AP Calculus BC

Course Overview

This course is aligned to the topics covered in the Calculus BC course description. This includes the topics of function, graphs, and their limits; derivatives and their applications; integrals and their applications; and polynomial approximation and series.

Textbook

Finney, Ross L., Franklin Demana, Bert Waits, and Daniel Kennedy. *Calculus: Graphical Numerical, Algebraic. 3rd Ed.* Pearson/Prentice Hall 2007

AP Calculus BC Course Planner

Dates are approximate and may change form year to year. Topics generally correspond to text chapters.

Prerequisites for Calculus (One Week)

- Use of a graphing calculator including intersection points and roots of functions
- Lines
- Trigonometry including trigonometric identities and the unit circle
- Rational, exponential, logarithmic, and trigonometric functions
- Graphing and function analysis

Limits and Their Properties (Two Week)

- Continuity
- One sided limits
- Formal definition of a limit
- Finding limits algebraically and graphically
- Properties of limits
- Techniques for evaluating limits
- Infinite limits
- Intermediate value theorem

Differentiation (Four Weeks)

- The tangent line problem
- Local linearity
- Definition of the derivative/difference quotient/notation
- Basic differentiation rules
- Derivative as a rate of change or slope
- Product and quotient rules
- Chain Rule
- Derivatives of trigonometric functions
- Higher order derivatives and notation
- Implicit differentiation

- Derivatives of inverse trigonometric functions
- Derivatives of exponential and logarithmic functions

Applications of Differentiation (Three Weeks)

- Absolute and relative extrema
- Rolle's Theorem
- Mean Value Theorem
- The first derivative test/ Increasing and decreasing functions
- The second derivative test/ Concavity and inflection
- Relationship between functions f',f'' and their graphs
- Limits at ∞ and horizontal asymptotes
- Graphing without a calculator
- Related rates
- Position, speed, and acceleration with derivatives
- Optimization Problems
- Business and economics applications
- Linearization and Newton's Method
- Differentials

The Definite Integral (Three Weeks)

- Estimating with finite sums
- Antiderivatives and basic integration rules
- Sigma notation; area; upper, lower, and midpoint sums
- Riemann Sums
- Trapezoid Rule
- Simpson's method
- First and second Fundamental Theorems of Calculus
- Average value of a function
- Graphical analysis of antiderivatives

Differential Equations and Mathematical Modeling (Three Weeks)

- Euler's Method
- Slope Fields
- Integration by parts
- Product rule for integration
- Integrals involving powers of trigonometric functions
- Trigonometric Substitution
- Inverse trigonometric and logarithmic functions
- Partial fractions
- Integration by tables and other techniques
- Evaluation of improper integrals
- Logistic differential equations
- Growth and decay
- Periodic and continuous compound interest

Applications of the Definite Integral (Three Weeks)

- Integral as accumulation of net change
- Areas in the plane
- Volume using disk, washer, and shell methods
- Finding volume using known cross sections
- Arc length and surface of revolution
- Work
- Fluid pressure and fluid force

Sequences, Improper Integrals, and L'Hôpital's Rule (Three Weeks)

- Arithmetic and geometric sequences
- Graphing a sequence
- Limits of sequences
- Indeterminate forms $0/0$, ∞/∞, $\infty*0$, $\infty-\infty$, 1^{∞},0^0, ∞^0
- L'Hôpital's Rule
- Relative rates of growth
- Improper integrals
- Tests for convergence and divergence

Infinite Series (Four Weeks)

- Series
- Convergence
- The integral test and p-series
- Comparisons test
- Alternating series test
- Ratio and root tests
- Taylor polynomials and approximations
- Power series
- Taylor and MacLaurin series
- Radius of convergence
- Endpoint convergence

Plane Curves, Parametric Equations, and Polar Coordinates (Two Weeks)

- Plane curves and parametric equations
- First and second derivatives of parametric equations
- Polar coordinates and polar graphs
- Derivatives of polar functions
- Area and arc length and polar coordinates
- Vector valued functions
- Derivatives of vector functions
- Velocity, acceleration, and speed
- Differentiation and integration of vector valued functions
- Velocity and acceleration of vector valued functions

- Hyperbolic Functions

AP Test Review (3-4 Weeks)

Teaching Strategies

Instruction is mainly lecture based in a direct instruction format. Students are also given the opportunity to explore problems in small groups or to complete discovery activities. The discovery activities lead into the day's topic of discussion or into further areas of exploration. Precise vocabulary is stressed and practiced daily through lecture and during discussion. Method and the ability to explain and rationalize methods of solution is emphasized throughout the class. Concepts and problems are presented and analyzed from multiple representations: graphical, numerical, and algebraic, and verbal.

Assessment

Assessment is both formative and summative. Students are given the opportunity to check independent work and solutions among small peer groups. This allows students to reflect upon their own learning and develop new strategies for learning that are less teacher directed. Following small grouping, students may be asked to verbally (orally or in writing) explain their methods or why a particular solution was incorrect.

Tests and quizzes are used to evaluate progress and to direct future instruction based on the strengths and needs of the students. Quizzes are unscheduled and unannounced; they are based upon recent topics and are used to assess effectiveness of instruction through student progress. Quizzes are used to direct instruction and assess the need for review and reteaching of taught concepts .

Formal unit tests follow each unit of study (i.e. applications of derivatives). Exams portions that do not allow calculator use foster the need for pen and paper skill. This also allows students to see their progress and level of knowledge independent of any technological aids. Embedded in assessments and review activities are the verbal explanation of methods and rationale by students.

Assessment follows the guidelines of the Calculus BC in terms of calculator use. During exams that allow calculator use, care is taken to allow students to use the four capabilities allowed to be referenced from the AP Calculus course description: "plot the graph of a function within an arbitrary viewing window, find the zeros of functions (solve equations numerically), numerically calculate the derivative of a function, and numerically calculate the value of a definite integral."

Graphing Calculators

Students are required to have a graphing calculator, preferably the TI-89 or similarly equipped model. They have learned requisite skills in the previous Calculus AB class: graphing in an arbitrary window, find zeros of a function, and calculate derivatives and definite integrals numerically. Care is taken to introduce students to polar and parametric modes of computation and graphing that they may be unfamiliar with.

Students are routinely encouraged to use the calculator as an exploratory tool. Often graphical methods are used to help confirm analytical solutions, but they must be able to mathematically explain justification too. Care is taken to note that graphical solutions are typically not exact and often not precise enough.

Students may find techniques and concepts through experimentation that will lead them to an analytic solution that may not have occurred to them otherwise. Students also use them to visualize problems including the graphs and relationships among functions and their derivatives or their integrals. They also find numerical derivatives and integrals. Students use the calculator to find the reasonableness of a result, such as finding a root graphically to compare to a numerical solution. They are encouraged to express verbally why or in what manner a graphical solution is related to an analytical one.

Graphers are used to quickly calculate and display slope fields to give a second interpretation of a family of antiderivatives. Students also explore the relationships amongst functions and their higher order derivatives to gain insight into the relationships between them; this is especially useful when covering position, velocity, and acceleration functions.

Analytical Methods
Throughout the book and the course, students are given opportunities to solve problems analytically. They learn strategies and organization necessary to solve complex application problems, whether the problems are presented from an analytical, verbal, or graphical viewpoint.

Graphical Methods
Graphical methods, especially involving the use of a graphing calculator are used throughout the course. Graphical methods are used to connect concepts of functions and their derivatives as well as definite integrals and their areas. Students also develop conceptual understanding of concepts and relationships through graphical explorations. Students link graphical methods to analytical and numerical methods. A graphical interpretation is presented to reinforce concepts, such as limiting a secant line to show a tangent and derivative. Graphical methods are also used to link the unfamiliar parametric and polar systems to the more familiar rectangular.

Numerical Methods
Numerical and tabular methods are used for approximations and to introduce topics. These are used so that students begin to create their own connection before a concept is formally presented. Examples are topics such as the introduction to limits or rectangle sums to approximate area, or Newton's method. Euler's method is another example of a numerical method used. With numerical methods, care is taken analyze the error of numerically derived solutions compared to an analytic method.

Verbal Methods
Verbal skills are formatively assessed and taught throughout the course. This occurs both orally and through written methods. Students are encouraged to explain concepts and processes verbally. Students must be able to enumerate the multiple representations of a problem or concepts as well as the advantages and disadvantages of each method. Students learn to justify their work on both independent work and assessments.

AP Review

Released test questions are used to create assessments. Students given timed response questions periodically throughout the year. Students must write the explanation of how they solved the problem as well as verbalize the method and solution in small groups and in front of the entire class. Students are graded based upon the rubric for the AP test. Students also grade peer work in order to become familiar with grading standards and expectations.

Free response packets are also given as a review. Students work independently out of class, and then they collaborate in class, discussing their approaches. This gives immediate feedback and allows students to properly complete or append their responses even if the original solution was incorrect.

In the weeks leading up to the AP test, released test questions are used directly in order to familiarize the students with the test format and structure of questions. Students practice by taking a full length practice test outside of class time in order to prepare their expectations for the actual AP exam. Through these strategies, students become familiarized with the test format, time constraints, and scoring rubrics.