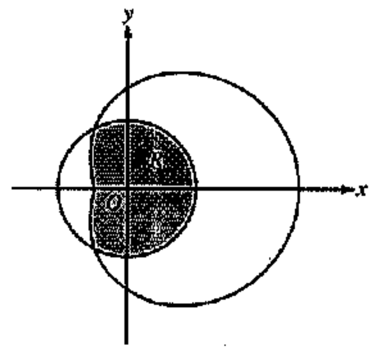
Ch. 10 CW

AP® CALCULUS BC
2007 SCORING GUIDELINES

Question 3



The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

(a) Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2\cos\theta$, as shaded in the figure above. Find the area of R.

(b) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos\theta$ has position $(x(t), y(t))$ at time t, with $\theta = 0$ when $t = 0$. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

(c) For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

(a) Area $= \frac{2}{3}\pi(2)^2 + \frac{1}{2}\int_{2\pi/3}^{4\pi/3}(3 + 2\cos\theta)^2\,d\theta$ $= 10.370$	4 : { 1 : area of circular sector 2 : integral for section of limaçon 1 : integrand 1 : limits and constant 1 : answer }
(b) $\left.\frac{dr}{dt}\right\|_{\theta=\pi/3} = \left.\frac{dr}{d\theta}\right\|_{\theta=\pi/3} = -1.732$ The particle is moving closer to the origin, since $\frac{dr}{dt} < 0$ and $r > 0$ when $\theta = \frac{\pi}{3}$.	2 : { 1 : $\left.\frac{dr}{dt}\right\|_{\theta=\pi/3}$ 1 : interpretation }
(c) $y = r\sin\theta = (3 + 2\cos\theta)\sin\theta$ $\left.\frac{dy}{dt}\right\|_{\theta=\pi/3} = \left.\frac{dy}{d\theta}\right\|_{\theta=\pi/3} = 0.5$ The particle is moving away from the x-axis, since $\frac{dy}{dt} > 0$ and $y > 0$ when $\theta = \frac{\pi}{3}$.	3 : { 1 : expression for y in terms of θ 1 : $\left.\frac{dy}{dt}\right\|_{\theta=\pi/3}$ 1: interpretation }

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2010 SCORING GUIDELINES

Question 3

A particle is moving along a curve so that its position at time t is $(x(t), y(t))$, where $x(t) = t^2 - 4t + 8$ and $y(t)$ is not explicitly given. Both x and y are measured in meters, and t is measured in seconds. It is known that $\frac{dy}{dt} = te^{t-3} - 1$.

(a) Find the speed of the particle at time $t = 3$ seconds.

(b) Find the total distance traveled by the particle for $0 \le t \le 4$ seconds.

(c) Find the time t, $0 \le t \le 4$, when the line tangent to the path of the particle is horizontal. Is the direction of motion of the particle toward the left or toward the right at that time? Give a reason for your answer.

(d) There is a point with x-coordinate 5 through which the particle passes twice. Find each of the following.

(i) The two values of t when that occurs

(ii) The slopes of the lines tangent to the particle's path at that point

(iii) The y-coordinate of that point, given $y(2) = 3 + \frac{1}{e}$

(a) Speed $= \sqrt{(x'(3))^2 + (y'(3))^2} = 2.828$ meters per second	1 : answer
(b) $x'(t) = 2t - 4$ Distance $= \int_0^4 \sqrt{(2t-4)^2 + \left(te^{t-3} - 1\right)^2}\,dt = 11.587$ or 11.588 meters	2 : { 1 : integral 1 : answer
(c) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 0$ when $te^{t-3} - 1 = 0$ and $2t - 4 \ne 0$ This occurs at $t = 2.20794$. Since $x'(2.20794) > 0$, the particle is moving toward the right at time $t = 2.207$ or 2.208.	3 : { 1 : considers $\frac{dy}{dx} = 0$ 1 : $t = 2.207$ or 2.208 1 : direction of motion with reason
(d) $x(t) = 5$ at $t = 1$ and $t = 3$ At time $t = 1$, the slope is $\left.\frac{dy}{dx}\right\|_{t=1} = \left.\frac{dy/dt}{dx/dt}\right\|_{t=1} = 0.432$. At time $t = 3$, the slope is $\left.\frac{dy}{dx}\right\|_{t=3} = \left.\frac{dy/dt}{dx/dt}\right\|_{t=3} = 1$. $y(1) = y(3) = 3 + \frac{1}{e} + \int_2^3 \frac{dy}{dt}\,dt = 4$	3 : { 1 : $t = 1$ and $t = 3$ 1 : slopes 1 : y-coordinate

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2011 SCORING GUIDELINES

Question 1

At time t, a particle moving in the xy-plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \geq 0$, $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin(t^2)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$.

(a) Find the speed of the particle at time $t = 3$, and find the acceleration vector of the particle at time $t = 3$.

(b) Find the slope of the line tangent to the path of the particle at time $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

(a) Speed $= \sqrt{(x'(3))^2 + (y'(3))^2} = 13.006$ or 13.007

Acceleration $= \langle x''(3), y''(3) \rangle$
$= \langle 4, -5.466 \rangle$ or $\langle 4, -5.467 \rangle$

$2 : \begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration} \end{cases}$

(b) Slope $= \frac{y'(3)}{x'(3)} = 0.031$ or 0.032

1 : answer

(c) $x(3) = 0 + \int_0^3 \frac{dx}{dt}\,dt = 21$

$y(3) = -4 + \int_0^3 \frac{dy}{dt}\,dt = -3.226$

At time $t = 3$, the particle is at position $(21, -3.226)$.

$4 : \begin{cases} 2 : x\text{-coordinate} \\ \quad 1 : \text{integral} \\ \quad 1 : \text{answer} \\ 2 : y\text{-coordinate} \\ \quad 1 : \text{integral} \\ \quad 1 : \text{answer} \end{cases}$

(d) Distance $= \int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = 21.091$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

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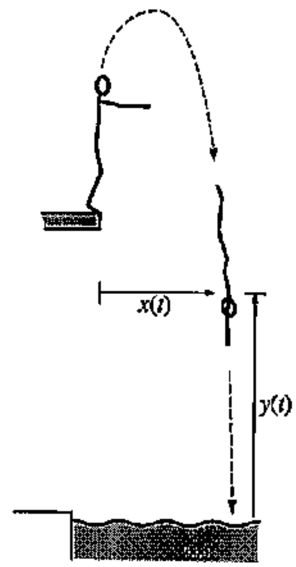
2009 SCORING GUIDELINES

Question 3

A diver leaps from the edge of a diving platform into a pool below. The figure above shows the initial position of the diver and her position at a later time. At time t seconds after she leaps, the horizontal distance from the front edge of the platform to the diver's shoulders is given by $x(t)$, and the vertical distance from the water surface to her shoulders is given by $y(t)$, where $x(t)$ and $y(t)$ are measured in meters. Suppose that the diver's shoulders are 11.4 meters above the water when she makes her leap and that

$$\frac{dx}{dt} = 0.8 \text{ and } \frac{dy}{dt} = 3.6 - 9.8t,$$

for $0 \le t \le A$, where A is the time that the diver's shoulders enter the water.



Note: Figure not drawn to scale.

(a) Find the maximum vertical distance from the water surface to the diver's shoulders.

(b) Find A, the time that the diver's shoulders enter the water.

(c) Find the total distance traveled by the diver's shoulders from the time she leaps from the platform until the time her shoulders enter the water.

(d) Find the angle θ, $0 < \theta < \frac{\pi}{2}$, between the path of the diver and the water at the instant the diver's shoulders enter the water.

(a) $\frac{dy}{dt} = 0$ only when $t = 0.36735$. Let $b = 0.36735$.

The maximum vertical distance from the water surface to the diver's shoulders is

$$y(b) = 11.4 + \int_0^b \frac{dy}{dt}\,dt = 12.061 \text{ meters.}$$

Alternatively, $y(t) = 11.4 + 3.6t - 4.9t^2$, so $y(b) = 12.061$ meters.

3 : { 1 : considers $\frac{dy}{dt} = 0$; 1 : integral or $y(t)$; 1 : answer }

(b) $y(A) = 11.4 + \int_0^A \frac{dy}{dt}\,dt = 11.4 + 3.6A - 4.9A^2 = 0$ when $A = 1.936$ seconds.

2 : { 1 : equation; 1 : answer }

(c) $\int_0^A \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = 12.946$ meters

2 : { 1 : integral; 1 : answer }

(d) At time A, $\frac{dy}{dx} = \left.\frac{dy/dt}{dx/dt}\right|_{t=A} = -19.21913.$

The angle between the path of the diver and the water is $\tan^{-1}(19.21913) = 1.518$ or 1.519.

2 : { 1 : $\frac{dy}{dx}$ at time A; 1 : answer }

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2009 SCORING GUIDELINES (Form B)

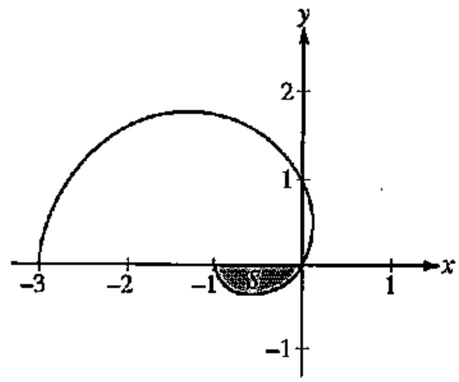
Question 4

The graph of the polar curve $r = 1 - 2\cos\theta$ for $0 \le \theta \le \pi$ is shown above. Let S be the shaded region in the third quadrant bounded by the curve and the x-axis.

(a) Write an integral expression for the area of S.

(b) Write expressions for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of θ.

(c) Write an equation in terms of x and y for the line tangent to the graph of the polar curve at the point where $\theta = \frac{\pi}{2}$. Show the computations that lead to your answer.



(a) $r(0) = -1$; $r(\theta) = 0$ when $\theta = \frac{\pi}{3}$.

Area of $S = \frac{1}{2}\int_0^{\pi/3} (1 - 2\cos\theta)^2 \, d\theta$

$2: \begin{cases} 1: \text{limits and constant} \\ 1: \text{integrand} \end{cases}$

(b) $x = r\cos\theta$ and $y = r\sin\theta$

$$\frac{dr}{d\theta} = 2\sin\theta$$

$$\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta = 4\sin\theta\cos\theta - \sin\theta$$

$$\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta = 2\sin^2\theta + (1 - 2\cos\theta)\cos\theta$$

$4: \begin{cases} 1: \text{uses } x = r\cos\theta \text{ and } y = r\sin\theta \\ 1: \frac{dr}{d\theta} \\ 2: \text{answer} \end{cases}$

(c) When $\theta = \frac{\pi}{2}$, we have $x = 0$, $y = 1$.

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \left.\frac{dy/d\theta}{dx/d\theta}\right|_{\theta=\frac{\pi}{2}} = -2$$

The tangent line is given by $y = 1 - 2x$.

$3: \begin{cases} 1: \text{values for } x \text{ and } y \\ 1: \text{expression for } \frac{dy}{dx} \\ 1: \text{tangent line equation} \end{cases}$