific data for student subgroups traditionally underserved.
Data Commitments	• Develop clear data commitments: when will assessments be given? Who will collect the information? Who will create data visuals that are easy to read?
Data Analysis	• Review data on a regular basis with the Every Ed Team (comprised of representative staff to support general education including students with exceptional needs and English learners) • Communicate data at staff meetings including time to brainstorm the next steps. • Establish a relationship between data outcomes and practices or strategies that were implemented.

Distance Learning Curriculum and Instructional Guidance for Mathematics

Per the provisions of Senate Bill 98 of 2020, commonly referred to as the “Budget Trailer Bill,” the Sacramento County superintendent of schools, under the direction of the executive director of the State Board of Education, developed distance learning curriculum and instructional guidance for mathematics, English language arts, and English language development that includes, but is not limited to, a framework for addressing critical standards, guidance and resources for formative and diagnostic assessment, guidance on recommended aggregate time for instruction and independent work by grade span, and guidance on embedding social emotional supports for pupils into distance learning curricula. The guidance is scheduled to be adopted by the State Board of Education in May of 2021. [A summary of the guidance will be included here and link to complete guidance will be added here upon SBE adoption.]

Conclusion

Many technologies have the potential to support rich and deep mathematical learning for all students. Some provide contexts and representations of mathematical ideas (and tools for interacting with them) that help students deepen their understanding and their practice of mathematics. Others are not discipline-specific but support student-centered pedagogy consistent with this framework’s Chapter 2: Teaching for Equity and Engagement. As new technologies emerge, it is crucial that mathematical learning goals drive their use, that the tools support all learners, and that implementation be supported with high-quality professional learning opportunities for educators.

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