Name KEY Honors Pre-Calc Hour 1 4/5 Review Chapter 4 Test

Complete the following WITHOUT a calculator.

1. Use the Remainder Theorem to find the remainder when the polynomial $16x^5 - 32x^4 - 81X + 162$ is divided by $x - 2$. State whether the binomial is a factor of the polynomial. Explain your answer.

   Remainder 0 Yes or No Yes

   Explaination Remainder is zero so $x-2$ is a factor (Factor Thm)

   $$\begin{array}{r|rrrrrr} 2 & 16 & -32 & 0 & 0 & -81 & 162 \\ & & 32 & 0 & 0 & 0 & -162 \\ \hline & 16 & 0 & 0 & 0 & -81 & 0 \end{array}$$

   At most how many real solutions would $16x^5 - 32x^4 - 81X + 162$ have? 5.

   At least how many real solutions would $16x^5 - 32x^4 - 81X + 162$ have? 1.

   The left and right hand behavior of $16x^5 - 32x^4 - 81X + 162$ is as $x \to -\infty, y \to -\infty$; as $x \to \infty, y \to \infty$.

2. Use Synthetic Division to divide $8x^4 - 20x^3 - 14x^2 + 8x + 1$ by $x + 1$.

   Solution $8x^3 - 28x^2 + 14x - 6 + \frac{7}{x+1}$

   $$\begin{array}{r|rrrrr} -1 & 8 & -20 & -14 & 8 & 1 \\ & & -8 & 28 & -14 & 6 \\ \hline & 8 & -28 & 14 & -6 & 7 \end{array}$$

   Find $f(-1)$ for the function in problem two. $f(-1) =$ 7.

   Is $x = -1$ a root of $f(x)$? No yes or no

   Explain answer Remainder is 7 not 0, so -1 is not a root. It must $f(-1) = 0$.

3. Use Long Division to find the other factor of $x^3 + 3x^2 - 8x - 24$ if $x^2 - 8$ is a factor.

   Other factor $x + 3$

   $$\begin{array}{r} x + 3 \\ x^2 + 0x - 8 \overline{\big)\, x^3 + 3x^2 - 8x - 24} \\ -x^3 \mp 0x^2 \pm 8x \\ \hline 3x^2 + 0x - 24 \\ -3x^2 + 0x \pm 24 \\ \hline 0 \end{array}$$

4. Solve $3x^2 - 24x = -30$ by Completing the Square.

   Solution $x = 4 \pm \sqrt{6}$

   $$3x^2 - 24x = -30$$
   $$x^2 - 8x = -10$$
   $$x^2 - 8x + 16 = -10 + 16$$
   $$(x-4)^2 = 6$$
   $$x - 4 = \pm\sqrt{6}$$
   $$\boxed{x = 4 \pm \sqrt{6}}$$

5. Given the following roots of $f(x)$, build the factors of $f(x)$, then use the factors to build a polynomial $f(x)$ of the least degree.
Roots $x = 5$; $x = -3$; $x = 2$
Factors of $f(x)$ $(x-5)(x+3)(x-2)$
Polynomial $f(x) =$ $x^3-4x^2-11x+30$

$(x-5)(x+3)(x-2)$
$(x^2-2x-15)(x-2)$
$x^3-2x^2-15x-2x^2+4x+30 = x^3-4x^2-11x+30$

6. Given the following roots of $f(x)$, build the factors of $f(x)$, then use the factors to build a polynomial $f(x)$ of the least degree.
Roots $x = 5i$; $x = -\sqrt{3}$
Factors of $f(x)$ $(x-5i)(x+5i)(x+\sqrt{3})$ or $f(x)=(x-5i)(x+5i)(x+\sqrt{3})(x-\sqrt{3})$
Polynomial $f(x) =$ $x^3+(\sqrt{3})x^2+25x+25\sqrt{3}$ or $f(x) = x^4+22x^2-75$

if real coefficients
$(x-5i)(x+5i)(x+\sqrt{3})$
$(x^2+25)(x+\sqrt{3})$
$x^3+\sqrt{3}x^2+25x+25\sqrt{3}$

if rational coefficients
$(x-5i)(x+5i)(x+\sqrt{3})(x-\sqrt{3})$
$(x^2+25)(x^2-3)$
$x^4-3x^2+25x^2-75 = x^4+22x^2-75$

7. Find the discriminant of $2x^2 - 4x + 9 = 0$ and describe the nature of the roots.
Value of discriminant -56 Nature of the roots 2 imag. conjugates
$a=2$ $b=-4$ $c=9$
$D = b^2-4ac$
$= 16-4(2)(9) = 16-72 = -56$

8. Solve $-3x^2 + 4 = 0$ by using the Quadratic Formula.
Solution $x = \pm\frac{2\sqrt{3}}{3}$
$a=-3$ $b=0$ $c=4$
$-3x^2=-4$
$x^2=\frac{4}{3} \Rightarrow x=\pm\sqrt{\frac{4}{3}} = \pm\frac{2\sqrt{3}}{3}$

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$= \frac{-0\pm\sqrt{0^2-4(-3)(4)}}{2(-3)} = \frac{\pm\sqrt{48}}{-6} = \frac{\pm 4\sqrt{3}}{-6} = \boxed{\pm\frac{2\sqrt{3}}{3}}$

9. List the possible rational roots of $2x^3 + 17x^2 + 23x - 42 = 0$

Solution $\pm1, \pm2, \pm3, \pm6, \pm7, \pm14, \pm21, \pm42, \pm\frac{3}{2}, \pm\frac{7}{2}, \pm\frac{21}{2}$

$p: \pm1, \pm2, \pm3, \pm6, \pm7, \pm14, \pm21, \pm42$
$q: \pm1, \pm2$
$\frac{p}{q}$: $p, \pm\frac{3}{2}, \pm\frac{7}{2}, \pm\frac{21}{2}$

10. Find the number of possible negative real zeros for $x^4 - 5x^3 + 2x^2 - 12x + 6$
Solution 0 – real

$-: f(-x) = x^4+5x^3+2x^2+12x+6$ 0 – real

11. Solve $\frac{2x-5}{x} + \frac{4x-1}{x+2} = \frac{-3x-8}{x^2+2x} = \frac{-3x-8}{x(x+2)}$ D.R. $x \neq 0, x \neq -2$

Solution $x = -\frac{2}{3},\ x = \frac{1}{2}$

$(2x-5)(x+2) + (4x-1)(x) = (-3x-8)$

$2x^2 - x - 10 + 4x^2 - x = -3x - 8$

$6x^2 + x - 2 = 0$

$(3x+2)(2x-1) = 0$

$\boxed{x = -\frac{2}{3} \quad x = \frac{1}{2}}$

✓ $y_1 = \frac{2x-5}{x} + \frac{4x-1}{x+2}$ graph

$y_2 = \frac{-3x-8}{x^2+2x}$ graph

2nd calc intersect

12. Solve $x - \frac{6}{x} + 5 = 0$ D.R. $x \neq 0$

Solution $x = -6,\ x = 1$

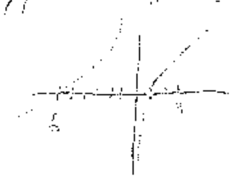
$x^2 - 6 + 5x = 0$

$x^2 + 5x - 6 = 0$

$(x+6)(x-1) = 0$

$\boxed{x = -6 \quad x = 1}$

✓ $y_1 = x - \frac{6}{x} + 5$ graph



13. Solve $\sqrt[3]{x+2} = \sqrt[3]{5x}$

Solution $x = \frac{1}{2}$

$x + 2 = 5x$

$2 = 4x$

$\boxed{x = \frac{1}{2}}$

✓ $\sqrt[3]{\frac{1}{2}+2} = \sqrt[3]{5(\frac{1}{2})}$ T

14. Solve $5 + \sqrt{x+2} = 8 + \sqrt{x+7}$

Solution no solution

✓ $5 + \sqrt{\frac{-14}{9} + \frac{18}{9}} = 8 + \sqrt{\frac{-14}{9} + \frac{63}{9}}$

$5 + \frac{2}{3} = 8 + \frac{7}{3}$ F

$\boxed{\therefore \text{no solution}}$

$\sqrt{x+2} = 3 + \sqrt{x+7}$

$x + 2 = 9 + 6\sqrt{x+7} + x + 7$

$-14 = 6\sqrt{x+7}$

$196 = 36(x+7)$

$196 = 36x + 252$

$-56 = 36x$

$\frac{-56}{36} = x = \frac{-14}{9}$

Name ___KEY___ Hour __1/4-15__ Review Chapter 4 Test

Complete the following. You MAY USE A calculator.

1. Approximate the real zeros of $f(x) = x^3 - 5x^2 + 2$ to the nearest tenth.

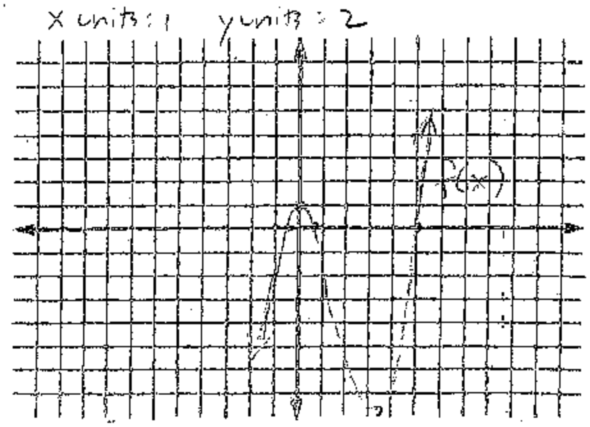
   Solution $x \approx -0.6, x \approx 1.0, x \approx 5.0$

   Sketch the graph.

   – On calc graph $y_1 = f(x)$
   $x\ [-2, 8]$ $y\ [-20, 10]$

   – 2nd → calc → intersect (3 times)

   $\boxed{x \approx -0.6 \quad x \approx 1.0 \quad x \approx 5.0}$



2. Between what two integers are the zeros of the function $2x^4 + 3x^2 - 20$.

   Solution Between -2 and -1, 1 and 2

   Sketch the graph.

   – On calc graph $y_1 = 2x^4 + 3x^2 - 20$
   $x: [-5, 5]$ $y\ [-30, 10]$

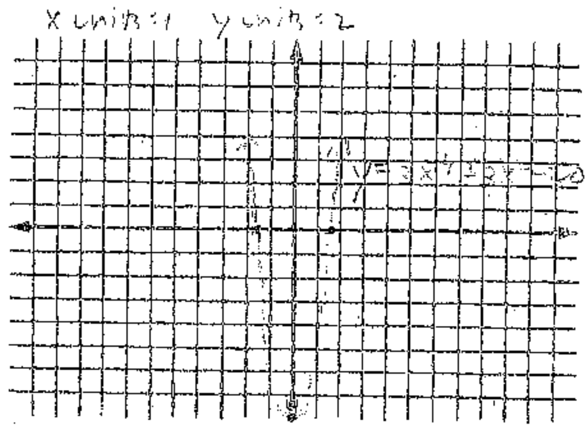
   Examination of graph shows real zeros between -2 and -1, 1 and 2

   2nd → table

x	-2	-1	0	1	2
y	24	-15	-20	-15	24

   root root

   Alternative justification using table



3. Determine the rational roots of $2x^3 + 3x^2 - 17x + 12 = 0$

   Solutions $x = -4, 1, \frac{3}{2}$

   Sketch the graph

   – On calc graph $y_1 = 2x^3 + 3x^2 - 17x + 12$
   $x\ [-5, 5]$ $y\ [-10, 50]$

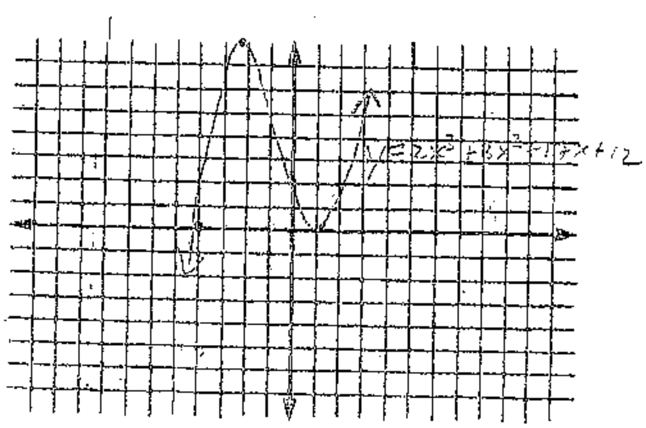
   From graph, pick zero of -4. Detail of graph near 1 indicates 2 add'l rational roots.

   $$\begin{array}{r|rrrr} -4 & 2 & 3 & -17 & 12 \\ & & -8 & 20 & -12 \\ \hline & 2 & -5 & 3 & 0 \end{array}$$ ← verifies zero

   $2x^2 - 5x + 3 = 0$

   $(2x - 3)(x - 1) = 0$

   $x = \frac{3}{2}$ $x = 1$ $\therefore \boxed{x = -4, 1, \frac{3}{2}}$



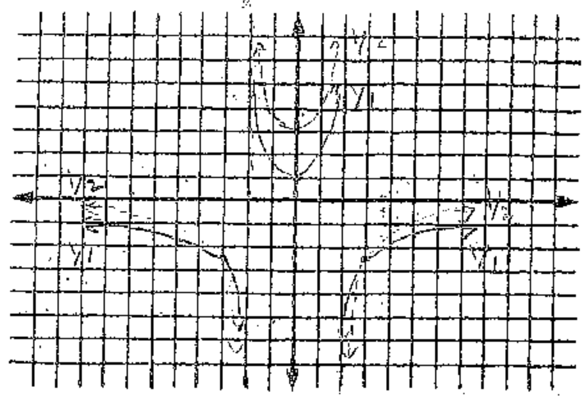
✓ $$\begin{array}{r|rrrr} 1 & 2 & 3 & -17 & 12 \\ & & 2 & 5 & -12 \\ \hline & 2 & 5 & -12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} \frac{3}{2} & 2 & 3 & -17 & 12 \\ & & 3 & 9 & -12 \\ \hline & 2 & 6 & -8 & 0 \end{array}$$

4. Solve $\frac{2}{2+x} + \frac{x}{2-x} < \frac{13}{4-x^2} = \frac{13}{(2+x)(2-x)}$ D.R. $x \neq \pm 2$

Solution $(-\infty,-3)\cup(-2,2)\cup(3,\infty)$

Sketch the graph



solid $y_1 = \frac{2}{2+x} + \frac{x}{2-x}$

dotted $y_2 = \frac{13}{4-x^2}$

x-units: 1 y-units: 1

On calc, graph $y_1 = \frac{2}{2+x} + \frac{x}{2-x}$, $y_2 = \frac{13}{4-x^2}$

X [-5, 5] Y [-7, 7]

Use 2nd → calc → intersect on three sections of graph created by V.A.'s @ $x = \pm 2$. I in left section, $x = -3$. In center section, $\emptyset$. I right section $x = 3$.

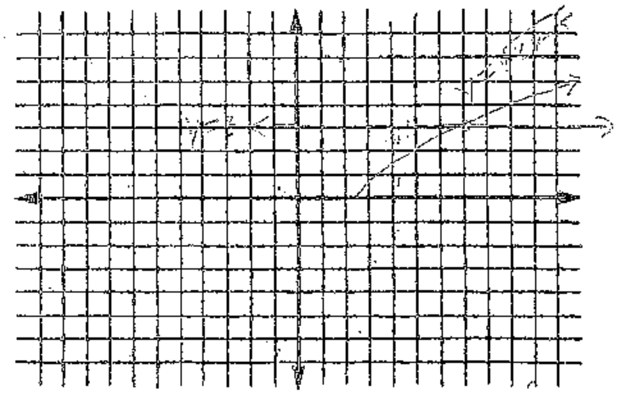
$\therefore (-\infty,-3)\cup(-2,2)\cup(3,\infty)$

Observe which graph is on top in each section

5. Solve $\sqrt{2x-5} + 8 \geq 11$ $2x-5 \geq 0$, $x \geq \frac{5}{2}$

Solution $[7,\infty)$

Sketch the graph X [0, 10] Y [0, 5]



x unit: 1 y unit: 1

On calc, graph $y_1 = \sqrt{2x-5}$, $y_2 = 3$

Looking for $y_1 = \sqrt{2x-5}$ above $y_2 = 3$.

Use 2nd → calc → intersect @ (7, 3)

$\therefore$ $x \geq 7$ or $[7,\infty)$

✓ $\sqrt{2x-5} \geq 3$

$2x-5 \geq 9$

$2x \geq 14$

$x \geq 7$

6. Write a polynomial function with integral coefficients to model the set of data below

4	4.5	5	5.5	6	6.5	7	7.5	8	8.5
7.3	11.2	12.1	11.2	8	6.2	3.5	2.5	2.2	5.7

stat → edit → input data

2nd → catalog → diagnostic on

stat → calc → choose regression

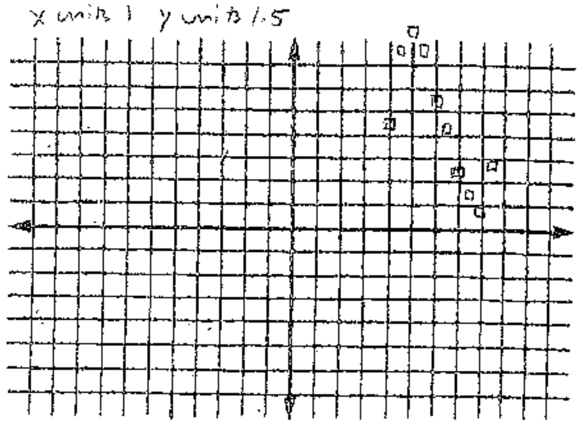
x^3 – $r^2 \approx 0.99244$ x^4 – $r^2 \approx 0.99270$

Solution See below

Sketch the data 1, Stat plot X [0, 10] Y [0, 15]

2nd → stat plot → on → graph

x unit 1 y unit 1.5



$f(x) \approx -0.0217x^4 + 1.5600x^3 - 24.2199x^2 + 135.5576x - 241.6645$