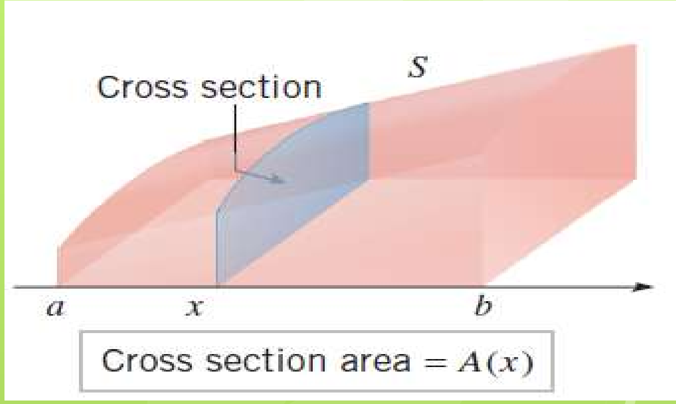
# Section 6.2

Volumes by Slicing:  Disks and Washers

# All graphics are attributed to:

- *Calculus,10/E* by Howard Anton, Irl Bivens, and Stephen Davis
  

# General Idea/Definition of Volume

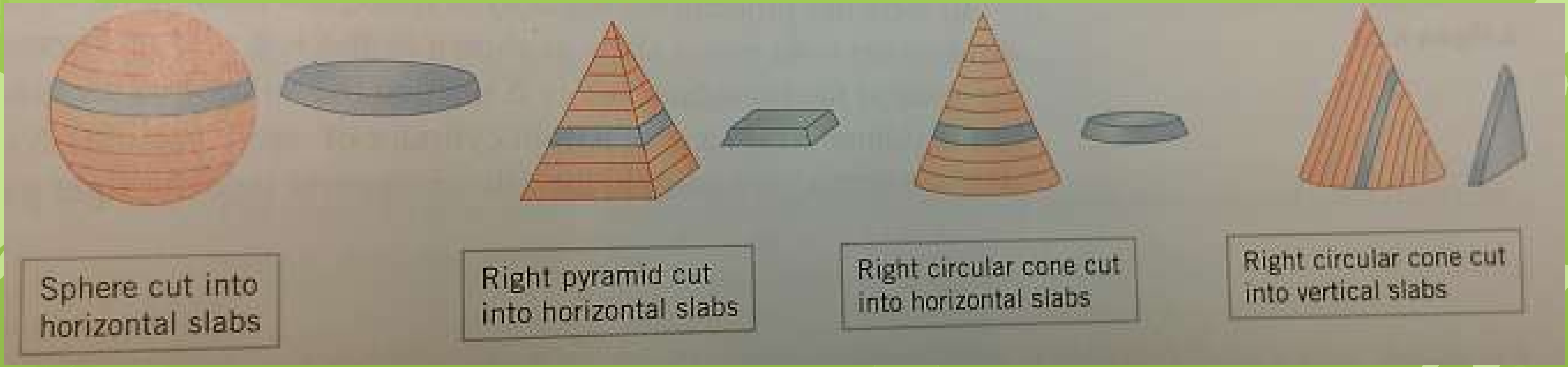



- In order to find volume we are first going to slice a three dimensional space (such as the one seen at the right) into infinitesimally narrow slices of area.
- Then, find the area of each slice separately.
- Next, find the sum of all of the areas (which will bring in the Sigma notation).
- Calculate the limit as the number of slices approaches infinity to get an accurate measure of volume.

**6.2.1 PROBLEM** Let $S$ be a solid that extends along the $x$-axis and is bounded on the left and right, respectively, by the planes that are perpendicular to the $x$-axis at $x = a$ and $x = b$ (Figure 6.2.5). Find the volume $V$ of the solid, assuming that its cross-sectional area $A(x)$ is known at each $x$ in the interval $[a, b]$.

# Which Area Formula?

- The formula depends upon the shape of the cross-section.  It could use a circle, square, triangle, etc.



*The volume of a solid can be obtained by integrating the cross-sectional area from one end of the solid to the other.*

**6.2.2 VOLUME FORMULA** Let $S$ be a solid bounded by two parallel planes perpendicular to the $x$-axis at $x = a$ and $x = b$. If, for each $x$ in $[a, b]$, the cross-sectional area of $S$ perpendicular to the $x$-axis is $A(x)$, then the volume of the solid is

$$V = \int_a^b A(x)\,dx \tag{3}$$

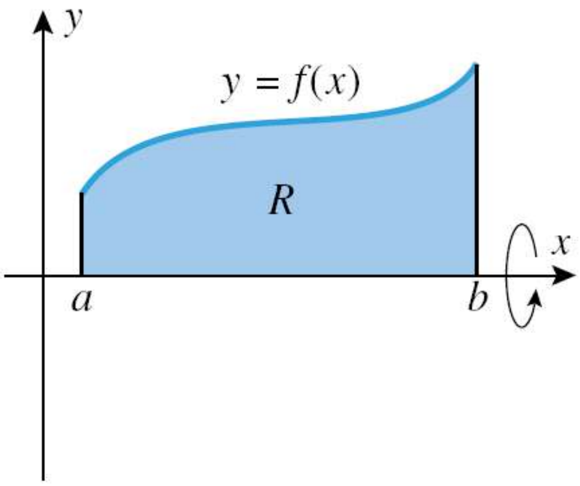
provided $A(x)$ is integrable.

# Rotating Area using Discs to Find Volume

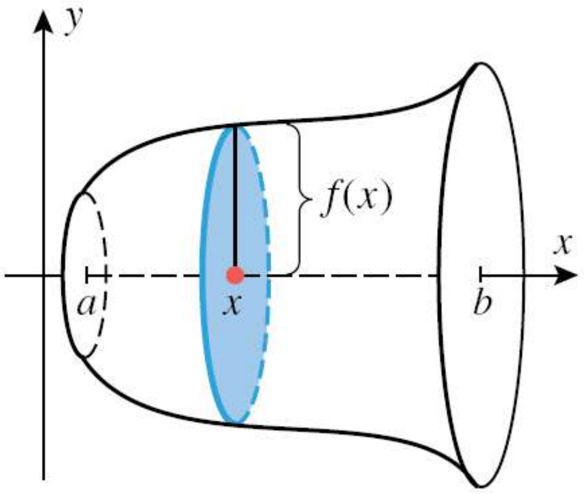
- One of the simplest examples of a solid with congruent cross sections is
- We are going to use all of the same ideas from Section 6.1 and more, so I am going to remind you of the process.
- The difference is that after we find the area of each strip, we are going to rotate it around the x-axis, y-axis, or another line in order to find the resulting volume.

- These notes are incomplete, so make sure you leave room to add things next class.
- If you prefer, you may take notes from the book.

**6.2.4 PROBLEM** Let $f$ be continuous and nonnegative on $[a, b]$, and let $R$ be the region that is bounded above by $y = f(x)$, below by the $x$-axis, and on the sides by the lines $x = a$ and $x = b$ (Figure 6.2.9$a$). Find the volume of the solid of revolution that is generated by revolving the region $R$ about the $x$-axis.




(a)




(b)

$$V = \int_a^b \pi[f(x)]^2\,dx$$

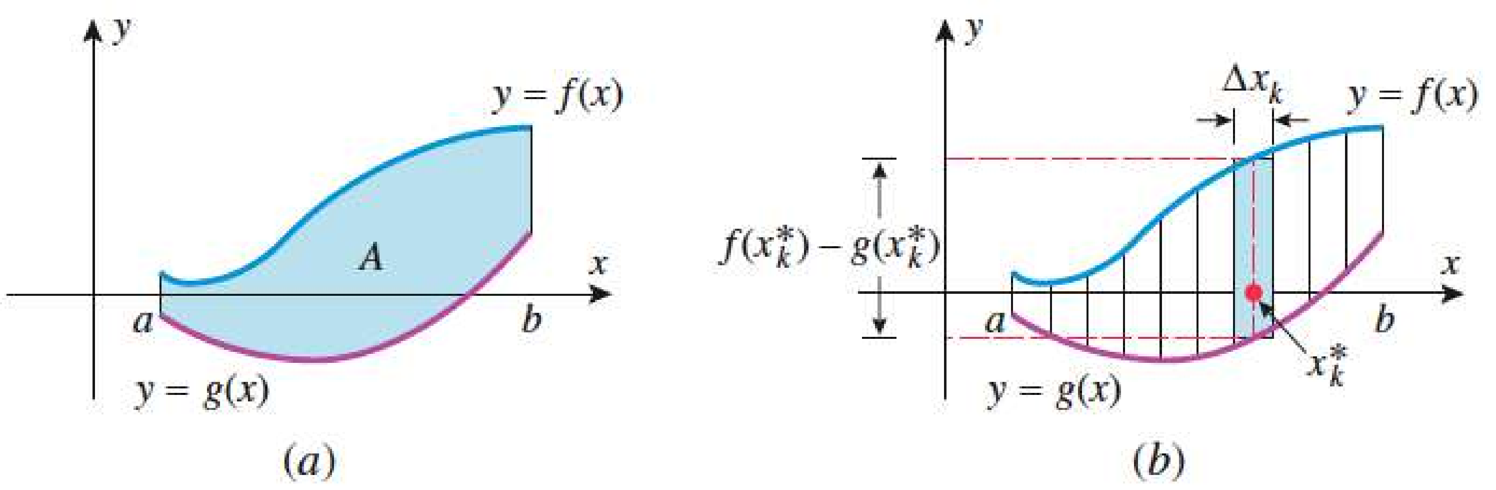
- Proceed to next slide

# Rotating Area using Washers to Find Volume

- We are going to use all of the same ideas from Section 6.1 and more, so I am going to remind you of the process.
- The difference is that after we find the area of each strip, we are going to rotate it around the x-axis, y-axis, or another line in order to find the resulting volume.

# Remember: Area Between y = f(x) and y = g(x)

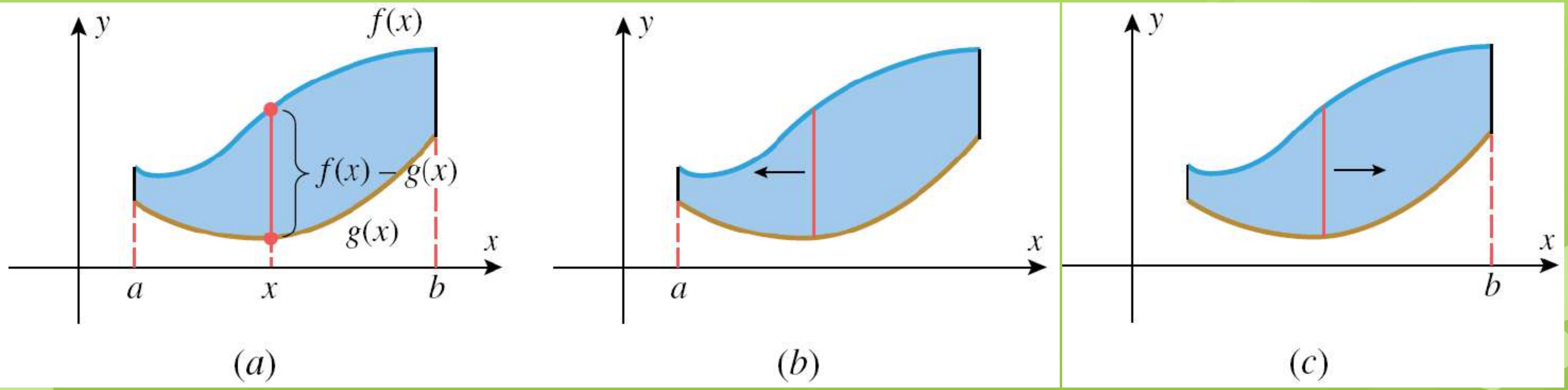
- To find the area between two curves, we will divide the interval [a,b] into n subintervals (like we did in section 5.4) which subdivides the area region into n strips (see diagram below).



# Area Between y = f(x) and y = g(x) continued

- To find the height of each rectangle, subtract the function output values $f(x_k*) - g(x_k*)$. The base is $\Delta x_k$.
- Therefore, the area of each strip is base * height = $\Delta x_k * [f(x_k*) - g(x_k*)]$.
- We do not want the area of one strip, we want the sum of the areas of all of the strips. That is why we need the sigma.
- Also, we want the limit as the number of rectangles "n" increases to approach infinity, in order to get an accurate area.

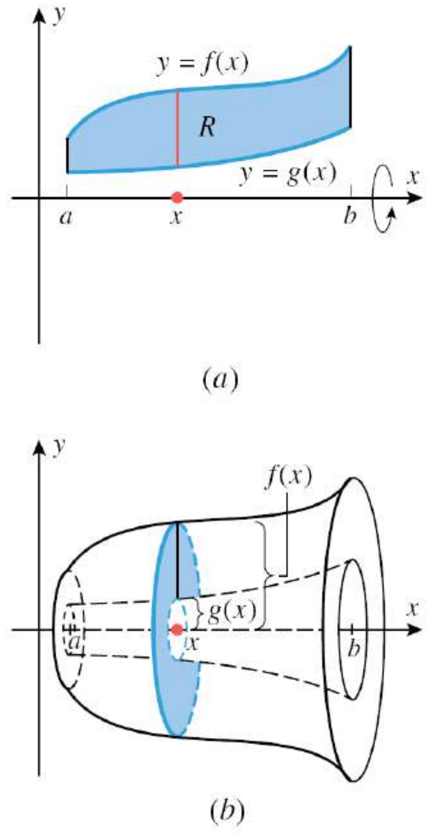
# Picture of Steps Two and Three From Previous Slide:



# Volume

- To find the height of each rectangular strip, subtract the function output values $f(x_k^*) - g(x_k^*)$. The base is $\Delta x_k$.
- Therefore, the area of each strip is base * height = $\Delta x_k$ * $[f(x_k^*) - g(x_k^*)]$.
- We do not want the area of one strip, we want the <u>sum</u> of the areas of all of the strips. That is why we need the sigma.
- Also, we want the limit as the number of rectangles “n” increases to approach infinity, in order to get an accurate area.

**6.2.5 PROBLEM** Let $f$ and $g$ be continuous and nonnegative on $[a, b]$, and suppose that $f(x) \geq g(x)$ for all $x$ in the interval $[a, b]$. Let $R$ be the region that is bounded above by $y = f(x)$, below by $y = g(x)$, and on the sides by the lines $x = a$ and $x = b$ (Figure 6.2.12$a$). Find the volume of the solid of revolution that is generated by revolving the region $R$ about the $x$-axis (Figure 6.2.12$b$).




$$V = \int_a^b \pi([f(x)]^2 - [g(x)]^2)\, dx$$

# Y - Orientation

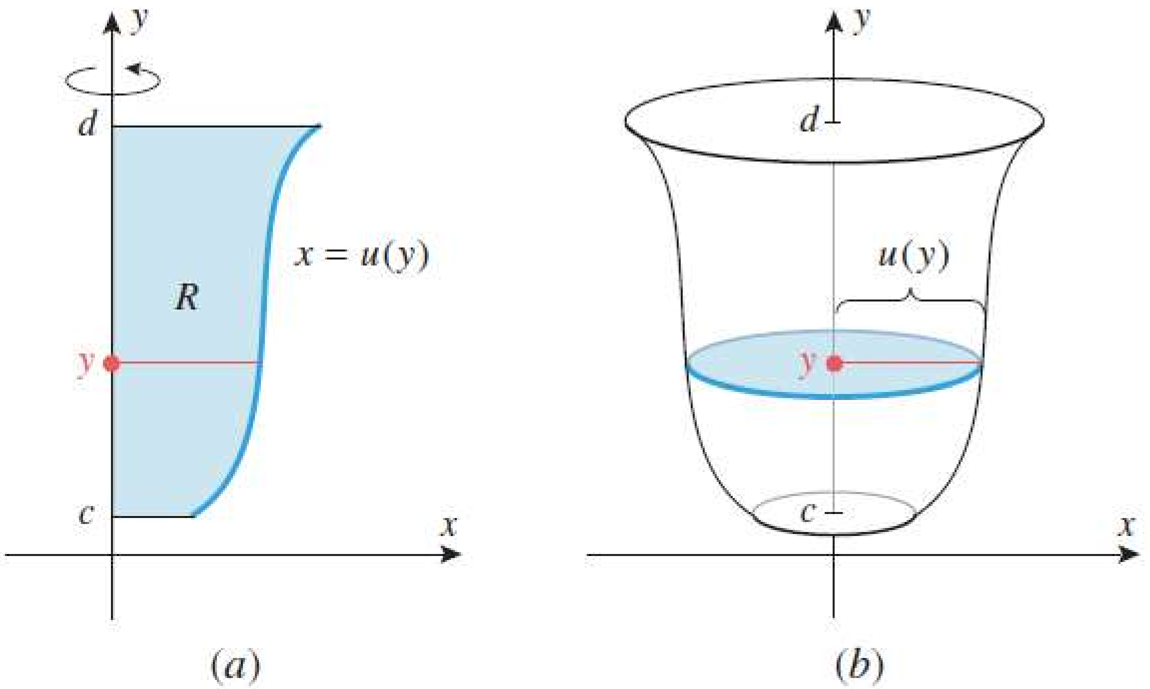
**6.2.3 VOLUME FORMULA** Let $S$ be a solid bounded by two parallel planes perpendicular to the $y$-axis at $y = c$ and $y = d$. If, for each $y$ in $[c, d]$, the cross-sectional area of $S$ perpendicular to the $y$-axis is $A(y)$, then the volume of the solid is

$$V = \int_c^d A(y)\, dy \tag{4}$$

provided $A(y)$ is integrable.

$$V = \int_c^d \pi[u(y)]^2\,dy$$

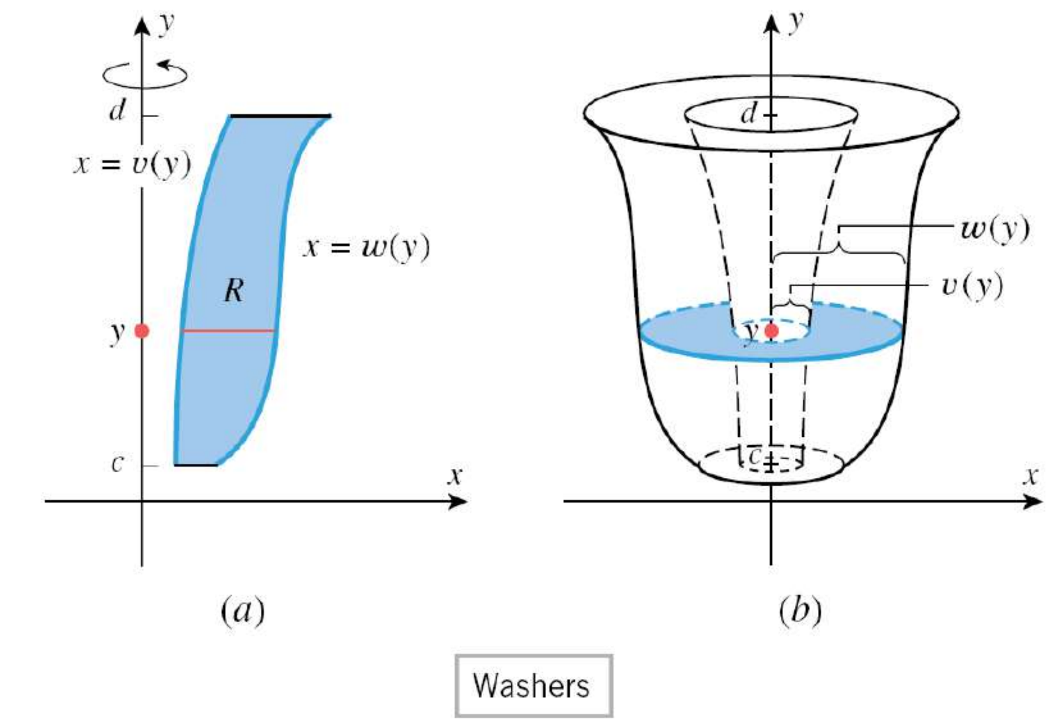
Disks




Disks

$$V = \int_c^d \pi([w(y)]^2 - [v(y)]^2)\,dy$$

Washers



- That is all for now.