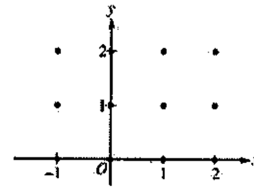
Sem 1 Ch. 8 Hw

AP® CALCULUS BC
2005 SCORING GUIDELINES

Question 4

Consider the differential equation $\frac{dy}{dx} = 2x - y$.



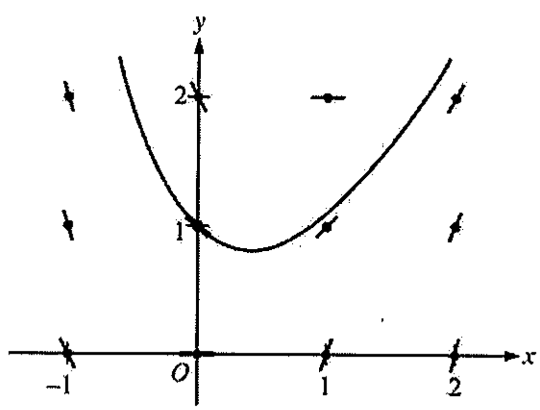
(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and sketch the solution curve that passes through the point $(0, 1)$. **(Note: Use the axes provided in the pink test booklet.)**

(b) The solution curve that passes through the point $(0, 1)$ has a local minimum at $x = \ln\left(\frac{3}{2}\right)$. What is the y-coordinate of this local minimum?

(c) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.4)$. Show the work that leads to your answer.

(d) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine whether the approximation found in part (c) is less than or greater than $f(-0.4)$. Explain your reasoning.

(a)



$3: \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \\ 1 : \text{curve through } (0, 1) \end{cases}$

(b) $\frac{dy}{dx} = 0$ when $2x = y$

The y-coordinate is $2\ln\left(\frac{3}{2}\right)$.

$2: \begin{cases} 1 : \text{sets } \frac{dy}{dx} = 0 \\ 1 : \text{answer} \end{cases}$

(c) $f(-0.2) \approx f(0) + f'(0)(-0.2)$
$= 1 + (-1)(-0.2) = 1.2$

$f(-0.4) \approx f(-0.2) + f'(-0.2)(-0.2)$
$\approx 1.2 + (-1.6)(-0.2) = 1.52$

$2: \begin{cases} 1 : \text{Euler's method with two steps} \\ 1 : \text{Euler approximation to } f(-0.4) \end{cases}$

(d) $\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx} = 2 - 2x + y$

$\frac{d^2y}{dx^2}$ is positive in quadrant II because $x < 0$ and $y > 0$.

$1.52 < f(-0.4)$ since all solution curves in quadrant II are concave up.

$2: \begin{cases} 1 : \frac{d^2y}{dx^2} \\ 1 : \text{answer with reason} \end{cases}$

AP® CALCULUS BC
2004 SCORING GUIDELINES (Form B)

Question 5

Let g be the function given by $g(x) = \frac{1}{\sqrt{x}}$.

(a) Find the average value of g on the closed interval $[1, 4]$.

(b) Let S be the solid generated when the region bounded by the graph of $y = g(x)$, the vertical lines $x = 1$ and $x = 4$, and the x-axis is revolved about the x-axis. Find the volume of S.

(c) For the solid S, given in part (b), find the average value of the areas of the cross sections perpendicular to the x-axis.

(d) The average value of a function f on the unbounded interval $[a, \infty)$ is defined to be $\lim_{b\to\infty}\left[\frac{\int_a^b f(x)\,dx}{b-a}\right]$. Show that the improper integral $\int_4^\infty g(x)\,dx$ is divergent, but the average value of g on the interval $[4, \infty)$ is finite.

(a) $\frac{1}{3}\int_1^4 \frac{1}{\sqrt{x}}\,dx = \frac{1}{3}\cdot 2\sqrt{x}\Big|_1^4 = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$

2 : { 1 : integral; 1 : antidifferentiation and evaluation }

(b) Volume $= \pi\int_1^4 \frac{1}{x}\,dx = \pi \ln x\Big|_1^4 = \pi \ln 4$

2 : { 1 : integral; 1 : antidifferentiation and evaluation }

(c) The cross section at x has area $\pi\left(\frac{1}{\sqrt{x}}\right)^2 = \frac{\pi}{x}$

Average value $= \frac{1}{3}\int_1^4 \frac{\pi}{x}\,dx = \frac{1}{3}\pi \ln 4$

1 : answer

(d) $\int_4^\infty g(x)\,dx = \lim_{b\to\infty}\int_4^b \frac{1}{\sqrt{x}}\,dx = \lim_{b\to\infty}\left(2\sqrt{b} - 4\right) = \infty$

This limit is not finite, so the integral is divergent.

$$\frac{\int_4^b g(x)\,dx}{b-4} = \frac{1}{b-4}\int_4^b \frac{1}{\sqrt{x}}\,dx = \frac{2\sqrt{b}-4}{b-4}$$

$$\lim_{b\to\infty}\frac{2\sqrt{b}-4}{b-4} = 0$$

4 : { 1 : $\int_4^b g(x)\,dx = 2\sqrt{b} - 4$; 1 : indicates integral diverges; 1 : $\frac{1}{b-4}\int_4^b g(x)\,dx = \frac{2\sqrt{b}-4}{b-4}$; 1 : finite limit as $b \to \infty$ }

AP® CALCULUS BC
2007 SCORING GUIDELINES (Form B)

Question 5

Consider the differential equation $\frac{dy}{dx} = 3x + 2y + 1$.

(a) Find $\frac{d^2y}{dx^2}$ in terms of x and y.

(b) Find the values of the constants m, b, and r for which $y = mx + b + e^{rx}$ is a solution to the differential equation.

(c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = -2$. Use Euler's method, starting at $x = 0$ with a step size of $\frac{1}{2}$, to approximate $f(1)$. Show the work that leads to your answer.

(d) Let $y = g(x)$ be another solution to the differential equation with the initial condition $g(0) = k$, where k is a constant. Euler's method, starting at $x = 0$ with a step size of 1, gives the approximation $g(1) \approx 0$. Find the value of k.

(a) $\frac{d^2y}{dx^2} = 3 + 2\frac{dy}{dx} = 3 + 2(3x + 2y + 1) = 6x + 4y + 5$	$2: \begin{cases} 1: 3 + 2\frac{dy}{dx} \\ 1: \text{answer} \end{cases}$
(b) If $y = mx + b + e^{rx}$ is a solution, then $m + re^{rx} = 3x + 2\left(mx + b + e^{rx}\right) + 1$. If $r \neq 0$: $m = 2b + 1$, $r = 2$, $0 = 3 + 2m$, so $m = -\frac{3}{2}$, $r = 2$, and $b = -\frac{5}{4}$. OR If $r = 0$: $m = 2b + 3$, $r = 0$, $0 = 3 + 2m$, so $m = -\frac{3}{2}$, $r = 0$, $b = -\frac{9}{4}$.	$3: \begin{cases} 1: \frac{dy}{dx} = m + re^{rx} \\ 1: \text{value for } r \\ 1: \text{values for } m \text{ and } b \end{cases}$
(c) $f\left(\frac{1}{2}\right) \approx f(0) + f'(0) \cdot \frac{1}{2} = -2 + (-3) \cdot \frac{1}{2} = -\frac{7}{2}$ $f'\left(\frac{1}{2}\right) \approx 3\left(\frac{1}{2}\right) + 2\left(-\frac{7}{2}\right) + 1 = -\frac{9}{2}$ $f(1) \approx f\left(\frac{1}{2}\right) + f'\left(\frac{1}{2}\right) \cdot \frac{1}{2} = -\frac{7}{2} + \left(-\frac{9}{2}\right) \cdot \frac{1}{2} = -\frac{23}{4}$	$2: \begin{cases} 1: \text{Euler's method with 2 steps} \\ 1: \text{Euler's approximation for } f(1) \end{cases}$
(d) $g'(0) = 3 \cdot 0 + 2 \cdot k + 1 = 2k + 1$ $g(1) \approx g(0) + g'(0) \cdot 1 = k + (2k + 1) = 3k + 1 = 0$ $k = -\frac{1}{3}$	$2: \begin{cases} 1: g(0) + g'(0) \cdot 1 \\ 1: \text{value of } k \end{cases}$