Are you ready for Calculus? Solutions

1.a. $$\frac{x(x^2-9)}{(x-4)(x-3)} = \frac{x(x-3)(x+3)}{(x-4)(x-3)} = \frac{x(x+3)}{x-4}$$

1.b. $$\frac{x^2-2x-8}{x^3+x^2-2x} = \frac{(x-4)(x+2)}{x(x^2+x-2)} = \frac{(x-4)(x+2)}{x(x-1)(x+2)} = \frac{x-4}{x(x-1)}$$

1.c. $$\left[\frac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}\right]\frac{25x^2}{25x^2} = \frac{(25x-5x^2)}{(25-x^2)} = \frac{5x(5-x)}{(5-x)(5+x)} = \frac{5x}{5+x}$$

1.d. $$\left[\frac{9-\frac{1}{x^2}}{3+\frac{1}{x}}\right]\frac{x^2}{x^2} = \frac{9x^2-1}{3x^2+x} = \frac{(3x-1)(3x+1)}{x(3x+1)} = \frac{3x-1}{x}$$

2.a. $$\left(\frac{2}{\sqrt{3}+\sqrt{2}}\right)\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right) = \frac{2\left(\sqrt{3}-\sqrt{2}\right)}{3-2} = 2\left(\sqrt{3}-\sqrt{2}\right)$$

2.b. $$\left(\frac{4}{1-\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{1+\sqrt{5}}\right) = \frac{4\left(1+\sqrt{5}\right)}{1+\sqrt{5}-\sqrt{5}-5} = \frac{4\left(1+\sqrt{5}\right)}{-4} = -\left(1+\sqrt{5}\right) = -1-\sqrt{5}$$

2.c. $$\left(\frac{1}{\left(1+\sqrt{3}\right)-\sqrt{5}}\right)\left(\frac{\left(1+\sqrt{3}\right)+\sqrt{5}}{\left(1+\sqrt{3}\right)+\sqrt{5}}\right) = \frac{\left(1+\sqrt{3}\right)+\sqrt{5}}{\left(1+\sqrt{3}\right)^2-\left(\sqrt{5}\right)^2} = \frac{\left(1+\sqrt{3}\right)+\sqrt{5}}{1+2\sqrt{3}+3-5}$$

$$= \left(\frac{1+\sqrt{3}+\sqrt{5}}{-1+2\sqrt{3}}\right)\left(\frac{-1-2\sqrt{3}}{-1-2\sqrt{3}}\right) = \frac{-1-2\sqrt{3}-1\sqrt{3}-6-1\sqrt{5}-2\sqrt{15}}{1-12}$$

$$= \frac{-1-2\sqrt{3}-1\sqrt{3}-6-1\sqrt{5}-2\sqrt{15}}{1-12} = \frac{-7-3\sqrt{3}-\sqrt{5}-2\sqrt{15}}{-11}$$

$$= \frac{7+3\sqrt{3}+\sqrt{5}+2\sqrt{15}}{11}$$

3.a. $$\frac{\left(2a^2\right)^3}{b} = \frac{8a^6}{b} \quad \left[= 8a^6b^{-1}\right]$$

3.b. $$\sqrt{9ab^3} = \left(9ab^3\right)^{\frac{1}{2}} = 3a^{\frac{1}{2}}b^{\frac{3}{2}}$$

3.c. $$\frac{\frac{2a}{b}}{\frac{3}{a}} = \frac{2a}{b}\cdot\frac{a}{3} = \frac{2a^2}{3b} \quad \left[= \frac{2}{3}a^2b^{-1}\right]$$

3.d. $$\frac{a(b-1)}{b(b-1)} = \frac{a}{b} \quad \left[= ab^{-1}\right]$$

3.e. $$\frac{b}{a\sqrt{a}} = \frac{b}{a^{\frac{3}{2}}} \quad \left[= a^{-\frac{3}{2}}b\right]$$

3.f. $$\frac{a^{\frac{4}{3}}b^{\frac{3}{2}}}{ba^{\frac{1}{2}}} = \frac{a^{\frac{8}{6}}b^{\frac{3}{2}}}{ba^{\frac{3}{6}}} = a^{\frac{5}{6}}b^{\frac{1}{2}}$$

4.a. $5^{x+1} = 5^2$

$x+1=2$

$x=1$

4.b. $3^{-1} = 3^{2x+2}$

$2x+2 = -1$

$2x = -3$

$x = -\frac{3}{2}$

4.c. $x = 2^3 = 8$

4.d. $\log_3 x^2 = \log_3 4^2 - \log_3 5^4$

$\log_3 x^2 = \log_3 \frac{4^2}{5^4}$

$x^2 = \frac{4^2}{5^4}$

$x = \pm\frac{4}{5^2} = \pm\frac{4}{25}$

5.a. $\log_2 \frac{5(x^2-1)}{x-1} = \log_2 \frac{5(x-1)(x+1)}{x-1} = \log_2 5(x+1)$

5.b. $\log_4 9^2 - \log_2 3 = \frac{\log_2 9^2}{\log_2 4} - \log_2 3 = \frac{\log_2 9^2}{2} - \log_2 3 = \log_2 (9^2)^{\frac{1}{2}} - \log_2 3$

$= \log_2 9 - \log_2 3 = \log_2 \frac{9}{3} = \log_2 3$

5.c. $3^{\log_3 5^2} = 25$

6.a. $\frac{1}{2}$

6.b. $\log_{10}(10^{-x}) = -x$

6.c. $\log_{10}(\sqrt{x})^2 + \log_{10}\left(x^{\frac{1}{3}}\right)^3 = \log_{10} x + \log_{10} x = 2\log_{10} x \ [= \log_{10} x^2]$

7.a. $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$\frac{x}{a} = 1 - \frac{y}{b} - \frac{z}{c}$

$x = a\left[1 - \frac{y}{b} - \frac{z}{c}\right]$

$a = \frac{x}{\left[1 - \frac{y}{b} - \frac{z}{c}\right]}\left[\frac{bc}{bc}\right]$

$a = \frac{bcx}{bc - cy - bz}$

7.b. $\frac{V}{2} = ab + bc + ca$

$\frac{V}{2} - bc = ab + ac$

$\frac{V}{2} - bc = a(b+c)$

$a = \left[\frac{\frac{V}{2} - bc}{b+c}\right]\left[\frac{2}{2}\right]$

$a = \frac{V - 2bc}{2(b+c)}$

7.c. $A = 2\pi r^2 + 2\pi r h$

$$0 = 2\pi r^2 + 2\pi r h - A$$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 2\pi \quad b = 2\pi h, \; c = -A$

$$r = \frac{-2\pi h \pm \sqrt{(2\pi h)^2 - 4(2\pi)(-A)}}{2(2\pi)}$$

$$= \frac{-2\pi h \pm \sqrt{4\pi^2 h^2 + 8A\pi}}{4\pi}$$

$$= \frac{-\pi h \pm \sqrt{\pi^2 h^2 + 2A\pi}}{2\pi}$$

only asks for positive r so

$$r = \frac{-\pi h + \sqrt{\pi^2 h^2 + 2A\pi}}{2\pi}$$

7.d. $A = P + nrP$

$$A = P(1 + nr)$$

$$P = \frac{A}{1 + nr}$$

7.e. $2x - y = xd + 2yd$

$$2x - y = d(x + 2y)$$

$$d = \frac{2x - y}{x + 2y}$$

8.a. $y = (x^2 + 4x + 4) + 3 - 4$

$$y = (x^2 + 4x + 4) + 3 - 4$$

$$y = (x + 2)^2 - 1$$

$$y + 1 = (x + 2)^2$$

$$y - (-1) = (x - (-2))^2$$

7.f. $\frac{2x}{4\pi} + \frac{1 - x}{2} = 0$

$$2\pi \left[\frac{x}{2\pi} + \frac{1 - x}{2} = 0 \right]$$

$$x + \pi(1 - x) = 0$$

$$x + \pi - x\pi = 0$$

$$x - x\pi = -\pi$$

$$x(1 - \pi) = -\pi$$

$$x = \frac{-\pi}{1 - \pi} = \frac{\pi}{\pi - 1}$$

8.b. $-2y = 3x^2 + 3x$

$$-2y = 3(x^2 + x + \quad)$$

$$-2y = 3\left(x^2 + x + \frac{1}{4}\right) - \frac{3}{4}$$

$$-2y + \frac{3}{4} = 3\left(x + \frac{1}{2}\right)^2$$

$$-2\left(y - \frac{3}{8}\right) = 3\left(x + \frac{1}{2}\right)^2$$

$$y - \frac{3}{8} = -\frac{3}{2}\left(x - \left(-\frac{1}{2}\right)\right)^2$$

8.c. $x + 9 = 9\left(y^2 - \frac{2}{3}y \qquad \right)$

$x + 9 = 9\left(y^2 - \frac{2}{3}y + \frac{1}{9}\right) - 1$

$x + 10 = 9\left(y^2 - \frac{2}{3}y + \frac{1}{9}\right)$

$x - (-10) = 9\left(y - \frac{1}{3}\right)^2$

9.a. $x^6 - 16x^4 = x^4\left(x^2 - 16\right) = x^4\left(x + 4\right)\left(x - 4\right)$

9.b. $4x^3 - 8x^2 - 25x + 50 = 4x^2(x - 2) - 25(x - 2) = (x - 2)(4x^2 - 25) = (x - 2)(2x + 5)(2x - 5)$

9.c. $8x^3 + 27 = (2x + 3)(4x^2 - 6x + 9)$

$a^3 + b^3 = (a + b)(a^2 + ab + b^3)$

9.d. $x^4 - 1 = (x^2 + 1)(x^2 - 1)$

$= (x^2 + 1)(x + 1)(x - 1)$

10.a. $x^6 - 16x^4 = 0$

$x^4\left(x^2 - 16\right) = 0$

$x^4\left(x + 4\right)\left(x - 4\right) = 0$

$x^4 = 0$ or $x + 4 = 0$ or $x - 4 = 0$

$x = 0$ $\quad x = -4$ $\quad x = 4$

$0, \pm 4$

10.b. $(x - 2)(2x + 5)(2x - 5) = 0$

$x - 2 = 0$ or $2x + 5 = 0$ or $2x - 5 = 0$ $\qquad 2, \pm\frac{5}{2}$

$x = 2$ $\qquad x = -\frac{5}{2}$ $\qquad x = \frac{5}{2}$

10.c. $(2x + 3)(4x^2 - 6x + 9) = 0$

$2x + 3 = 0$ or $4x^2 - 6x + 9 = 0$

$x = -\frac{3}{2}$ $\qquad \sqrt{b^2 - 4ac} = \sqrt{(-6)^2 - 4(4)9} = \sqrt{36 - 72}$

the square root of a negative is not real

11.a. $3\sin^2 x = \cos^2 x$

$\frac{\sin^2 x}{\cos^2 x} = \frac{1}{3}$

$\tan^2 x = \frac{1}{3} \quad \Rightarrow \quad \tan x = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3} \qquad \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6},$

11.b. $(1-\sin^2 x)-\sin^2 x = \text{sun } x$

$$1 - 2\sin^2 x - \sin x = 0$$

$$0 = 2\sin^2 x + \sin x - 1$$

$$0 = (2\sin x - 1)(\sin x + 1)$$

$2\sin x - 1 = 0$ or $\sin x + 1 = 0$ $(-\pi < x < \pi)$

$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\qquad$ $x\left(=\frac{3\pi}{2}\right) = -\frac{\pi}{2}$

11.c. $\tan x + \sec x = 2\cos x$

$$\cos x\left[\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x\right]$$

$$\sin x + 1 = 2\cos^2 x$$

$$\sin x + 1 = 2(1-\sin^2 x)$$

$$\sin x + 1 = 2 - 2\sin^2 x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$2\sin x - 1 = 0$ or $\sin x + 1 = 0$

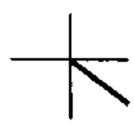
$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = -1$

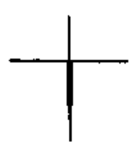
$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\qquad$ $x = \frac{3\pi}{2}$ (tan x and sec x are undefined here)

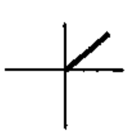
$x = \frac{\pi}{6} + 2k\pi$ or $\frac{5\pi}{6} + 2k\pi$ where k is any integer

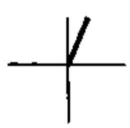
12. a.  $\frac{\sqrt{3}}{2}$

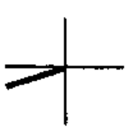
12. b.  $-\frac{\sqrt{2}}{2}$

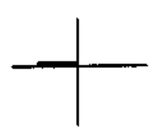
12. c. $\tan^{-1} x$ is defined for $-\frac{\pi}{2} < x < \frac{\pi}{2}$  $-\frac{\pi}{4}$

12. d. $\sin^{-1} x$ is defined for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$  $-\frac{\pi}{2}$

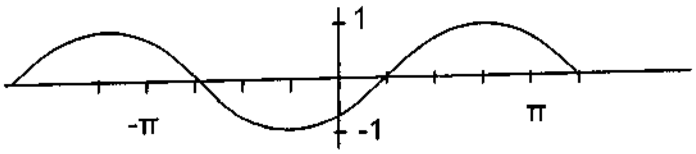
12. e.  $\frac{\sqrt{2}}{2}$

12. f.  $\frac{\pi}{3}$

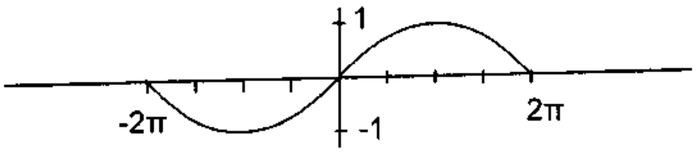
12. g.  $-\frac{\sqrt{3}}{3}$

12. h. $\cos^{-1} x$ is defined for $0 \le x \le \pi$  π

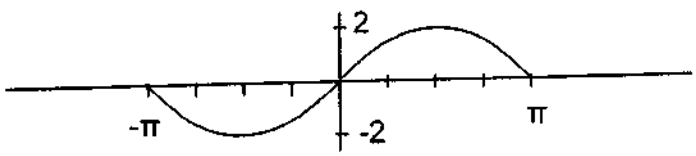
13. a. $\sin\left(x - \frac{\pi}{4}\right)$ $b = 1$ $c = \frac{\pi}{4}$ phase shift $= \frac{c}{b} = \frac{\frac{\pi}{4}}{1} = \frac{\pi}{4}$ to right



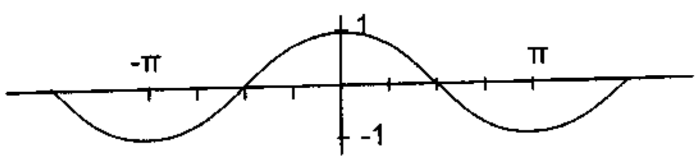
13. b. $\sin\frac{x}{2}$ horizontal stretch 2 times



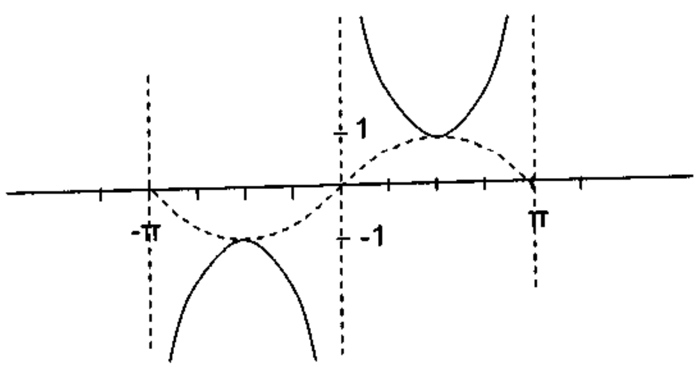
13. c. $2\sin x$ magnitude increased 2 times



13. d. $\cos x$



13. e. $\frac{1}{\sin x} = \csc x$ Draw sin x curve to help draw csc x curve, $-\pi$, 0, and π are asymptotes



14. a. $4x^2 + 12x + 3 = 0$ *not* factorable - use quadratic formula

$a = 4$ $b = 12$ $c = 3$

$$x = \frac{-12 \pm \sqrt{12^2 - 4(4)3}}{2(4)} = \frac{-12 \pm \sqrt{144 - 48}}{8} = \frac{-12 \pm \sqrt{96}}{8} = \frac{-12 \pm 4\sqrt{6}}{8} \quad \frac{-3 \pm \sqrt{6}}{2}$$

14. b. $(2x+1)(x+2) = 5$

$$2x^2 + 4x + x + 2 = 5$$

$$2x^2 + 5x + 2 - 5 = 0$$

$$2x^2 + 5x - 3 = 0$$

$$(2x-1)(x+3) = 0$$

$2x - 1 = 0$ or $x + 3 = 0$ $\qquad -3, \frac{1}{2}$, (-2 is not in the domain)

$x = \frac{1}{2}$ $\qquad x = -3$

14. c. $$x(x+1)\left[\frac{x+1}{x} - \frac{x}{x+1} = 0\right]$$

$$(x+1)^2 - x^2 = 0$$

$$x^2 + 2x + 1 - x^2 = 0$$

$$2x + 1 = 0$$

$x = -\frac{1}{2}$ (x = 0 and -1 are not in the domain)

15. a. -2 |

1	-4	1	0	-7	1
	-2	12	-26	52	-90
1	-6	13	-26	45	-89

Remainder -89

15. b.

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
x^3+1 & x^5 - x^4 + x^3 + 2x^2 - x + 4 \\
 & \underline{x^5 \qquad\qquad + x^2} \\
 & -x^4 + x^3 + x^2 - x + 4 \\
 & \underline{-x^4 \qquad\qquad -x} \\
 & x^3 + x^2 \quad + 4 \\
 & \underline{x^3 \qquad\quad + 1} \\
 & x^2 \quad + 3
\end{array}
$$

Remainder $x^2 + 3$

16. a. 2 |

12	-23	-3	2
	24	2	-2
12	1	-1	0

$(12x^2 + x - 1) = (4x - 1)(3x + 1)$

$12x^3 - 23x^2 - 3x + 2 = (x - 2)(4x - 1)(3x + 1)$ $\qquad \left\{2, \frac{1}{4}, -\frac{1}{3}\right\}$

16. b. Rational zero (root) theorem p = ±1 q = ±12, ±6, ±4, ±3, ±2, ±1

$$\frac{p}{q} = \pm\frac{1}{12}, \pm\frac{1}{6}, \pm\frac{1}{4}, \pm\frac{1}{3}, \pm\frac{1}{2}, \pm1$$

½	12	8	-1	-1	
		6	7		
	12	14	6		Not a solution

-½	12	8	-1	-1
		-6	-1	1
	12	2	-2	0

$(12x^2 + 2x - 2) = 2(6x^2 + 1x - 1) = 2(3x - 1)(2x + 1)$

$$\left\{-\frac{1}{2}, \frac{1}{3}, -\frac{1}{2}\right\}$$

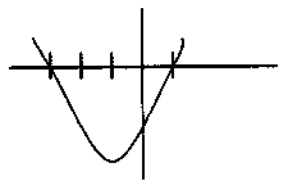
17. a. $x^2 + 2x - 3 \le 0$

$(x - 1)(x + 3) \le 0$

$x - 1 = 0$ or $x + 3 = 0$

$x = 1$ $x = -3$

zeros -3, 1



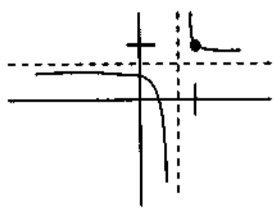
[-3, 1] or $-3 \le x \le 1$

17. b. If $3x - 2 > 0$ $x > \frac{2}{3}$ $2x - 1 \le 3x - 2$

$1 \le x$

If $3x - 2 < 0$ $x < \frac{2}{3}$ ~~$2x - 1 \ge 3x - 2$~~

~~$1 \ge x$~~

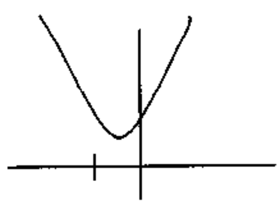


$x \ge 1$ or $x < \frac{2}{3}$

17. c. $(x^2 + x + \quad) + 1 > 0$

$\left(x^2 + x + \frac{1}{4}\right) + 1 - \frac{1}{4} > 0$

$\left(x^2 + x + \frac{1}{4}\right) + \frac{3}{4} > 0$



never less than zero, so all x

18. a. $-1 \le -x + 4 \le 1$

$\underline{-4 \qquad -4 \qquad -4}$

$-5 \le -x \le -3$

$5 \ge x \ge 3$

18. b. $5x - 2 = 8$ or $-(5x - 2) = 8$

$\underline{+2 \quad +2}$ $-5x + 2 = 8$

$5x = 10$ $\underline{-2 \quad -2}$

$x = 2$ $-5x = 6$

$x = -\frac{6}{5}$

18.c. $2x+1 = x+3$ or $-(2x+1) = x+3$

$x = 2$ $\qquad$ $-2x - 1 = x+3$

$-4 = 3x$

$x = -\frac{4}{3}$

19.a. $m = \frac{-4-3}{2--1} = \frac{-7}{3}$ $\qquad$ $y+4 = \frac{-7}{3}(x-2)$

$3(y+4) = -7(x-2)$

$3y+12 = -7x+14$ $\qquad$ or $\qquad$ $3y = -7x+2$

$7x + 3y = 2$ $\qquad$ $y = -\frac{7}{3}x + \frac{2}{3}$

19.b. $2x - 3y + 5 = 0$ $\qquad$ $\perp m = \frac{-3}{2}$ $\qquad$ $y-2 = \frac{-3}{2}(x+1)$

$-3y = -2x - 5 \quad 0$ $\qquad$ $2(y-2) = -3(x+1)$

$y = \frac{2}{3}x + \frac{5}{3} \Rightarrow m = \frac{2}{3}$ $\qquad$ $2y-4 = -3x-3$

or $m = -\frac{A}{B} = -\frac{2}{-3} = \frac{2}{3}$ $\qquad$ $3x + 2y = 1$

19.c. $\left(\frac{-1+3}{2}, \frac{4+2}{2}\right) = (1,3)$ $\qquad$ $y=3$

20.a. $y = 3x - 7$ $\quad$ Substitute $\quad$ $x + 5(3x-7) + 3 = 0$

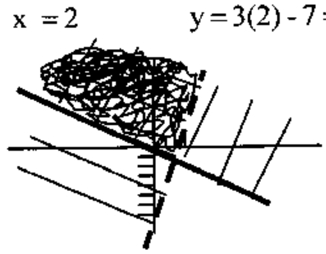
$x + 15x - 35 + 3 = 0$

$16x - 32 = 0$

$x = 2$ $\qquad$ $y = 3(2) - 7 = -1$ $\qquad$ $(2, -1)$

20.b. $y \geq 3x - 7$ and $5y \geq -x - 3$

$y \geq -\frac{x}{5} - \frac{3}{5}$



21.a. $(x-h)^2 + (y-k)^2 = R^2$ where (h, k) is the center of the circle and R is the radius

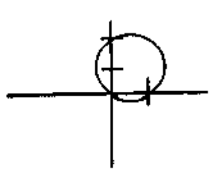
$(x-1)^2 + (y-2)^2 = R^2$

$(-2-1)^2 + (-1-2)^2 = R^2$

$9 + 9 = R^2$

$18 = R^2 \Rightarrow (x-1)^2 + (y-2)^2 = 18$

21. b. Uses a Geometry rule that says the segment perpendicular to the midpoint of a chord must be a diameter of the circle (i. e. go through the center of the circle)



The midpoint of (0, 0) & (0, 2) is (0, 1)
and the midpoint of (0, 0) & (1, 0) is (½, 0)
so the center of the circle is (½, 1)

continued next page

21.b. cont. $(x-h)^2 + (y-k)^2 = R^2$ where (h, k) is the center of the circle and R is the radius

$$\left(x-\frac{1}{2}\right)^2 + (y-1)^2 = R^2 \quad \text{using } (0,0) \text{ as the point on the circle}$$

$$\frac{1}{4} + 1 = R^2$$

$$\frac{5}{4} = R^2 \Rightarrow \left(x-\frac{1}{2}\right)^2 + (y-1)^2 = \frac{5}{4}$$

22.a. $(x-h)^2 + (y-k)^2 = R^2$ where (h, k) is the center of the circle and R is the radius

$$(x^2 + 6x \quad) + (y^2 - 4y \quad) + 3 = 0 \quad \text{Complete the squares}$$

$$(x^2 + 6x + 9) + (y^2 - 4y + 4) + 3 - 9 - 4 = 0$$

$$(x + 3)^2 + (y - 2)^2 - 10 = 0$$

$$(x + 3)^2 + (y - 2)^2 = 10 \Rightarrow \text{center } (-3, 2) \text{ \& Radius } \sqrt{10}$$

b. The slope of the radius from (-3,2) to (-2,5) is

$m = \frac{5-2}{-2--3} = \frac{3}{1} = 3$ to be tangent to the circle a line must be perpendicular to the radius

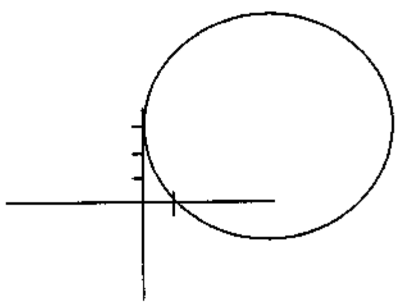
so the slope of the tangent line is $-\frac{1}{3}$

$$(y-5) = -\frac{1}{3}(x+2)$$

$$3y - 15 = -x - 2$$

$$x + 3y = 13$$

23.



The center is on $y = 3$ or $k = 3$

two points on the circle is (1,0) & (0,3)

$$(1-h)^2 + (0-3)^2 = R^2$$

$$(0-h)^2 + (3-3)^2 = R^2$$

$$(1-h)^2 + (0-3)^2 = (0-h)^2 + (3-3)^2$$

$$1 - 2h + h^2 + 9 = h^2$$

$$10 - 2h = 0$$

$$10 = 2h$$

$$h = 5$$

b. Center is (5,3)

$$(0-5)^2 + (3-3)^2 = R^2$$

$$25 = R^2$$

$$(x-5)^2 + (y-3)^2 = 25$$

a. Call the other x-intercept (a, 0)

$$(a-5)^2 + (0-3)^2 = 25$$

$$a^2 - 10a + 25 + 9 = 25$$

$$a^2 - 10a + 9 = 0$$

$$(a-9)(a-1) = 0 \Rightarrow a = 1 \text{ or } 9 \quad (9,0)$$

24. Distance from A = $\sqrt{(x+1)^2 + (y-1)^2}$ and Distance from B = $\sqrt{(x-2)^2 + (y+1)^2}$

Distance from A = 3 (Distance from B)

$$\sqrt{(x+1)^2 + (y-1)^2} = 3\sqrt{(x-2)^2 + (y+1)^2}$$

$$(x+1)^2 + (y-1)^2 = 9\left((x-2)^2 + (y+1)^2\right)$$

$$\left[x^2 + 2x + 1 + y^2 - 2y + 1\right] = 9\left[x^2 - 4x + 4 + y^2 + 2y + 1\right]$$

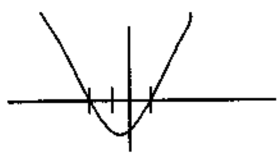
$$x^2 + 2x + 1 + y^2 - 2y + 1 = 9x^2 - 36x + 36 + 9y^2 + 18y + 9$$

$$0 = 8x^2 + 8y^2 - 38x + 20y + 43$$

25.a. $x^2 + x - 2 > 0$

$(x+2)(x-1) > 0$

Domain $(-\infty, -2)$ and $(1, \infty)$



25. b. i. Domain all real numbers, Range 7

25. b. ii. Domain all real numbers except x = ½ (vertical asymptote),

Range all real numbers except $\frac{5}{2}$ (horizontal asymptote)

26. $x > 0$ $f(x) = \frac{x}{x} = 1$ $x < 0$ $f(x) = \frac{x}{-x} = -1$ $x = 0$ f(x) is undefined

Domain all real numbers except 0, Range {-1,1}

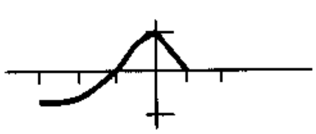
27.a. $$\frac{[2(x+h)+3]-[2x+3]}{h} = \frac{2x+2h+3-2x-3}{h} = \frac{2h}{h} = 2$$

27.b. $$\frac{\left[\frac{1}{x+h+1}\right]-\left[\frac{1}{x+1}\right]}{h} = \frac{(x+h+1)(x+1)}{(x+h+1)(x+1)}\left[\frac{\left[\frac{1}{x+h+1}\right]-\left[\frac{1}{x+1}\right]}{h}\right] = \frac{(x+1)-(x+h+1)}{h(x+h+1)(x+1)}$$

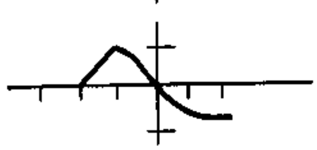
$$= \frac{x+1-x-h-1}{h(x+h+1)(x+1)} = \frac{-h}{h(x+h+1)(x+1)} = \frac{-1}{(x+h+1)(x+1)}$$

27.c. $$\frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h} = \frac{2xh + h^2}{h} = 2x + h$$

28. a. Shifts one unit to left

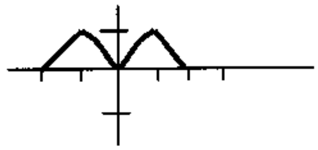


28. b. Reflects across y-axis

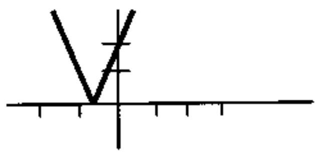


28. c. all positive y values

28. d. negative x-axis is reflection of positive x-axis



29. a. Draw y = 3x + 2 – do not go below the x-axis, reflect across $x = -\frac{2}{3}$

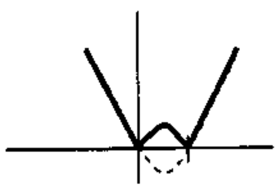


29. b.

$$(x^2 + x + \quad)$$

$$\left(x^2 + x + \frac{1}{4}\right) + -\frac{1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}$$



30. a. Less than 4 indicates the parabola faces down

$$y = -(x + 1)(x - 3) = -(x^2 - 2x - 3) = -(x^2 - 2x - \quad) + 3$$

$$= -(x^2 - 2x + 1) + 3 + 1$$

$$= -(x^2 - 1)^2 + 4$$

30. b.

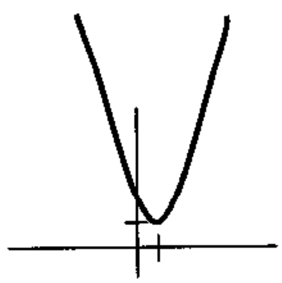
$$y = 2x^2 - 4x + 3$$

$$y = 2(x^2 - 2x \qquad) + 3$$

$$y = 2(x^2 - 2x + 1) + 3 - 2$$

$$y = 2(x - 1)^2 + 1$$

vertex (1,1) goes up 2x faster than x^2



31. a. $x = t + 1$ $\quad t = x - 1$

$$y = (x - 1)^2 - (x - 1)$$

$$y = x^2 - 2x + 1 - x + 1$$

$$y = x^2 - 3x + 2$$

31. b. $x = \sqrt[3]{t} - 1$ $\quad \sqrt[3]{t} = x + 1$ $\quad t = (x + 1)^3$

$$y = (x + 1)^3\left[(x + 1)^3 - 1\right]$$ Acceptable Answer

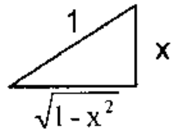
$$y = (x + 1)^3\left[x^3 + 3x^2 + 3x + 1 - 1\right]$$

$$y = (x + 1)^3\left[x^3 + 3x^2 + 3x\right]$$

$$y = x(x + 1)^3\left[x^2 + 3x + 3\right]$$

31.c. $x = \sin t$ $t = \sin^{-1} x$ $y = \cos(\sin^{-1} x)$

$$y = \sqrt{1 - x^2}$$
$$y^2 = 1 - x^2$$
$$x^2 + y^2 = 1$$



32.a. $x = 2y + 3$

$$2y = x - 3$$
$$y = \frac{x-3}{2}$$

32.b. $x = \frac{y+2}{5y-1}$

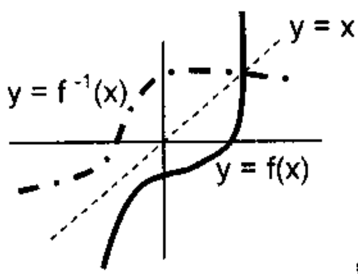
$$(5y-1)x = y+2$$
$$5yx - x = y + 2$$
$$5yx - y = x + 2$$
$$y(5x-1) = x + 2$$
$$y = \frac{x+2}{5x-1}$$

32.c. $x = y^2 + 2y - 1$

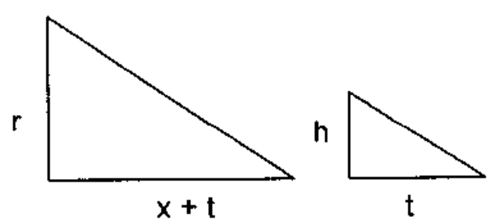
$$0 = y^2 + 2y - 1 - x$$
$$0 = y^2 + 2y - (1+x) \qquad a = 1, b = 2, c = -(1+x)$$
$$y = \frac{-2 \pm \sqrt{4 - 4(-(1+x))}}{2} = \frac{-2 \pm \sqrt{4+4+4x}}{2} = \frac{-2 \pm \sqrt{8+4x}}{2} = \frac{-2 \pm \sqrt{4(2+x)}}{2} = -1 \pm \sqrt{(2+x)}$$

When $x = 0$ in the original equation $y = -1$, so x is limited to $x > -1$ for the inverse.

33. reflects across y = x



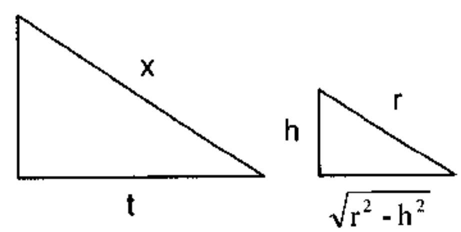
34. a.



Similar Triangles $\frac{x+t}{t} = \frac{r}{h}$

$$x + t = \frac{rt}{h}$$
$$x = \frac{rt}{h} - t$$
$$x = \frac{rt}{h} - \frac{th}{h}$$
$$x = \frac{t(r-h)}{h}$$

b.



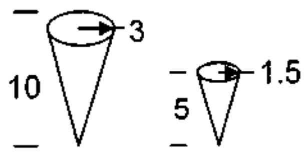
Similar Triangles $\frac{x}{r} = \frac{t}{\sqrt{r^2 - h^2}}$

$$x = \frac{rt}{\sqrt{r^2 - h^2}}$$

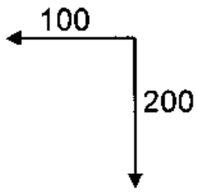
35. a. Area of the square $= (2r)^2 = 4r^2$ Area of the square $= \pi r^2$

Area inside the square but outside the circle $= 4r^2 - \pi r^2$ ratio $= \dfrac{4r^2 - \pi r^2}{4r^2} = \dfrac{4-\pi}{4} = 1 - \dfrac{\pi}{4}$

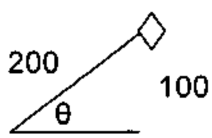
35. b. perimeter $= r + 2r + r + \dfrac{2\pi r}{2} = 4r + \pi r$



35. c. Area $= \pi\ (1.5)^2 = 2.25\,\pi$



35. d. Distance $= \sqrt{100^2 + 200^2} = \sqrt{100^2 + 2^2\,(100^2)} + \sqrt{(1+4)\,(100^2)} = 100\sqrt{5}$ km



35. e. $\sin\theta = \dfrac{100}{200} \Rightarrow \theta = \sin^{-1}\left(\dfrac{100}{200}\right) = \dfrac{\pi}{6}$ or 30°