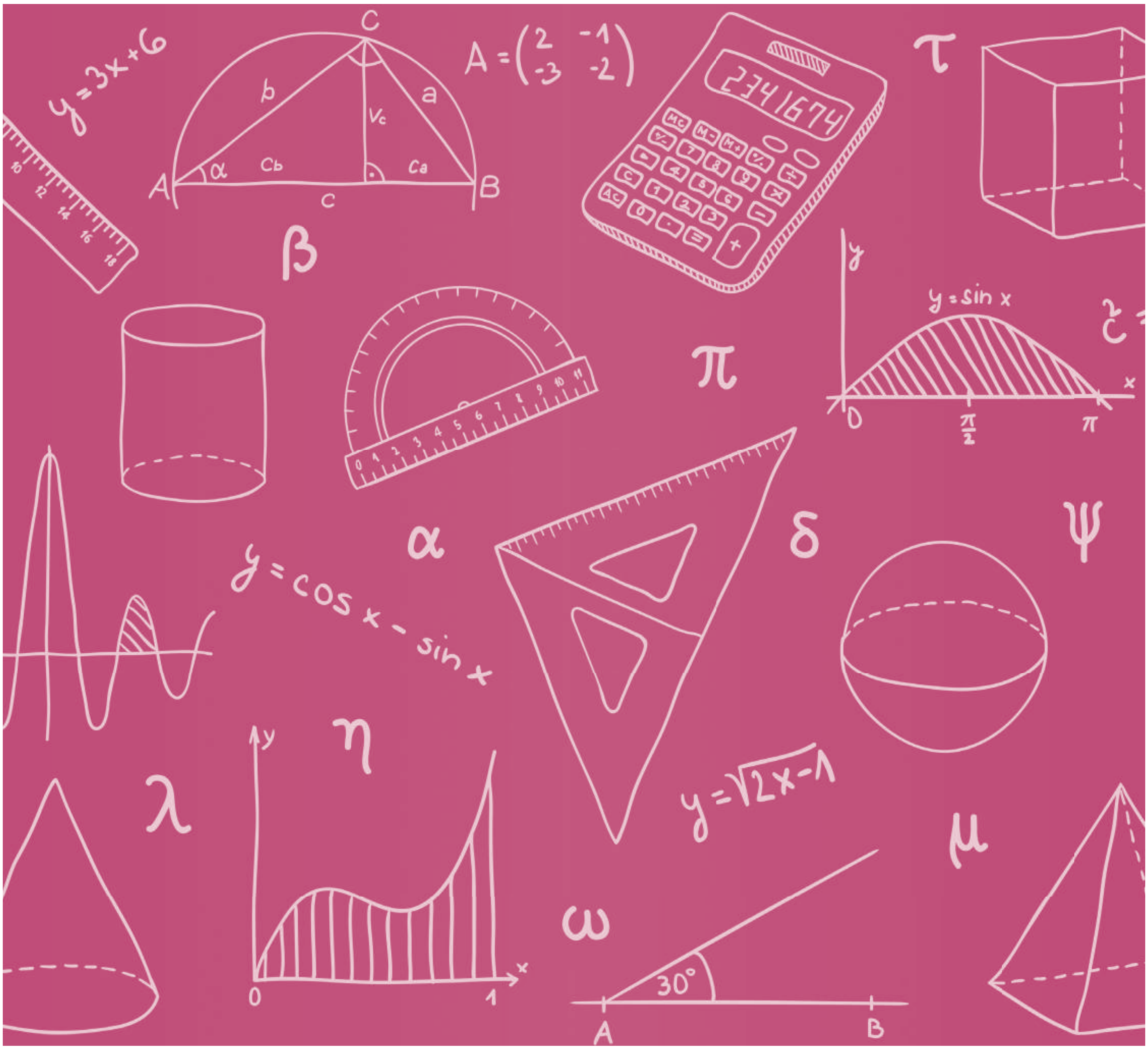
GRADE
3

LESSONS FOR LEARNING

FOR THE COMMON CORE STATE STANDARDS IN MATHEMATICS



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NC DEPARTMENT OF PUBLIC INSTRUCTION

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M0910

Third Grade – Standards

1. **Developing understanding of multiplication and division and strategies for multiplication and division within 100** – Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.
2. **Developing understanding of fractions, especially unit fractions (fractions with numerator 1)** – Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket; but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.
3. **Developing understanding of the structure of rectangular arrays and of area** – Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.
4. **Describing and analyzing two-dimensional shapes** – Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

MATHEMATICAL PRACTICES

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

OPERATIONS AND ALGEBRAIC THINKING

Represent and solve problems involving multiplication and division.

3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (Note: See Glossary, Table 2.)

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ☐ ÷ 3, 6 × 6 = ?.*

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5 Apply properties of operations as strategies to multiply and divide. (Note: Students need not use formal terms for these properties.) *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

3.OA.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (Note: This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order – Order of Operations.)

3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

NUMBER AND OPERATIONS IN BASE TEN

Use place value understanding and properties of operations to perform multi-digit arithmetic. (Note: A range of algorithms may be used.)

3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

NUMBER AND OPERATIONS - FRACTIONS

Note: Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Develop understanding of fractions as numbers.

3.NF.1 Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.

b. Represent a fraction *a/b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line.

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

MEASUREMENT AND DATA

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

3.MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Note: Excludes compound units such as cm3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Note: Excludes multiplicative comparison problems – problems involving notions of "times as much"; see Glossary, Table 2.)

Represent and interpret data.

3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

3.MD.7 Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

GEOMETRY

Reason with shapes and their attributes.

3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Game: Playing Circles and Stars

Common Core Standard:

Represent and solve problems involving multiplication and division.

3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.

Additional/Supporting Standard(s):

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5 Apply properties of operations as strategies to multiply and divide.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them
2. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision

Student Outcomes:

- I can see multiplication as combining equal groups.
- I can use repeated addition as a strategy to find the total number of stars.
- I can use strategies to find the total number of stars.
- I can record my answer by writing an equation.

Materials:

- One six-sided die or a spinner for each pair of children
- Game Recording Sheet per player
- Data chart for Class

Advance Preparation:

Circles and Stars is a game that gives children a visual interpretation of multiplication and repeated addition. The game also helps students see multiplication as the combining of equal-sized groups that can be represented with a multiplication equation.

Directions:

- The teacher should model this game by inviting a student to play as a partner.
- This game can be introduced to the entire class. The teacher might play a game with another student to model the game. Use modeling and questioning to ensure understanding.
  Teacher: Player One rolls the die and draws the corresponding number of circles at the top on the top row of the recording sheet. If the player rolls a 4, the player will draw 4 circles
- **Student:** Player Two rolls the die and draws the corresponding number of circles in the round one box on his/her recording sheet. If player two rolls a 1, the player will draw 1 circle.

- **Teacher:** Player One rolls the die a second time and draws the corresponding number of stars in each circle. (An option may be to draw Xs which are easier to draw.) If a player rolls a 2, the player will draw 2 stars in each circle. (See Player One's (Jack) recording sheet.) Player records both the addition and multiplication equation for each round. Many students will need to record both equations to move them from repeated addition to multiplication. Later, they might just record for multiplication.
- **Student:** Player 2 rolls a second time and draws the corresponding number of stars on his/her recording sheet. Player 2 records equations.
- **Teacher and Student:** Each player should record his/her name and partner's name in the top left corner of the player's recording sheet. At the end of the game, each player will total his/her total stars for each round and record the total in the top left box. Record partner's score and record the difference between the two scores.

Circle and Stars Recording Sheet

Jack's total ____ Partner's total ____ Difference ____	** ** ** ** 2 + 2 + 2 + 2 =8 4 x 2 = 8			

- Both players repeat these steps until all boxes are filled.
- Each player finds the total number of stars on his/her game sheet. Players need to check each other's work.
- Each player records own score plus partner's score. Find the difference and record.
- Many students will need to record both equations to move them from repeated addition to multiplication. Later, they might just record for multiplication.

Questions to Pose:

Before:

- If your die had a zero and your first roll was a 5 and your second roll was a zero? Explain how your would record?

During:

- What numbers did you represent in different ways? Compare with your partner. Explain.
- What other observations did you make as you were playing this game? Explain.

After:

- What is the fewest number of stars you can get in one round? Explain.
- What is the greater number of stars you can get in one round? Explain.

Possible Misconceptions/Suggestions:

Possible Misconceptions	Suggestions
Students get confused when thinking about the number of groups and the number in each groups Students add the two numbers without thinking about number of groups	Students need to explain what the circles represent and what the stars in each circle represent.

Special Notes:

Extensions:

1. **Game Play**
   - Play the game using a 1-9 die or 0-9 spinner.
   - Players record multiplication and the related division fact followed by an explanation.

     4 x 2 = 8 4 circles with 2 stars in each circle

     8 ÷ 2 = 4 8 stars divided into 4 groups There are 2 stars in each group.
2. **Class Data Chart** (Prepare before lesson. See attached page.)
   - Teacher should list all numbers 1-36 on a chart using column format. (Thirty six is the largest product possible product using two (1-6) dice.
   - **Show the class the Circle and Stars Data Chart**.
   - Teacher selects one student and the student's recording sheet. Teacher and student model how to use tally marks to record the student's scores for each round on the Class Data Chart. Tell students they are to play more games of Circles and Stars. As students complete their recording sheet, ask students to make tally marks on the class chart to show the number of stars he/she had for each **round**. Suggest that if one partner reads each score, the other partner can record tally marks on the Class Data Chart. Encourage students to play many games.
3. **Discuss the data**

   After all students have played several games and recorded their products for each round on the class chart, elicit students in conversations about the data. Ask: "Why did I write the numbers 1-36 on the chart?"

   a. Are there numbers that are impossible using a (1-6) die? Explain.
   b. Why do some numbers have more tally marks than other numbers? Explain.
   c. What are the ways to get 2 as an answer? Ways for 6? Ways for 12? (Students might think about this with a partner or in small groups. Record equations.)
   d. Which number(s) 1-36 has the most combinations using two 1-6 dice? What numbers can I skip count by to say this number? (Relate numbers on dice to factors in multiplication equations.
   e. *You can skip count by both factors and land on the number, which is the product.* Is this always true? Ask students to test this idea. Some may want to test larger numbers.
   f. Is there a product that can only be represented one way? Why? Explain.
   g. What other observations do you notice about the data?
   h. How might this data be useful for thinking about multiplication combinations (facts)?

Adapted from *About Teaching Mathematics by Marilyn Burns.*

Circles and Stars – Data Collection

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Circles and Stars Recording Sheet

Name ____________________________ Date __________

Players Total __________________ Partner's Total __________ Difference _____________	Round 1	Round 2	Round 3
Round 4	**Round 5**	**Round 6**	**Round 7**

Total Number of Stars __________________________

Solving Division Problems

Common Core Standard:

Represent and solve problems involving multiplication and division.

3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

Additional/Supporting Standard:

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (Note: See Glossary, Table 2.)

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

Student Outcomes:

- I can solve division word problems and explain how I solved them.
- I can retell a division word problem and act it out with counters.
- I can draw a representation of how I solved a division word problem.
- I can write an equation to represent a division word problem.

Materials:

- Materials such as cubes, tiles, counters to put into groups
- Blackline for each student. There are three different blackline division problem masters.

Advance Preparation:

- Decide how to pair students to work together.
- Present the story problems on the board, document camera, white board.
- This lesson is designed so that division problems are shared with the whole class. Partners work on the problems and then solution strategies are shared. After sharing strategies, students solve the division problems on the blackline master. The format of this lesson can be repeated many times using different division problems to promote class discussion and sharing of strategies. Three different blackline masters are provided and can be completed over the course of several lessons.

Directions:

1. Give each pair of students about 20 objects.
2. Explain that you are going to tell a story and they are going to act out the problem with the given materials. Post the problem – write it on the board or on chart paper.
3. Problem 1: A bag has 18 jellybeans. Laura and her friend want to share them equally. How many jellybeans will Laura and her friend get?
4. Have 1-2 students retell the problem in their own words.
5. Ask the students what they know. Examples of responses are they know the total number of jellybeans in each group?
6. Have partners solve the problem with the given materials. Partners also draw a solution strategy to illustrate how they used objects to solve the problem.
7. As students work with partners to solve the problem, the teacher checks with each group, asking questions such as those written in the "Questions to Pose." This is formative assessment. Students who struggle may benefit from retelling the problem. It may help some students to solve an easier problem first, such as six jellybeans shared by two friends. See if they can apply solving the easier problem to the posted problem.
8. After students have solved the problem, ask 2-3 pairs to share their solution strategy. Students may say that they first got 18 beans and drew faces/circles for Laura and her friend. Then they gave each girl an equal number (9) of jellybeans. Some may say they gave a fair-share, one jellybean at a time. Others may share that they knew 9+9 = 18 so each girl received 9 beans.
9. Ask students what equation could we write to match this problem. (18 ÷2 = ?) or (2 x ? =18)
10. Share another division problem. Problem 2: I bought 28 pencils. The pencils came in packs. There were 4 pencils in a pack. How many packs did I buy? Post the problem—write it on the board or on chart paper.
11. Have 1-2 students retell the problem in their own words.
12. Ask the students what they know. Examples of responses are they know the total number of pencils (28) and how many pencils are in a pack (4). Ask the students what they need to know—how many packs did I buy?
13. Have partners solve the problem with the given materials. Partners also draw a solution strategy to illustrate what they did with the objects.
14. After students have solved the problem, ask 2-3 pairs to share their solution strategy. Students may say that they first got 28 counters and counted out 4 counters at a time. Four counters were placed in a group. After all the counters are grouped they counted the number of groups (7).
15. Ask students what equation could we write to match this problem. (28 ÷4 = ?) or (4 x ? = 28)
16. Both problems are posted. Ask students to talk with their partners about how the problems are alike and how they are different. After students have shared ideas ask some to share with the entire class. Possible responses: Both problems are division. In both problems you know the total. The problems are about fair shares. On one problem you know the number of groups but need to know how many are in a group. In the other problem you know how many are in the group but need to know the number of groups.
17. The teacher can determine if the class is ready to work independently with partners or needs additional whole class work. Pose additional problem if the class needs more time for working and discussing as a whole class.
18. If students are ready have them complete the problems on the handout. Explain that they are to use the materials to "act out" the problem and draw a representation of their solution strategy.

Questions to Pose:

Before:

What does equal groups mean?
How can we make sure we give out fair-shares?

During:

After reading a problem, what do we know about the story?
After reading a problem, what do we need to know?
How do we use the counters to help solve the problem?
How many groups are there?
How many are in each group?
What is the total amount?

After:

How are these two problems alike?
How are these two problems different?
How can multiplication facts help you solve these problems?

Possible Misconceptions/Suggestions:

Possible Misconceptions	Suggestions
Students add the two numbers or multiply the two numbers.	Have students retell the story problem. Have them use the materials to act out the problem. If students still cannot understand what is happening in the problem, change the numbers so all numbers are one digit. Act out the problem.
Students may not be able to explain how the problems are alike and different.	Talk to the students about groups and the number in each group. Read problems together and talk about if it tells the number of groups or the number in each group. Demonstrate with counters. Share ideas on how to represent the problem on paper. Talk about the total. Read the problem(s) and identify the total.

Special Notes:

- After students are comfortable with division problems, have students write problems. The teacher shares some of the student problems as examples for the class to solve.
- If some students are having difficulty with solving the problems, change the numbers in the problems. Keep the total 12 or less.

Solutions:

Answers to the problems presented to the whole class are found in the description of the lesson.

Solving Division Problems

Name ____

Solve each problem and show your solution strategy.

1. There are 25 desks in the classroom. The teacher puts them in groups of 5. How many groups of desks are in the classroom?

2. Four friends are given a pack of trading cards to share equally. The pack contains 32 cards. How many cards should each person get?

3. Mario has 49 flowers. He wants to put them in bouquets of 7 flowers each. How many bouquets will Mario be able to make?

Solving Division Problems

Name ____

Solve each problem and show your solution strategy.

1. Nine children are building toy bikes. They have 36 toy wheels to share equally. How many toy wheels will each child get?

2. Mrs. Allen's class made 54 muffins for the bake sale. They put the muffin in bags to sell. They put 6 muffins in each bag. How many bags of muffins did they have to sell?

3. My mother gave me a pack of 45 new pencils. When I get to school, I want to share them equally among 8 of my friends and me. How many pencils will each of us get?

Solving Division Problems

Name ____

Solve each problem and show your solution strategy.

1. Five children have one book of 20 movie tickets to share equally. Each movie costs one ticket. How many movies can each child see?

2. I was drawing triangles. I counted 21 sides on all of the triangles I drew. Each triangle has 3 sides. How many triangles did I draw?

3. I bought 20 cups of yogurt. Each pack of yogurt has 4 cups. How many packs did I buy?

Seeing Arrays as Equal Groups

Common Core Standard:

Represent and solve problems involving multiplication and division.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Additional/Supporting Standard(s):

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.5 Apply properties of operations as strategies to multiply and divide.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure

Student Outcomes:

- I can work with partners to find different arrangements that add to 100.
- I can count to 100 by counting and combining groups to get to 100.
- I can use what I know about money (nickels, dimes, quarters, half-dollar).
- I can skip count to get to 100.
- I can combine groups using multiplication and addition.
- I can multiply by 2s, 5s, 10s.

Materials:

- **One Hundred Hungry Ants**, *by Elinor J. Pinczes (Houghton Mifflin, 1993*
- 100 objects per groups (color tiles, 2 color counters, paper clips, pennies)
- dot paper (each dot represents an ant)
- tape, scissors

Advance Preparation:

- 100 items for each group
- If possible find a copy of the book, *One Hundred Hungry Ants* or access **YouTube** *(http://www.youtube.com/watch?v=kmdSUHPwJtc or other sites.*
  If you do not have a copy of *One Hundred Ants* and you cannot access **You Tube**, skip to step B in this lesson.

Directions:

Activity A: (Begin here if you have the book or access to YouTube.)

If you have the book, *One Hundred Hungry Ants*, read the book aloud to the class.

1. Each time the ants rearrange themselves, ask students to predict what the next arrangement might be. If you do NOT have the book or access to YouTube, you might read about the book online.

Lesson:

2. Show an array to the whole group. Ask students to describe the arrangement of counters or tiles. Ask students to build the array on blank paper. Ask students to describe the arrangement. Be sure students are able to describe rows and columns. Ask students to rotate the array by turning the paper. Ask: What do you notice? (It is important that students focus on equal groups. Students should be able to skip count by the number in each group or use a combination of doubles and other counting strategies. Some students may count by ones. This is an opportunity for formative assessment.
3. Tell students, we are going to imagine how 100 ants might march in different rows.
4. Ask students, "How many different arrangements do you think we could make with 100 counters or 100 tiles?"
5. Ask students to work in groups to use cubes or counters to build an arrangement to show 5 equal groups with 20 cubes or counters in each group. Groups should be in straight rows and columns.
6. Different groups share solution strategies.
7. Students should show how they know they have exactly 100 counters.
8. Students work in groups of four. Each group should represent each array on dot paper by thinking about each dot as an ant. Draw a frame around an array to illustrate the arrangement. Label each array. (See the end of the lesson for blackline master of dot paper.) Students who represent 1 x 100 and 2 x 50 will have to cut and paste the dot paper to show the array.
9. Each group of students should be able to explain how they know they have found all possible arrangements.

Questions to Pose:

Before:

How would you describe your array? How many objects did you use in your arrangement?
If you could rotate your arrangement, you would see 20 groups of 5 objects each. Explain why?

During:

If an array of four rows of twenty-five is possible, then is an array of twenty- five rows of four also possible? Explain your reasoning?

After:

- How many different arrangements did you find for the 100 Hungry Ants?
- Explain how arrays with the same numbers but in different order—for example, twenty lines of five (20 x 5) and five lines of twenty (5 x 20)—would be different formations in the story
- How do some arrays relate to other arrays with the same number in reverse?

Possible Misconceptions/Suggestions:

Possible Misconceptions	Suggestions
Students may not know that 5 x 20 is the same amount as 20 x 5	Students need many opportunities to rotate an array to see the commutative property.
When counting in equal groups, students lose track.	Students might separate the rows and columns to give them more space to count objects or to more easily see one equal group at a time.
Students may have difficulty seeing that each arrangement can be rotated to show the commutative property.	Students need many opportunities to find the total number of objects when building arrays. Students can learn by listening to other students as they share strategies.

Special Notes:

- It is important for third grade students to count large amounts of objects. Students are more likely to realize that grouping objects is a more efficient way to find a total.
- Teachers should notice and name the strategies students are using.
- Teachers ask questions that provoke children to think, articulate their thinking, and sometimes try a new strategy; and we extend their thinking.

Solutions:

1 x *100; 2* x *50; 4* x *25; 5* x *20; 10* x *10* and the reverse.
100 x1; 50 x 2; 25 x 4; 20 x 5 (commutative property)

Adapted from Math Solutions professional development.

Arrow Cards

Common Core Standard:

Use place value understanding and properties of operations to perform multi-digit arithmetic. (Note: A range of algorithms may be used.)

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

Student Outcomes:

- I can add within 1000 with ease by using a strategy based on place value (collecting hundreds, collecting the tens, and collecting the ones, and when necessary, composing ten ones to make a ten or composing ten tens to make a hundred).
- I can use arrow cards as a tool to help me add using place value strategies.
- I can use a number line to add within 1000.

Materials:

- Arrow Cards (One set per partner on card stock if preferred)
- Zip plastic bag to store cards
- Paper to draw a number line
- Paper to chart a problem to be displayed
- Large paper or a way to display a number line

Advance Preparation:

- There is a growing body of research to suggest the importance of the number line as a tool for helping children develop greater flexibility in mental arithmetic as they construct mathematical meaning, develop number sense, come to understand number relationships, and develop powerful strategies for addition and subtraction. The number line can do much more than simply help children count to 100. The number line can be used as a tool to help children function well with the various operations. The number line is a powerful visual tool for adding and subtracting.
- Arrow cards will need to be copied, cut and stored in bags prior to beginning this task. This can be time consuming, but the value of these cards and the flexibility of their use is worth the time spent in preparation.
- Students may have had opportunities to use these cards in previous grades and may understand how to line the arrows up to show the value. If not, the teacher will need to spend time "playing" with these materials prior to instruction.

Directions:

1. Arrow cards are distributed to students and a quick warm up is done to make sure students understand how to use the cards. For the warm up, ask students to show you 53 and review the hidden 50 in 53, Ask students to show you 438 and review the hidden 400 and 30 in 438. Have students show 85 and 58. What is the difference in these two numbers? Which is larger? How do you know?
2. Depending upon the students' prior knowledge and work with arrow cards, the teacher may need to spend more time working with place value understanding and composing and decomposing numbers before moving to using place value strategies for addition and subtraction. See the second grade lesson on arrow cards if needed to build understanding of place value before using this lesson.
3. Present pairs of students with the problem 457 + 246= ____ . Ask students to use the arrow cards to solve this problem. Allow students to struggle with the problem as the teacher circulates and poses additional questions to the students. When students begin to finish have them share their solution with another set of partners and then compare solution strategies. After lots of small group discussion, pull students together and discuss how they solved the problem. Using some type of projection device, walk through the strategies used to solve the problem. Share the following solution strategy with the students. 457 is the same as 400 + 50 + 7 and 246 is the same as 200 + 40 + 6 when the numbers are decomposed. Be sure students see the visual model of this by separating their arrow cards , have students then combine the hundreds, tens, and ones so that they are seeing 400 + 200, 50 + 40, and 7 + 6. Exchange the hundreds for the total of 600, exchange the tens for a total of 90, and exchange the ones for a total of 13. Now put these cards together so that we have 600 + 90 + 13. When students put these cards together it will look like this:

6	0	0
	9	0
	1	3

   Now as students begin to total the problem they will see that the 90 and 10 will need to be composed into a hundred. When this happens a hundred is added to the hundreds place so the cards will now look like this:

7	0	0
	0	0
	0	3

   and the sum total becomes 703.
4. If the teacher feels that students need more guidance in walking through the next problem, it can be done as a whole group. If students were successful in completing the first problem in partners then continue to allow the students to persevere in solving the next problem.
5. Give students the problem 586 + 346 = _____. Students **MUST** use the arrow cards to solve the problem. Some students will know how to use the traditional algorithm to solve this problem and will want to quickly use their abstract understanding to solve the problem. Encourage (even require) the students to use the arrow cards so they begin to understand what is happening with the algorithm.

6. Repeat the process of sharing the strategies partners used and be sure the place value strategy of composing number is modeled as shown:

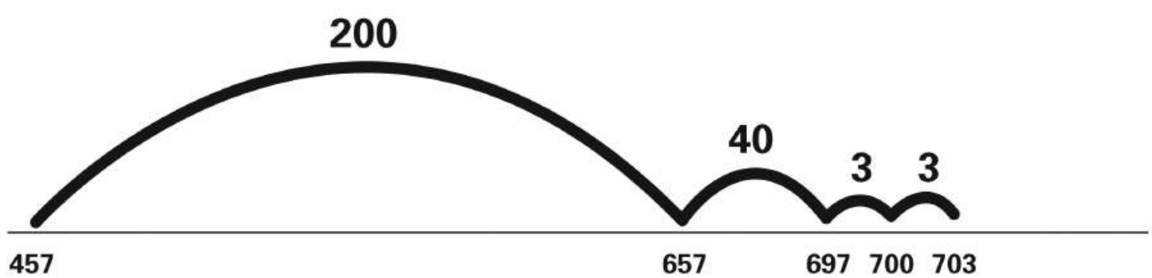
8	0	0
1	2	0
	1	2

When cards are totaled:

9	0	0
	2	0
	1	2

for a sum total of 932.

7. Continue this process with several problems and continue to require the use of arrow cards.
8. Using the problem we began with of 457 + 246= ____ ask students how it could be modeled on a number line. This should generate lots of discussion and again will depend on the prior experience students have had with number lines. Some good questions to lead this discussion would include, "Does our number line have to start at 1?" "Does a number line have to have every number marked on it?" "What could we begin with to solve this problem?" "What are the increments we could use to jump on the number line?" The teacher should model and "try" the suggestions the students give for ways to solve the problem. One efficient way to solve it problem would be:



After solving this problem, have students go back to the problems they solved earlier and create a number line to represent their solution strategy. Discuss the place value understandings used in both of these methods.

9. Ask pairs of students to come up with a problem they would like to solve, this problem would need to be approved by the teacher and may depend on the understanding of the students. It would need to be a problem that could be solved by using the arrow cards and could be represented on a number line. This would limit students in using a number over 1000 (because of the arrow cards). Have pairs of students chart their problem and solution in a way that could be displayed in the classroom. Ask students to display the solution with the arrow cards and the number line on the walls and have students do a "Gallery Walk" to look at each others problems. Encourage students to make their solutions very clear and to include pictures, models, and words so that a second grader could understand how they solved the problem.

Questions to Pose:

Before:

Can you show me how to make 325, 457, 289, 58, 399, etc. with the arrow cards.
What does the hidden zero represent?
Explain why the zero is important.

During:

How can you use your arrow cards to help you solve this problem?
Is there more than one way to represent that number?
Can you show me another way to make that number?
Does our number line have to start at 1?
Does a number line have to have every number marked on it?
What could we begin with to solve this problem?
What are the increments we could use to jump on the number line?
What have we already learned that can help us with the number line?
How is understanding place value going to help you solve this problem?

After:

What strategy did you find most helpful in solving the problems?
How are the arrow cards and the number line alike as a strategy?
What would you say to a second grader if you were trying to teach them how to add using arrow cards?
What would you say to a second grader if you were trying to teach them to add using a number line?

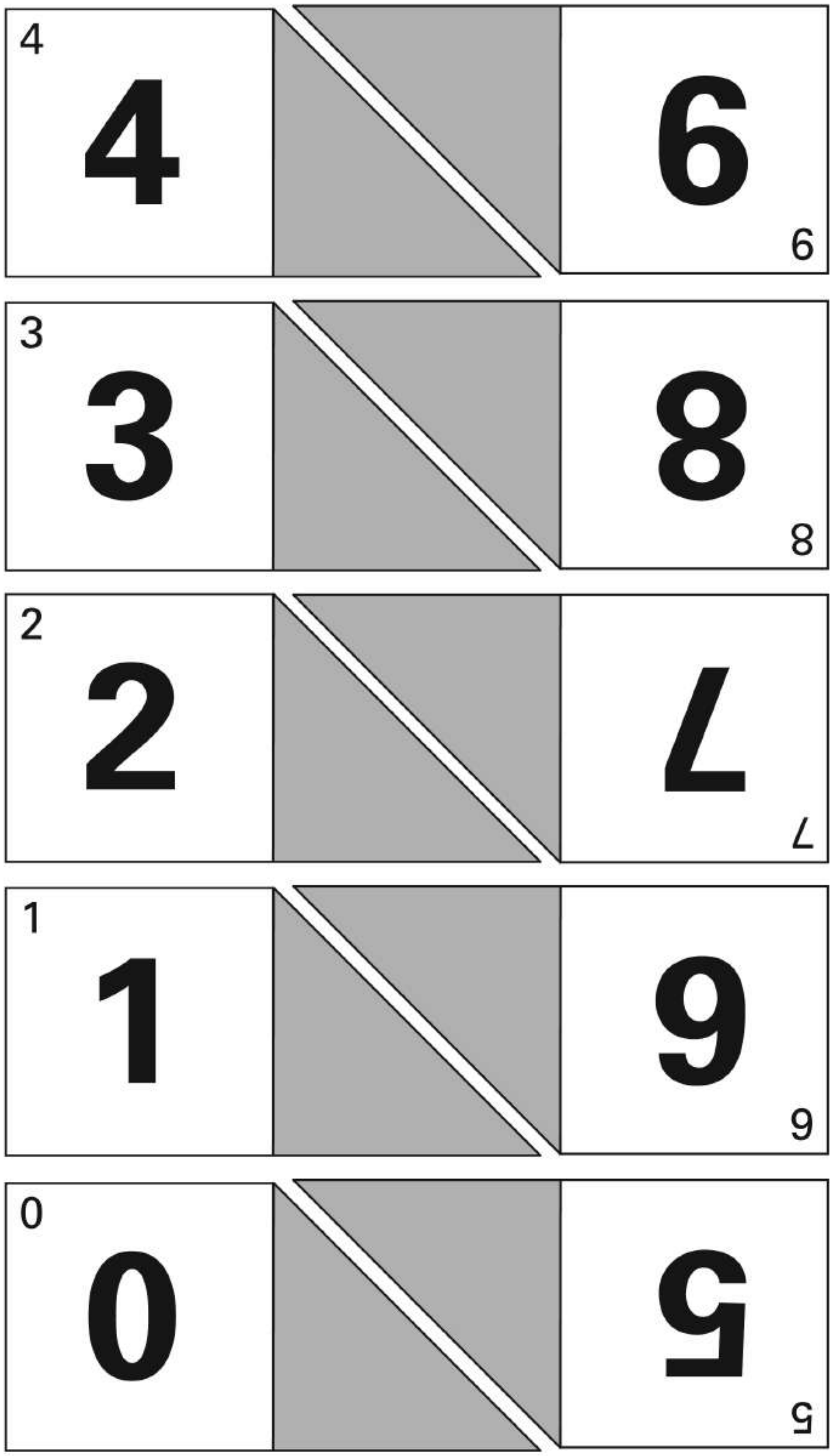
Possible Misconceptions/Suggestions:

Possible Misconceptions	**Suggestions**
Students may not have a conceptual understanding of place value so that they would think 234 is 2+3+4 rather than 200+30+4 and may not see the relevance of the zeros.	Have students build numbers with base ten blocks and compare those numbers to the arrow cards so that the students can see why the zero is important. This may take several conversations and a variety of models until students begin to understand.
Students with limited or no experience with an empty number line may want to put each number on the number line. Students may not know how to space the numbers on the number line so that they represent the quantity of the number correctly.	Begin with smaller numbers for these students so they can see the actual space between numbers on a number line that is marked with appropriate spacing. Refer to the second grade lesson titled "The Human Number Line."

Special Notes:

This task is very involved and may take several days depending on the prior experiences and the place value understandings of the students. This lesson only involves addition but the standard also addresses subtraction.

Adapted from Partners for Mathematical Learning, 2009



40	90
30	80
20	70
10	60
00	50

400 4	0	0

300 3	0	0

200 2	0	0

100 1	0	0

000 0	0	0

500 **5**	**0**	**0**	
600 **6**	**0**	**0**	
700 **7**	**0**	**0**	
800 **8**	**0**	**0**	
900 **9**	**0**	**0**	

Hexagon Sandwiches

Common Core Standard:

Develop understanding of fractions as numbers.

3.NF.1 Understand a fraction 1/*b* as the quantity formed by 1 part when *a* whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by a parts of size 1/*b*

Additional/Supporting Standard(s):

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure

Student Outcomes:

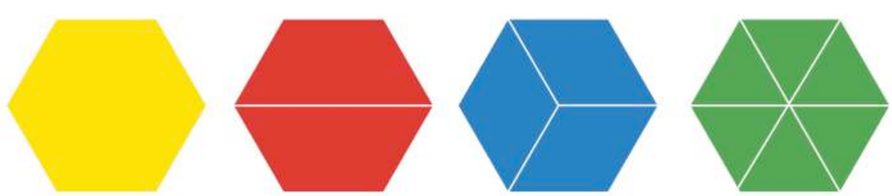
- If the hexagon is the whole, I can use unit fractions to find the value for other pattern blocks.
- I can use different pattern blocks to make a combination that equals one whole.
- I can combine unit fractions and prove they are equivalent to one whole.

Materials:

- Pattern Blocks (hexagons, trapezoids, triangles, blue rhombuses)
- Hexagonal Grid paper
- Colored pencils or crayons to match the color of pattern block pieces

Advance Preparation:

- If students have not used pattern blocks, provide students with time to explore the relationships among the pattern blocks.
- Allow students to discover and share relationships among the pattern blocks (hexagon, triangles, trapezoids blue rhombuses.)
- Students need prior experiences to identify and count unit fractions to equal a whole.



Directions for Activity:
Teacher: Show students the yellow hexagon. "Let's say that this yellow pattern block is the bottom of a sandwich."

- Tell students they are going to make toppings that exactly cover each Hexagon Sandwich.
- A hexagon sandwich is an open-faced sandwich (It doesn't have a top.)
- The sandwich has a hexagon on the bottom.
- The topping for each sandwich is made with other pattern blocks that fit exactly on the hexagon.
- Remember you can combine different shapes.
- Each hexagon sandwich must have a different combination of blocks for the topping

Questions to Pose:
Before:

- Can someone show me two pattern blocks that will equal one whole?
- What pattern block is a half of a hexagon sandwich?
- Can someone show a combination of different-size blocks to make a whole?

During:

- Who can share a different combination to make a topping?

After:

- How many different solutions did you find when "building toppings for hexagon sandwiches"?
- How do you know you have found all the possible solutions?

Possible Misconceptions/Suggestions:

- All students should use models to build toppings.
- Students may not realize that a topping may be made from repeating the same shape. Example: 6 green triangles
- Students may not realize that 2 trapezoids are equal to one whole, not 2 wholes.
- Students may not understand that if two red trapezoids combine to make a whole, each piece is ½.
- Students may see that a different arrangement of the same pattern block is a different amount of the same topping. (See Special Note below.)

Special Notes:
Consider one solution combination made with the same pieces to be the same solution. (Teacher may need to model this when students are sharing solution strategies.)

Extension:
Students who finish quickly may investigate all the different ways to cover a sandwich that is made of two yellow hexagons. The two yellow hexagons are considered to be 1 whole sandwich. Students use combinations of halves, thirds, and sixths to find ways to build a topping for each new sandwich. Students record, using a Hexagon Grid. Students record fractional parts. One yellow hexagon is ½ of the whole. A trapezoid is ¼ of a whole, a rhombus is 1/6 of the whole and a triangle is 1/12 of the whole.

Solutions:
Students share different solutions. There are 8 possible solutions. If students place a hexagon as a topping, then there are 9 solutions.

Hexagon Sandwiches

Materials:

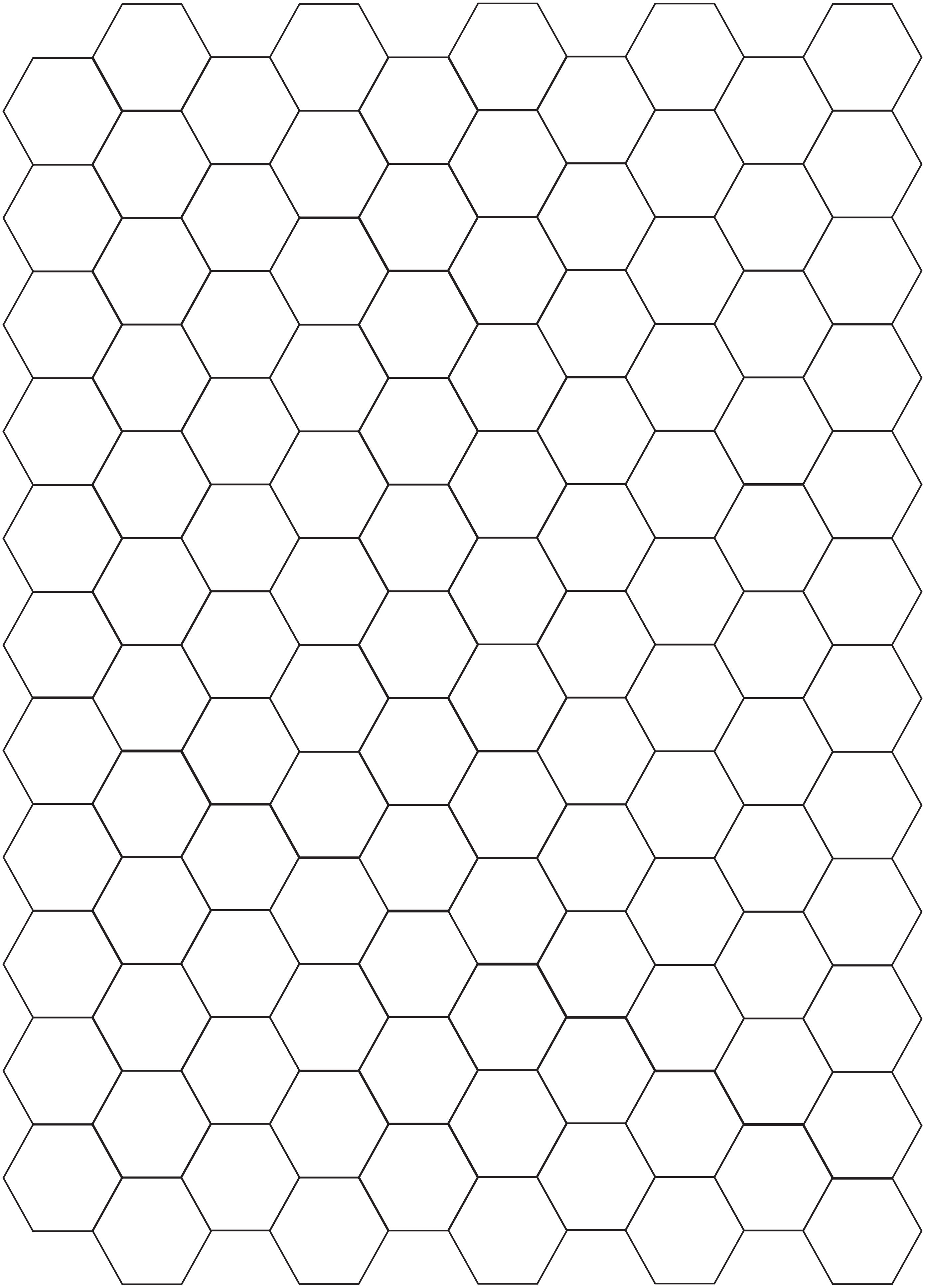
Pattern Blocks (hexagons, trapezoids, triangles, blue rhombuses)
Hexagon Grid paper (per student)
Colored pencils or crayons to match the color of pattern block pieces

Your Task:

- Use different combinations of pattern blocks that will build toppings for each yellow hexagon sandwich.

hexagon (bottom of a sandwich) without topping

- How many different sandwiches can you make?
- To be different, a sandwich must use a different set of blocks from all other sandwiches.
- Using your "Hexagon Recording Sheet", record each solution by tracing each of the different pattern blocks you are using to completely cover a hexagon sandwich.
- On each solution, record fractional parts, using unit fractions.
  Example: 1/2 + 1/6 + 1/6 + 1/6 = 1 sandwich
- How will you know when you have found all of the different ways to create toppings for the sandwiches? (no repeats and none missing)
- Share solution strategies in groups and with the whole class.
- How do you know when you have found all of the possible ways to add toppings to the hexagon sandwiches?



Solving Equal Sharing Problems

Common Core Standard:

Develop understanding of fractions as numbers.

3.NF.1 Understand a fraction 1/*b* as the quantity formed by 1 part when *a* whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by a parts of size 1/*b*.

Additional/Supporting Standard:

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

Student Outcomes:

- I can solve equal sharing problems and explain how I solved them.
- I can retell the problem.
- I can create or show fractional parts by drawing, cutting, folding, and splitting things.
- I can compare the size of a fractional part to the whole unit.
- I can justify my answer.

Materials:

- Materials such as cubes, tiles, counters, scissors, blank paper, etc.

Advance Preparation:

- Decide how to pair students to work together.
- Students need opportunities to count by unit fractions; compare fractional parts;
- Have materials available as students should select manipulatives or "draw pictures" to share strategies.
- Present a story problem on the board, document camera, white board.
- This lesson is designed so that fraction problems shared with the whole class have answers that are mixed numbers and fractions less than 1.
- Introduce the symbol for ½ once students learn to differentiate between halves, fourths and thirds.
- After sharing strategies, students solve fraction problems on the blackline master.
  The format of this lesson can be repeated many times using different equal groups problems to promote class discussion and sharing of strategies. Three different blackline masters are provided and can be completed over the course of several lessons.

Directions:

1. Explain that students are going to solve one equal group problem.
2. Students work with a partner to solve each equal-sharing problem.
3. Students will be expected to share their solution strategies and justify their answers.
4. Problem posed by the teacher: Three students want to share 7 candy bars so that each child gets the same amount. How much candy bar can each child get? Label your answer and show each child's share. (If 3 children want to share 8 candy bars so that each child gets the same amount, the answer can be 2 2/8 candy bar Or 2 and 1/4 candy bar.
5. As students work with partners to solve the problem, the teacher checks with each group, asking questions such as those written in the "Questions to Pose." This is formative assessment. Students who struggle may benefit from retelling the problem.
6. Teacher selects several "pairs" to share and explain their solution strategy. (Teacher might select students who have models/drawing/cut-out etc. strategies and then move to a student with a more abstract solution. (Document camera or overhead projectors can be use to share strategies.)
7. Students label fractional parts.
8. Students solve problems on the blackline masters. Students may work with partners. All students must show their solution strategies for each problem.
9. The teacher can determine if the class is ready to work independently with partners or needs additional whole class work. Pose additional problem(s) if the class needs more time for working and discussing as a whole class.
10. If students are ready have them complete the problems on the handout. Explain that they are to use the materials to "act out" the problem and draw a representation of their solution strategy.
11. Each student should be engaged in solving and discussing solution strategies at an appropriate level or strategy.

Questions to Pose:

Before:

What does it mean to have an equal share?
How can we make sure we give a fair-share?

During:

After reading a problem, what do we know?
After reading a problem, what do we need to know?
How might you use cubes, counters, drawings, etc. to help solve the problem?

After:

How are the different solution strategies alike? How are they different?
Students should be able to justify thinking.

Possible Misconceptions/Suggestions:

Possible Misconceptions	Suggestions
Students do not make sense of the problem and may add numbers and find the sum. Students might see the leftovers as remainders and will not fair share the leftovers.	Have students retell the story problem. Have them use the materials to act out the problem. If students still cannot understand what is happening in the problem, change the problem so half of something will part of the answer.
Students may not understand that each student has the same whole and the same fractional part.	Students should be able to prove the shares are equal.

Special Notes:

- After students are comfortable with fraction problems, students may write their own problems.
- The teacher may share some of the student problems as examples for the class to solve.
- It is important that students label fractional parts and can also show fractional parts are equivalent to the whole.

Solutions:

Class problem: Three students want to share 7 candy bars so that each child gets the same amount. How many candy bars can each child get?

Answer: Each student will receive 2 candy bars and 1/3 of another candy bar or 2 1/3 candy bar. Students should have drawings, cut-out fractional parts or written explanations to show their solution.

Blackline Master: 5 2/4 or 5 1/2   3 3/6 or 3 1/2

2 2/6 or 2 1/3   2 2/4 or 2 1/2

Solving Equal Sharing Problems

Name ____

Solve each problem. Show your solution strategy using drawings, numbers, and words. Record and label your answer.

1. Four boys want to share 22 cookies so that each boy gets the same amount of cookies. How much cookie should each boy receive?

2. Jeremy has 21 cheese sticks. He is going to share them equally with five other friends. How much cheese stick will Jeremy and his five friends receive?

Solving Equal Sharing Problems

Name ____

Solve each problem. Show your solution strategy using drawings, numbers, and words. Record and label your answer.

1. Six girls want to share 14 cupcakes equally. How many cupcake would each girl get?

2. Four children want to share 10 chocolate bar so that everyone gets the same amount. How much chocolate bar can each child have?

Is This Duck "One-Half Red?"

Common Core Standard:

Develop understanding of fractions as numbers.

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

Additional Standards: 3.NF.1

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.

Student Outcomes:

- I can find the value of pattern blocks when the yellow hexagon is equivalent to one whole.
- I can label fractional parts of the duck using unit fractions.
- I can combine unit fractions to find the value of the duck.
- I can explain my solution strategies to others.

Materials:

- Copy of worksheets for each student
- Triangle grid paper (Each page could be cut in half.)
- Pattern blocks (hexagons, triangles, trapezoids, blue rhombus)
- Colored pencils

Advance Preparation:

- The teacher decides if students will work with a partner or in small groups.
- Each student should complete the activity even if students are working with a partner.
- Ask students: What do you know about area?
- Students have prior experiences finding relationships among pattern blocks. Ask: If the hexagon is one whole, discuss with your group the relationships among other pattern blocks.
- Students have prior experience counting unit fractions.

Directions for Activity:

Teacher: (Show the duck as shown on the student page.) "This is a design I made with pattern blocks. I used several different colors and shapes in my design of a duck. All of the pieces in my design create the whole duck."

Teacher: My questions for you: "Is the duck exactly one half red? Is the duck greater than one half red? Is the duck less than one half red?"

1. You will use pattern blocks to build this duck on Triangle Grid Paper.
2. After building the duck, remove each pattern block one at a time and color the shape the same color as the pattern block. Example, "If you remove a triangle, color the triangle space green. Continue until each pattern block is removed and the area of the each pattern block is colored.
3. Each student should label each pattern block shape using unit fractions.
4. Each student circles the sentence with the correct answer. (Students working with a partner or in a small group should come to consensus.)
5. Using a document camera, students share their solution strategies and justify their answer.

Questions to Pose:

(Beginning) What relationships did you find among the pattern blocks? How might you find the value of the duck if a hexagon is equal to 1?
(Middle) What equivalences were you able to find?
(Ending) Explain your solution strategy to another group.

Possible Misconceptions/Suggestions:

Students may count and find that 4 pattern blocks are red rather than seeing the fractional equivalency of the red trapezoid. Students may not consider each pattern block as part of one duck. Students may not know fraction equivalences. Students may not be able to combine unit fractions to decide the value of the duck.

Solution:

The duck is equivalent to 4 hexagons and 1/6 of another hexagon. The duck has only 2 red hexagons. The duck has 2 hexagons and 1/6 of a hexagon (blue, yellow and green). The duck is less than ½ red. The duck would be one half red if there was **not** one extra green triangle.

Special Notes:

Is This Duck One-Half Red?

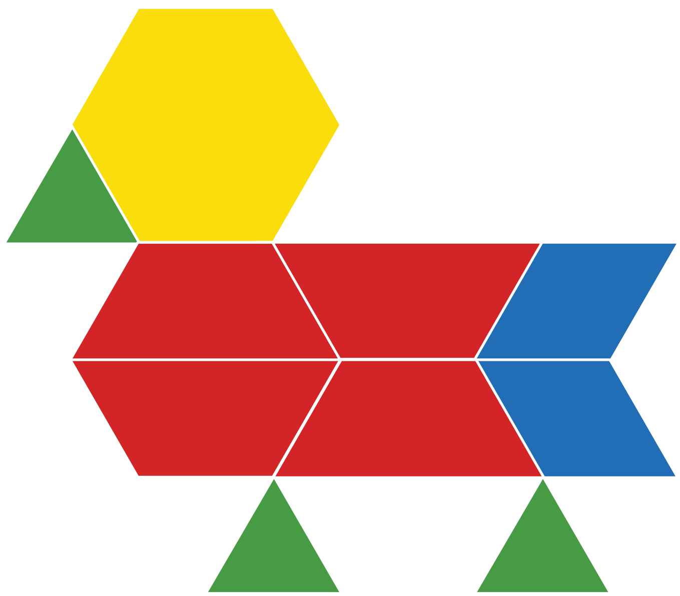
1. Use Pattern Blocks to build the duck below on triangle grid paper.

2. As you remove a pattern block, color the space that holds each block.

   Examples:
   - Remove a yellow hexagon and color the hexagon on the paper yellow.
   - Remove a trapezoid and color the trapezoid on the paper red.

   Continue until you have a complete picture of the duck.

3. Using unit fractions, label each colored shape.

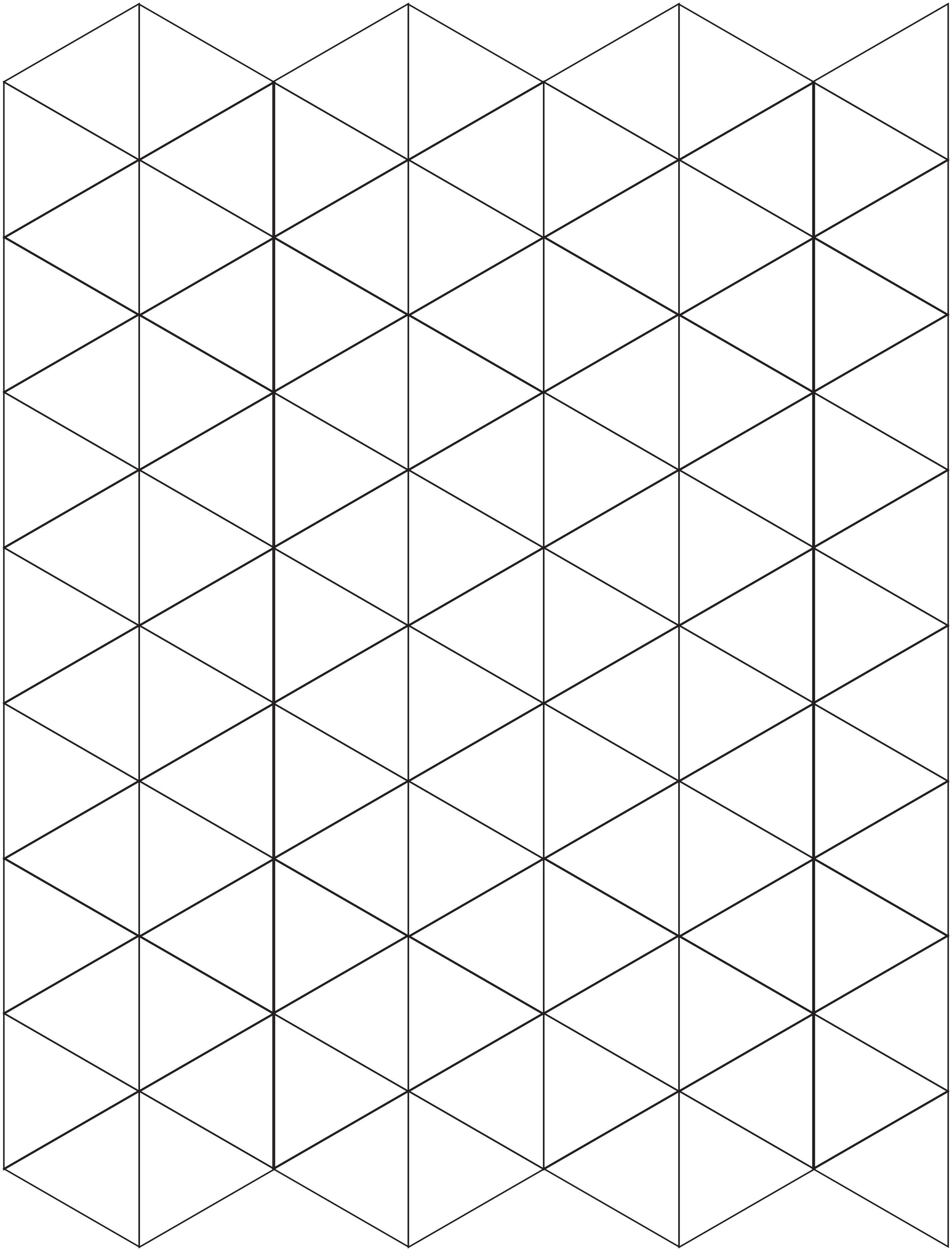


4. Is this duck exactly one-half red?

   Circle the statement below that answers the question, “Is the duck exactly one-half red?”

   - The duck is **exactly** one-half red.
   - The duck is **less than** one-half red.
   - The duck is **greater than** one-half red.

5. Using numbers and words, explain why you chose the answer you circled above.



Fractional Parts of Rectangles

Common Core Standard:

Develop understanding of fractions as numbers

3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

Additional/Supporting Standard(s):

3.NF.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by *a* parts of size 1/*b*.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure

Student Outcomes:

- I can construct rectangles using square tiles.
- I can count unit fractions to name fractional parts.
- I can recognize equivalent fractions.
- I can find different ways to represent the same fractional part.
- I can connect my model to area, when using square units.

Materials:

- Square tiles (green, yellow, red, and blue) Each group will need a total of 30+ square tiles
- One-inch grid paper
- Colored pencils
- Tape/Scissors (Students or groups may need tape and scissors for some of their models.)

Advance Preparation:

- Depending on prior experiences, teacher may assign partners or small groups for this task.
- Students should have prior experiences naming and identifying fractional parts of a whole.
- Students should have experiences with counting and comparing unit fractions.
- Students can identify, compare and identify equivalent fractions.

Directions for Activity:

- Students will use color tiles to build rectangles based on specified fractional parts.
- Each student will record his/her group's solution(s) for Activity 1 and Activity 2.

Activity 1A:

- Each student will use square tiles to build a rectangle that is 1/2 red. If working with a partner or in small groups, each student should build the model the group has decided to build.
- Each student(s) should record the solution on one-inch grid paper by coloring squares to match the rectangle.
- Using fraction notation, label the fractional parts of your rectangle.
- Students share solutions for the whole class, proving their rectangles are exactly one-half red.

Activity 1B:

- Each student will build a rectangle with a different area that is 1/2 red. Challenge groups to find different solution strategies for 1/2 red.
- Show your solution on one-inch grid paper by coloring squares to match your rectangle.
- Using fraction notation, label the fractional parts of your rectangle.
- Find ways to prove your new rectangle is also 1/2 red.

Activity 2A:

- Use square tiles to build a rectangle that is 1/2 red, 1/4 yellow, and 1/4 green.
- Show your solution on one-inch grid paper by coloring squares to match your rectangle.
- Using fraction notation, label the fractional parts of your rectangle.
- Prove your new rectangle is 1/2 red, 1/4 yellow, and 1/4 green.

Activity 2B:

- Find at least one other rectangle with a different area that is 1/2 red, 1/4 yellow, and 1/4 green.
- Show your solution on one-inch grid paper by coloring squares to match your rectangle.
- Using fraction notation, label the fractional parts of your rectangle.
- Prove your new rectangle is 1/2 red, 1/4 yellow, and 1/4 green.

Questions to Pose:

Before:

- What strategies might you use to determine one half?

During:

- How did you decide the number of tiles needed to build a different rectangle that is ½ red?
- How many solutions strategies might you find for building rectangles that are ½ red?
- Explain how the numbers in the fractions relate to the different tiles you used to create your rectangles.

After:

- Explain how Activity#1 is different from Activity #2.
- How did you decide how many tiles you need to build the rectangle in Activity #2?
- What strategies did you use to find solutions?
- Why is Activity #2 more difficult?
- Did anyone find more than one or two solutions for Activity #2?

Possible Misconceptions/Suggestions:

Students may rearrange the tiles to find a different solution strategy. This would be a good opportunity to show that ½ red can be arranged in different ways but the area of ½ red remains the same.

Students mat not see there are infinite solutions for building a rectangle where ½ of a rectangle is red. (Please note the pattern described below.)

Special Notes:

- Some students may need to spend more time and solve Activity #1 in different ways. Students might also find solutions for ½ yellow, etc.

Extension:

- Students might build a rectangle of 4 tiles. (1/2 or 2 of the tiles will be red.)
- Students might build a rectangle of 6 tiles: (1/2 or 3 of the tiles will be red.)
- Many students may see this pattern and see that 1/2 of any even number could be constructed with red tiles.
- Some children may use their knowledge of equivalent fractions to make additional models. For example, to build rectangles that are one-half red, children may first create fractions equivalent to 1/2, such as 2/4 and 3/6, and then see if rectangles can be formed using the number of tiles indicated by the denominator. If so, they may realize that the numerator is the number of red tiles that should be used.

Challenge:

- Some children may use their knowledge of equivalent fractions to build additional rectangles.
- For example, to build rectangles that are two-thirds red, children may first create fractions equivalent to 2/3, such as 4/6 and 6/9, and then see if rectangles can be formed using the number of tiles indicated by the denominator. If so, students may realize that the numerator is the number of red tiles that should be used. Using a document camera, students can show and describe this pattern.

Additional Activities:

Be sure to build, record and label your solution.

1. 1/2 red, 1/4 yellow, 1/8 blue, 1/8 green
2. 1/8 red, 3/8 green, 1/2 yellow
3. 2/5 red, 3/5 blue
4. 1/3 yellow, 2/3 red
5. 1/3 blue, 1/3 green, 1/6 yellow, 1/6 red

Solutions:

Activity 1

Part 1 and Part 2: 2 red tiles, 1 blue, 1green or 4 red tiles, 2 green tiles and 2 blue tiles
Solutions will vary. There are infinite solutions. Students can share solutions in table groups. Groups may join with other groups, to compare and justify strategies and solutions.

Activity 2

Part 1 and Part 2: 2 red, 1 green, 1 yellow**;** 4 red, 2 green, 2 yellow (infinite solutions)

Finding Fractional Parts of Rectangles

Materials:

- Square tiles (green, yellow, red, and blue) Each group will need a total of 30 or more square tiles
- One-inch grid paper
- Colored pencils
- Tape/Scissors (available)

Activity 1:

Part 1

- Working with a partner, use square tiles to build a rectangle that is ½ red. If working with a partner, each person should build the same model.
- Each student(s) should label the rectangle as ½ red. Record the solution on one-inch grid paper by coloring squares to match the rectangle.
- Using fraction notation, label the fractional parts of your rectangle.
- Find ways to prove that your rectangle is exactly one-half red.

Part 2

- Working with a partner, each student will build a rectangle with a different area that is ½ red.
- Show your solution on one-inch grid paper by coloring squares to match your rectangle.
- Using fraction notation, label the fractional parts of your rectangle.
- Find ways to prove your new rectangle is also ½ red.
- Adding more tiles ½ red.
- Show each solution on one-inch grid paper by coloring square to match your rectangles.
- Find ways to prove each new rectangle is also ½ red.

Activity 2:

Part 1

- Use square tiles to build a rectangle that is 1/2 red, 1/4 yellow, and 1/4 green.
- Show your solution on one-inch grid paper by coloring squares to match your rectangle.
- Using fraction notation, label the fractional parts of your rectangle.
- Prove your new rectangle is 1/2 red, 1/4 yellow, and 1/4 green.

Part 2

- Find at least one other rectangle with a different area that is 1/2 red, 1/4 yellow, and 1/4 green.
- Show your solution on one-inch grid paper by coloring squares to match your rectangle.
- Using fraction notation, label the fractional parts of your rectangle.
- Prove your new rectangle is 1/2 red, 1/4 yellow, and 1/4 green.

Name of Task. Doubling, Halving, Tripling

Common Core Standard:

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

Additional/Supporting Standard(s):

Represent and solve problems involving multiplication and division.

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriately tools strategically.
6. Attend to precision.
7. Look for and make use of structure.

Student Outcomes:

- I can draw rectangles and label the length and the width.
- I can find and label the perimeter of rectangles.
- I can find and label the area of rectangles.
- I can double and triple numbers.

Materials:

- One-inch grid paper (one or two per student)
- Colored pencils or thin markers to trace rectangles
- Problems for students – blackline master

Advance Preparation:

- Students should have prior experiences finding and labeling area and perimeter.
- Students should have prior experiences measuring in inches.

Directions:

1. Show the table below to students.
2. Using one inch grid paper and a document camera, draw the rectangle that has a length of 3 inches and a width of 2 inches. Teacher and student add labels to the rectangles. (width -2 inches and length -3 inches)
3. Ask: How might you find the perimeter of the rectangle? (Find measurement for each side.) 2 + 2 + 3 + 3 = 10 or 2 x 2 + 2 x 3 = 10
4. Ask: How might you find the area of the rectangle?
   Students might count square units.
   Students might multiply the length by the width (2 x 3 = 6 square units).
5. How might the rectangle look if you double the width and also double the length?
6. Students draw their new rectangle on one-inch grid paper. Ask students what does it mean to double the length and the width? Students label the length and the width of the new rectangle. Also, find the area.
7. Ask students to describe the rectangle they would draw next, if they were to triple the length (3 inches) and the width (2 inches.)
8. Ask students to triple the size of the original rectangle. 2 (width) and 3 (length)
9. Draw the new rectangle on the one-inch grid paper. Find the perimeter and area of the new rectangle.

	Original Size	Double the Size	Triple the Size
Length	3	6	9

	Original Size	Double the Size	Triple the Size
Width	2	4	6

Questions to Pose:

<u>Before:</u>

How can you find the perimeter of a rectangle on one-inch each grid paper?
How can you find the area of a rectangle on one-inch grid paper

<u>During:</u>

How did you find the perimeter of the rectangle after you tripled the length and the width of the rectangle?
How did you find the area of the rectangle after you tripled the length and the width?

<u>After:</u>

Describe and share your solution to another rectangle you doubled? Tripled?
Did anyone reduce the length and width of a rectangle and make it half of its original perimeter?
Can you share your strategy and describe how you reduced the rectangle?

Possible Misconceptions/Suggestions:

Possible Misconceptions	**Suggestions**
Students confuse area and perimeter Students may not know how to find perimeter and area. Students label area without using square units.	Students need to see perimeter as linear- the outside or boundary around a garden, etc. Provide students with contextual problems

Special Notes:

Students might reduce the size of the rectangle and make it half its original perimeter, they should divide the length and the width by two.

Solutions:

Doubling and Halving Rectangles

Name: ____

Materials:

- One-inch grid paper (one or two per student)
- Colored pencils or thin markers to trace rectangles

Problem to solve: Show your work.

1. Jeremy found a picture on the Internet. The dimensions of Jeremy's picture are:
   Width: 4 inches   Height: 6 inches
   - Using one-inch grid paper, draw and label a rectangle to show the length and width of Jeremy's picture.
   - Find and label the perimeter of Jeremy's picture. Show your solution strategy
   - Find and label the area of Jeremy's picture. Show your solution strategy.
2. Jeremy decided he needed a larger picture than the 4" x 6" picture to hang on his bedroom wall. He decided to double the picture.
   - Using one-inch grid paper, draw and label a rectangle to show the length and width of the larger picture.
   - Find and label the perimeter of Jeremy's picture. Show your solution strategy.
   - Find and label the area of Jeremy's picture. Show your solution strategy.
3. Jeremy decided he needed a smaller picture to carry in his pocket. He decided to halve the dimensions of the original picture. The dimensions of the original picture are: Width: 4 inches   Height: 6 inches
   - Using one-inch grid paper, draw and label a rectangle to show the length and width of Jeremy's SMALLER picture.
   - Find and label the perimeter of Jeremy's picture. Show your solution strategy.
   - Find and label the area of Jeremy's picture. Show your solution strategy.

Challenge Problem: Decide on the dimensions for a new picture. Write your new dimensions: Width ______   Height ______

Using one-inch grid paper, draw and label rectangles to show:

a. Length and Width of your new picture:
b. Double the Length and Width of the new picture.
c. Triple the Length and Width of the new picture.
d. Find the Length and Width if you halve the new picture.
