

Read the example problems listed for each assignment – these will help you on the specific homework problems assigned. You are responsible for making a meaningful attempt on each homework question and checking your work either online or in the back of the textbook. If you get stuck, the underlined problems have full solutions worked out online, and the answers to the odd-numbered problems are in the back of the textbook. If you still have questions after checking your answers, then come in at lunch by appointment, after school, or during intervention as class time will not be spent going over the homework.

**Assignment 1: Review Topics Handout**

**Assignment 2: Review Topics Handout**

**Assignment 3: Conceptual Understanding of Limits**

Read p61 Example 1, and p63 Example 2. Solve Pages 64–67: #1, 6, 7, 22, 31

**Assignment 4: Limits-Graphically and Numerically**

Read p70-71 Examples 2-3. Solve Page 74: #4, 1, 5, 6

**Assignment 5: Limits That Do Not Exist**

Read p72-73 Examples 7-8. Handout, and also Pages 75–76: #18, 19, 22, 23, 43, 57

**Assignment 6: Evaluating Limits Analytically**

Read p78 Examples 2-3. Solve Page 80: #6, 3, 5, 13, 18, 23, 28, 27, 29, 30

**Assignment 7: Continuity**

Read p82-83 Example 2, and p86 Example 3. Solve Pages 88–89: (ignore one-sided continuity part of directions) #5, 24, 23, 49, 51 (sketch graph only), 71, 70, 80, 79

**Assignment 8: Finding Limits Algebraically**

Read pp92-93 Examples 1,3,5,6.

Solve Pages 94–95: (ignore instructions, just find the limits algebraically.) #2, 3, 16, 22, 21, 25, 39, 51, 54

**Assignment 10: Infinite Limits**

Read pp102-103 Examples 2-4. Solve Pages 105: #1, 8, 9, 12, 13, 14, 16, 18, 17, 20, 23, 25

**Assignment 11: IVT (Intermediate Value Theorem)**

Read p107 Examples 1-2. Solve Page 109: #6, 2, 21-24

**Assignment 12: Review Handout**

**AP As We Go**

**Test**

**Homework Heading**

Assignment Number

Name

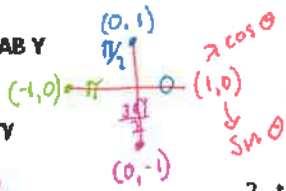
Period

Page Number Problem Numbers

2019-2020

Trigonometry

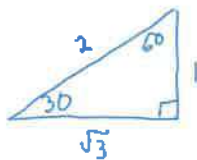
Unit 1 Lesson 1: Review



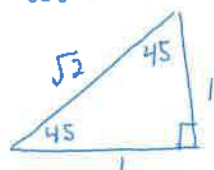
- 1.  $\cos 0 = 1$
- 2.  $\tan \frac{\pi}{2} = \frac{0}{0}$  und
- 3.  $\sin \pi = 0$

- 4.  $\cos \frac{3\pi}{2} = 0$
- 5.  $\tan \pi = \frac{0}{-1} = 0$
- 6.  $\sin \frac{\pi}{2} = 1$

7.  $\sin \frac{\pi}{3} = \frac{O}{H} = \frac{\sqrt{3}}{2}$   
*Sin 60*



8.  $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$  or  $\frac{\sqrt{2}}{2}$   
*cos 45*



9.  $\tan \frac{\pi}{6} = \frac{O}{A} = \frac{1}{\sqrt{3}}$   
*tan 30*

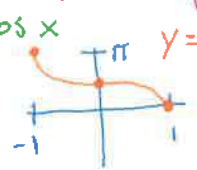
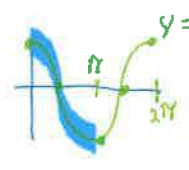
Now, you should be able to give the sin, cos, tan, csc, sec, cot, of any angle.

8.  $\sec \frac{5\pi}{4} = -\sqrt{2}$   
 $\cos \frac{5\pi}{4} = -\frac{1}{\sqrt{2}}$   
 $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

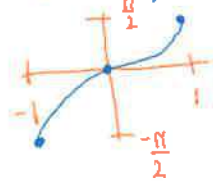
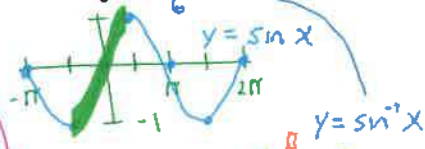
9.  $\cot \frac{4\pi}{3} = \frac{1}{\sqrt{3}}$   
 $\cot \frac{\pi}{3} = \frac{1}{\sqrt{3}}$   
*cot 60*

10.  $\csc \frac{\pi}{4} = \sqrt{2}$   
 $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

11.  $\arccos -\frac{1}{2} = \frac{2\pi}{3}$  | sol  
 $\cos^{-1}(-\frac{1}{2}) =$   
 what angle has a cos ratio of  $-\frac{1}{2}$ ?



12.  $\arcsin -\frac{1}{2} = -\frac{\pi}{6}$



13.  $\tan^{-1} 0 = 0^\circ$  or  $0$  rad  
 what angle has a tan ratio of  $\frac{0}{1}$ ?  $\frac{y}{x}$

Identify what type of equation each is and then solve by hand and check using your calculator.

1.  $x^2 + 7x = 9$  quadratic

$$x^2 + 7x - 9 = 0$$

cant factor

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-9)}}{2 \cdot 1}$$

$$x = \frac{-7 \pm \sqrt{85}}{2}$$

2.  $\left(\frac{3}{x} + \frac{2}{5} = 1\right) \frac{5x}{1}$  Rational

$$15 + 2x = 5x$$

$$15 = 3x$$

$$x = 5$$

3.  $\ln(-8x) = \ln(x+1) + \ln(3x+5)$

log

 $\log A + \log B$   
 $\log AB$ 

$$\ln(-8x) = \ln(x+1)(3x+5)$$

$$-8x = (x+1)(3x+5)$$

$$-8x = 3x^2 + 8x + 5$$

$$0 = 3x^2 + 16x + 5$$

$$0 = (3x+1)(x+5)$$

$$3x+1=0 \quad x+5=0$$

$$x = -\frac{1}{3} \quad x = -5$$

5.  $2 \sin(x) + 1 = 2$  for  $0 \leq x \leq 2\pi$

$$2 \sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$x = \sin^{-1} \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

4.  $4^x = 8^{3x+5}$  Exponential

$$\log_2 4^x = \log_2 8^{3x+5}$$

$$x \log_2 4 = (3x+5) \log_2 8$$

$$x \cdot 2 = (3x+5) \cdot 3$$

$$2x = 9x + 15$$

$$-7x = 15$$

$$x = -\frac{15}{7}$$

6.  $\cos(2x) = -\frac{1}{2}$  for  $0 \leq x \leq 2\pi$

let  $u = 2x$

$$\cos u = -\frac{1}{2}$$

$$u = \cos^{-1} -\frac{1}{2}$$

$$u = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$$

$$2x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$

Assignment 1: Handout

## Unit #1 Assignment 1

Solve each equation below by hand and then check by solving them with your calculator.

1.  $2x^2 + 5x = 1$

2.  $\frac{8}{x} + \frac{5}{8} = 2$

3.  $\ln(x) = \ln(9x-1) - \ln(3x+5)$

4.  $8^{x+3} = 4^{3x-2}$

5.  $3 \sin(2x) - 1 = -1$  for  $0 \leq x \leq 2\pi$

6.  $\cos(4x) = -\frac{1}{2}$  for  $0 \leq x \leq 2\pi$

$$y = x^5$$

Give an example of each type of *function*. Write each so that y is implicitly defined as a function of x.

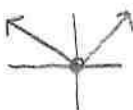
**Polynomial:**  $y = 4$

**Linear:**  $y = 2x + 7, y = \frac{1}{2}x$  linear is 1st power

**Quadratic:**  $y = 3x^2 + 2x - 4, y = \underline{x^2}$

**Algebraic:**  $y = x\sqrt{x+2}$   $x+2 \geq 0$   $x \geq -2$  restriction on domain


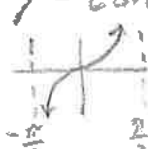
**Absolute Value:**  $y = 2|x+4| - 5$   $y = |x|$

**Piecewise:**  $y = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$  

**Rational:**  $y = \frac{x+7}{x-3}$   $x-3 \neq 0$   $x \neq 3$

**Exponential:**  $y = 2^{x-3} + 4, y = e^x, y = 2^x$  Domain  $x \in \mathbb{R}$   
Range  $y > 0$

**Logarithmic:**  $y = \log_2 x$   $x > 0$  restricted range  $y \in \mathbb{R}$   
 $y = \ln x$

**Trigonometric:**  $y = \sin x$   $y = \cos x$   $y = \tan x$   
  
D:  $x \in \mathbb{R}$  R:  $-1 \leq y \leq 1$   $|y| \leq 1$   
  
D:  $x \neq \frac{\pi}{2} + \pi k$   $k \in \mathbb{Z}$

**Inverse Function:**

$y = 2x + 1$   
 $x = 2y + 1$   $x - 1 = 2y$   
 $\frac{x-1}{2} = y$

## Graphing

Determine the intercepts of each equation.  $x$ -int: sub 0 for  $y$        $y$ -int: sub 0 for  $x$

1.  $3x + 2y = 9$

$$\begin{aligned} x\text{-int} \\ 3x + 2 \cdot 0 &= 9 \\ 3x &= 9 \\ x &= 3 \quad (3, 0) \end{aligned}$$

$$\begin{aligned} y\text{-int} \\ 3 \cdot 0 + 2y &= 9 \quad y = \frac{9}{2} \quad (0, \frac{9}{2}) \\ 2y &= 9 \end{aligned}$$

2.  $y = x^2 + 9x + 8$

$$\begin{aligned} x\text{-int} \\ 0 &= x^2 + 9x + 8 \\ 0 &= (x+8)(x+1) \\ x &= -8, -1 \quad (-8, 0) \\ & \quad (-1, 0) \end{aligned}$$

$$\begin{aligned} y\text{-int} \\ y &= 0^2 + 9 \cdot 0 + 8 \\ y &= 8 \quad (0, 8) \end{aligned}$$

3.  $y = x^3 - 5$

$$\begin{aligned} x\text{-int} \\ 0 &= x^3 - 5 \\ 5 &= x^3 \\ x &= \sqrt[3]{5} \quad (\sqrt[3]{5}, 0) \end{aligned}$$

$$\begin{aligned} y\text{-int} \\ y &= 0^3 - 5 \\ y &= -5 \quad (0, -5) \end{aligned}$$

4.  $y = \frac{2x+4}{x-7}$

$$\begin{aligned} x\text{-int} \\ \frac{0}{1} &= \frac{2x+4}{x-7} \quad (-2, 0) \\ 0 &= 2x+4 \\ x &= -2 \end{aligned}$$

$$\begin{aligned} y\text{-int} \\ y &= \frac{2 \cdot 0 + 4}{0 - 7} \\ y &= -\frac{4}{7} \quad (0, -\frac{4}{7}) \end{aligned}$$

5.  $y = \cos x$

$$\begin{aligned} x\text{-int} \\ 0 &= \cos x \\ \cos^{-1} 0 &= x \\ x &= \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \dots \end{aligned}$$

$$\begin{aligned} y\text{-int} \\ y &= \cos 0 \\ y &= 1 \quad (0, 1) \end{aligned}$$

**Transformations**

**Translations:** If  $y = f(x)$  and  $c > 0$ , then describe each type of translation.

$y = f(x - c)$ : right  $c$        $y = f(x + c)$ : left  $c$   
 $y = f(x) - c$ : down  $c$        $y = f(x) + c$ : up  $c$


**Reflections:** If  $y = f(x)$ , then describe each type of reflection.

$y = -f(x)$ : reflect over  $x$ -axis       $y = f(-x)$ : reflect over  $y$ -axis       $y = -f(-x)$ : reflect over  $x$  and  $y$ -axis

**Scaling (dilations):** If  $y = f(x)$  and  $c > 0$ , then describe each type of scaling.

$y = f(cx)$ , if  $c > 1$ : Horizontal Compress  $y = x^2$        $y = f(cx)$ , if  $0 < c < 1$ : Horizontal Stretch  
 $y = cf(x)$ , if  $c > 1$ : Vertical stretch       $y = cf(x)$ , if  $0 < c < 1$ : Vertical Compress

**Other Facts**

**Vertical Line Test for Functions:**  Passes VLT  $\Rightarrow$  Function      Does not pass horizontal line test  $\Rightarrow$  inverse is not a function.


**Horizontal and Vertical Lines:**  
 $y = \#$  zero slope       $x = \#$  undefined slope

**Positive and Negative Slope:**  
 rise to right      fall to right

**Parallel and Perpendicular:**  
 same slope      neg reciprocal slopes  $\frac{2}{3}$  and  $-\frac{3}{2}$

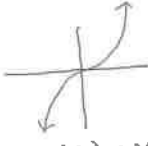
**Increasing vs Decreasing:** 

**Symmetry:**



$y = x^2$

$y = 1^2 = 1$        $y = (-1)^2 = 1$



$f(x) = x^3$

$y$ -axis symmetry      odd Function  
 Even Function      Origin Symmetry  
 $f(x) = f(-x)$        $-f(x) = f(-x)$   
 $-f(1) = -1$        $f(-1) = -1$

## Formulas

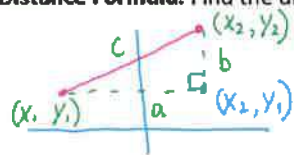
Pythagorean Theorem:  $a^2 + b^2 = c^2$

Trigonometry:  $\sin^2 \theta + \cos^2 \theta = 1$

**Slope Formula:** Find the slope of the line that passes through the points (3, 7) and (-8, 5).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = \frac{5-7}{-8-3} = \frac{-2}{-11} = \frac{2}{11}$$

**Distance Formula:** Find the distance between the points  $(x_1, y_1)$  and  $(x_2, y_2)$ .



$$a^2 + b^2 = c^2$$

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d$$

$$d = \sqrt{(3 - (-8))^2 + (7 - 5)^2}$$

$$d = \sqrt{125}$$

$$d = 5\sqrt{5}$$

**Point-Slope Formula:**

Find the equation of the line that passes through the point  $(-4, -7)$  with a slope of  $\frac{7}{11}$ .

$$y - y_1 = m(x - x_1)$$

$$y - (-7) = \frac{7}{11}(x - (-4))$$

DONE

**Quadratic Formula:** Determine the roots of  $y = 3x^2 + 9x - 5$  using the quadratic formula.

$$x = \frac{-9 \pm \sqrt{9^2 - 4(3)(-5)}}{2 \cdot 3}$$

$$x = \frac{-9 \pm \sqrt{141}}{6} \leftarrow \text{discriminant is } > 0 \text{ so 2 real roots}$$

$$\text{disc} > 0 \Rightarrow 2 \text{ real roots}$$

$$\text{disc} < 0 \Rightarrow 2 \text{ imag roots}$$

$$\text{disc} = 0 \Rightarrow 1 \text{ real root (bounces)}$$

## Assignment 2: Handout



Unit 1 Lesson 3

Topic: Conceptual Understanding of Limits

Goal: Compare **average rate of change** to **instantaneous rate of change** in order to understand limits. *at a moment in time*  
*can be neg*  $\downarrow$  *Pos*  $\downarrow$  *slope*

Velocity: speed and direction of an object—the speed limit is velocity (70 mph means 70 miles per hour). You are cruising to San Diego in your Bug adorned with flowers. As you drive you try to maintain your speed as you do not want to get a ticket. You record your distance at different times. The results are listed in the table.

*To San Diego vel is pos Return Home vel is neg but speed still pos*

Time (in minutes)	0	5	12	15	21	34	47
Distance (in miles)	0	5.7	13.28	17.28	25.3	40.9	53.9

Average Velocity:  $\frac{\text{distance traveled}}{\text{time interval}} = \frac{\Delta d}{\Delta t}$  (If you graphed these values it would be the **slope between the points**).  
*change in distance*  $\rightarrow$   $\Delta d$   
*change in time*  $\rightarrow$   $\Delta t$

Find the average velocity for each time interval, miles per minute.

*[a, b]*

$[0, 5] \rightarrow \frac{5.7 - 0}{5 - 0} = 1.14 \text{ mi/min}$       $[5, 12] \rightarrow \frac{13.28 - 5.7}{12 - 5} = 1.08 \text{ mi/min}$       $[12, 15] \rightarrow \frac{17.28 - 13.28}{15 - 12} = 1.33 \text{ mi/min}$

$[15, 21] \rightarrow \frac{25.3 - 17.28}{21 - 15} = 1.34 \text{ mi/min}$       $[21, 34] \rightarrow \frac{40.9 - 25.3}{34 - 21} = 1.2 \text{ mi/min}$       $[34, 47] \rightarrow \frac{53.9 - 40.9}{47 - 34} = 1 \text{ mi/min}$

Find the average velocity for each time interval, miles per hour.

$[0, 5] \rightarrow \frac{1.14 \text{ mi}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 68.4 \text{ mi/hr}$       $[5, 12] \rightarrow \frac{1.08 \text{ mi}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 64.8 \text{ mi/hr}$       $[12, 15] \rightarrow \frac{1.33 \text{ mi}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 79.8 \text{ mi/hr}$

$[15, 21] \rightarrow \frac{1.34 \text{ mi}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 80.22 \text{ mi/hr}$       $[21, 34] \rightarrow \frac{1.2 \text{ mi}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 72 \text{ mi/hr}$       $[34, 47] \rightarrow \frac{1 \text{ mi}}{\text{min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 60 \text{ mi/hr}$

**Average Velocity** = *Slope of line between 2 points*

In what time periods were you running the risk of getting a ticket?

Note:

As you drive, when you look at your speedometer that is instantaneous velocity. It is constantly changing.

The formula for the distance traveled by a falling body is given by  $s(t) = 4.9t^2$ , where  $t$  is time and  $s(t)$  is meters. We can use this to find the average velocity of a falling object.

$$\text{Average Velocity} = \frac{\text{distance traveled}}{\text{length of time interval}} = \frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

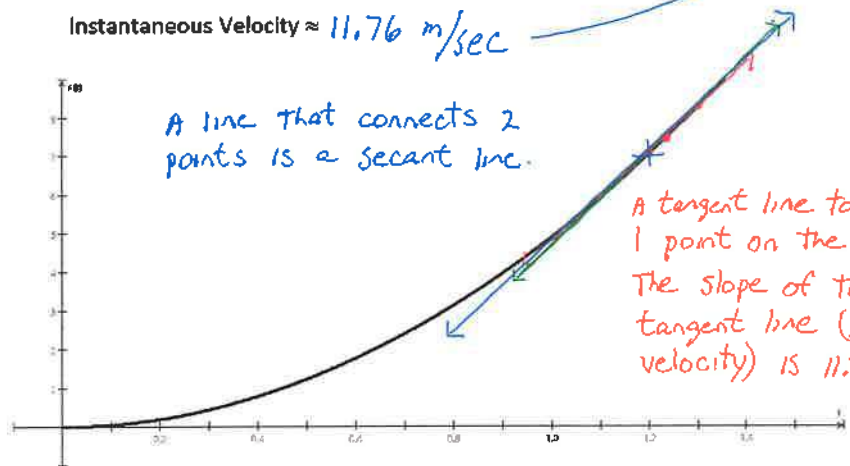
Complete the table.

Time Intervals (seconds)	[1.2, 1.3]	[1.2, 1.205]	[1.2, 1.201]	[1.2, 1.2001]	[1.2, 1.20001]
Average Velocity (meters per second)	12.25 m/sec	11.7845 m/sec	11.7649 m/sec	11.76049 m/sec	$s(1.20001) = 7.0561$ 11.760049

This interval is so small that the points seem to sit on each other.

slope

Instantaneous Velocity  $\approx 11.76$  m/sec



A line that connects 2 points is a secant line.

A tangent line touches 1 point on the graph. The slope of the tangent line (or the velocity) is 11.76 m/sec.

$$\begin{aligned} s(1.2) &= 4.9(1.2)^2 = 7.056 \text{ m} \\ s(1.3) &= 4.9(1.3)^2 = 8.281 \text{ m} \\ \frac{s(1.3) - s(1.2)}{1.3 - 1.2} &= \frac{8.281 - 7.056}{1.3 - 1.2} = 12.25 \text{ m/sec} \end{aligned}$$

$$\begin{aligned} s(1.205) &= 4.9(1.205)^2 = 7.1149225 \text{ m} \\ s(1.2) &= 7.056 \text{ m} \\ \frac{s(1.205) - s(1.2)}{1.205 - 1.2} &= \frac{7.1149225 - 7.056}{1.205 - 1.2} = 11.7845 \text{ m/sec} \end{aligned}$$

$$\begin{aligned} s(1.201) &= 4.9(1.201)^2 = 7.0678 \text{ m} \\ s(1.2) &= 7.056 \text{ m} \\ \frac{s(1.201) - s(1.2)}{1.201 - 1.2} &= \frac{7.0678 - 7.056}{1.201 - 1.2} = 11.7649 \text{ m/sec} \end{aligned}$$

$$\begin{aligned} s(1.2001) &= 7.0572 \\ s(1.2) &= 7.056 \\ \frac{s(1.2001) - s(1.2)}{1.2001 - 1.2} &= \frac{7.0572 - 7.056}{1.2001 - 1.2} = 11.760049 \text{ m/sec} \end{aligned}$$

## Average vs Instantaneous

As the time interval decreases, the average velocity approaches the instantaneous velocity. The instantaneous rate of change is the limit of the average rate of change.

## The Secant Line vs Tangent Line

2 points = average velocity      1 point = instantaneous velocity

As the time interval decreases, the secant line approaches the tangent line. The tangent line is the limit of the secant line.

Assignment #3: Read p. 61 Example 1, and p. 63 Example 2. Solve Pages 64-67: #1, 6, 7, 22, 31

## Warm Up

## Lesson 4

1. For what value(s) is each function undefined? Give the domain of each function.

A.  $y = \frac{2x-4}{x+7} \rightarrow$  undefined at  $x = -7$

Domain  
 $x \in \mathbb{R}, x \neq -7$

B.  $y = \sqrt{x-1}$  undefined when  $x < 1$

$x-1 \geq 0$   
 $x \geq 1$

Domain  
 $x \geq 1$  or  $[1, \infty)$

2. Find the equation of the line that passes through the points (3.1, 4.7) and (3.01, 5.29). Give your answer in point-slope form.

$$m = \frac{5.29 - 4.7}{3.01 - 3.1}$$

$$m = \frac{0.59}{-0.09}$$

$$m = \frac{-59}{9}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4.7 = \frac{-59}{9}(x - 3.1)$$

Unit 1 Lesson 4

Topic: Limits-Graphically and Numerically

Goal: Find the limit of a function both graphically and numerically.

**Limit:** The value that  $y$  approaches as the value of  $x$  gets closer to a given number.

$\frac{0}{0}$  implies it is a **Removable Discontinuity**—we call  $\frac{0}{0}$  **Indeterminate**.

$\frac{\#}{0}$  is not indeterminate, it is **undefined**—vertical line

Hole is a removable discontinuity.

$$f(2) = \frac{2^2 - 4}{2 - 2} = \frac{0}{0}$$

implies remove disc so more work is needed.

If  $f(x) = \frac{x^2 - 4}{x - 2}$ , then  $\lim_{x \rightarrow 2} f(x) =$

$$\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{(x-2)(x+2)}{x-2}$$

$$= \lim_{x \rightarrow 2} x + 2$$

$$= 2 + 2$$

$$\lim_{x \rightarrow 2} f(x) = 4$$

now plug in 2

Graphically Finding Limits

If  $f(x) = \frac{x^2 - 4}{x - 2}$ , then  $\lim_{x \rightarrow 2} f(x) = ?$

(This is asking you to find the limit as  $x$  approaches 2.)

We will sketch the graph of this function to get an idea of what is happening.

What are the restrictions on the domain?

$$x - 2 \neq 0 \quad x \neq 2 \quad D: x \in \mathbb{R}, x \neq 2$$

What are the  $x$ -intercepts?

$$\frac{0}{1} = \frac{x^2 - 4}{x - 2} \quad 0 = x^2 - 4 \quad x = \pm 2 \quad \text{so } x = -2$$

$$4 = x^2 \quad \text{but } x \neq 2 \quad (-2, 0)$$

What is the  $y$ -intercept?

$$y = \frac{0^2 - 4}{0 - 2} \quad y = 2 \quad (0, 2)$$

What are the vertical asymptotes or holes?

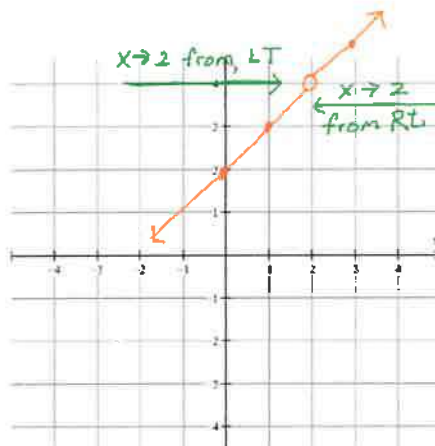
$$f(x) = \frac{(x+2)(x-2)}{(x-2)} \quad \text{so hole at } x = 2$$

$$f(x) = x + 2 \quad f(2) = 2 + 2 = 4 \quad (2, 4) \text{ hole}$$

remove  $x-2$

What is the horizontal asymptote?

deg num > deg denom so no HA



$\lim_{x \rightarrow 2} f(x)$   
As  $x$  approaches 2 from the left and the right the value of  $f(x)$  approaches 4

Numerically Finding Limits

If  $f(x) = \frac{x^2-4}{x-2}$ , then  $\lim_{x \rightarrow 2} f(x) = ?$  Plug in 1st  $\lim_{x \rightarrow 2} f(x) = \frac{2^2-4}{2-2} = \frac{0}{0}$  more work!

Using the function  $f(x) = \frac{x^2-4}{x-2}$ , complete the tables.

$x \rightarrow 2^-$  (x is approaching 2 from the left)

x	1	1.5	1.9	1.99	1.999
f(x)	3	3.5	3.9	3.99	3.999

$x \rightarrow 2^+$  (x is approaching 2 from the right)

x	3	2.5	2.01	2.001	2.0001
f(x)	5	4.5	4.01	4.001	4.0001

$x \rightarrow 2^- f(x) \rightarrow 4$

$x \rightarrow 2^+ f(x) \rightarrow 4$

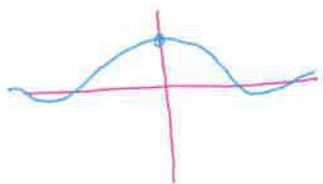
Each of these is referred to as a one-sided limit.

$\lim_{x \rightarrow 2} f(x) = 4$

{This is read as: The limit as x approaches 2 is 4 }

If  $f(x) = \frac{\sin x}{x}$ , then  $\lim_{x \rightarrow 0} f(x) = ?$   $\frac{\sin 0}{0} = \frac{0}{0}$  more work!

Graphically



Numerically

x approaches from LT		x approaches from RT	
$\lim_{x \rightarrow 0^-} f(x)$		$\lim_{x \rightarrow 0^+} f(x)$	
-1	0.84	1	0.84
-0.5	0.95	0.5	0.95
-0.1	0.99	0.1	0.99
-0.01	1	0.01	1
-0.001	1	0.001	1

Therefore  $\lim_{x \rightarrow 0} f(x) = 1$

f(x) approaches 1 as x approaches 0 but f(0) may not equal 1.

AP Calculus AB Y

Unit #1: Review, Limits and Continuity

Complete the table and use the result to estimate the limit.  $\lim_{x \rightarrow -3} \frac{\sqrt{1-x}-2}{x+3} = \frac{\sqrt{1-(-3)}-2}{-3+3} = \frac{2-2}{0} = \frac{0}{0}$  more work

x	-3.1	-3.01	-3.001	-3	-2.999	-2.99	-2.9
f(x)	-0.2485	-0.2498	-0.25	Undefined	-0.25	-0.2502	-0.2516

so  $\lim_{x \rightarrow -3} f(x) = -0.25$

plug in Find  $\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = \frac{e^0 - 1}{0} = \frac{0}{0}$  More work!

Numerically and then verify it by looking at the graph.

h	1	-0.1	-0.001	0	0.001	0.1	1
f(h)	0.63212	0.95	0.9995	Und	1.0005	1.0517	1.7183

$\lim_{h \rightarrow 0^-} f(h) = 1$  and  $\lim_{h \rightarrow 0^+} f(h) = 1$   $\therefore \lim_{h \rightarrow 0} f(h) = 1$

Assignment #4: Read p. 70-71 Examples 2-3. Solve Page 74: #4, 1, 5, 6

Warm Up

Lesson 5

No Calculator.

Graph  $f(x) = \begin{cases} x^2 + 1, & x < 1 \\ -\frac{2}{3}x + \frac{8}{3}, & x \geq 1 \end{cases}$

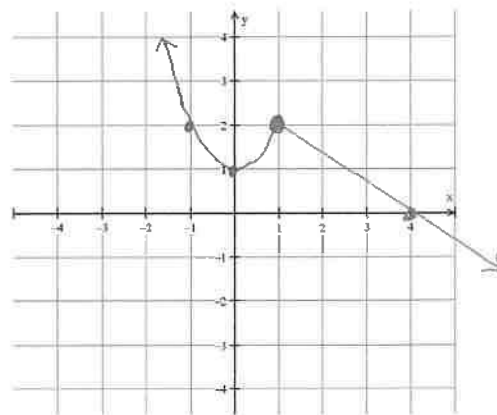
then find  $\lim_{x \rightarrow 1} f(x) = 2$

As x gets closer to 1 from the left and the right, y gets closer to 2.

$x=1 \quad f(1) = -\frac{2}{3}(1) + \frac{8}{3}$   
 $= \frac{6}{3}$   
 $= 2$

(1,2)

now use slope of  $-\frac{2}{3}$



Unit 1 Lesson 5

Topic: **Limits That Do Not Exist**

Goal: To be able to recognize and illustrate the three ways a limit can fail.

1.  $f(x)$  approaches a different number from the left and right.

*Jump Discontinuity*

$$\lim_{x \rightarrow 0^-} f(x) = -2$$

$$\lim_{x \rightarrow 0^+} f(x) = 1$$

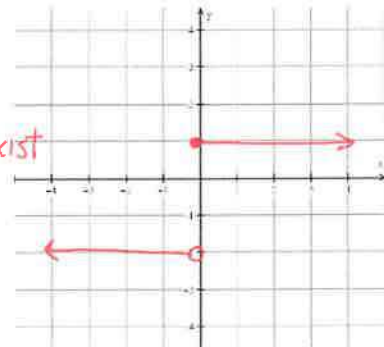
$$f(x) = \begin{cases} -2 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

$$\lim_{x \rightarrow 0} f(x) = \text{Does Not Exist DNE}$$

*Answer*

$$\lim_{x \rightarrow 0^-} f(x) \neq \lim_{x \rightarrow 0^+} f(x)$$

*Reason*



2.  $f(x)$  increases or decreases without bound as  $x$  approaches a given value.

Decreases without bound, means it goes to  $-\infty$ .

Increases without bound, means it goes to  $+\infty$ .

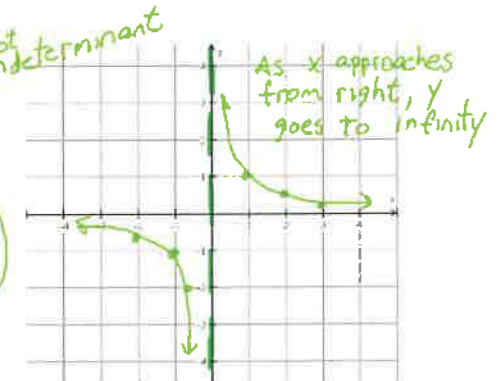
Note: You get  $1/0$  here which is undefined. This means you have a vertical asymptote.

$$f(x) = \frac{1}{x} \quad f(0) = \frac{1}{0} \text{ not indeterminate}$$

$$\lim_{x \rightarrow 0} f(x) = \text{DNE}$$

$$\lim_{x \rightarrow 0^+} f(x) = \infty \text{ DNE}$$

$$\lim_{x \rightarrow 0^-} f(x) = -\infty \text{ DNE}$$

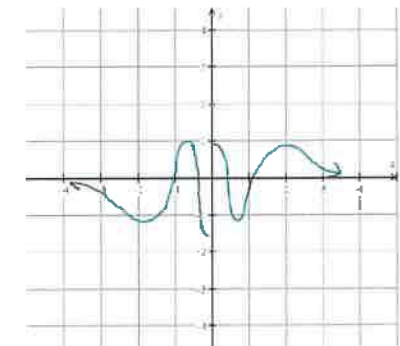


3.  $f(x)$  oscillates between two fixed values as  $x$  approaches a given value.

*Graphing Calc*

$$f(x) = \sin \frac{1}{x} \quad x \neq 0$$

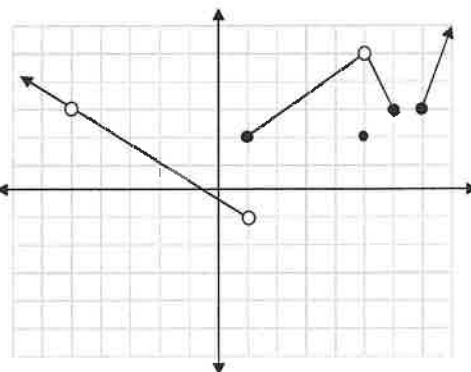
$$\lim_{x \rightarrow 0} f(x) = \text{DNE}$$



The graph to the right represents the function  $f(x)$ . Find each of the following:

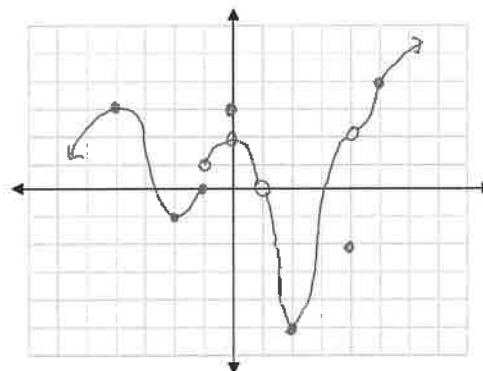
$f(1) = 2$        $f(5) = 2$        $f(6) = 3$   
 $f(6.5) = \text{und}$        $f(-1) = \frac{1}{3}$        $f(7) = 3$   
 $f(-5) = \text{und}$        $f(0) = -\frac{1}{3}$   
 $\lim_{x \rightarrow 5} f(x) = 5$        $\lim_{x \rightarrow 6} f(x) = \text{DNE}$        $\lim_{x \rightarrow 1} f(x) = \text{DNE}$   
 $\lim_{x \rightarrow 1} f(x) = \frac{1}{3}$        $\lim_{x \rightarrow 7} f(x) = \text{DNE}$        $\lim_{x \rightarrow 5} f(x) = 3$   
 $\lim_{x \rightarrow 0} f(x) = -\frac{1}{3}$

$\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$   
 $\lim_{x \rightarrow 6^-} f(x) = 3$        $\lim_{x \rightarrow 6^+} f(x) = \text{DNE}$



Create a graph that has the following properties.

$f(-4) = 3$        $f(-2) = -1$        $f(-1) = 0$        $f(0) = 3$   
 $f(1) = \text{undefined}$        $f(2) = -5$        $f(4) = -2$        $f(5) = 4$   
 $\lim_{x \rightarrow -4} f(x) = 3$        $\lim_{x \rightarrow -2} f(x) = -1$        $\lim_{x \rightarrow -1} f(x) = 0$        $\lim_{x \rightarrow -1} f(x) = 1$        $\lim_{x \rightarrow 0} f(x) = 2$   
 $\lim_{x \rightarrow 1} f(x) = 0$        $\lim_{x \rightarrow 2} f(x) = -5$        $\lim_{x \rightarrow 4} f(x) = 2$        $\lim_{x \rightarrow 4} f(x) = 2$        $\lim_{x \rightarrow 5} f(x) = 4$



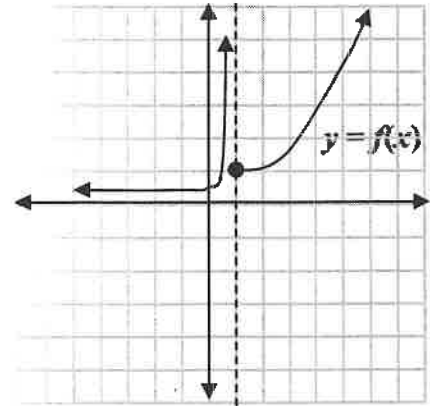
Assignment #5: Read p. 72-73 Examples 7-8. Handout, and p. 75-76: #18, 19, 22, 23, 43, 57



Unit #1 .

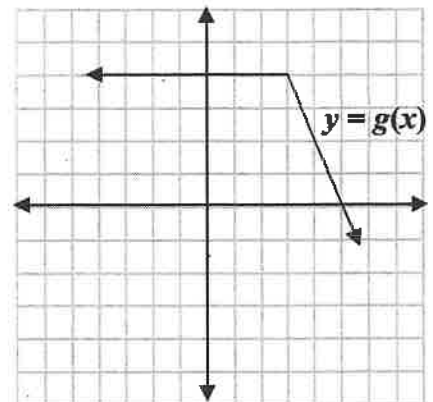
Assignment #5: Handout  
Limits Graphically  
For the functions graphed find each limit.

1.  $\lim_{x \rightarrow 1^+} f(x) = 1$       2.  $\lim_{x \rightarrow 1^-} f(x) = \infty$       3.  $f(1) = 1$   
 4.  $\lim_{x \rightarrow 1} f(x) = \text{DNE}$       5.  $\lim_{x \rightarrow 3} f(x) = 1/2$       6.  $\lim_{x \rightarrow \infty} f(x) = \infty$   
 7.  $\lim_{x \rightarrow \infty} f(x) = 0$



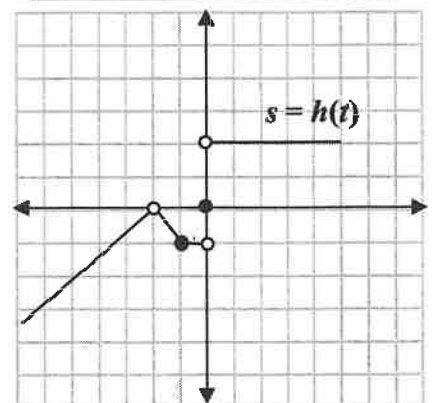
Use for exercises 1-7

8.  $\lim_{x \rightarrow 3^+} g(x) = 4$       9.  $\lim_{x \rightarrow 3^-} g(x) = 4$       10.  $g(3) = 4$   
 11.  $\lim_{x \rightarrow 3} g(x) = 4$       12.  $\lim_{x \rightarrow 2} g(x) = 4$       13.  $\lim_{x \rightarrow \infty} g(x) = -\infty$   
 14.  $\lim_{x \rightarrow \infty} g(x) = 4$       15.  $\lim_{x \rightarrow 5} g(x) = 0$       16.  $g(5) = 0$   
 17.  $g(4.8) = 1/2$



Use for exercises 8-17

18.  $\lim_{x \rightarrow 2^+} h(t) = 0$       19.  $\lim_{x \rightarrow 2^-} h(t) = 0$       20.  $h(-2) = \text{NOT Def}$   
 21.  $\lim_{x \rightarrow 2} h(t) = 0$       22.  $\lim_{x \rightarrow -1^+} h(t) = -1$       23.  $\lim_{x \rightarrow -1^-} h(t) = -1$   
 24.  $h(-1) = -1$       25.  $\lim_{x \rightarrow -1} h(t) = -1$       26.  $\lim_{x \rightarrow 0^+} h(t) = 2$   
 27.  $\lim_{x \rightarrow 0^-} h(t) = -1$       28.  $\lim_{x \rightarrow 0} h(t) = \text{DNE}$       29.  $h(0) = 0$   
jump disc  
 30.  $\lim_{x \rightarrow 1/2} h(t) = -1$       31.  $\lim_{x \rightarrow 7/8} h(t) = 2$



Use for exercises 18-31

## Warm Up

## Lesson 6

Simplify:

A.  $\log_3 81 = 4$

$$C. \frac{3 + \frac{4}{e^x} \frac{e^x}{1}}{\frac{2}{e^x} \frac{e^x}{1}} = \frac{3e^x + 4}{2}$$

B.  $\frac{3}{x} - \frac{2}{x+1}$

$$\frac{x+1}{x+1} \cdot \frac{3}{x} - \frac{2}{x+1} \cdot \frac{x}{x}$$

$$\frac{3x+3 - 2x}{x(x+1)}$$

$$\frac{x+3}{x(x+1)} \quad x \neq 0, -1$$

D.  $\frac{5}{2+\sqrt{x}} \cdot \frac{2-\sqrt{x}}{2-\sqrt{x}}$  conjugate

$$\frac{10 - 5\sqrt{x}}{4 - 2\sqrt{x} + 2\sqrt{x} - x}$$

$$\frac{10 - 5\sqrt{x}}{4 - x}$$

 $x \geq 0$ 

and

$2 + \sqrt{x} \neq 0$

$\sqrt{x} \neq -2$

no sol

## Unit 1 Lesson 6

## Topic: Evaluating Limits Analytically

Goal: Evaluate limits analytically.



$$1. \quad \lim_{x \rightarrow 2} 3 = 3 \quad \lim_{x \rightarrow -10} 3 = 3$$

$$2. \quad \lim_{x \rightarrow 2} x = 2 \quad \lim_{x \rightarrow 1} x = 1 \quad \lim_{x \rightarrow -1} x = -1$$

Plugging in 2

## Properties of Limits

$$1. \quad \lim_{x \rightarrow 2} x = 2 \quad \lim_{x \rightarrow 2} 3x = 3 \cdot 2 = 6 \quad \text{Scalar Multiple}$$

$$2. \quad \lim_{x \rightarrow 2} x^2 = 2^2 = 4 \quad \lim_{x \rightarrow 2} x = 2 \quad \lim_{x \rightarrow 2} x^2 + x = 2^2 + 2 = 4 + 2 = 6 \quad \text{(Sum or Difference)}$$

$$3. \quad \lim_{x \rightarrow 2} (x^2 + 1) = 2^2 + 1 = 5 \quad \lim_{x \rightarrow 2} (x - 1) = 2 - 1 = 1 \quad \lim_{x \rightarrow 2} (x^2 + 1)(x - 1) = 5(1) = 5 \quad \text{(Product)}$$

$$4. \quad \lim_{x \rightarrow 2} (x^2 + 1) = 5 \quad \lim_{x \rightarrow 2} (x - 1) = 1 \quad \lim_{x \rightarrow 2} \frac{(x^2 + 1)}{(x - 1)} = \frac{5}{1} = 5 \quad \text{(Quotient)}$$

$$5. \quad \lim_{x \rightarrow 2} (x^2 + 1)^6 = (2^2 + 1)^6 = 5^6$$

AP Calculus AB Y

Find each limit.

plug in the number

$$\begin{aligned} \lim_{x \rightarrow 4} (x^4 - 3x^2 + 7)^3 &= \\ &= (1^4 - 3 \cdot 1^2 + 7)^3 \\ &= 5^3 \end{aligned}$$

$$\lim_{x \rightarrow -1} \sqrt[3]{x} = \sqrt[3]{-1} = -1$$

$$\begin{aligned} \lim_{x \rightarrow \pi} \cos x &= \cos \pi \\ &= -1 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{4}} \tan x &= \tan \frac{\pi}{4} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{(x^2 - 2)^2}{\sqrt{x+3}} &= \\ &= \frac{(2^2 - 2)^2}{\sqrt{2+3}} \\ &= \frac{4}{\sqrt{5}} \text{ or } \frac{4}{5^{1/2}} \end{aligned}$$

$$\lim_{x \rightarrow 2} \sqrt[3]{x} = \sqrt[3]{2} \text{ or } 2^{1/3}$$

$$\begin{aligned} \lim_{x \rightarrow -1} \sqrt{x} &= \sqrt{-1} \\ &= \text{DNE} \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow \pi} \sin x &= \sin \pi \\ &= 0 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{2}} \tan x &= \tan \frac{\pi}{2} \\ &= \frac{1}{0} \text{ und} \\ &= \text{SO DNE} \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 1} \frac{3^x}{\sqrt[3]{2x-1}} &= \\ &= \frac{3^1}{\sqrt[3]{2 \cdot 1 - 1}} \\ &= \frac{3}{\sqrt[3]{1}} \\ &= 3 \end{aligned}$$

Unit #1: Review, Limits and Continuity

$$\lim_{x \rightarrow 4} \sqrt{x} = \sqrt{4} = 2$$

$$\begin{aligned} \lim_{x \rightarrow 1} \sqrt{x^4 + 1} &= \sqrt{1^4 + 1} \\ &= \sqrt{2} \text{ or } 2^{1/2} \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{3}} \cos x &= \cos \frac{\pi}{3} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow \frac{3\pi}{4}} \cos x &= \cos \frac{3\pi}{4} \\ &= -\frac{1}{\sqrt{2}} \text{ or } -\frac{\sqrt{2}}{2} \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 16} \frac{\log_4 x}{e^x} &= \frac{\log_4 16}{e^{16}} \\ &= \frac{2}{e^{16}} \end{aligned}$$

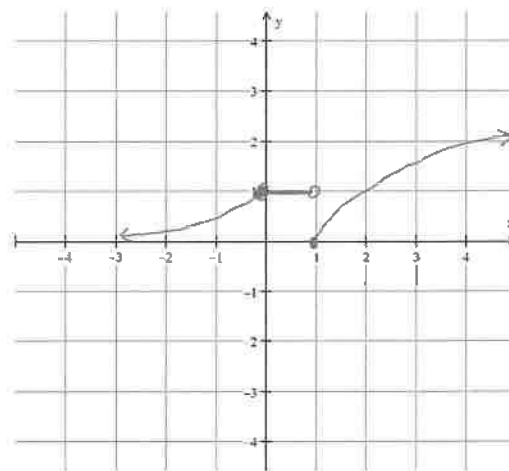
Assignment #6: Read p. 78 Examples 2-3. Solve p. 80: #6, 3, 5, 13, 18, 23, 28, 27, 29, 30

## Warm Up

## Lesson 7

Graph the piecewise function.

$$f(x) = \begin{cases} e^x, & x < 0 \\ 1, & 0 \leq x < 1 \\ \ln x, & x \geq 1 \end{cases}$$



Find:

A.  $\lim_{x \rightarrow 0} f(x) = 1$

B.  $\lim_{x \rightarrow 1} f(x) = \text{DNE}$       Jump  
Disc.       $\lim_{x \rightarrow 1^-} f(x) = 1$        $\lim_{x \rightarrow 1^+} f(x) = 0$

C.  $\lim_{x \rightarrow 1/2} f(x) = 1$

D.  $\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \ln x = \ln 2$

## Unit 1 Lesson 7

## Topic: Continuity

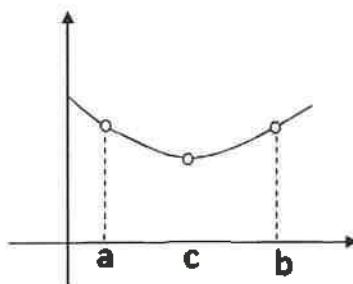
Goal: To be able to determine if a function is continuous at a point and if it is not continuous, explain why.

Continuity: A function is continuous at  $c$  (a point) if the following three conditions are met:

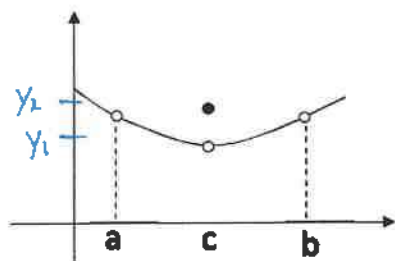
1.  $f(c)$  is defined.
2.  $\lim_{x \rightarrow c} f(x)$  exists.
3.  $\lim_{x \rightarrow c} f(x) = f(c)$

To be continuous on an open interval  $(a, b)$ , the function must be continuous at each point on the interval. Any function that is continuous from  $(-\infty, +\infty)$  is everywhere continuous.

Examples of a function **Not Continuous** at a point  $c$  and why they are not.



$f(c)$  not defined  
 $\Rightarrow f(x)$  not continuous

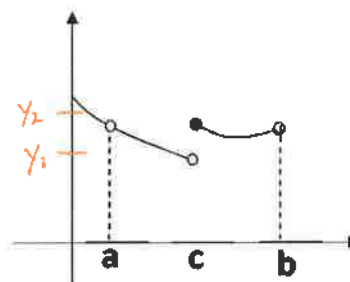


$f(c)$  is defined  $f(c) = y_2$

$$\lim_{x \rightarrow c} f(x) = y_1$$

$$\lim_{x \rightarrow c} f(x) \neq f(c)$$

$\therefore f(x)$  is not continuous



$f(c)$  is defined  $f(c) = y_2$

$$\lim_{x \rightarrow c} f(x) = \text{DNE}$$

b/c

$$\lim_{x \rightarrow c^-} f(x) = y_1 \text{ and}$$

$$\lim_{x \rightarrow c^+} f(x) = y_2$$

AP Calculus AB Y

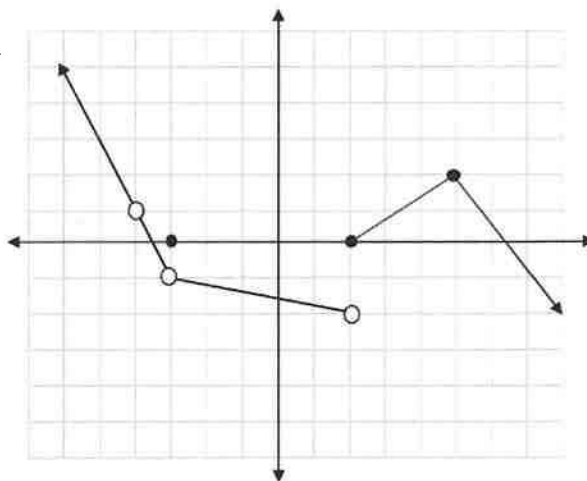
Unit #1: Review, Limits and Continuity

Determine at what points the function is continuous and at what points it is not continuous. At every point where it is not continuous, state specifically why it is not continuous. Also state what type of discontinuity it is and whether it is right or left continuous at each point.

$x = -4$  not continuous b/c  $f(-4)$  is not defined

$x = -3$  Not continuous b/c  $\lim_{x \rightarrow 3} f(x) \neq f(3)$

$x = 2$  Not continuous b/c  $\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$



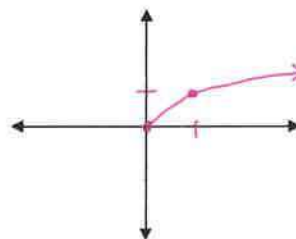
One sided Limits

$\lim_{x \rightarrow 2^-} f(x) = -2$  or  $\lim_{x \rightarrow 2^+} f(x) = 0$

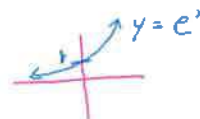
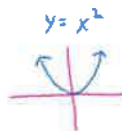
One-Sided Continuity

$f(x) = \sqrt{x}, x \geq 0$

$\lim_{x \rightarrow 0^-} f(x) = \text{DNE}$   
There is no graph to the left of 0.



Let  $f(x) = x^2$  and  $g(x) = e^x$ . Both are continuous for all  $x$ .



the following are continuous:

A.  $f(x)g(x)$   
 $x^2e^x$   
mult

B.  $f(x) + g(x)$   
 $x^2 + e^x$   
addition

C.  $f(x) - g(x)$   
 $x^2 - e^x$   
sub

D.  $7f(x)$   
 $7 \cdot x^2$   
scalar mult

Assignment 7: Read p. 82-83 Example 2, and p. 86 Example 3. Solve p. 88-89: (Ignore one-sided continuity part of directions) #5, 24, 23, 49, 51 (sketch graph only), 71, 70, 80, 79

## Warm Up

## Lesson 8

Find each limit.

$$1. \quad \lim_{x \rightarrow \frac{\pi}{4}} (\sin x) = \sin \frac{\pi}{4} \\ = \frac{1}{\sqrt{2}}$$

$$2. \quad \lim_{a \rightarrow 11} \frac{a^2 - 121}{a - 11} = \frac{11^2 - 121}{11 - 11} = \frac{0}{0} \text{ more work}$$

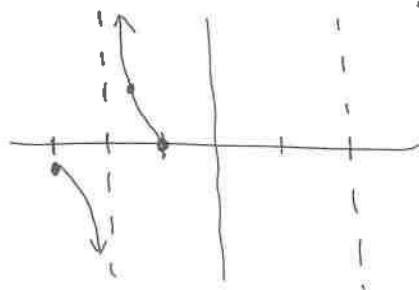
$$\lim_{a \rightarrow 11} \frac{\cancel{(a-11)}(a+11)}{\cancel{a-11}} = \lim_{a \rightarrow 11} a + 11 \\ = 22$$

$$3. \quad \lim_{h \rightarrow 5} \frac{h-5}{h-9} = \frac{5-5}{5-9} \\ = \frac{0}{-4} = 0$$

$$4. \quad \lim_{t \rightarrow -2} \frac{t^2 - 4}{t + 1} = \frac{(-2)^2 - 4}{-2 + 1} \\ = \frac{0}{-1} = 0$$

$$5. \quad \lim_{t \rightarrow -2} \frac{t+1}{t^2-4} = \frac{-2+1}{(-2)^2-4} = \frac{-1}{0} \text{ DNE}$$

$$f(t) = \frac{t+1}{t^2-4} = \frac{t+1}{(t-2)(t+2)}$$

asymptotes  $t = \pm 2$ 

$$f(-1) = 0$$

$$f(-1.5) = \frac{-0.5}{-1.75} = \text{pos}$$

$$f(-3) = \frac{-2}{5}$$



## Unit 1 Lesson 8

## Topic: Finding Limits Algebraically

Goal: Find the limit of a function using various algebraic manipulations-tricks.

Let  $f(x) = \frac{x^2-9}{x+3}$ ,  $x \neq -3$  Let  $g(x) = x-3$  Graph both on calculator.

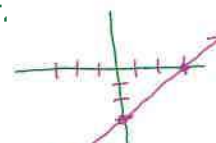
What is the difference?

Factoring

$$\lim_{x \rightarrow -3} \frac{x^2-9}{x+3} = \frac{(-3)^2-9}{-3+3} = \frac{0}{0}$$

*more work*

$$\lim_{x \rightarrow -3} \frac{(x+3)(x-3)}{x+3} = -6$$

 $f(x)$  has a hole at  $(-3, -6)$ 

$$\lim_{x \rightarrow 3} \frac{x^2-9}{x-3} = \frac{3^2-9}{3-3} = \frac{0}{0} \quad \lim_{x \rightarrow 3} \frac{(x+3)(x-3)}{x-3} = 6$$

$$\lim_{x \rightarrow 6} \frac{x^2-8x+12}{x-6} = \frac{6^2-8 \cdot 6+12}{6-6} = \frac{0}{0} \quad \lim_{x \rightarrow 6} \frac{(x-6)(x-2)}{x-6} = 4$$

$$\lim_{x \rightarrow -4} \frac{x^3+64}{x+4} = \frac{(-4)^3+64}{-4+4} = \frac{0}{0}$$

$$\begin{array}{r} x^2-4x+16 \\ x+4 \overline{) x^3+0x^2+0x+64} \\ \underline{-(x^3+4x^2)} \phantom{+0x} \\ -4x^2+0x \phantom{+64} \\ \underline{-(-4x^2-16x)} \phantom{+64} \\ 16x+64 \\ \underline{-(16x+64)} \\ 0 \end{array}$$

$$\lim_{x \rightarrow -4} \frac{(x+4)(x^2-4x+16)}{x+4} = (-4)^2-4(-4)+16 = 48$$

Dividing Out and Rationalizing

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+1}-1}{x}; x \neq 0 \quad \frac{\sqrt{0+1}-1}{0} = \frac{0}{0}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sqrt{x+1}-1}{x} & \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1} = \lim_{x \rightarrow 0} \frac{x+1+\sqrt{x+1}-\sqrt{x+1}-1}{x[\sqrt{x+1}+1]} \\ & = \lim_{x \rightarrow 0} \frac{x}{x[\sqrt{x+1}+1]} = \frac{1}{\sqrt{0+1}+1} = \frac{1}{2} \end{aligned}$$

*conjugate*

## Complex Fractions

$$\lim_{x \rightarrow 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}; x \neq 0 \quad \frac{\frac{1}{0+4} - \frac{1}{4}}{0} = \frac{0}{0} \quad \text{still need to plug in to see if you get } \frac{0}{0}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\left(\frac{1}{x+4} - \frac{1}{4}\right) \frac{4(x+4)}{4(x+4)}}{x} &= \lim_{x \rightarrow 0} \frac{4 - (x+4)}{4x(x+4)} = \lim_{x \rightarrow 0} \frac{-x}{4x(x+4)} \\ &= \frac{-1}{4(0+4)} = \frac{-1}{16} \end{aligned}$$

## Try These

$$\lim_{x \rightarrow -1} \frac{x+1}{x^3+1} = \frac{-1+1}{(-1)^3+1} = \frac{0}{0}$$

$$\lim_{x \rightarrow 8} \frac{\sqrt{2x+3}}{x-7} = \frac{\sqrt{2 \cdot 8 + 3}}{8-7} = \frac{\sqrt{19}}{1} = \sqrt{19}$$

$$\begin{aligned} \lim_{x \rightarrow -1} \frac{x+1}{(x+1)(x^2-x+1)} &= \frac{1}{(-1)^2 - (-1) + 1} \\ &= \frac{1}{3} \end{aligned}$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{x+2} - \frac{1}{2}}{x} = \frac{\frac{1}{0+2} - \frac{1}{2}}{0} = \frac{0}{0}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\left(\frac{1}{x+2} - \frac{1}{2}\right) \frac{2(x+2)}{2(x+2)}}{x} &= \lim_{x \rightarrow 0} \frac{2 - (x+2)}{2x(x+2)} \\ &= \lim_{x \rightarrow 0} \frac{-x}{2x(x+2)} \\ &= \frac{-1}{2(0+2)} = \frac{-1}{4} \end{aligned}$$

Assignment 8: Read p. 92-93 Examples 1,3,5,6. Solve p. 94-95: (Ignore instructions, just find the limits algebraically.) #2, 3, 16, 22, 21, 25, 39, 51, 54

## Warm Up

## Lesson 10

Give these values without a calculator.

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos \pi = -1$$

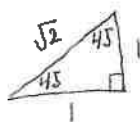
$$\csc \frac{\pi}{4} = \sqrt{2}$$

$$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\tan^{-1} \left( \frac{1}{1} \right) = \frac{\pi}{4}$$

$$\sin^{-1} \left( \frac{1}{2} \right) = \frac{\pi}{6}$$

$$\cos^{-1} \left( \frac{1}{2} \right) = \frac{\pi}{3}$$



If  $\tan \theta = \frac{3}{4}$ , then  $\sin \theta =$

$$= \frac{y}{r}$$

$$3^2 + 4^2 = r^2$$

$$25 = r^2$$

$$r = \sqrt{25}$$

$$\sin \theta = \frac{3}{5}$$

If  $f(x) \leq 3$  and  $f(x) \geq 3$ , then  $f(x) = 3$

Unit 1 Lesson 10

Topic: Infinite Limits

Goal: Find limits that are unbounded (go off to  $-\infty$  or  $+\infty$ ).

\* use table in calculator

Three ways to find infinite limits:

1. Plug in the values.
2. Make a table.
3. Look at graph.

Definition of a horizontal asymptote: The limit as  $x$  approaches  $\pm\infty$ .

Definition of a vertical asymptote: If  $f(x)$  approaches infinity (or negative infinity) as  $x$  approaches  $c$  from the right or left, then the line  $x = c$  is a vertical asymptote of the graph of  $f$ .

Vertical Asymptotes

Look at all of the limits associated with this function at  $x = 1$ .

$$f(x) = \frac{1}{x-1}$$

SKIP

Find each limit.  $\rightarrow$  deg num < deg denom

$$\lim_{x \rightarrow \infty} \frac{3}{x-2} = 0$$

x	10	100	1000	10,000	As $x \rightarrow \infty$
f(x)	3/8	3/98	3/998	3/9,998	$y \rightarrow 0$

$$\lim_{x \rightarrow \infty} \frac{x+3}{x-2} = 1 \quad \rightarrow \text{deg num} = \text{deg denom}$$

x	10	100	1000	As $x \rightarrow \infty$
f(x)	1.625	1.05	1.005	$y \rightarrow 1$

$$\lim_{x \rightarrow \infty} \frac{x^2+3}{x-2} = \text{DNE} \quad \rightarrow \text{deg num} > \text{deg denom}$$

x	10	100	1000	As $x \rightarrow \infty$
f(x)	12.815	102.07	1002.007	$y \rightarrow \infty$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2+3}}{x-2} = -1 \quad \rightarrow \text{Think about } \frac{\sqrt{x^2}}{x} = \frac{|x|}{x}$$

x	-10	-100	-1000	As $x \rightarrow -\infty$
f(x)	-0.846	-0.981	-0.998	$y \rightarrow -1$

Find each infinite limit.

$$\lim_{x \rightarrow \infty} (x+3) = \infty \text{ DNE}$$



$$\lim_{x \rightarrow -\infty} (x+3) = -\infty \text{ DNE}$$

$$\lim_{x \rightarrow \infty} (x^2 + 1.3) = \text{DNE}$$

$$\lim_{x \rightarrow -\infty} (x^2 + 1.3) = \text{DNE}$$

As  $x \rightarrow \infty$   $y \rightarrow \infty$ 

$$\lim_{x \rightarrow \infty} \frac{x^2 - 98}{3x^2 + 47} = \frac{1}{3}$$

$$\lim_{x \rightarrow -\infty} \frac{x^2 - 98}{3x^2 + 47} = \frac{1}{3}$$

Deg num = deg denom

$$\lim_{x \rightarrow \infty} \frac{5x^3 + 1}{7x^3 - 257} = \frac{5}{7}$$

$$\lim_{x \rightarrow -\infty} \frac{5x^3 + 1}{7x^3 - 257} = \frac{5}{7}$$

$$\lim_{x \rightarrow \infty} \frac{x + 0.5}{19x^2 - 45} = 0$$

$$\lim_{x \rightarrow \infty} \frac{e^x - 6}{e^x - 4} = \frac{0 - 6}{0 - 4}$$

$$e^{-\infty} \rightarrow 0$$

Deg num &lt; deg denom

$$= \frac{3}{2}$$

$$\lim_{x \rightarrow \infty} \frac{x+1}{\sqrt{3x^2+5}} = \frac{1}{\sqrt{3}}$$

$$\lim_{x \rightarrow \infty} \frac{3x+1}{\sqrt{x^2+5}} = -3$$

$$\text{Think: } \frac{x}{|x|\sqrt{3}}$$

$$\text{Think: } \frac{3x}{|x|} = \frac{-3\infty}{\infty}$$

$$\lim_{x \rightarrow \infty} \frac{x^2+1}{\sqrt{x^4+5}} = 1$$

$$\lim_{x \rightarrow \infty} \frac{x^2+1}{\sqrt{x^4+5}} = 1$$

$$\text{Think: } \frac{x^2}{|x^2|}$$

Assignment 10: Read p. 102-103 Examples 2-4. Solve p. 105: #1, 8, 9, 12, 13, 14, 16, 18, 17, 20, 23, 25

## Warm Up

## Lesson 11

Solve each without a calculator.

$$\frac{\sin x = \cos x}{\cos x} \quad 0 \leq x \leq 2\pi$$

$$\tan x = 1$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$

$$2 \sin(2x) + 2 = 2 \quad 0 \leq x \leq 2\pi$$

$$2 \sin 2x = 0$$

$$\sin 2x = 0$$

$$u = 2x$$

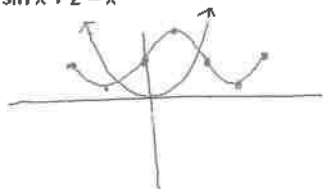
$$\sin u = 0$$

$$u = 0, \pi, 2\pi, 3\pi$$

$$2x = 0, \pi, 2\pi, 3\pi \quad x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

Solve using a calc.

$$\sin x + 2 = x^2$$



$$x = -1.0626$$

$$x = 1.667$$

## Unit 1 Lesson 11

Topic: **IVT (Intermediate Value Theorem)**

**Goal:** Use the Intermediate Value Theorem to prove a function does or does not have a value at a given value of  $x$ .

IVT: If a function is continuous, then you cannot skip any numbers. y-values

Let  $f(x)$  be a continuous function on the interval  $[-4, 13]$ . Some selected values of  $f(x)$  are given in the table.

$x$	-4	-2.3	0	3	5.6	7.1	9	11.4	13
$f(x)$	11.9	2.7	-6.1	-3.8	1.1	5.2	2.1	-0.2	6.7

}  $y=0$ 
}  $y=0$ 
}  $y=0$

Is there a solution to  $f(x) = 0$ ? Explain.  $y=0$

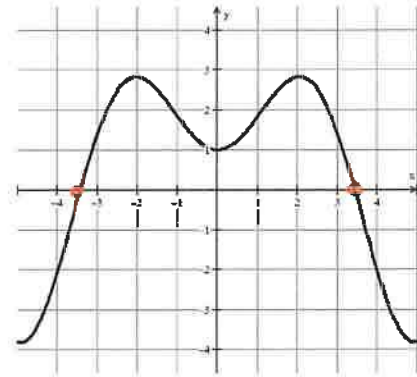
Yes. Since  $f(x)$  is continuous and  $f(x)$  goes from pos to neg or neg to pos on  $[-2.3, 0] \cup [3, 5.6] \cup [9, 11.4]$ , then there exists an  $x$  in those intervals such that  $f(x) = 0$ .  
IVT

On what interval does  $f(x) = 9$ ? Explain.

Since  $f(x)$  is continuous and  $f(-4) = 11.9$  and  $f(-2.3) = 2.7$ , then there exists an  $x \in [-4, -2.3]$  such that  $f(x) = 9$ .  
IVT

*continuous function*

The graph of  $f(x) = x \sin(x) + 1$  is shown at the right.



On the interval  $[-4, 4]$ , does  $f(x) = 0$ ? Explain.

Yes. Since  $f(x)$  is continuous on  $[-4, 4]$  and  $f(-4) < 0$  and  $f(-3) > 0$  then there exists a  $c \in [-4, -3]$  such that  $f(c) = 0$ .

If  $f(-3.5) < 0$ , what do you now know about the solution to  $f(x) = 0$  on the interval  $[-4, -3]$ ?

Since  $f(x)$  is cont and  $f(-3.5) < 0$  and  $f(-3) > 0$  then there exists a  $c \in [-3.5, -3]$  such that  $f(c) = 0$ .

Show using IVT that there is a solution to  $2^x + 3^x = 4^x$ .

*all continuous*

$$2^x + 3^x - 4^x = 0$$

Since you are equal to zero, find 1 pos and 1 neg sol.

Since the function is continuous with  $f(0) = 1$  and  $f(2) = -3$  then there exists a  $c \in [0, 2]$  such that  $f(c) = 0$

$$x=0$$

$$2^0 + 3^0 - 4^0$$

$$1 + 1 - 1$$

$$1 \quad f(0) = 1$$

$$x=2$$

$$2^2 + 3^2 - 4^2$$

$$4 + 9 - 16$$

$$-3 \quad f(2) = -3$$

See HW + change this example.  
and second part of top example

Assignment 11: Read p. 107 Examples 1-2. Solve p. 109: #6, 2, 21-24

Assignment 12: Review Handout