

AP Calculus BC

Syllabus

PRIMARY TEXTBOOK

Larson, Ron, Bruce H. Edwards, and Robert P. Hostetler. *Calculus, 8th Edition*. Boston: Houghton Mifflin.

COURSE OUTLINE

Unit 1: Limits and their Properties (3 blocks)

- Introduction to Limits
- Graphical and Numerical Analysis of Limits
- Properties of Limits
- Evaluation of Limits, Algebraic and Trigonometric Techniques
- One-sided Limits and Limits that Do Not Exist
- Infinite Limits and Asymptotic Behavior
- Continuity and the Intermediate Value Theorem

Unit 2: Differentiation (5 blocks)

- Graphical and Analytical Definition of Derivative (Limit-based)
- Physical Interpretation of Derivative; Derivative as a Rate of Change
- Continuity and Differentiability
- Basic Differentiation (piecewise, absolute value, numerically defined, exponential, logarithmic, trigonometric, and inverse trigonometric functions)
- Product Rule, Quotient Rule, Chain Rule
- Implicit Differentiation
- Higher Order Derivatives
- Differentiation of Parametric Equations
- Differentiation of Vector-valued Functions
- Analysis of Polar Curves using Differentiation

Unit 3: Applications of Differentiation (9 blocks)

- Indeterminate Forms and L'Hopital's Rule
- Finding the Equation of the Tangent Line and Tangent Line Approximation
- First Derivative and Increasing/Decreasing Intervals
- Second Derivative and Concavity and Points of Inflection
- Relative Extrema
- Absolute Extrema and the Extreme Value Theorem
- Rolle's Theorem and Mean Value Theorem
- Curve Sketching, Graphical Analysis with Derivatives
- Relationships among Graphs of $f(x), f'(x), f''(x)$
- Related Rates Application
- Optimization
- Position, Velocity, and Acceleration Problems
- Parametric Equations and Vectors; Motion along a curve, position, velocity, acceleration, and speed

Unit 4: Integration (10 blocks)

- Riemann Sums and Area under a Curve; Left, Right, and Midpoint Sums
- Fundamental Theorem of Calculus
- Antiderivatives and Indefinite Integration
- Definite Integral as a Limit of Riemann Sums
- Properties of the Definite Integral
- Integration using U-Substitution and Change of Variables
- Integral-Valued Functions and their Derivatives
- Derivative of the Composite of an Integral-Valued Function and another Function
- Integral of a Rate of Change Function to Represent Accumulated Change
- Mean Value Theorem for Integrals
- Average Value of a Function
- Numerical Integration; Trapezoidal Rule
- Acceleration, Velocity, Position, and Distance Traveled
- Integration of Parametric Equations
- Integration of Vector Valued Functions
- Integration of Polar Functions

Unit 5: Advanced Integration Techniques (7 blocks)

- Integration by Parts
- Integration using Partial Fractions
- Integration by Trigonometric Substitution
- Improper Integrals

Unit 6: Differential Equations (5 blocks)

- Slope Fields
- Euler's Methods
- Solving Separable Differential Equations
- Exponential Functions; Growth and Decay Applications
- Writing, Interpreting, and Solving Logistic Models Expressed as Differential Equations
- Solving First Order Linear Differential Equations

Unit 7: Applications of Integration (5 blocks)

- Area Between Curves
- Solids of Revolution; Disk/Washer Method
- Volume of Solids with Similar Cross Sections
- Arc Length and Area of a Surface of Revolution
- Area and Arc Length of Polar Curves

Unit 8: Infinite Series (8 blocks)

- Sequences, Convergence and Divergence
- Series as a Sequence of Partial Sums
- Series, Convergence and Divergence
- Geometric Series with Decimal Expansion and Applications
- N-th Term Test for Divergence
- Integral Test; Geometric Representation with Rectangular Areas
- P-Series; Harmonic Series
- Direct and Limit Comparison Tests for Series
- Alternating Series Test and Alternating Series Remainder
- Ratio and Root Tests

Unit 9: Taylor and Maclaurin Series (12 blocks)

- Power Series and Functions Defined by Power Series
- Radius and Interval of Convergence
- Taylor and Maclaurin Series
- Maclaurin Series for $e^x, \sin x, \cos x, \frac{1}{1-x}$
- Manipulation of Series to Form New Series using Substitution, Differentiation, and Antidifferentiation
- Taylor Polynomial Approximations
- Error Bounds (Alternating Series and Lagrange Error Bound)

TEACHING STRATEGIES

Course Overview and Rule of Four

The AP Calculus BC course follows the detailed topic outline presented above. Throughout the year, the course encourages student discovery of concepts, making sense of problems, constructing viable arguments to justify answers, and making connections between various topics. Teaching strategies allow students ample time for discovering new concepts via class discussions and projects; utilizing technology to explore patterns and visual representations; independently analyzing topics through homework and problem sets; and using reasoning to support conclusions and reflect. The structure of the class follows a “you do, we do, I do” philosophy where the students are first expected to explore a topic through various activities *before* learning formal definitions and theorems. Involving students in their learning in this way leads to a sense of accomplishment from persevering through a problem, thus building confidence in their mathematical abilities. There will also be heavy emphasis on calculus applications, particularly for differentiation and integration, so students appreciate the value of such topics beyond the classroom. Furthermore, students are given extensive opportunities to work with problems presented in the rule of four – graphically, numerically, analytically, and verbally. For example, when exploring the topic of limits and functions, students will first work with calculators to graph a function and analyze limit patterns from a table, and then they will verbally define and numerically evaluate limits. Or for the unit on differential equations, students will solve problems analytically, graphically represent the differential equation on a slope field, use a tabular and numerical method for Euler’s Method, and verbally make connections between the analytic and approximation techniques.

Technology Use

Students have available the TI-Inspire graphing calculator for classroom use and are highly encouraged to have one for personal use at home. Graphing calculators are an extremely valuable tool for investigating calculus topics, and thus will be used for student discovery of concepts and understanding of analytical processes. The visual representation of calculators gives students the opportunity to make connections with functions and limits both graphically and numerically. Students use calculators as a problem-solving tool to interpret and support their findings, demonstrating conceptual understanding of the topics. By the AP exam, students are expected to identify when calculator use is appropriate and strategic in solving a problem as well as be comfortable plotting the graph of a function, finding zeroes of a function, numerically calculating the derivative of a function, and numerically calculating the value of a definite integral.

Mathematical Communication

Students will be expected to verbalize understanding of mathematical ideas, both orally and in written form. The ability to justify answers effectively using mathematical language demonstrates a stronger understanding of concepts. Warm-ups, tests, quizzes, and other forms of assessments will ask students to explain procedures and justify answers in a written response. Students will work often in small groups and pairs to facilitate discussion as they work together to make sense of problems and present a team response. Students are encouraged to hold discussions about homework and projects outside of class, via personal interaction and online discussion boards, in an effort to expand cooperative learning beyond the classroom.

Assessments

Students will be assessed both formatively and summatively. Formative assessments include problem sets, homework assignments, exploration projects, class assignments, and daily warm-ups in the style of AP questions. Summative assessments are presented as cumulative unit tests. Reflecting the structure of the AP exam, assessments include calculator and non-calculator sections, as well as both multiple choice and free response questions. All summative assessments are also timed in order to prepare students for AP exam conditions. Grading follows AP scoring guidelines (particularly for free response questions) so that students understand what a “complete” solution entails. Students are given multiple opportunities to demonstrate mastery of any particular topic.

AP Exam Review

The school is on an alternating-day block schedule. Students have class every other day for 90 minutes. The school year calendar is constructed so that students have 73 to 75 blocks of class instruction before the AP exam date. There is also a rotating study hall time designated for each class every other week. This study hall time can be used for additional AP exam preparation, particularly in the second semester. The course outline allows about three weeks of class time to review for the exam using released exam materials, review books purchased for class use, and textbook exercises. Allowing students to practice time management, presenting a broad review of major concepts, and repeated exposure to AP released exams will give them more confidence when presented with the actual exam.