Unit #3 Assignments

You are responsible for doing all of the homework and checking your work. If you get stuck, the solutions are worked out at the end of the unit and the odd numbered exercises are also available online through the textbook publisher. If you still have questions on the homework problems after going over the solutions, then come in before school, at lunch by appointment, afterschool, or during intervention as class time will not be devoted to going over the homework.

Any student who does not have the homework done prior to a test is not eligible for a retake of that test.

Unit #3—Trigonometric Identities and Formulas

Assignment #1: 7.3 Addition Formulas—Sine and Cosine
____________ Page 677 #'s 13-16, 35, 37, 38, 47(a and b only), 55, 57, 63

Assignment #2: 7.3 Addition Formulas—Tangent
____________ Page 677: 39, 41, 47c, 49, 59, 60

Assignment #3: 7.4 Double-Angle Formulas
____________ Page 688: 7, 9, 23, 25, 57, 59, 61

Assignment #4: 7.4 More Fun
____________ Page 690: 85-93 odds

Assignment #5: 7.4 Extra Practice
____________ Handout

Assignment #6: 7.6 Solving Trigonometric Equations
____________ Handout

Assignment #7: Solving Trigonometric Inequalities
____________ Handout

Assignment #8: Review

____

Test

Trigonometric Formulas

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

Reciprocal Identities

$\sin x = \frac{1}{\csc x}$ $\csc x = \frac{1}{\sin x}$

$\cos x = \frac{1}{\sec x}$ $\sec x = \frac{1}{\cos}$

$\tan x = \frac{1}{\cot x}$ $\cot x = \frac{1}{\tan x}$

Reduction Identities

$\sin(-x) = -\sin x$ $\csc(-x) = -\csc x$

$\cos(-x) = \cos x$ $\sec(-x) = \sec x$

$\tan(-x) = -\tan x$ $\cot(-x) = -\cot x$

Sum and Difference Formulas

$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$

$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$

$\tan(\alpha \pm \beta) = \frac{tan \quad \pm tan}{1 \mp tan\, \alpha\, tan\, \beta}$

Double -Angle Formulas

$\sin 2\alpha = 2 \sin \alpha \cos \alpha$

$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$
$= 2 \cos^2 \alpha - 1$
$= 1 - 2 \sin^2 \alpha$

$\tan 2\alpha = \frac{2\, ta}{1 - tan^2\, \alpha}$

Cofunction Identities

$\sin x = \cos\left(\frac{\pi}{2} - x\right)$ $\csc x = \sec\left(\frac{\pi}{2} - x\right)$

$\cos x = \sin\left(\frac{\pi}{2} - x\right)$ $\sec x = \csc\left(\frac{\pi}{2} - x\right)$

$\tan x = \cot\left(\frac{\pi}{2} - x\right)$ $\cot x = \tan\left(\frac{\pi}{2} - x\right)$

Topic: 7.3 Sum and Difference Formulas for Cosine and Sine

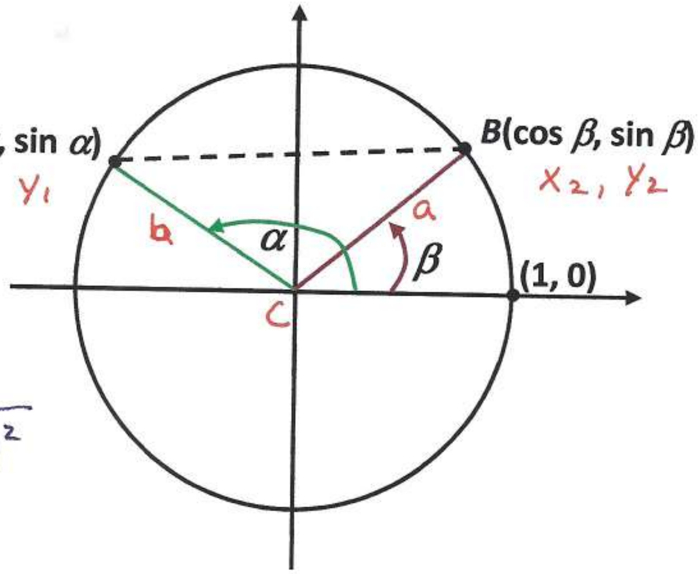
Goal: Apply the sum and difference formulas to find the exact values of sine and cosine.

Law of Cosines $c^2 = a^2 + b^2 - 2ab\cos C$

(AB)² = $1^2 + 1^2 - 2(1)(1)\cos(\alpha - \beta)$

$(AB)^2 = 2 - 2\cos(\alpha - \beta)$

sub for $(AB)^2$



Distance Formula $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(AB)² = $(x_2 - x_1)^2 + (y_2 - y_1)^2$

$(AB)^2 = (\cos\beta - \cos\alpha)^2 + (\sin\beta - \sin\alpha)^2$

$= \cos^2\beta - 2\cos\beta\cos\alpha + \cos^2\alpha + \sin^2\beta - 2\sin\beta\sin\alpha + \sin^2\alpha$

$= 2 - 2\cos\beta\cos\alpha - 2\sin\beta\sin\alpha$

$2 - 2\cos(\alpha - \beta) = 2 - 2\cos\beta\cos\alpha - 2\sin\beta\sin\alpha$

$-2\cos(\alpha - \beta) = -2\cos\beta\cos\alpha - 2\sin\beta\sin\alpha$

$\cos(\alpha - \beta) = \cos\beta\cos\alpha + \sin\beta\sin\alpha$

Thus,

cos (α – β) = $\cos\alpha\cos\beta + \sin\alpha\sin\beta$

cos (α + β) = $\cos\alpha\cos\beta - \sin\alpha\sin\beta$

sin (α + β) = $\sin\alpha\cos\beta + \cos\alpha\sin\beta$

sin (α – β) = $\sin\alpha\cos\beta - \cos\alpha\sin\beta$

1. Find the exact value of sin 15°.

$\sin(45° - 30°) =$ (A = 45°, B = 30°)

$\sin 45 \cos 30 - \cos 45 \sin 30$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$\frac{\sqrt{6} - \sqrt{2}}{4}$

[redacted]

2. Find the exact value of cos 75°.

3. [redacted] Find the exact value of:

$\cos 50° \cos 10° - \sin 50° \sin 10°$ (A = 50°, B = 10°) $= \cos(A + B)$

$= \cos(50 + 10) = \cos 60 = \frac{1}{2}$

$\cos 40° \cos 10° + \sin 40° \sin 10° = \cos(40 - 10) = \cos 30 = \frac{\sqrt{3}}{2}$

Simplify

4. [redacted] $\cos\left(\frac{3\pi}{2} + x\right) + \cos\left(\frac{3\pi}{2} - x\right)$ (A + B, A − B)

$\cos\frac{3\pi}{2}\cos x - \sin\frac{3\pi}{2}\sin x + \cos\frac{3\pi}{2}\cos x + \sin\frac{3\pi}{2}\sin x$

$0 \cdot \cos x - (-1)\sin x + 0 \cdot \cos x + (-1)\sin x$

$\sin x + -\sin x$

0

$x^2+y^2=r^2$ $y=4, r=5$, so $x=3$ → $\alpha \in Q1$ $\beta \in Q2$

5. [redacted] Suppose that the sin $\alpha = \frac{4}{5}$ and sin $\beta = \frac{5}{13}$, where $0 < \alpha < \frac{\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$.

$x^2+5^2=13^2$ → $y=5, r=13$, so $x=-12$ in Q2

Find:

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

need $\cos\alpha$ and $\cos\beta$

$= \frac{3}{5}\left(-\frac{12}{13}\right) - \frac{4}{5}\left(\frac{5}{13}\right)$

$= \frac{-36-20}{65} = -\frac{56}{65}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{4}{5}\left(\frac{-12}{13}\right) + \frac{3}{5}\left(\frac{5}{13}\right)$

$= \frac{-48+15}{65}$

$= \frac{-33}{65}$

$\cos(\alpha-\beta)$

$\sin(\alpha-\beta)$

Based upon your results, which quadrant is the angle $\alpha + \beta$ in? Justify your conclusion.

Q3 b/c $\cos(\alpha+\beta)$ is neg and $\sin(\alpha+\beta)$ is neg.

[redacted]

Assignment #1: 7.3 Page 677 #'s 13-16, 35, 37, 38, 47(a and b only), 55, 57, 63

Topic: 7.3 Sum and Difference formulas for Tangent and Cofunction Formulas

Goal: Apply the sum and difference formulas for tangent to find the exact values of tangent.

tan (α + β) = $\dfrac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta} \cdot \dfrac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}}$

$$= \dfrac{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} - \dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

$$= \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

tan (α ± β) = $\dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$x^2 + y^2 = r^2$ α: $1^2 + 3^2 = r^2$, $\sqrt{10} = r$ β: $1^2 + 2^2 = r^2$, $\sqrt{5} = r$

1. Suppose $\tan\alpha = \frac{1}{3}$ $\frac{y}{x}$ and $\tan\beta = \frac{1}{2}$ $\frac{y}{x}$

Find:

$\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \left[\dfrac{\frac{1}{3} + \frac{1}{2}}{1 - \frac{1}{3}\cdot\frac{1}{2}}\right]\dfrac{\frac{6}{1}}{\frac{6}{1}}$

$= \dfrac{2+3}{6-1} = \dfrac{5}{5} = 1$

$\tan(\alpha - \beta) = \left[\dfrac{\frac{1}{3} - \frac{1}{2}}{1 + \frac{1}{3}\cdot\frac{1}{2}}\right]\dfrac{\frac{6}{1}}{\frac{6}{1}}$

$= \dfrac{2-3}{6+1} = \dfrac{-1}{7}$

$\sin(\alpha - \beta)$

[redacted]

$\sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\frac{1}{\sqrt{10}}\cdot\frac{2}{\sqrt{5}} - \frac{3}{\sqrt{10}}\cdot\frac{1}{\sqrt{5}}$

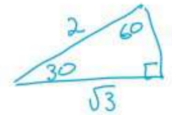
$\frac{2}{\sqrt{50}} - \frac{3}{\sqrt{50}} = \frac{-1}{\sqrt{50}} = \frac{-1}{5\sqrt{2}}$

$\sin\alpha = \frac{1}{\sqrt{10}}$ $\sec\alpha = \frac{\sqrt{10}}{3}$ $\cot\alpha = 3$

$\cos\beta = \frac{2}{\sqrt{5}}$ $\csc\beta = \sqrt{5}$ $\cot\beta = 2$

2. Find the exact value of $\tan 15°$.

$\tan(45-30) = \dfrac{\tan 45 - \tan 30}{1 + \tan 45\tan 30}$



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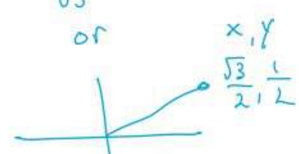
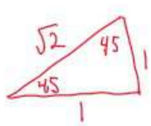


$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$



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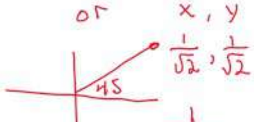


$\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

$= \left[\dfrac{1 - \frac{1}{\sqrt{3}}}{1 + 1\left(\frac{1}{\sqrt{3}}\right)}\right]\dfrac{\frac{\sqrt{3}}{1}}{\frac{\sqrt{3}}{1}}$

$= \left(\dfrac{\sqrt{3}-1}{\sqrt{3}+1}\right)\left(\dfrac{\sqrt{3}-1}{\sqrt{3}-1}\right) = \dfrac{3 - 2\sqrt{3} + 1}{3 - 1}$

$= \dfrac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$

Cofunction Identities

$\sin x = \cos\left(\frac{\pi}{2} - x\right)$ $\csc x = \sec\left(\frac{\pi}{2} - x\right)$

$\cos x = \sin\left(\frac{\pi}{2} - x\right)$ $\sec x = \csc\left(\frac{\pi}{2} - x\right)$

$\tan x = \cot\left(\frac{\pi}{2} - x\right)$ $\cot x = \tan\left(\frac{\pi}{2} - x\right)$ Can also be used with degrees where $\frac{\pi}{2} = 90^o$

Solve using cofunction identities:

1) $\cos\left(42^o + \frac{2x}{3}\right) = sin(78^o)$

$= \cos(90 - 78)$

$\cos\left(42^\circ + \frac{2x}{3}\right) = \cos 12^\circ$

$42^\circ + \frac{2x}{3} = 12^\circ$

$-42 \qquad -42$

$\frac{2x}{3} = -30^\circ$

$2x = -90^\circ$

$x = -45^\circ$

2) $\tan\left(\frac{\pi}{6} + 2x\right) = \cot\left(\frac{\pi}{12} - 4x\right)$

$= \tan\left[\frac{\pi}{2} - \left(\frac{\pi}{12} - 4x\right)\right]$

$= \tan\left(\frac{\pi}{2} - \frac{\pi}{12} + 4x\right)$

$\tan\left(\frac{\pi}{6} + 2x\right) = \tan\left(\frac{5\pi}{12} + 4x\right)$

$\frac{\pi}{6} + 2x = \frac{5\pi}{12} + 4x$

$-\frac{5\pi}{12} \quad -2x \qquad -\frac{5\pi}{12} \quad -2x$

$\left(-\frac{3\pi}{12} = 2x\right)\frac{1}{2}$

$-\frac{3\pi}{24} = x$

$-\frac{\pi}{8} = x$

Assignment #2: 7.3 Page 677: 39, 41, 49, 59, 60 and worksheet 7.3

Name : ____________________

7.3 Solve - Cofunction Identities

Solve using cofunction identities.

1) $\sin \frac{9x}{2} = \cos\left(15^0 + \frac{x}{2}\right)$

2) $\tan \frac{\pi}{12} = \cot\left(x - \frac{\pi}{36}\right)$

3) $\csc (x + 20^0) = \sec 25^0$

4) $\cos\left(4x + \frac{\pi}{10}\right) = \sin\left(9x - \frac{8\pi}{45}\right)$

5) $\sec (3x + 8^0) = \csc (22^0 + 7x)$

6) $\sec\left(\frac{\pi}{5} - x\right) = \csc\left(5x + \frac{\pi}{20}\right)$

7) $\tan 7x = \cot (21x + 6^0)$

8) $\sin\left(\frac{13\pi}{60} + \frac{x}{2}\right) = \cos \frac{5\pi}{18}$

Topic: 7.4 Double-Angle Formulas

Goal: Apply the double angle and half–angle formulas to find the exact values.

Double Angle Formulas

$\sin 2\alpha = \sin(\alpha + \alpha) =$ $\sin A \cos B + \cos B \sin A$

$\sin\alpha\cos\alpha + \cos\alpha\sin\alpha$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = \cos(\alpha + \alpha) =$ $\cos A \cos B - \sin A \sin B$

$\cos\alpha\cos\alpha - \sin\alpha\sin\alpha$

$\cos^2\alpha + \sin^2\alpha = 1$

$\cos^2\alpha = 1 - \sin^2\alpha$

1) $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

2) $\cos 2\alpha = 1 - \sin^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha$

$\sin^2\alpha = 1 - \cos^2\alpha$

3) $\cos 2\alpha = \cos^2\alpha - (1 - \cos^2\alpha) = 2\cos^2\alpha - 1$

$\tan 2\alpha = \tan(\alpha + \alpha) = \dfrac{\tan\alpha + \tan\alpha}{1 - \tan\alpha\tan\alpha}$

$= \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$

1. Simplify

a) $2 \sin 10° \cos 10°$

$= \sin 2\alpha$

$= \sin(2 \cdot 10)$

$= \sin 20°$

b) $\cos^2 15° - \sin^2 15°$

$= \cos 2\alpha$

$= \cos 2(15)$

$= \cos 30°$

$= \frac{\sqrt{3}}{2}$

c) $1 - 2\sin^2 x$

$= \cos 2\alpha$

$= \cos 2x$

d) $2 \sin 3\alpha \cos 3\alpha$

$= \sin 2A$ where $A = 3\alpha$

$= \sin 2 \cdot 3\alpha$

$= \sin 6\alpha$

e) $3 \cos 25° \sin 25°$ $\quad 2\cos\alpha \sin\alpha$

$\frac{3}{2}(2 \cos 25° \sin 25°)$

$\frac{3}{2} \sin 2(25)$

$\frac{3}{2} \sin 50°$

f) $\cos^2 115° + \sin^2 115° = 1$

not minus!

$\cos^2\alpha + \sin^2\alpha = 1$

[redacted]

2. [redacted] If $tan\ A = \frac{1}{2}$, find $cos\ 2\ A$ and $tan\ 2\ A$.

$\cos 2A = 2\cos^2 A - 1$

$1^2 + 2^2 = r^2$

$\sqrt{5} = r$

$\cos A = \frac{2}{\sqrt{5}}$

$\cos 2A = 2\left(\frac{2}{\sqrt{5}}\right)^2 - 1$

$= \frac{8}{5} - 1 = \frac{3}{5}$

$\tan 2A = \dfrac{2\left(\frac{1}{2}\right)}{1 - \left(\frac{1}{2}\right)^2}$

$= \dfrac{1}{1 - \frac{1}{4}}$

$= \dfrac{1}{\frac{3}{4}} = \frac{4}{3}$

[redacted]

Assignment #3: 7.4 Page 688: 7, 9, 23, 25, 57, 59 ,61

Topic: 7.4 Formulas

Goal: Apply the formulas to expressions to derive other formulas.

[redacted]

$$\sin 4x = \sin\overset{A\ +\ B}{(2x + 2x)} = \sin 2x \cos 2x + \cos 2x \sin 2x$$

$$= 2 \sin 2x \cos 2x$$

$$= 2(2 \sin x \cos x)(\cos^2 x - \sin^2 x)$$

$$= 4 \sin x \cos x (\cos^2 x - \sin^2 x)$$

[redacted]

Prove: $\frac{1-\cos 2A}{1+\cos 2A} = \tan^2 A$

$\cos 2A = 1 - 2\sin^2 A$

$\cos 2A = 2\cos^2 A - 1$

$$\frac{1-(1-2\sin^2 A)}{1+(2\cos^2 A - 1)} = \tan^2 A$$

$$\frac{2\sin^2 A}{2\cos^2 A} = \tan^2 A$$

$$\tan^2 A = \tan^2 A$$

Assignment #4: 7.4 Page 690: 85-93 odds

Assignment #5: Extra Practice Handout

1. Suppose $\cos\alpha = \frac{1}{2}$ and $\sin\beta = \frac{3}{4}$, where $0 < \alpha < \beta < \frac{\pi}{2}$, Find:

a) $\sin(\alpha + \beta) =$

b) $\cos(\alpha + \beta) =$

c) $\tan(\alpha + \beta) =$

d) $\cos(\alpha - \beta) =$

e) $\sin(\alpha - \beta) =$

f) $\tan(\alpha - \beta) =$

g) $\sin 2\alpha =$

h) $\cos 2\alpha =$

i) $\tan 2\beta =$

2. Suppose $\cos \alpha = \frac{2}{3}$ and $\sin \beta = \frac{1}{5}$, where $0 < \alpha < \frac{\pi}{2} < \beta < \pi$, Find:

a) $\sin(\alpha + \beta) =$

b) $\cos(\alpha + \beta) =$

c) $\tan(\alpha + \beta) =$

d) $\cos(\alpha - \beta) =$

e) $\sin(\alpha - \beta) =$

f) $\tan(\alpha - \beta) =$

g) $\sin 2\beta =$

h) $\cos 2\alpha =$

i) $\tan 2\alpha =$

3. Simplify $\cos(30^\circ + \theta) + \cos(30^\circ - \theta)$

4. Simplify $\sin(\pi + x) - \sin(\pi - x)$

5. Simplify $\cos 2x \cos x + \sin 2x \sin x$

6. Find the exact value of $\sin 105^\circ$

Topic: 7.6 Solving Trigonometric Equations

Goal: Solve trigonometric equations.

1. Solve: $\cos 2x = 1 - \sin x$ for $0 \leq x < 2\pi$

$1 - 2\sin^2 x = 1 - \sin x$

$+2\sin^2 x \quad +2\sin^2 x$

$0 = 2\sin^2 x - \sin x$

$0 = \sin x (2\sin x - 1)$

$\sin x = 0$

$x = 0, \pi$

$2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

2. Solve: $3 \cos 2x + \cos x = 2$ for $0 \leq x < 2\pi$

$3(2\cos^2 x - 1) + \cos x = 2$

$6\cos^2 x - 3 + \cos x = 2$

$6\cos^2 x + \cos x - 5 = 0$

$6A^2 + A - 5 = 0$

$(A + 1)(6A - 5) = 0$

$(\cos x + 1)(6\cos x - 5) = 0$

$\cos x + 1 = 0$

$\cos x = -1$

$x = \pi$

$6\cos x - 5 = 0$

$\cos x = \frac{5}{6}$ calc.

$x_{ref} = 0.586$ rad

cos pos in

Q1: 0.586

Q3: $\pi + 0.586$

3. Solve: $2 \sin 2x = 1$ for $0° \leq x < 360°$

$$\sin 2x = \frac{1}{2} \Rightarrow \sin \theta = \frac{1}{2}$$

$$\theta = 30°, 150°, 390°, 510°,$$

$$2x = 30°, 150°, 390°, 510°$$

$$x = 15°, 75°, 195°, 255°$$

4. Solve: $\tan 2x = 3 \tan x$ for $0 \leq x < 2\pi$

$$\frac{2\tan x}{1-\tan^2 x} = 3\tan x$$

$$2\tan x = 3\tan x - 3\tan^3 x$$

$$0 = \tan x - 3\tan^3 x$$

$$0 = \tan x\,(1 - 3\tan^2 x)$$

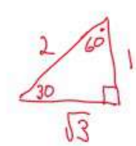
$\tan x = 0$

$x = 0°, 180°$

$1 - 3\tan^2 x = 0$

$\sqrt{\tan^2 x} = \sqrt{\frac{1}{3}}$

$\tan x = \pm\frac{1}{\sqrt{3}}$



$x = 30°$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Assignment #6: Handout

Solve the following equations of $0° \leq x < 360°$

1. $\cos 2x = \cos x$

2. $\sin^2 x = \sin x$

3. $\sin x = \cos x$

4. $\sin 3x = \cos 3x$

Topic: Solving Trigonometric Inequalities

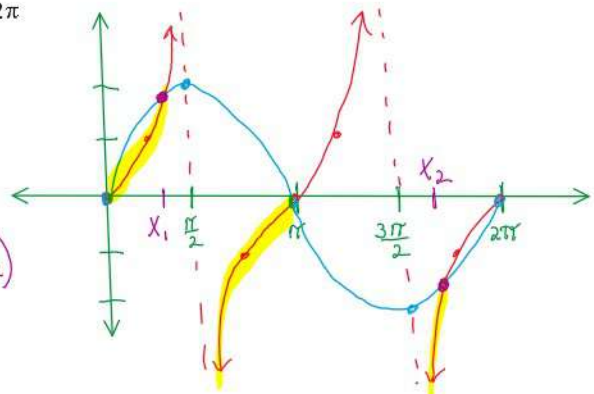
Goal: Solve trigonometric inequalities.

What is the difference between the solution to $x + 3 = 9$ and $x + 3 > 9$?

$x = 6$ — 1 sol $x > 6$ — inf sol

1. Solve: $\tan x < 2 \sin x$ for $0 \le x < 2\pi$

Per $= \pi$ Per $= 2\pi$ Amp $= 2$



Sol

$(0, x_1) \cup \left(\frac{\pi}{2}, \pi\right) \cup \left(\frac{3\pi}{2}, x_2\right)$

[redacted]

$\tan x = 2 \sin x$

$\frac{\sin x}{\cos x} = 2 \sin x$

$\sin x = 2 \sin x \cos x$

$0 = 2 \sin x \cos x - \sin x$

$0 = \sin x (2 \cos x - 1)$

$\sin x = 0$

$x = 0, \pi$

$2 \cos x - 1 = 0$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ (x_1, x_2)

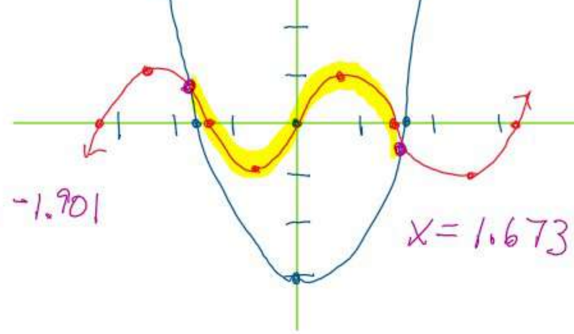
Sol

$\left(0, \frac{\pi}{3}\right) \cup \left(\frac{\pi}{2}, \pi\right) \cup \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)$

2. [redacted] Solve $\sin 2x \ge x^2 - 3$ (y_1, y_2)

cannot solve algebraically when trig functions are mixed with variables. Graph Calc.

To find intersection on calc: 2nd Calc 5



$x = -1.901$ $x = 1.673$

Assignment #7: Handout

For questions 1-3 solve each inequality for $0 \leq x < 2\pi$. Give answers to the nearest hundredth. You many use a graphing calculator to solve each.

1. $\cos x > \frac{1}{2}\sin x$

2. $\sin\left(3x - \frac{\pi}{2}\right) > 0$

3. $\cos x \leq \sin 2x$

4. Solve the equation $2\cos(x + 45°) = 1$ for $0° \leq x < 360°$ by using trigonometric identities. Round your answer to the nearest tenth if necessary. You may use a graphing calculator to solve these.

For questions 5 and 6 solve each equation for $0 \leq x < 2\pi$, by using trigonometric identities. Give answers to the nearest hundredth. You many use a graphing calculator to solve each.

5. $\sin x \cos x = \frac{1}{2}$

6. $\cos 2x = \sec x$

7. Solve the equation $\sin 2x = \ln(x + 1)$ for $-2\pi \leq x < 2\pi$ by using a graphing calculator. Give your answer to the nearest hundredth.

Assignment #8: Review

1. Solve sin 4x = cos 4x for $0 \le x < 2\pi$ both graphically and algebraically.

Per = $\frac{\pi}{2}$ both

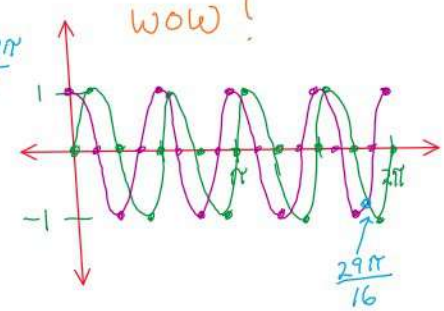
$\frac{\sin 4x}{\cos 4x} = \frac{\cos 4x}{\cos 4x}$

$\tan 4x = 1$

$4x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}, \frac{21\pi}{4}, \frac{25\pi}{4}, \frac{29\pi}{4}$

$x = \frac{\pi}{16}, \frac{5\pi}{16}, \frac{9\pi}{16}, \frac{13\pi}{16}, \frac{17\pi}{16}, \frac{21\pi}{16}, \frac{25\pi}{16}, \frac{29\pi}{16}$

$0 \le x < 2\pi$



2. Solve cos 2θ > sin θ for $0° \le \theta \le 360°$ both graphically and algebraically.

$1 - 2\sin^2\theta = \sin\theta$

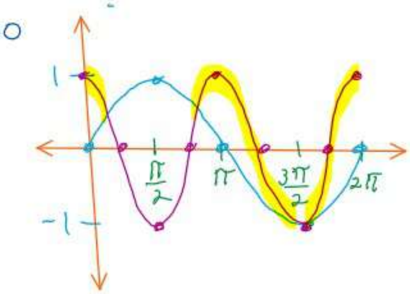
$0 = 2\sin^2\theta + \sin\theta - 1$

$0 = (2\sin\theta - 1)(\sin\theta + 1)$

$2\sin\theta - 1 = 0$ $\quad \sin\theta + 1 = 0$

$\sin\theta = \frac{1}{2}$ $\quad \sin\theta = -1$

$\theta = 30°, 150°$ $\quad \theta = 180°$



3. Give the exact value of tan 15°.

$\tan(45 - 30) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \left[\frac{\frac{\sqrt{2}}{2} - \frac{1}{\sqrt{3}}}{1 + \frac{\sqrt{2}}{2} \cdot \frac{1}{\sqrt{3}}}\right]\frac{\frac{2\sqrt{3}}{1}}{\frac{2\sqrt{3}}{1}} = \left[\frac{\sqrt{6} - 2}{2\sqrt{3} + \sqrt{2}}\right]\left[\frac{2\sqrt{3} - \sqrt{2}}{2\sqrt{3} - \sqrt{2}}\right]$

$= \frac{2\sqrt{18} - \sqrt{12} - 4\sqrt{3} - 2\sqrt{2}}{4\sqrt{9} - 2} = \frac{6\sqrt{2} - 2\sqrt{3} - 4\sqrt{3} - 2\sqrt{2}}{10}$

$= \frac{2\sqrt{2} - 3\sqrt{3}}{5}$

4. Evaluate $2\cos^2\frac{\pi}{12} - 1$. (A)

$\cos 2A = \cos 2\left(\frac{\pi}{12}\right)$

$= \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

Prove

5. [redacted] $\frac{\sin 2x}{1 - \cos x} = 2\cot x(1 + \cos x)$

$= \left[\frac{\sin 2x}{1 - \cos x}\right]\left[\frac{1 + \cos x}{1 + \cos x}\right]$

$= \frac{\sin 2x(1 + \cos x)}{1 - \cos^2 x}$

$= \frac{\sin 2x(1 + \cos x)}{\sin^2 x}$

$= \frac{2\cos x \sin x(1 + \cos x)}{\sin^2 x}$

$= \frac{2\cos x(1 + \cos x)}{\sin x}$

$= 2\cot x(1 + \cos x)$

6. Prove: $(1 + \cot^2 x)(1 - \cos 2x) = 2$

$\csc^2 x\left(1 - (1 - 2\sin^2 x)\right) =$

$\frac{1}{\sin^2 x} \cdot \frac{2\sin^2 x}{1} =$

$2 = 2$

Q1

7. ∠A is acute and tan A = $\frac{2}{3}$. Find: $y = 2 \quad x = 3 \quad 2^2 + 3^2 = r^2 \quad r = \sqrt{13}$

A. sin A $= \frac{2}{\sqrt{13}}$

B. cos 2A $= 1 - 2\sin^2 A = 1 - 2\left(\frac{2}{\sqrt{13}}\right)^2 = 1 - \frac{8}{13} = \frac{5}{13}$

C. tan [redacted] $2A = \frac{2\tan A}{1 - \tan^2 A} = \frac{2\left(\frac{2}{3}\right)}{1 - \left(\frac{2}{3}\right)^2} = \frac{\frac{4}{3}}{1 - \frac{4}{9}} = \frac{\frac{4}{3}}{\frac{5}{9}} = \frac{4}{3} \cdot \frac{9}{5} = \frac{12}{5}$

8. If ∠α and ∠β are both acute angles with sec α = $\frac{3}{2}$ and cot β = 2, then find

$r = 3 \quad x = 2 \qquad x = 2 \quad y = 1$

$\cos\alpha = \frac{2}{3} \quad \cos\beta = \frac{2}{\sqrt{5}}$

$\sin\alpha = \frac{\sqrt{5}}{3} \quad \sin\beta = \frac{1}{\sqrt{5}}$

A. sin 2α = $2\sin A\cos A$

$= 2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right) = \frac{4\sqrt{5}}{9}$

B. cos (α – β) = $\cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \frac{2}{3} \cdot \frac{2}{\sqrt{5}} + \frac{\sqrt{5}}{3} \cdot \frac{1}{\sqrt{5}} = \left[\frac{4 + \sqrt{5}}{3\sqrt{5}}\right]\frac{\sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{5} + 5}{15}$

C. tan (α – β) = $\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} = \frac{\frac{\sqrt{5}}{2} - \frac{1}{2}}{1 + \frac{\sqrt{5}}{2} \cdot \frac{1}{2}} = \frac{\left[\frac{\sqrt{5} - 1}{2}\right]}{\left[1 + \frac{\sqrt{5}}{4}\right]} \cdot \frac{\frac{4}{1}}{\frac{4}{1}} = \frac{2\sqrt{5} - 2}{4 + \sqrt{5}}$

D. sin (α + β) = $\sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \frac{\sqrt{5}}{3} \cdot \frac{2}{\sqrt{5}} - \frac{2}{3} \cdot \frac{1}{\sqrt{3}} = \frac{2}{3} - \frac{2}{3\sqrt{5}} = \frac{2\sqrt{5} - 2}{3\sqrt{5}} = \frac{10 - 2\sqrt{5}}{15}$

9. Simplify

A. $\sin(x+\pi)\cos(x+\pi)$

$\frac{1}{2}\left[2\sin(x+\pi)\cos(x+\pi)\right] = \frac{1}{2}\sin 2(x+\pi) = \frac{1}{2}\sin(2x+2\pi)$

$= \frac{1}{2}\sin(2x)$

A A

$2\sin A\cos A$

A − B A + B

B. $\sin(45^\circ - x) - \sin(45^\circ + x)$

$\sin(45^\circ)\cos x - \cos(45)\sin x - \left[\sin(45)\cos x + \cos(45)\sin x\right]$

0

C. $2\sin\frac{\pi}{12}\cos\frac{\pi}{12}$

$2\sin 2\left(\frac{\pi}{12}\right)$ $2\cdot\frac{1}{2}$

$2\sin\frac{\pi}{6}$ 1

D. $\cos^2(25^\circ) - \sin^2(25^\circ)$

$\cos 2(25^\circ)$

$\cos 50^\circ$

10. Solve the following system of inequalities using your calculator.

$y < \sin 2x$ and $y > \tan x$ for $0 \le x < \pi/2$

$\left(0, \frac{\pi}{4}\right)$