Lesson 2: Mean Value Theorem

AP Calculus

Mrs. Mongold

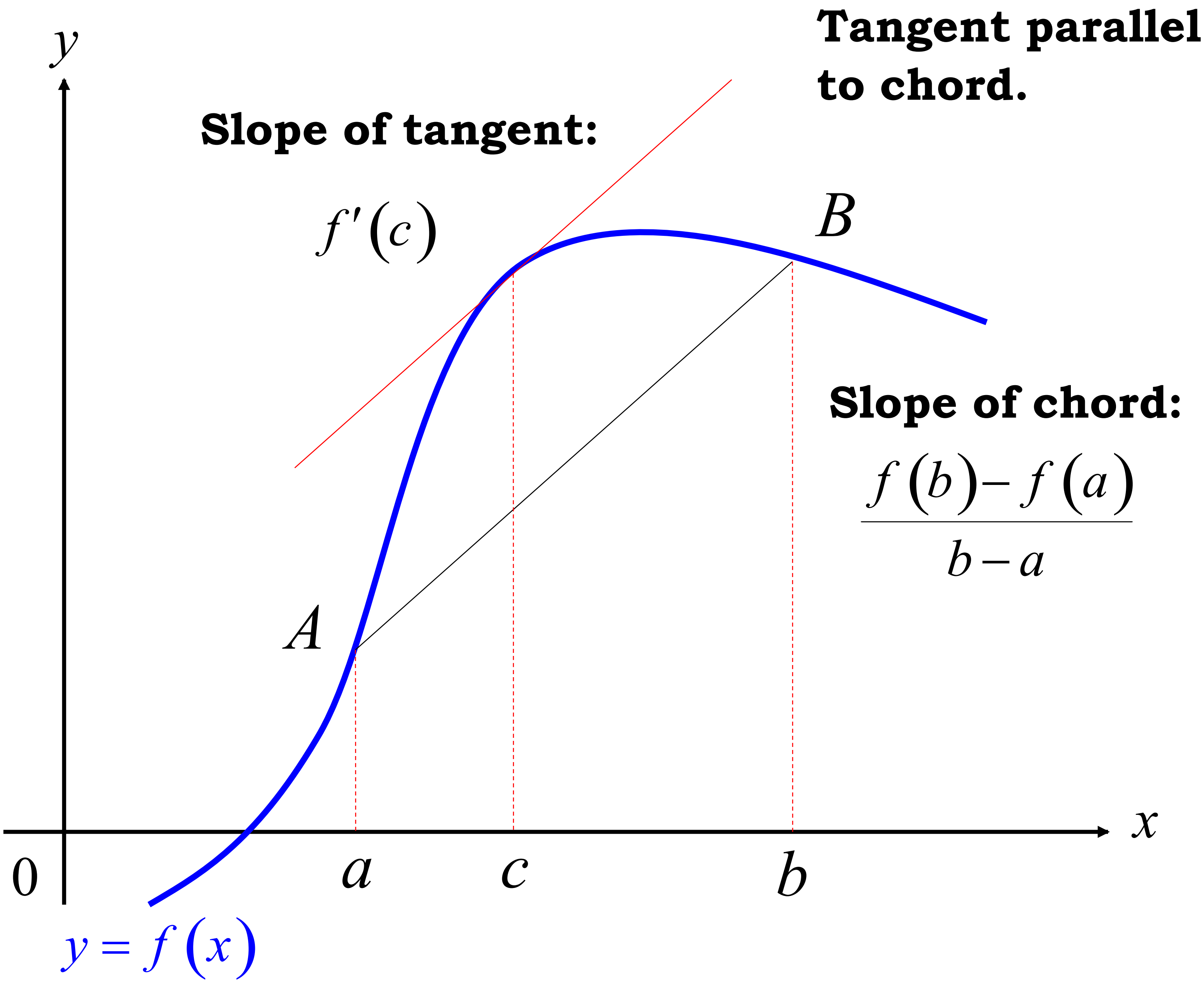
Mean Value Theorem for Derivatives

If $f(x)$ is continuous over $[a,b]$ and differentiable over (a,b), then at some point c between a and b:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

The Mean Value Theorem says that **at some point in the closed interval, the actual slope equals the average slope**.

rv

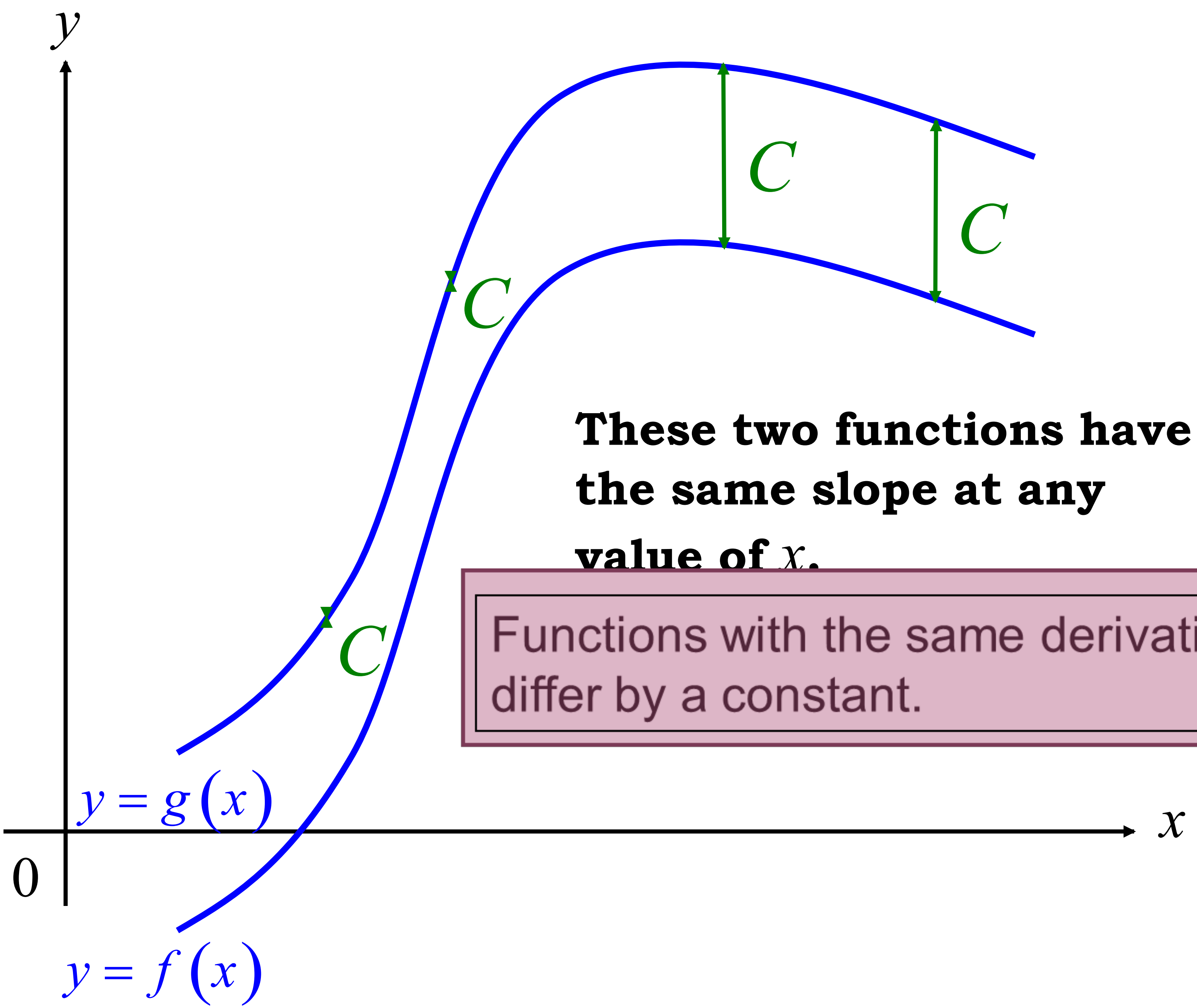


Tangent parallel to chord.
Slope of tangent:
$f'(c)$
B
Slope of chord:
$\frac{f(b)-f(a)}{b-a}$
A
y
x
0
a
c
b
$y=f(x)$

ns:

A function is <u>increasing</u> over an interval if the derivative is always positive.

A function is <u>decreasing</u> over an interval if the derivative is always negative.



These two functions have the same slope at any value of x.

Functions with the same derivative differ by a constant.

Example 1

- **Show f(x)=x^2 satisfies the hypothesis of the MVT on the interval [0,2] then find a solution c to the equation**

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Example 2

$$f(x) = \sqrt{1 - x^2}$$

A =(-1, f(-1)) and B =(1, f(1)). Find a tangent to f in the Interval (-1, 1) that is parallel to the secant AB

Example 3

- **If a car acceleration from zero takes 8 seconds to go 352 feet, its avg. velocity for the 8 sec. Interval is 352/8= 44 ft/sec or 30 mph. At some point during the acceleration the thm says, the speedometer must read exactly 30 mph**

Increasing vs. Decreasing

- **F is increasing on I if x, $x_1 < x_2$ then $f(x_1) < f(x_2)$**
- **F is decreasing on I if x, $x_1 < x_2$ then $f(x_1) > f(x_2)$**

- **F'>0 the f is increasing**
- **F'<0 then f is decreasing**

Example 4

- **Determine where $y = x^2$ rises and falls**

Example 5

- **Where is f(x) = $x^3 - 4x$ increases and decreases**

Consequences of MVT

- **Functions with f'=0 are constant**
  - **If f'(x) = 0 at each point of an interval I, then there is a constant C for which f(x)=C for all x in I.**
- **Functions with the same derivative differ by a constant**
  - **If f'(x)=g'(x) at each point of an interval I, then there is a constant C such that f(x)= g(x)+C**

Special Case of MVT is Rolle's Theorem

- **For a** *differentiable* **function, the derivative is 0 at the point where** f **changes direction. Thus, we expect there to be a point** c **where the tangent is horizontal. These ideas are precisely stated by Rolle's Theorem:**
- **Rolle's Theorem**
- **Let** f **be differentiable on** $(a\ b)$ **and continuous on** $[a\ b]$**. If** $f(a)=f(b)=0$**, then there is at least one point** c **in** $(a\ b)$ **for which** $f'(c)=0$**.**
- **Notice that both conditions on** f **are necessary. Without either one, the statement is false!**
- **For a** *discontinuous* **function, the conclusion of Rolle's Theorem may not hold: For a** *continuous, non-differentiable* **function, again this might not be the case:**

AP Test Tip

- **Probably will not say Mean Value Theorem the word "average" will probably be used to indirectly imply that the use of MVT is what they are looking for.**

Example 6:

Find the function $f(x)$ **whose derivative is** $\sin(x)$ **and whose graph passes through** $(0,2)$**.**

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

so: $$\frac{d}{dx}-\cos(x) = \sin(x)$$

$$\therefore f(x) = -\cos(x) + C$$

$$2 = -\cos(0) + C$$

Example 6:

Find the function $f(x)$ whose derivative is $\sin(x)$ and whose graph passes through $(0,2)$.

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

so: $$\frac{d}{dx}-\cos(x) = \sin(x)$$

$$\therefore f(x) = -\cos(x) + C$$

$$2 = -\cos(0) + C$$

$$2 = -1 + C$$

$$3 = C$$

Notice that we had to have initial values to determine the value of C.

$$f(x) = -\cos(x) + 3$$

The process of finding the original function from the derivative is so important that it has a name:

Antiderivative

A function $F(x)$ is an **antiderivative** of a function $f(x)$ if $F'(x) = f(x)$ for all x in the domain of f. The process of finding an antiderivative is **antidifferentiation**.

You will hear <u>much</u> more about antiderivatives in the futu

This section is just an introduction.

Example 7b: Find the velocity and position equations for a downward acceleration of 9.8 m/sec² and an initial velocity of 1 m/sec downward.

$$a(t) = 9.8$$

(We let down be positive.)

$$v(t) = 9.8t + C$$

acceleration.

$$1 = 9.8(0) + C$$

$$1 = C$$

$$v(t) = 9.8t + 1$$

Example 7b: Find the velocity and position equations for a downward acceleration of 9.8 m/sec^2 and an initial velocity of 1 m/sec downward.

$$a(t) = 9.8$$

$$v(t) = 9.8t + C$$

$$1 = 9.8(0) + C$$

$$1 = C$$

$$v(t) = 9.8t + 1$$

$$s(t) = \frac{9.8}{2}t^2 + t + C$$

The power rule in reverse: Increase the exponent by one and multiply by the reciprocal of the new exponent.

Example 7b: Find the velocity and position equations for a downward acceleration of 9.8 m/sec² and an initial velocity of 1 m/sec downward.

$$a(t) = 9.8$$

$$v(t) = 9.8t + C$$

$$1 = 9.8(0) + C$$

$$1 = C$$

$$v(t) = 9.8t + 1$$

$$s(t) = \frac{9.8}{2}t^2 + t + C$$

$$s(t) = 4.9t^2 + t + C$$

The initial position is zero at time zero

$$0 = 4.9(0)^2 + 0 + C$$

$$0 = C$$

$$s(t) = 4.9t^2 + t$$

Homework on Calendar