Ch. 10 Hw

AP® CALCULUS BC
2011 SCORING GUIDELINES (Form B)

Question 2

The polar curve r is given by $r(\theta) = 3\theta + \sin\theta$, where $0 \le \theta \le 2\pi$.

(a) Find the area in the second quadrant enclosed by the coordinate axes and the graph of r.

(b) For $\frac{\pi}{2} \le \theta \le \pi$, there is one point P on the polar curve r with x-coordinate -3. Find the angle θ that corresponds to point P. Find the y-coordinate of point P. Show the work that leads to your answers.

(c) A particle is traveling along the polar curve r so that its position at time t is $(x(t), y(t))$ and such that $\frac{d\theta}{dt} = 2$. Find $\frac{dy}{dt}$ at the instant that $\theta = \frac{2\pi}{3}$, and interpret the meaning of your answer in the context of the problem.

(a) Area $= \frac{1}{2}\int_{\pi/2}^{\pi} (r(\theta))^2 \, d\theta = 47.513$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \\ 1 : \text{answer} \end{cases}$

(b) $-3 = r(\theta)\cos\theta = (3\theta + \sin\theta)\cos\theta$
$\theta = 2.01692$
$y = r(\theta)\sin(\theta) = 6.272$

$3 : \begin{cases} 1 : \text{equation} \\ 1 : \text{value of } \theta \\ 1 : y\text{-coordinate} \end{cases}$

(c) $y = r(\theta)\sin\theta = (3\theta + \sin\theta)\sin\theta$

$$\left.\frac{dy}{dt}\right|_{\theta=2\pi/3} = \left[\frac{dy}{d\theta}\cdot\frac{d\theta}{dt}\right]_{\theta=2\pi/3} = -2.819$$

The y-coordinate of the particle is decreasing at a rate of 2.819.

$3 : \begin{cases} 1 : \text{uses chain rule} \\ 1 : \text{answer} \\ 1 : \text{interpretation} \end{cases}$

AP® CALCULUS BC
2010 SCORING GUIDELINES (Form B)

Question 2

The velocity vector of a particle moving in the plane has components given by

$$\frac{dx}{dt} = 14\cos\left(t^2\right)\sin\left(e^t\right) \text{ and } \frac{dy}{dt} = 1 + 2\sin\left(t^2\right), \text{ for } 0 \le t \le 1.5.$$

At time $t = 0$, the position of the particle is $(-2, 3)$.

(a) For $0 < t < 1.5$, find all values of t at which the line tangent to the path of the particle is vertical.

(b) Write an equation for the line tangent to the path of the particle at $t = 1$.

(c) Find the speed of the particle at $t = 1$.

(d) Find the acceleration vector of the particle at $t = 1$.

(a) The tangent line is vertical when $x'(t) = 0$ and $y'(t) \neq 0$.

On $0 < t < 1.5$, this happens at $t = 1.253$ and $t = 1.144$ or 1.145.

$$2 : \begin{cases} 1 : \text{sets } \frac{dx}{dy} = 0 \\ 1 : \text{answer} \end{cases}$$

(b) $\left.\frac{dy}{dx}\right|_{t=1} = \frac{y'(1)}{x'(1)} = 0.863447$

$x(1) = -2 + \int_0^1 x'(t)\,dt = 9.314695$

$y(1) = 3 + \int_0^1 y'(t)\,dt = 4.620537$

The line tangent to the path of the particle at $t = 1$ has equation $y = 4.621 + 0.863(x - 9.315)$.

$$4 : \begin{cases} 1 : \left.\frac{dy}{dx}\right|_{t=1} \\ 1 : x(1) \\ 1 : y(1) \\ 1 : \text{equation} \end{cases}$$

(c) Speed $= \sqrt{(x'(1))^2 + (y'(1))^2} = 4.105$

1 : answer

(d) Acceleration vector: $\langle x''(1), y''(1) \rangle = \langle -28.425, 2.161 \rangle$

$$2 : \begin{cases} 1 : x''(1) \\ 1 : y''(1) \end{cases}$$

AP® CALCULUS BC
2007 SCORING GUIDELINES (Form B)

Question 2

An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \arctan\left(\frac{t}{1+t}\right) \text{ and } \frac{dy}{dt} = \ln\left(t^2+1\right)$$

for $t \geq 0$. At time $t = 0$, the object is at position $(-3, -4)$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the speed of the object at time $t = 4$.

(b) Find the total distance traveled by the object over the time interval $0 \leq t \leq 4$.

(c) Find $x(4)$.

(d) For $t > 0$, there is a point on the curve where the line tangent to the curve has slope 2. At what time t is the object at this point? Find the acceleration vector at this point.

(a) Speed $= \sqrt{x'(4)^2 + y'(4)^2} = 2.912$	1 : speed at $t = 4$
(b) Distance $= \displaystyle\int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 6.423$	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(c) $x(4) = x(0) + \displaystyle\int_0^4 x'(t)\,dt$ $= -3 + 2.10794 = -0.892$	$3 : \begin{cases} 2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } x(0) = -3 \end{cases} \\ 1 : \text{answer} \end{cases}$
(d) The slope is 2, so $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2$, or $\ln\left(t^2+1\right) = 2\arctan\left(\frac{t}{1+t}\right)$. Since $t > 0$, $t = 1.35766$. At this time, the acceleration is $\langle x''(t), y''(t)\rangle\big\vert_{t=1.35766} = \langle 0.135, 0.955\rangle$.	$3 : \begin{cases} 1 : \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2 \\ 1 : t\text{-value} \\ 1 : \text{values for } x'' \text{ and } y'' \end{cases}$