Section 4.5

The Derivative in Graphing and Applications: "Applied Maximum and Minimum Problems"

All graphics are attributed to:

- *Calculus,10/E* by Howard Anton, Irl Bivens, and Stephen Davis

Introduction

- In this section we will show how the methods from the last section can be used to solve various applied optimization problems.
- Some of the problems will involve finite closed intervals while others will involve intervals that are not both finite and closed.
- Either way, the steps will be very similar except for step 4 on the next slide.

Problems Involving Finite Closed Intervals

A Procedure for Solving Applied Maximum and Minimum Problems

Step 1. Draw an appropriate figure and label the quantities relevant to the problem.

Step 2. Find a formula for the quantity to be maximized or minimized.

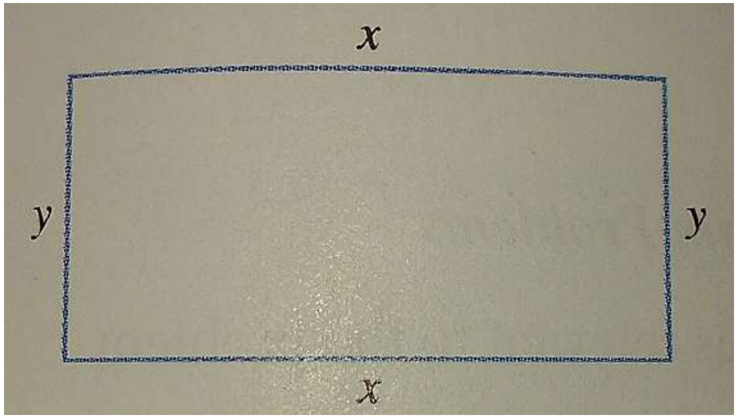
Step 3. Using the conditions stated in the problem to eliminate variables, express the quantity to be maximized or minimized as a function of one variable.

Step 4. Find the interval of possible values for this variable from the physical restrictions in the problem.

Step 5. If applicable, use the techniques of the preceding section to obtain the maximum or minimum.

Example

- A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 100 running feet of chicken wire is available for the fence?
- Step 1: Draw and label
- x=length of the rectangle (ft)
- y=width of the rectangle (ft)
- A=area of the rectangle (ft^2)



Example continued

- Step 2: Formula to maximize or minimize
- We want the largest possible area: A = x*y
- Step 3: Write that formula as a function of one variable
- Since perimeter is 100=2x+2y we can solve it for y=50-x and substitute into step 2: A = x*(50-x)
- Step 4: Interval of possible values
- Since there are only 100 feet of fence, the longest on side could be is 50 feet and the smallest is 0: $0 \le x \le 50$
- Step 5: Find the relative extrema
- Take the derivative, set=0, solve for x, determine whether the results are maximum(s) or minimum(s) (see next slide).

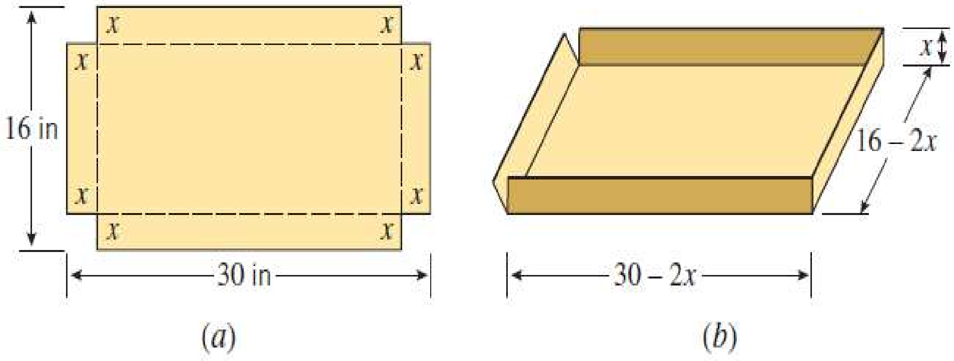
Example results

<table>
<tr>
<td>Step 5: Find the Relative Extrema</td>
<td>$A=x(50-x)=-x^2+50x$
$A'=-2x+50$
$0=-2x+50$
$2x=50$
$X=25$</td>
<td>Simplify
Take the derivative
Set derivative = 0
Solve for x</td>
</tr>
<tr>
<td></td>
<td>
<table>
<tr><td>type</td><td>#</td><td>Subst. in A</td><td>Result</td><td>Effect</td></tr>
<tr><td>min</td><td>0</td><td>0(50-0)</td><td>0</td><td>min</td></tr>
<tr><td>Extrema</td><td>25</td><td>25(50-25)</td><td>625</td><td>max</td></tr>
<tr><td>max</td><td>50</td><td>50(50-50)</td><td>0</td><td>min</td></tr>
</table>
</td>
<td>Don’t forget that the interval we found in step 4 was [0,50]</td>
</tr>
</table>

Example #2

- PLEASE FOLLOW THE STEPS!!!!!!☺
- An open box is to be made from a 16 inch by 30 inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides (see below). What size should the squares be to obtain a box with the largest volume?

- Step 1: Draw and label
- x=length (in.) of sides of the square
- V=volume (cubic inches) of the resulting box



<table>
<tr><td>Step 2:
Formula to max or min</td><td>V = L*W*H</td><td>We want the largest possible volume of a rectangular prism: V = L*W*H</td></tr>
<tr><td>Step 3:
Write as a function of one variable</td><td>V = (30-2x)(16-2x)(x)

V = $4x^3 - 92x^2 + 480x$</td><td>Look at the labels on the sides of the new box on the right of the last slide and do substitution.
FOIL and distribute</td></tr>
<tr><td>Step 4:
Interval of possible values</td><td>$0 \leq x \leq 8$</td><td>Since there are only 16 inches of cardboard along the width, the longest the height could be is 8 inches if you fold the 16 on half and the smallest is 0.</td></tr>
<tr><td>Step 5:
Find the relative extrema</td><td>$\frac{dv}{dt} = 12x^2 - 184x + 480$
0 = $4(3x^2 - 46x + 120)$
0 = 4(x-12)(3x-10)
x=12, x=10/3=3.333</td><td>Take the derivative.
Set=0.
Factor by grouping.
Solve for x.
Test critical points and endpoints in V</td></tr>
<tr><td></td><td><table>
<tr><td></td><td>#</td><td>Subst. in V</td><td>=</td><td>Effect</td></tr>
<tr><td>min</td><td>0</td><td>(30-2*0)(16-2*0)(0)</td><td>0</td><td>min</td></tr>
<tr><td>Extr</td><td>3.3</td><td>(30-2*3.3)(16-2*3.3)(3.3)</td><td>726</td><td>max</td></tr>
<tr><td></td><td>12</td><td>Not in interval</td><td>n/a</td><td></td></tr>
<tr><td>max</td><td>8</td><td>(30-2*8)(16-2*8)(8)</td><td>0</td><td>min</td></tr>
</table></td><td>Don't forget that the interval we found in step 4 was [0,8]</td></tr>
</table>

- Therefore, the greatest possible volume is about 726 cubic inches if each square has sides 10/3 in.

More Examples

- There are more examples in the book if you need to look at them for help with a specific problem.
- Please make sure you FOLLOW THE STEPS!
- Example 4 on page 278 is a good one to look at if you need help with a problem involving similar triangles:

King of the Hammers Race