CP Calculus Block 19

HW-Review p88,

84;90;96;108;114;

120;126;

CP Calculus Block 19

HW p98,x3's

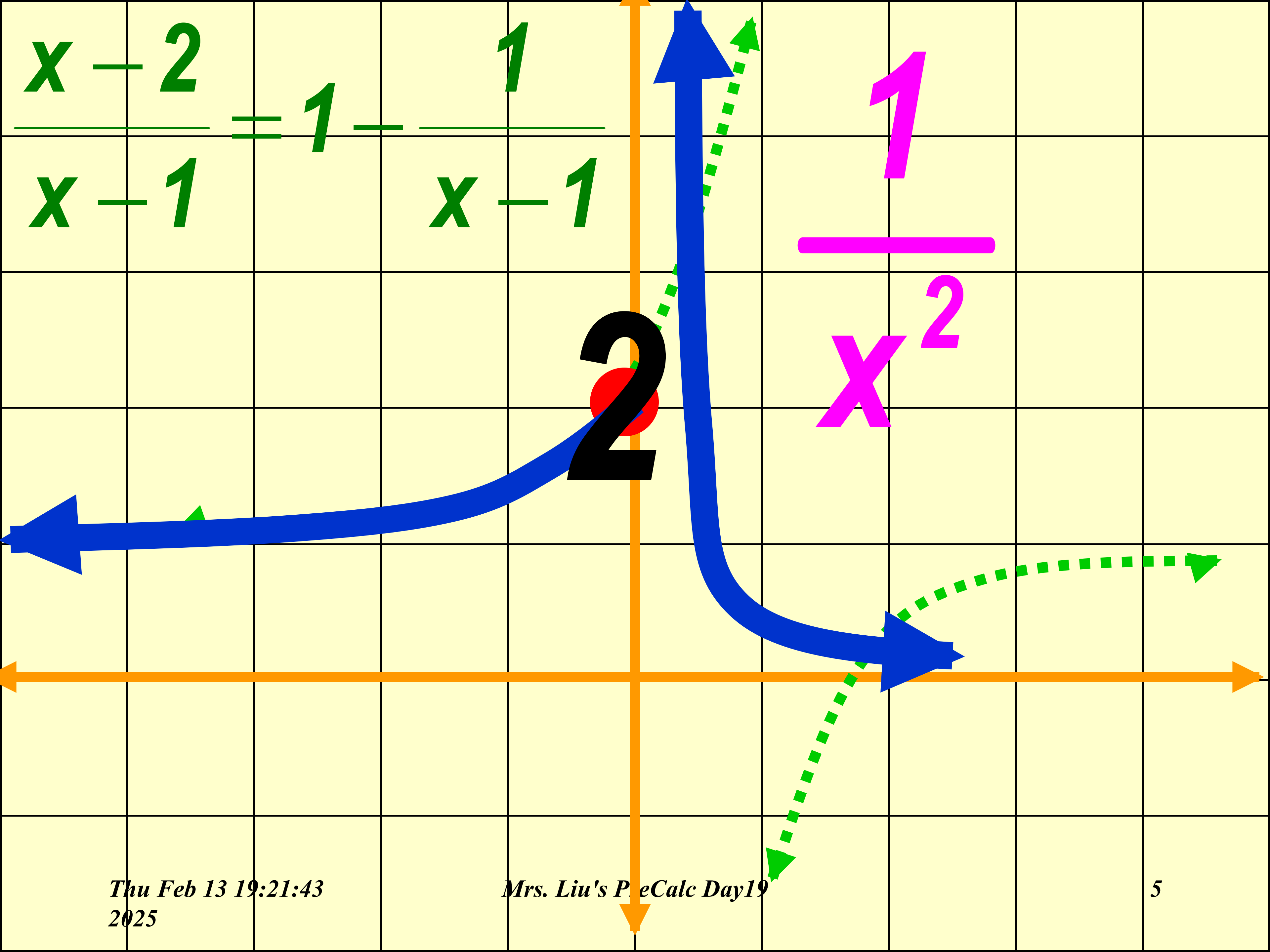
3-48

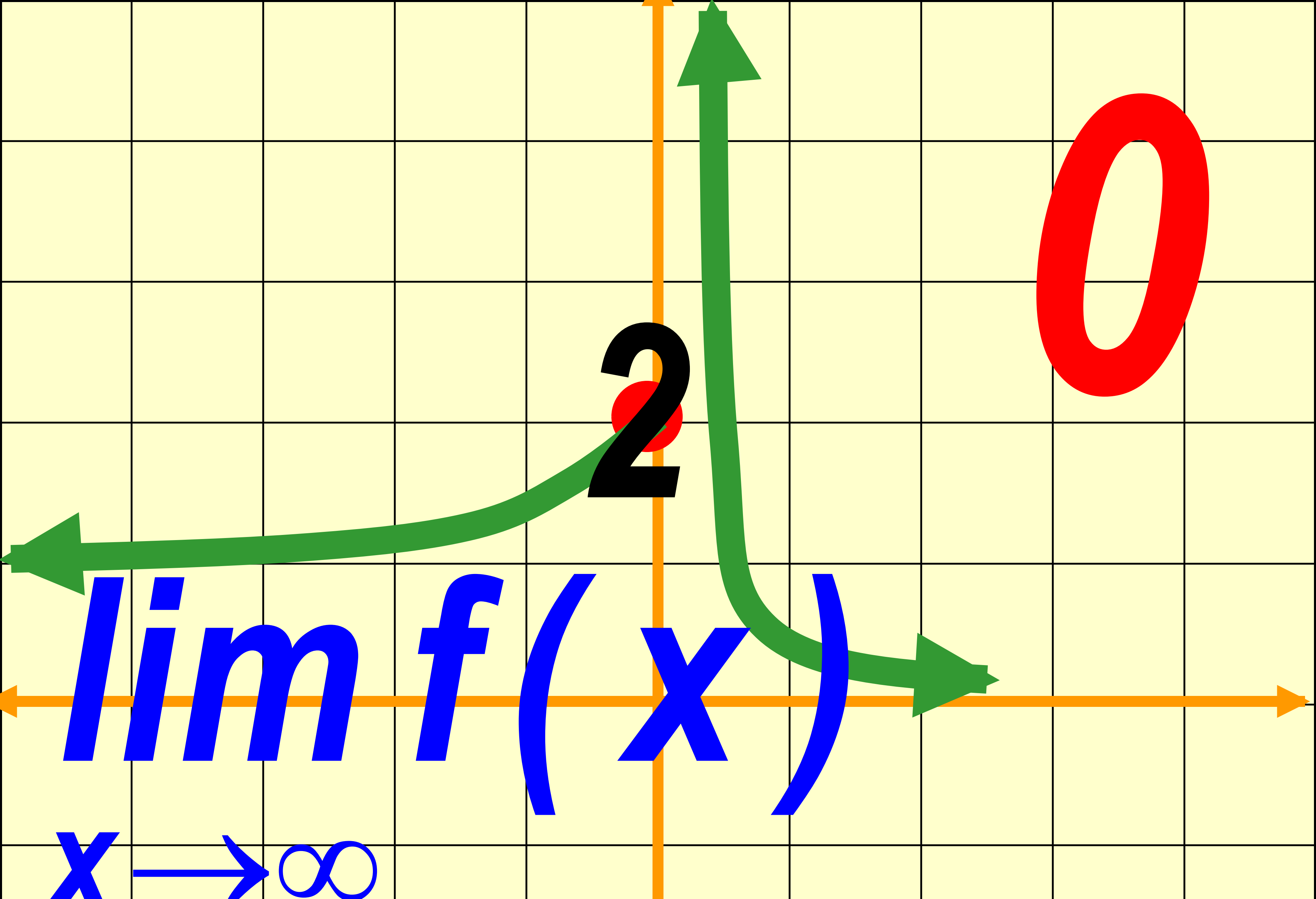
Ch 2.4 Continuity and One-sided Limit

Do Now:

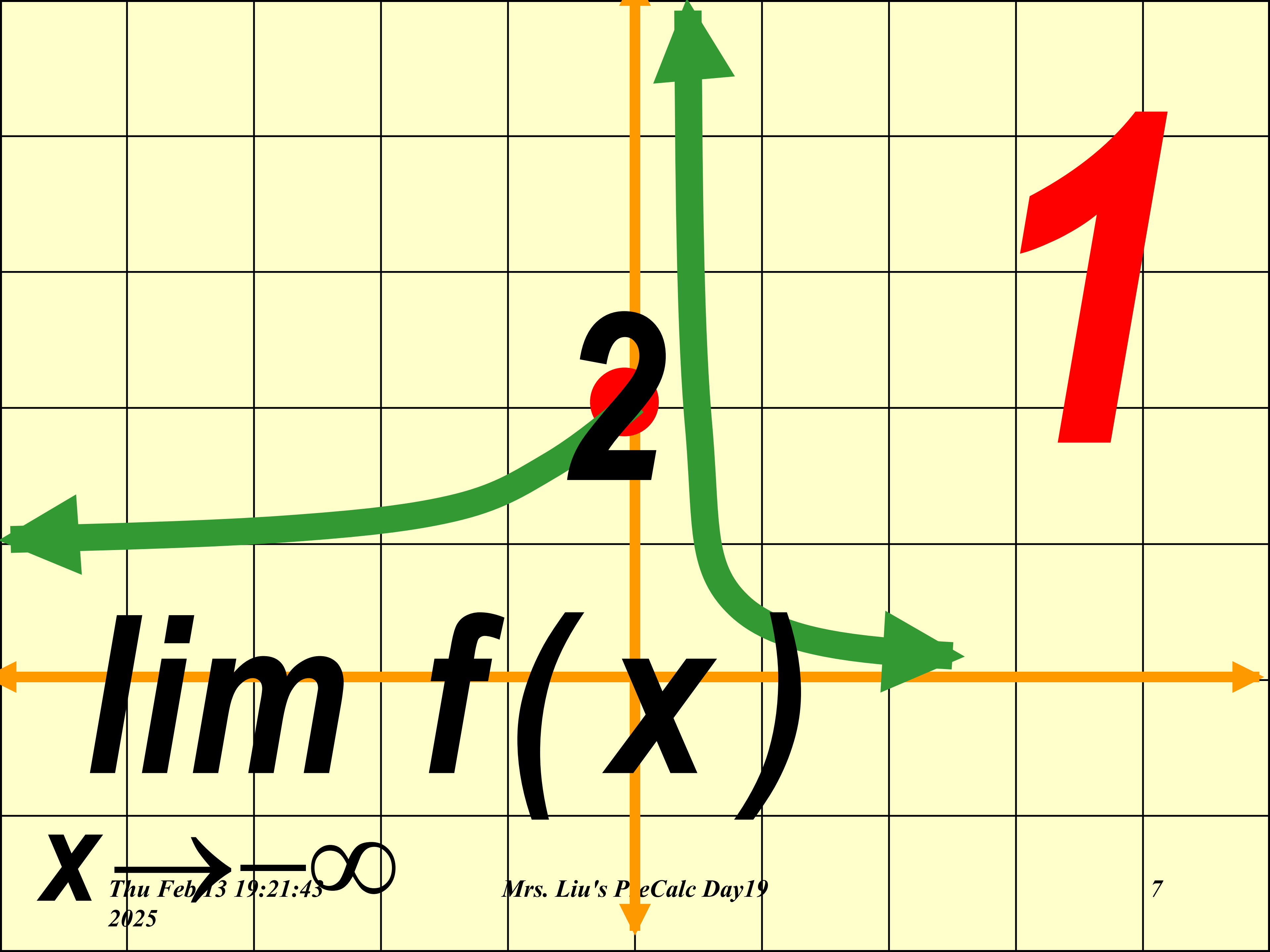
$$f(x)=\begin{cases}\dfrac{x-2}{x-1}, & x\le 0\\[2ex] \dfrac{1}{x^2}, & x>0\end{cases}$$

$$\frac{x-2}{x-1} = 1 - \frac{1}{x-1}$$



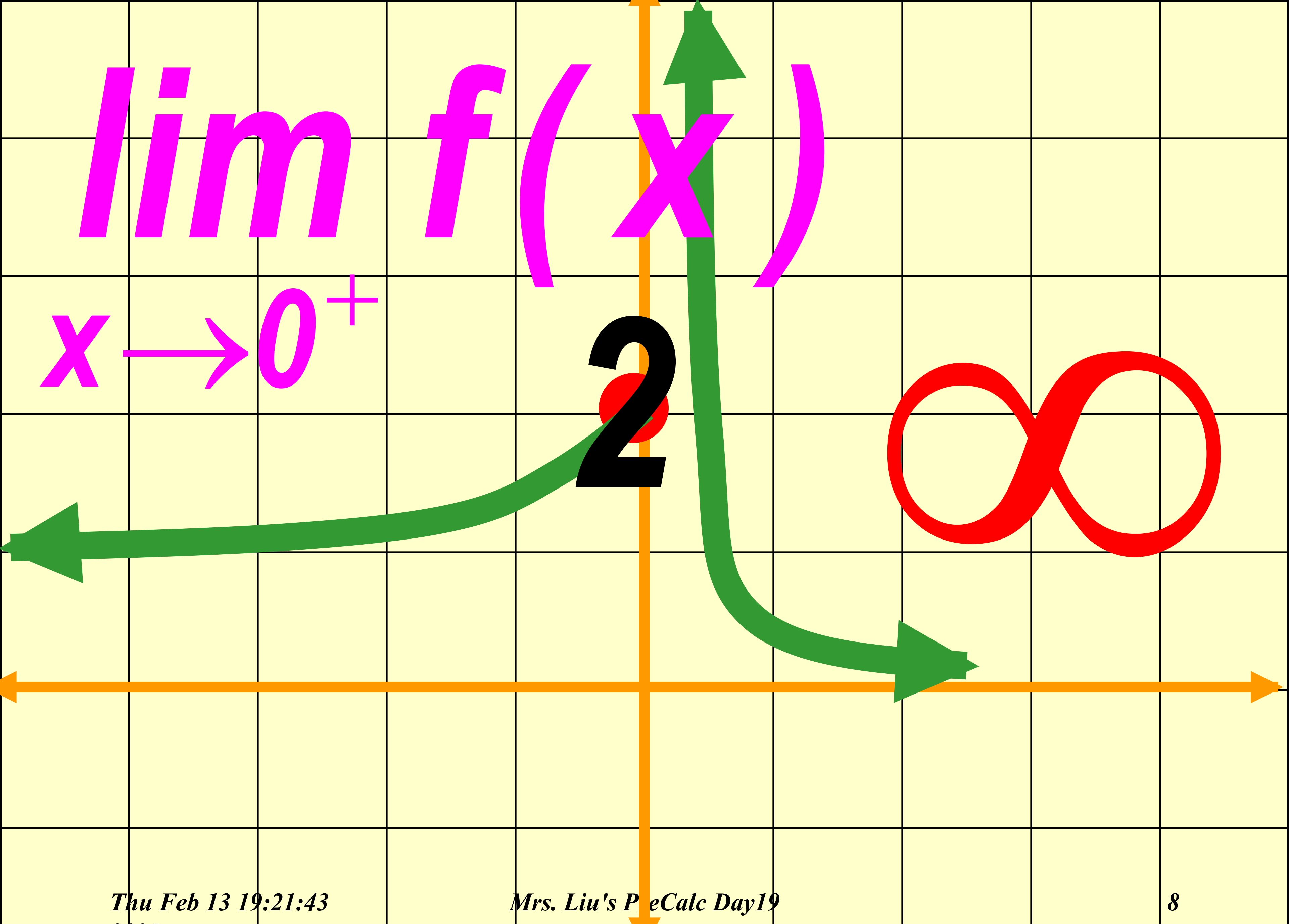


2
0
lim f(x)
x→∞

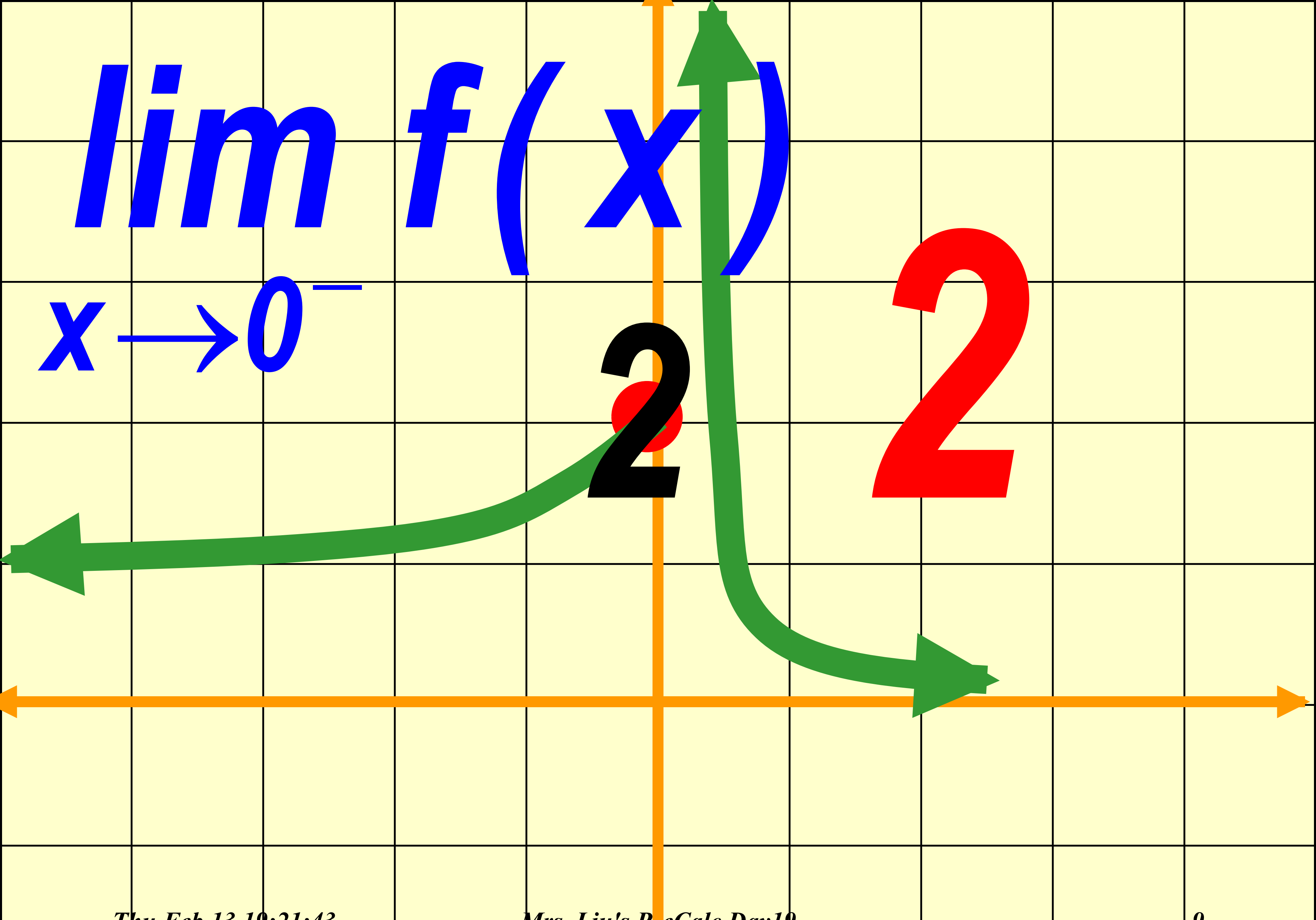


$$\lim_{x \to -\infty} f(x)$$

$$\lim_{x \to 0^+} f(x)$$



$$\lim_{x \to 0^-} f(x)$$



Objective 1:

Determine the continuity of the functions

Definition A:

Continuity at an interior point

$$\lim_{x \to c} f(x) = f(c)$$

Definition B:

Continuity on closed interval [a,b], left endpoint a or right endpoint b

$$\lim_{x \to a^{+}} f(x) = f(a)$$

$$\lim_{x \to b^{-}} f(x) = f(b)$$

Definition C: A function is continuous if it is continuous at each point of its domain

The Continuity Test

The function $y=f(x)$ is continuous at $x=c$ *if and only if* all of the following are true:

i) f(c) exitsts

ii) $\lim_{x \to c} f(x)$ exists

iii) $\lim_{x \to c} f(x) = f(c)$

Existence of Limit

$$\lim_{x \to c^{+}} f(x) = \lim_{x \to c^{-}} f(x)$$

$$= L$$

Properties of Continuity I

Scalar Multiple:

bf(x)

Properties of Continuity ii

Sum or difference

f + g, or f - g

Properties of Continuity iii

Product

$f(x)g(x)$

Properties of Continuity iv

Quotient

f(x)/g(x) $g(x) \neq 0$

Continuity of Composite Functions

$f(g(x))$ or $g(f(x))$

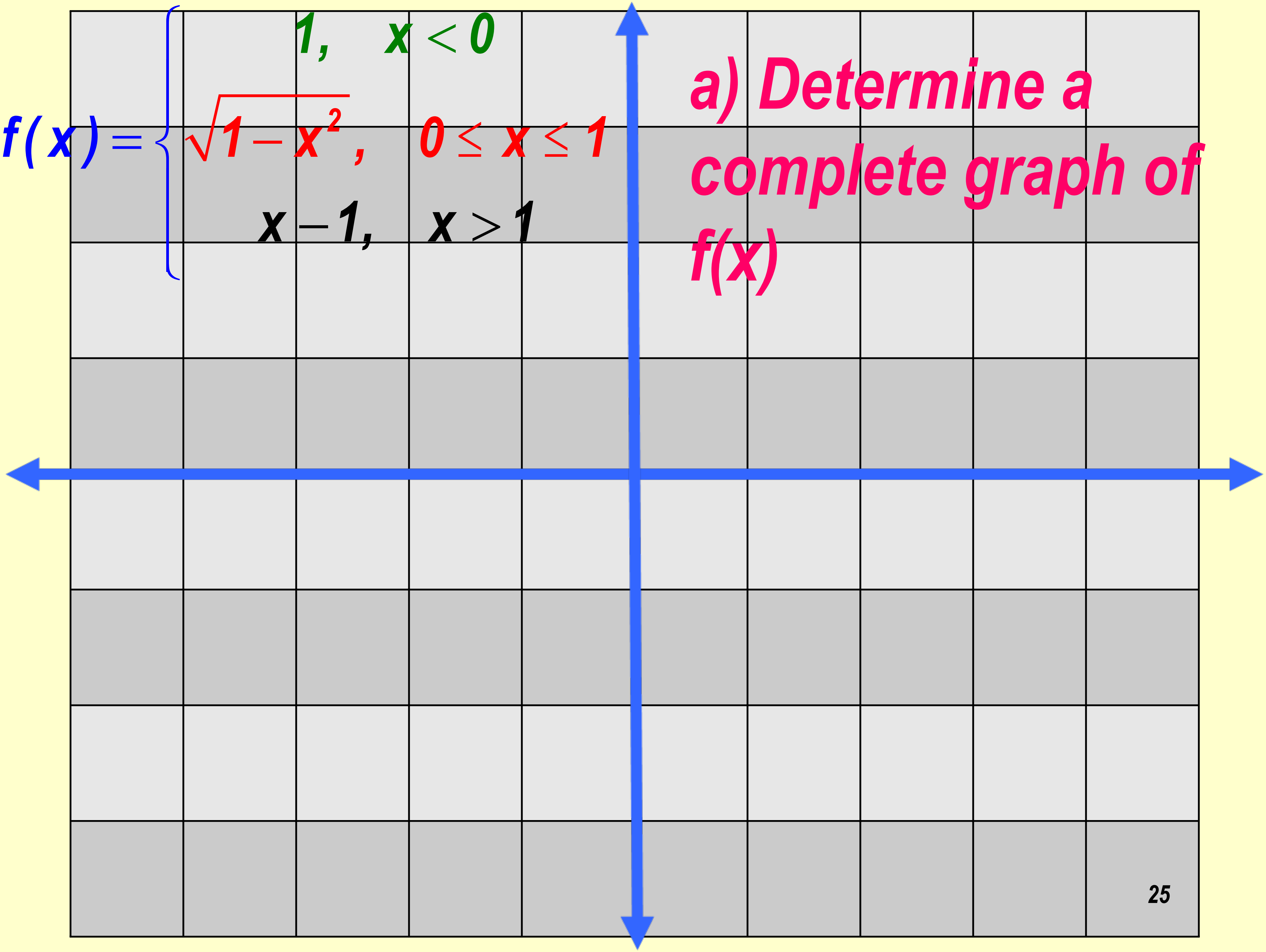
Theorem: ***If f is continuous at c and g is continuous at f©, then the composite g∘f is continuous at c.***

Example 0 Given f(x)

$$f(x) = \begin{cases} 1, & x < 0 \\ \sqrt{1 - x^2}, & 0 \le x \le 1 \\ x - 1, & x > 1 \end{cases}$$

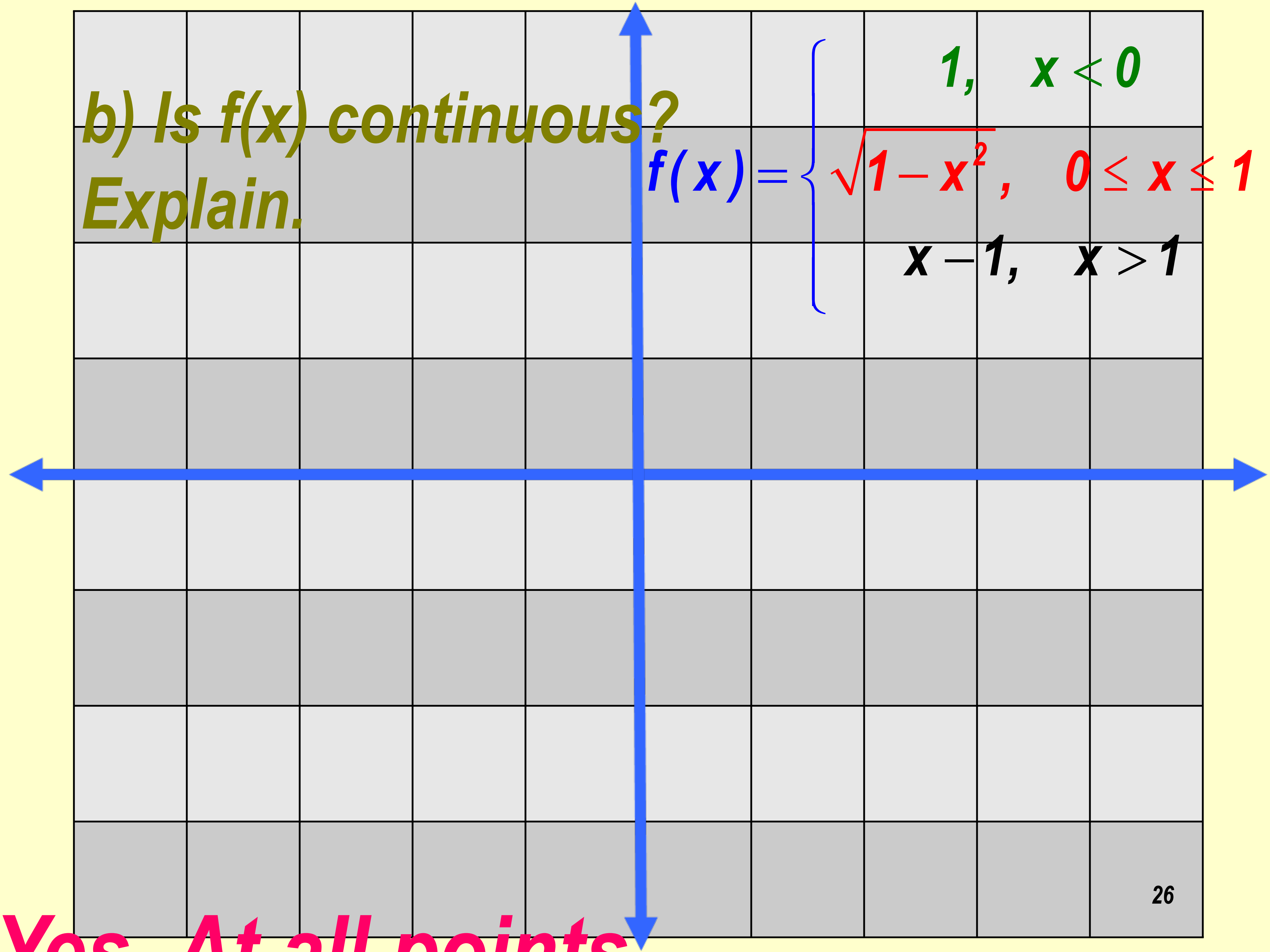
$$f(x) = \begin{cases} 1, & x < 0 \\ \sqrt{1 - x^2}, & 0 \leq x \leq 1 \\ x - 1, & x > 1 \end{cases}$$

a) Determine a complete graph of f(x)



b) Is f(x) continuous? Explain.

$$f(x)=\begin{cases} 1, & x<0 \\ \sqrt{1-x^2}, & 0\le x\le 1 \\ x-1, & x>1 \end{cases}$$



Yes. At all points

Example 1 ***Find the points at which the function is not continuous***

$$f(x) = \frac{1}{(x+2)^2}$$

$$x = -2$$

Example 2 Find the points at which the function is not continuous

$$x = -2, 5$$

$$f(x) = \frac{x+3}{x^2 - 3x - 10}$$

Example 3 Find the points at which the function is not continuous

$$f(x) = \frac{1}{x^2+1}$$

No discontinuities

Example 4 **Find the points at which the function is not continuous**

$$f(x) = |2x + 3|$$

No discontinuities

Example 5 ***Find the points at which the function is not continuous***

$$f(x) = \frac{|x|}{x}$$

$x = 0$

Example 6 ***Find the points at which the function is not continuous***

$x<1/3$

$$f(x)=\sqrt[4]{3x-1}$$

Example 7 ***Find the points at which the function is not continuous***

No discontinuities

$$f(x) = \sqrt[5]{2 - x}$$

Example 8 Define $g(x)$ so that $g(3)=6$ $g(x)=(x^2-9)/(x-3)$ is continuous at $x=3$

Example 9 Define $f(1)$ so that $f(x)=(x^2-1)/(x^2-1)$ is continuous at $x=1$

$f(1)=\frac{3}{2}$

Example 10 Given g(x)

$$g(x) = \begin{cases} x^3, & x < \frac{1}{2} \\ bx^2, & x \geq \frac{1}{2} \end{cases}$$

What value should be assigned to b to make g(x) continuous at x=1/2 ?

$$b = \frac{1}{2}$$

Example 11: Find the limit

$$\lim_{x\to 0} \tan x$$

0

Example 12: Find the limit

1

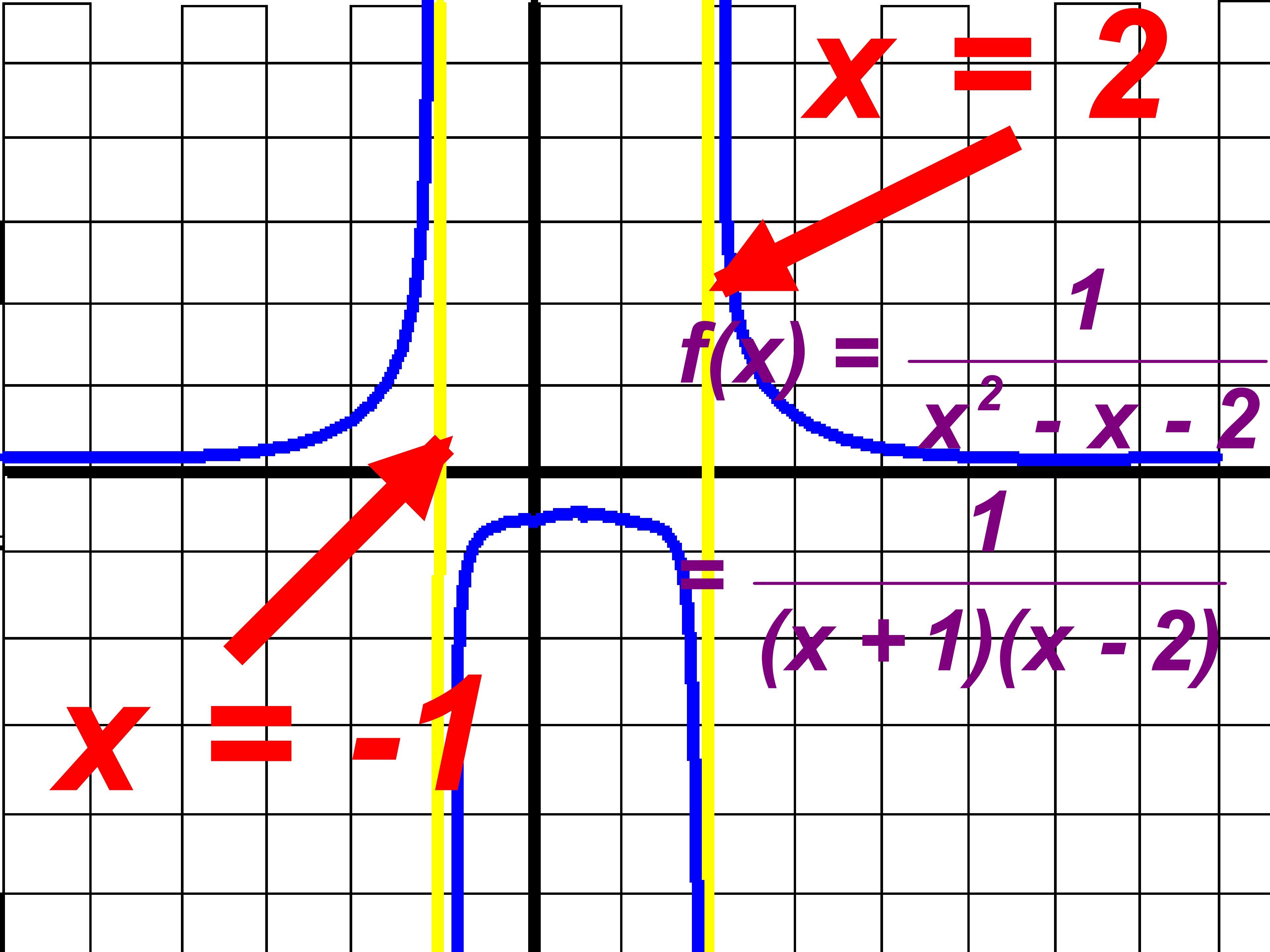
$$\lim_{x \to 0} \sin\left(\frac{\pi}{2}\cos(\tan x)\right)$$

Example 13: ***Find the vertical asymptote(s) of***

$$f(x) = \frac{3}{(x + 2)}$$

Example 14: ***Find the vertical asymptote(s) of***

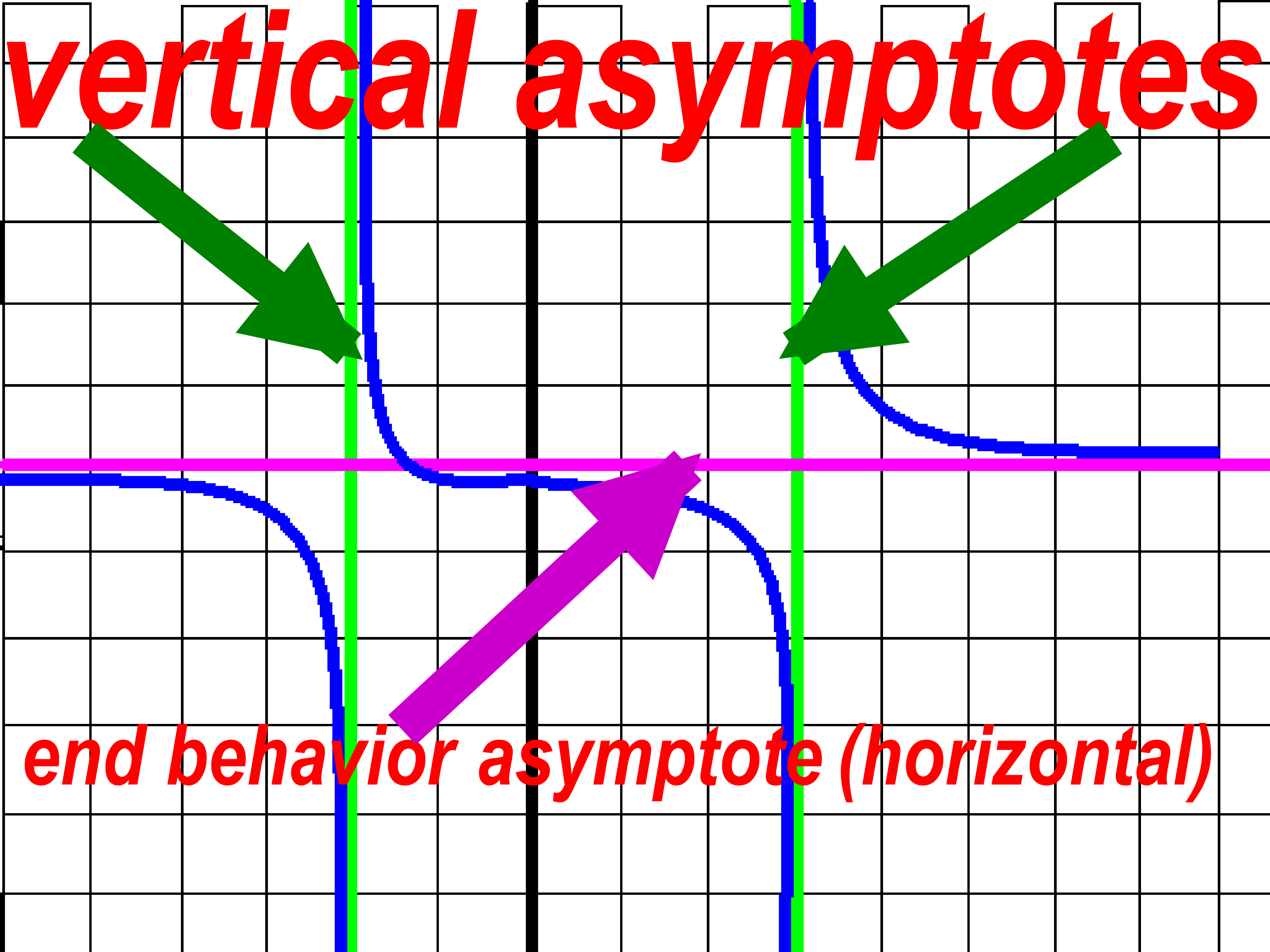
$$f(x) = \frac{1}{x^2 - x - 2}$$



x = 2
$f(x) = \frac{1}{x^2 - x - 2}$
$= \frac{1}{(x + 1)(x - 2)}$
x = -1

***Example 15:* Find the end behavior asymptote(s) of**

$$f(x) = \frac{x + 1}{x^2 - x - 6}$$


vertical asymptotes
end behavior asymptote (horizontal)

Example 16

a)2

Find $\lim_{x \to \pm\infty} f(x)$ if

b)2

$$f(x) = \frac{2x^3 + 7}{x^3 - x^2 + x + 7}$$

Example 17

Find $\lim_{x\to\pm\infty} f(x)$ if

$$f(x) = \frac{3x+7}{x^2-2}$$

a) 0

b) 0

Example 18

Find $\lim_{x\to\pm\infty} f(x)$ if

$$f(x)=\frac{x^4}{x^3+1}$$

a) ∞

b) $-\infty$

Example 19

Find $\lim_{x \to \pm\infty} f(x)$ if

a) 0

b) 0

$$f(x) = \frac{10x^5 + x^4 + 31}{x^6}$$

Example 20

Find $\lim_{x \to \pm\infty} f(x)$ if

$$a) -1$$

$$b) -1$$

$$f(x) = \frac{-x^4}{x^4 - 7x^3 + 7x^2 + 9}$$

Lesson quiz 1: Given

$$f(x)=\begin{cases} x^2-1, & -1\le x<0 \\ 2x, & 0\le x<1 \\ 1, & x=1 \\ -2x+4, & 1<x<2 \\ 0, & 2<x\le 3 \end{cases}$$

$$f(x)=\begin{cases} x^2-1, & -1\le x<0 \\ 2x, & 0\le x<1 \\ 1, & x=1 \\ -2x+4, & 1<x<2 \\ 0, & 2<x\le 3 \end{cases}$$

a) Does f(1) exist?

$$\text{yes}, f(1)=1$$

$$f(x)=\begin{cases} x^2-1, & -1\le x<0 \\ 2x, & 0\le x<1 \\ 1, & x=1 \\ -2x+4, & 1<x<2 \\ 0, & 2<x\le 3 \end{cases}$$

b) $\lim_{x\to 1} f(x)$ exist?

yes, $\lim_{x\to 1} f(x)=2$

$$f(x)=\begin{cases} x^2-1, & -1\le x<0 \\ 2x, & 0\le x<1 \\ 1, & x=1 \\ -2x+4, & 1<x<2 \\ 0, & 2<x\le 3 \end{cases}$$

c) Does $\lim_{x\to 1} f(x) = f(1)$?

No

$$f(x)=\begin{cases} x^2-1, & -1\le x<0 \\ 2x, & 0\le x<1 \\ 1, & x=1 \\ -2x+4, & 1<x<2 \\ 0, & 2<x\le 3 \end{cases}$$

d) Is f(x) continuous at x = 1?

No

$$f(x)=\begin{cases} x^2-1, & -1\le x<0 \\ 2x, & 0\le x<1 \\ 1, & x=1 \\ -2x+4, & 1<x<2 \\ 0, & 2<x\le 3 \end{cases}$$

e) At what values of x is $f(x)$ continuous?

all points in [-1, 3] except $x = 0, 1, 2$

$$f(x)=\begin{cases} x^2-1, & -1\le x<0 \\ 2x, & 0\le x<1 \\ 1, & x=1 \\ -2x+4, & 1<x<2 \\ 0, & 2<x\le 3 \end{cases}$$

f) How should h be defined to make h a continuous extension of f to the point x = 1?

$$h(x)=f(x)$$

$$x\neq 1, h(1)=2$$