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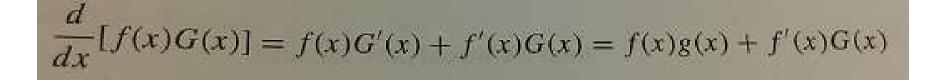
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INTRODUCTION

 In this section we will discuss an integration technique that is essentially an antiderivative of the formula for differentiating a product of two functions.

THE PRODUCT RULE AND INTEGRATION BY PARTS

- Since there is not a product rule for integration, we need to develop a general method for evaluating integrals of the form $\int f(x)g(x) dx$
- If we start with the product rule and work backwards (integration), it will help identify a pattern.



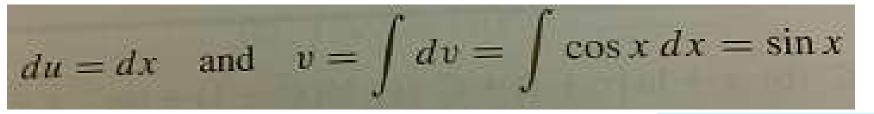
THE PRODUCT RULE AND INTEGRATION BY PARTS – CON'T

 This pattern produces the formula we use to make difficult integrations easier. We call the method integration by parts.

$$\int u\,dv = uv - \int v\,du$$

EXAMPLE #1

- Use integration by parts to evaluate $\int x \cos x \, dx$.
- The first step is to pick part of the expression x cos x to be u and another part to be dv (we will talk about strategies on a later slide).
- For now, let u = x and let dv = cosx dx. Then (2nd step)



The 3rd step is to apply the integration by parts formula

$$\int u\,dv = uv - \int v\,du$$

$$\int \underbrace{x \cos x \, dx}_{u} = \underbrace{x \sin x}_{u} - \int \underbrace{\sin x}_{v} \frac{dx}{du}_{u}$$
$$= x \sin x - (-\cos x) + C = x \sin x + \cos x + C$$

SOME GUIDELINES FOR CHOOSING U AND DV

- The goal is to choose u and dv to obtain a new integral that is easier than the original.
- The more you practice, the easier it is to pick u and dv correctly on your first try.
- One suggestion is to pick a part of the expression to be u that gets "easier/simpler/smaller" when you take its derivative.
- Another suggestion is to pick a part of the expression to be dv that you know how to integrate or that is easy to integrate.

EXAMPLE #2



- One choice is to let u=1 and dv = lnx dx.
- If we did that, we would have to take the integral of lnx dx which we do not know how to do.
- STEP 1: Therefore, we should let u=Inx since we know its derivative and let dv = dx.
- STEP 2: That gives us

$$du = \frac{1}{x} dx \qquad v = \int dx = x$$

STEP 3: Apply the formula

$$\int u\,dv = uv - \int v\,du$$

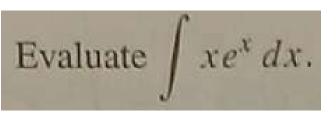
$$\int \ln x \, dx = \int u \, dv = uv - \int v \, du = x \ln x - \int dx = x \ln x - x + C$$

LIATE METHOD

- LIATE is an acronym for Logarithmic, Inverse trigonometric (which we have not done this year), Algebraic, Trigonometric, Exponential.
- If you have to take the integral of the product of two functions from different categories in the list, you will often be more successful if you select u to be the function whose category occurs earlier in the list.
- This method does not always work, but it works often enough to be a good rule of thumb.



LIATE EXAMPLE



- We could either make u = x (algebraic) or u = e^x (exponential).
 According to LIATE, since algebraic is earlier in the list, we should make u = x and dv = e^x dx. STEP 1
- STEP 2: That give us
- STEP 3: Apply the formula

$$du = dx$$
 and $v = \int e^x dx = e^x$
 $\int u \, dv = uv - \int v \, du$

$$\int xe^x dx = \int u dv = uv - \int v du = xe^x - \int e^x dx = xe^x - e^x + C$$



REPEATED INTEGRATION BY PARTS

It is sometimes necessary to use integration by parts more than once in the same problem:

Example 4 Evaluate
$$\int x^2 e^{-x} dx$$
.
Solution. Let
 $u = x^2$, $dv = e^{-x} dx$, $du = 2x dx$, $v = \int e^{-x} dx = -e^{-x}$

so that from (3)

$$\int x^2 e^{-x} dx = \int u \, dv = uv - \int v \, du$$
$$= x^2 (-e^{-x}) - \int -e^{-x} (2x) \, dx$$
$$= -x^2 e^{-x} + 2 \int x e^{-x} \, dx$$

The last integral is similar to the original except that we have replaced x^2 by x. Another integration by parts applied to $\int xe^{-x} dx$ will complete the problem. We let

$$u = x$$
, $dv = e^{-x} dx$, $du = dx$, $v = \int e^{-x} dx = -e^{-x}$

so that

$$\int xe^{-x} dx = x(-e^{-x}) - \int -e^{-x} dx = -xe^{-x} + \int e^{-x} dx = -xe^{-x} - e^{-x} + C$$

Finally, substituting this into the last line of (4) yields

$$\int x^2 e^{-x} dx = -x^2 e^{-x} + 2 \int x e^{-x} dx = -x^2 e^{-x} + 2(-x e^{-x} - e^{-x}) + C$$
$$= -(x^2 + 2x + 2)e^{-x} + C \blacktriangleleft$$

pg 494 in book may be easier to read



(4)

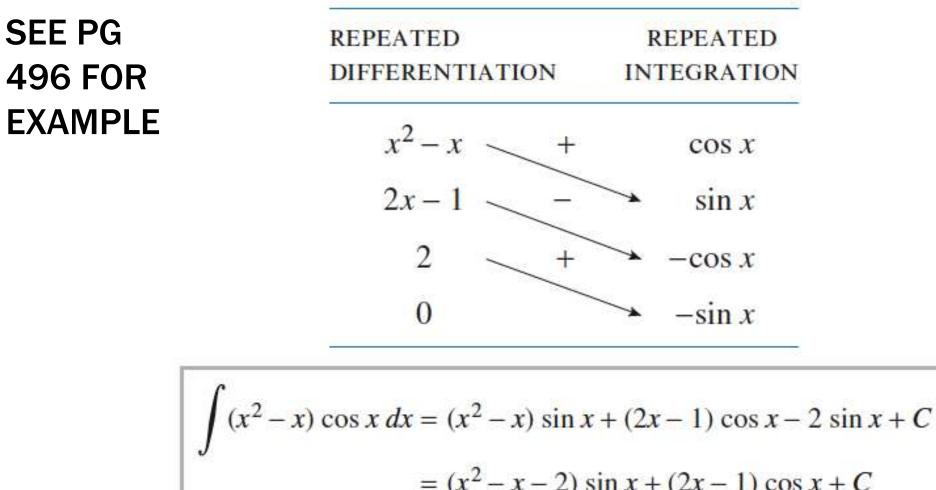
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IF YOU MUST DO INTEGRATION BY PARTS MORE THAN TWICE, YOU MAY WANT TO USE A TABLE



Tabular Integration by Parts

- Step 1. Differentiate p(x) repeatedly until you obtain 0, and list the results in the first column.
- Step 2. Integrate f(x) repeatedly and list the results in the second column.
- Step 3. Draw an arrow from each entry in the first column to the entry that is one row down in the second column.
- Step 4. Label the arrows with alternating + and signs, starting with a +.
- Step 5. For each arrow, form the product of the expressions at its tip and tail and then multiply that product by +1 or -1 in accordance with the sign on the arrow. Add the results to obtain the value of the integral.



$$= (x^2 - x - 2) \sin x + (2x - 1) \cos x + 0$$



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