SECTION 1.6

"Limits and Continuity":

Continuity of Trigonometric, Exponential, and Inverse Functions

ALL GRAPHICS ARE ATTRIBUTED TO:

- *Calculus,10/E* by Howard Anton, Irl Bivens, and Stephen Davis

CONTINUITY OF TRIGONOMETRIC FUNCTIONS

- sin x and cos x are continuous everywhere.
- tan x, cot x, csc x, and sec x are continuous everywhere except at their asymptotes.

1.6.2 THEOREM *If f is a one-to-one function that is continuous at each point of its domain, then f^{-1} is continuous at each point of its domain; that is, f^{-1} is continuous at each point in the range of f.*

- Therefore, $\sin^{-1} x$, $\cos^{-1} x$, and $\tan^{-1} x$ are only continuous on their own domains which are $(-\pi/2, \pi/2)$ for $\sin^{-1} x$ and $\tan^{-1} x$ and $(0,\pi)$ for $\cos^{-1} x$.

THE SQUEEZING THEOREM

- This theorem is confusing to many people. We just want the general idea this year and Theorem 1.6.5 on a later slide will give you the two most common uses for us this year.

1.6.4 THEOREM (*The Squeezing Theorem*) *Let f, g, and h be functions satisfying*

$$g(x) \leq f(x) \leq h(x)$$

for all x in some open interval containing the number c, with the possible exception that the inequalities need not hold at c. If g and h have the same limit as x approaches c, say

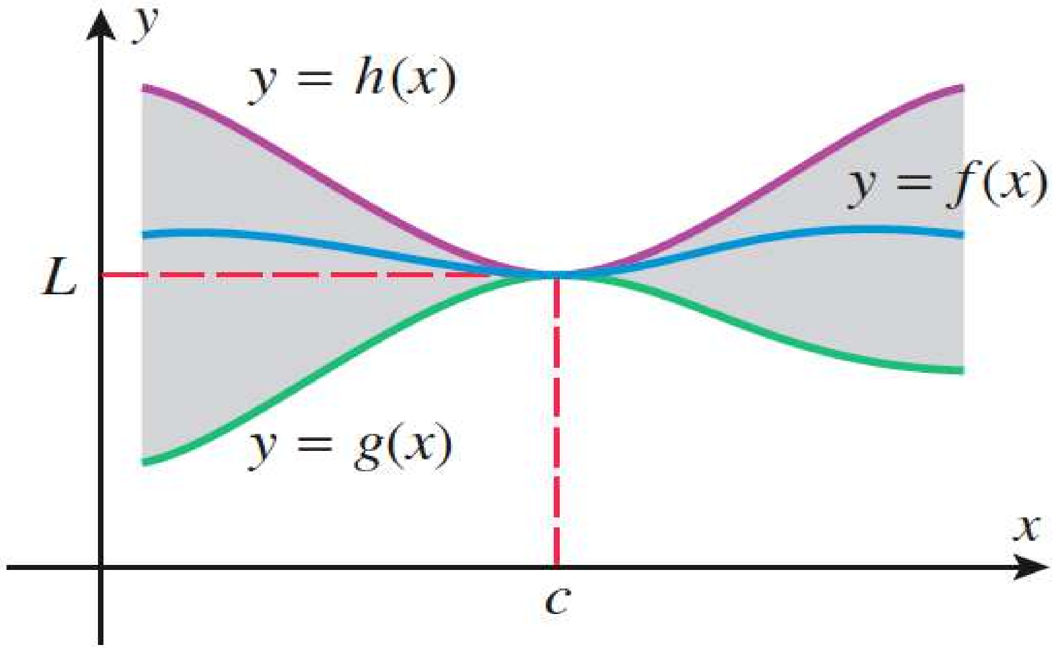
$$\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$$

then f also has this limit as x approaches c, that is,

$$\lim_{x \to c} f(x) = L$$

SQUEEZING THEOREM GRAPHICALLY

- There is no easy way to calculate some indeterminate type 0/0 limits algebraically so, for now, we will squeeze the function between two known functions to find its limit like in the graph below.



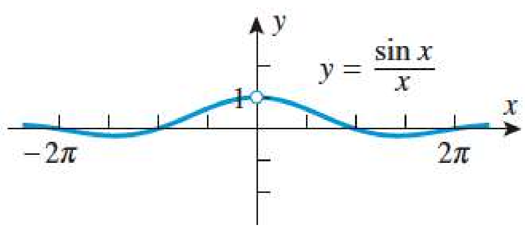
COMMON USES THIS YEAR

- These are the most common applications of the squeezing theorem that we will use this year.

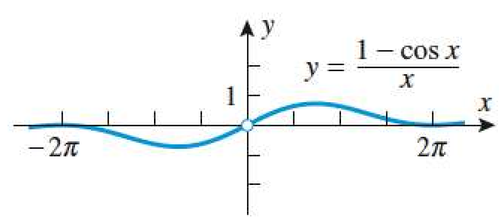
1.6.5 THEOREM

$$(a)\quad \lim_{x\to 0}\frac{\sin x}{x}=1 \qquad (b)\quad \lim_{x\to 0}\frac{1-\cos x}{x}=0$$

- They may make more sense if you look at their graphs and find the limits that way.



$$\lim_{x\to 0}\frac{\sin x}{x}=1$$



$$\lim_{x\to 0}\frac{1-\cos x}{x}=0$$

PROOFS AND EXAMPLES

- Theorem 1.6.5 is proven on page 123 if you are interested in how it works.
- Example 1 of how it is used:

Find $\lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \frac{\frac{\sin x}{\cos x}}{x}$ — Use tan x quotient identity to rewrite

$= \lim_{x \to 0} \frac{\sin x}{\cos x} * \frac{1}{x} = \lim_{x \to 0} \frac{\sin x}{x} * \frac{1}{\cos x}$ — Multiply by the reciprocal and rearrange

$= \lim_{x \to 0} \frac{\sin x}{x} * \lim_{x \to 0} \frac{1}{\cos x}$ — Apply the limit of the product=product of the limits rule

$= (1) * \frac{1}{\cos 0}$ — By the squeezing theorem and substitution

$= 1 * \frac{1}{1} = 1$

ANOTHER EXAMPLE

- Example 2:

Find $\lim_{\theta \to 0} \frac{\sin 2\theta}{\theta} = \lim_{\theta \to 0} \frac{\sin 2\theta}{\theta} * \frac{2}{2}$ Multiply top and bottom by 2 so that we can apply theorem

$= \lim_{\theta \to 0} \frac{2 * \sin 2\theta}{2\theta} = 2 * \lim_{x \to 0} \frac{\sin 2\theta}{2\theta}$ Multiply, rearrange, and apply limit property

$= 2 * (1) = 2$ By the squeezing theorem

SNOWMOBILING IN MAMMOTH