Calculus BC Plus Summer Packet

Welcome to BC Plus!

In order to complete the curriculum before the AP Calculus BC Exam in May, it is necessary to do some preparatory work this summer. As you already know, Chapter 1 in your Calculus book is a general review of Algebra II and Pre-Calculus concepts.

Attached to this letter is a table of contents, the summer assignment, and the answers. The summer assignment, or review packet, helps you to focus on the mathematical skills and content you will need to use in solving Calculus problems. These problems deal with skills and content that you studied in AP Calculus AB. Use your AB Calculus notes to help you solve the review problems.

You are responsible for completing this summer assignment. The review packet must be completed and evidence of your understanding is to be shown on the packet with answers placed in the spaces where provided. Complete work must be shown to justify your answer, graphs must be carefully drawn and labeled, and attempts must be made for each problem. **Do not use a calculator unless a problem specifically indicates for you to do so.** *If you use a calculator, you must set up what you entered into the calculator and what the calculator produced for you on your paper.*

Be prepared to turn in your completed summer assignment on the first day of class. We will spend the first few days going over the packet together as a class and then you will be tested on the material. The problem packet will be collected and thoroughly graded and count towards part of your first quarter grade. It is important to check your answers with those provided in this packet.

At this level, doing homework is more than just getting the problems done. The problems should be a learning experience. Take your time and make sure you understand the concepts behind each problem. Seek out help to deal with problems and concepts you find challenging. I recommend that you try to meet with other BC Plus students in small groups this summer to help each other. We are all in this together!

There are 2 Calculus books left in the Westport Library over the summer to use as a reference if necessary. You will also be able to download this summer assignment online from a link on the Staples High School website.

We are looking forward to seeing you in September. Good luck with the assignment!

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Contents

Formulas

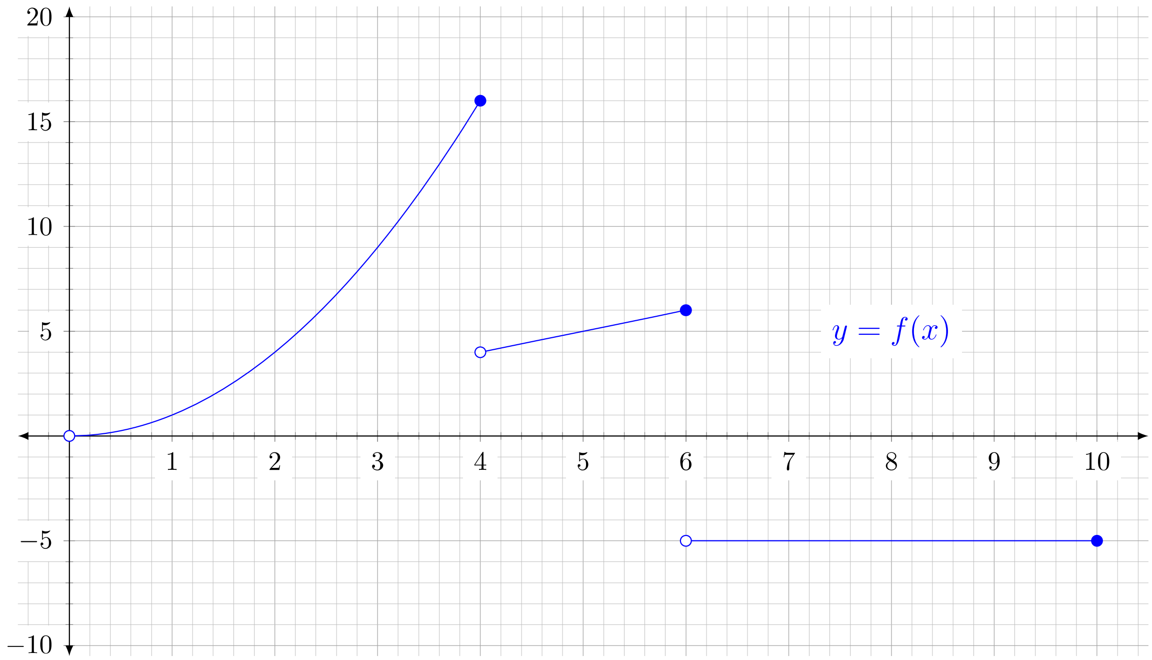
Double-Angle:
$\sin 2x = 2\sin x\cos x$
$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$

This packet is to be completed **without** the use of a calculator unless specifically indicated to do so for a particular question by the symbol.

Note: Although the packet is organized such that question types tend to fall under the chapters with which they are associated, some free-response questions may contain parts that involve concepts from later chapters.

Chapter 2: Limits and Continuity

1. Consider the following graph.



(a) $\lim_{x\to 4^+} f(x)$

(a) ____________

(b) $\lim_{x\to 4^-} f(x)$

(b) ____________

(c) $\lim_{x\to 0^+} f(x)$

(c) ____________

(d) $\lim_{x\to 0^-} f(x)$

(d) ____________

(e) $\lim_{x\to 6^+} f(x)$

(e) ____________

(f) $\lim_{x\to 6^-} f(x)$

(f) ____________

2. Evaluate the following limits.

(a) $\lim_{t \to 3} \frac{t^2 - 4t + 4}{t^2 - 4}$

(a) ______________

(b) $\lim_{x \to 0} \frac{3 \sin 4x}{\sin 3x}$

(b) ______________

3. Assume that $\lim_{x \to 4} f(x) = 2$ and $\lim_{x \to 4} g(x) = 5$.

(a) $\lim_{x \to 4} (g(x) + 1)$

(a) ______________

(b) $\lim_{x \to 4} x f(x)$

(b) ______________

(c) $\lim_{x \to 4} g^2(x)$

(c) ______________

(d) $\lim_{x \to 4} \frac{g(x)}{f(x) - 3}$

(d) ______________

4. Evaluate the following limits for x approaching infinity.

(a) $\lim_{x \to \infty} \frac{x^4 - 5}{x^2 + 12}$

(a) ______________

(b) $\lim\limits_{x\to\infty} \dfrac{x-5}{x^3}$

(b) ______________

(c) $\lim\limits_{x\to\infty} \dfrac{21x^5+3x^3+15}{40x^5+91x^4+21x^4}$

(c) ______________

5. For $f(x) = \dfrac{x^7+9}{4x^6-4}$, evaluate the following by **graphing** $f(x)$ on a calculator.

(a) $\lim\limits_{x\to\infty} f(x)$

(a) ______________

(b) $\lim\limits_{x\to-\infty} f(x)$

(b) ______________

(c) $\lim\limits_{x\to-1^+} f(x)$

(c) ______________

(d) $\lim\limits_{x\to-1^-} f(x)$

(d) ______________

6. Algebraically show that $\lim\limits_{x\to 0} \dfrac{\tan(x)}{x} = 1$.

7. Algebraically show that $\lim\limits_{x\to 0} \dfrac{1-\cos(x)}{x} = 0$.

Chapter 3: Derivatives

8. Take the derivative of each of the following expressions.

   (a) $x^3 + 2x^2 - \frac{x}{2} - 8$

   (a) ____________

   (b) $x^{-\frac{4}{3}} + 5x^{-\frac{1}{2}} + \ln x$

   (b) ____________

   (c) $\tan(4x)$

   (c) ____________

   (d) $\sin(x)\ \cos(x)$

   (d) ____________

   (e) $\sin^2(x)$

   (e) ____________

   (f) $\ln(2x + 1)$

   (f) ____________

   (g) 2^x

   (g) ____________

(h) e^{2x}

(h) ____________

(i) $(e^x)^2$

(i) ____________

(j) $\tan^{-1}(5x)$

(j) ____________

(k) $\cos(x)\ \csc(x)$

(k) ____________

9. Evaluate the following.

   (a) If $f(x) = x^2$ and $x \geq 0$, find the derivative of $f^{-1}(x)$ at x = 4.

   (a) ____________

   (b) If $g(x) = x^3 + 4x^2 - 5x - 20$, find $(g^{-1})'(0)$.

   (b) ____________

(c) $\dfrac{d}{dx}\displaystyle\int_{\pi}^{x}(t^{10} - 7t^{8})(\sin^{-1} t)\,dt$

(c) ______________

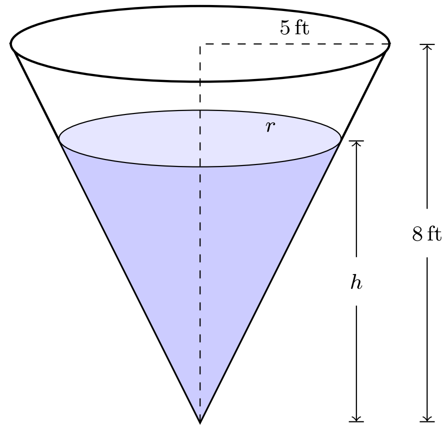
(d) $\dfrac{d}{dx}\displaystyle\int_{3}^{x^2}\sqrt{1 - t^4}\,dt$

(d) ______________

(e) $\dfrac{d}{dx}\displaystyle\int_{x}^{5x^3}\mathrm{e}^{\cos t}\,dt$

(e) ______________

Chapter 4: Applications of Derivatives

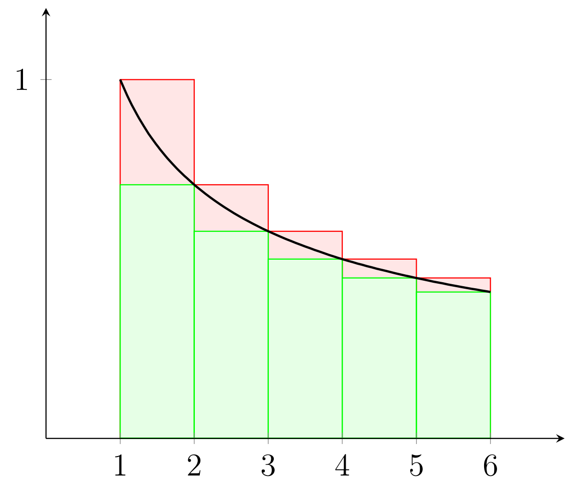


10. Water drains from a large, conical tank. The tank has a radius of 5 feet and a height of 8 feet. (The volume of a cone is given by $V = \dfrac{\pi r^2 h}{3}$.)

    (a) At what rate is the radius of the water in the tank changing when water is draining from the tank at a rate of $1.5\,\frac{\text{ft}^3}{\text{min}}$ and the height of the water in the tank, h, is 4 feet?

(a) ____________

Chapter 5: The Definite Integral



11. The above graph shows area approximations for the graph of $f(x) = x^{-0.5}$. $f(x)$ is defined on the interval [1, 6].

 (a) Using a left Riemann sum with step sizes of 1, approximate $\int_1^6 f(x)\,dx$. Is this approximation an underestimate or an overestimate? Explain your reasoning.

(a) ____________

(a) ____________

 (b) Using a midpoint Riemann sum with step sizes of 1, approximate $\int_1^6 f(x)\,dx$.

(b) ____________

 (c) Using a right Riemann sum with step sizes of 1, approximate $\int_1^6 f(x)\,dx$. Is this approximation an underestimate or an overestimate? Explain your reasoning.

(c) ____________

(c) ____________

12. Erika is playing golf in another universe where the laws of Newtonian mechanics do not apply. The velocity of her ball is modeled as a function of time by $V(t) = 2x^{0.65}e^{\cos(3x)}$ $\frac{\text{m}}{\text{s}}$ for $0 \leq t \leq 10$, where t is measured in seconds. As the ball travels through the air, a drag force acts upon the ball which places an acceleration on the ball opposite the direction of the ball's motion. $D(t)$ $\frac{\text{m}}{\text{s}^2}$ models the drag force acceleration as shown in the table below.

t (seconds)	0	2	5	7	9	10
$D(t)$ (meters/second^2)	0.11	0.74	1.25	1.87	2.40	2.91

Note: $V(t)$ accounts for the ball's velocity WITHOUT a drag force. $D(t)$ is NOT factored into the model for $V(t)$. You may also use a calculator for all parts.

(a) Estimate $D'(8)$. Show the work that leads to your answer. Indicate units of measure.

(a) ______________

(b) Assuming that a drag force is not acting on the ball, find the total distance the ball travels on the interval $1 \leq t \leq 4$. Indicate units of measure.

(b) ______________

(c) Using a right Riemann sum with five subintervals as indicated by the table, estimate the velocity opposite the direction of the ball's motion due to the drag force over the 10 seconds.

(c) ______________

(d) Using your answer from part (c), calculate the net velocity of the ball after 10 seconds.

(d) ______________

(e) For $5 \leq t \leq 10$, is there a time t when the ball's acceleration is the same as the acceleration due to the drag force in the opposite direction? Explain why or why not.

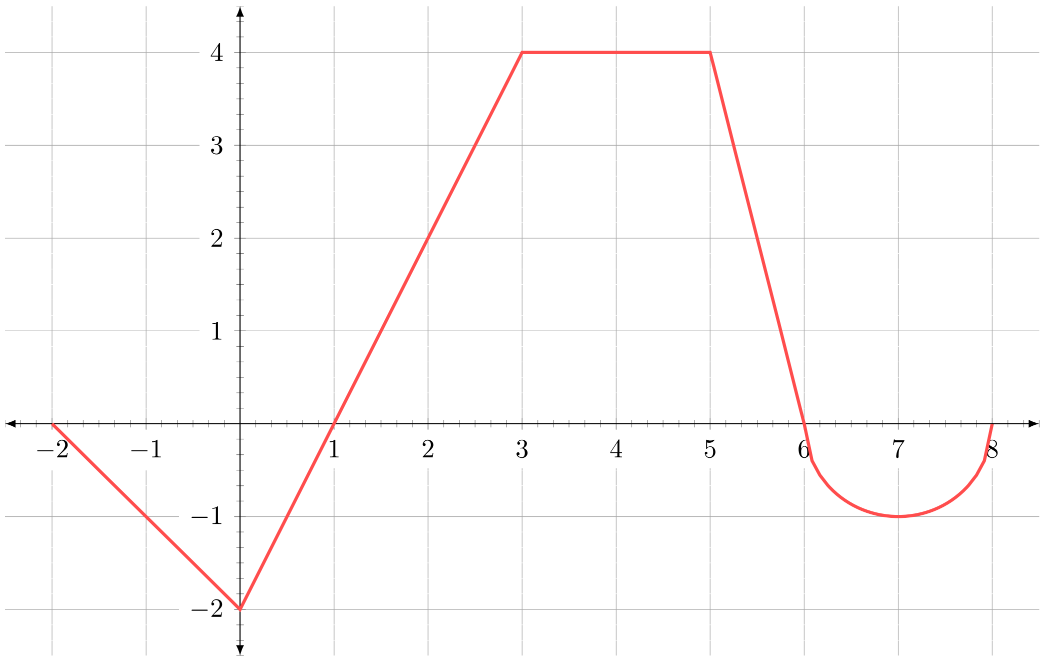
(e) ____________

(f) Let $D(t) = \dfrac{3t}{t^2+9} + 2$. Find $Q(9)$, the velocity due to the drag force after 9 seconds given that the initial velocity due to the drag force at $t = 0$ is 2.5 $\frac{\text{m}}{\text{s}}$.

(f) ____________

(g) Find a model for $Q(t)$, the velocity due to the drag force in the opposite direction of the ball's motion given that the initial velocity due to the drag force at $t = 0$ is 2.5 $\frac{\text{m}}{\text{s}}$.

(g) ____________



Graph of f

13. The figure above shows the graph of the piecewise-linear function f, which consists of a triangle, a trapezoid, and a semi-circle. For $-2 \leq x \leq 8$, the function g is defined by $g(x) = \int_4^x f(t)\,dt$.

(a) Find $g(2)$ and $g'(2)$.

(a) ____________

(a) ____________

(b) Does g have a relative minimum, maximum, or neither at $x = 1$? Explain your solution.

(b) ____________

(c) Determine the concavity of the graph of g at $x = 6$. Does g have a point of inflection at $x = 6$? Explain your solution.

(c) ____________

(d) Find the absolute minimum value of $g(x)$ on the interval $-2 < x < 8$.

(d) ____________

(e) Find the absolute maximum value of $g(x)$ on the interval $-2 \leq x \leq 8$. Explain your solution.

(e) ____________

Chapter 6: Differential Equations and Mathematical Modeling

14. Let $\dfrac{dy}{dx} = \dfrac{y+9}{3x^2-1}$ and $g(\sqrt{21}) = 5$.

(a) Find an equation for the line tangent to the curve at (2, 1).

(a) ______________

(b) Find $\dfrac{d^2y}{dx^2}$ in terms of x and y.

(b) ______________

(c) Let $y = g(x)$ represent a particular solution to the modified separable differential equation, $\dfrac{dy}{dx} = \dfrac{x(y+9)}{3x^2+1}$. Find $g(x)$.

(c) ______________

15. Evaluate each of the following expressions using geometry, known antiderivatives, u-substitution, integration by parts, and partial fraction decomposition..

(a) $\displaystyle\int \mathrm{e}^x \cos(x)\,dx$

(a) ____________

(b) $\displaystyle\int x^2 \sin(x)\,dx$

(b) ____________

(c) $\displaystyle\int \sin(x)\,\mathrm{e}^{\cos(x)}\,dx$

(c) ____________

(d) $\int_{-2}^{2} \sqrt{4-x^2}\,dx \to$ Sketch a graph to help in solving.

(d) ____________

(e) $\int \frac{2x}{1-4x^2}\,dx$

(e) ____________

(f) $\int \frac{4x}{3x^2+x-2}\,dx$

(f) ____________

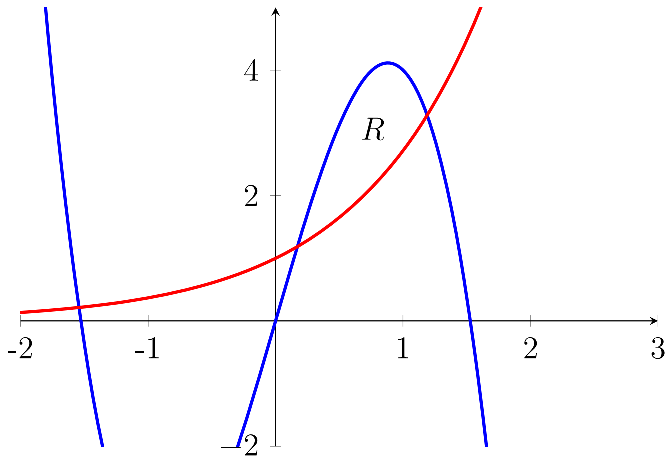
(g) $\int \sin^2(x)\,dx$

(g) ____________

(h) $\int \frac{8}{x^2+3x-10}\,dx$

(h) ____________

Chapter 7: Applications of Definite Integrals



16. Let R be the region enclosed by $f(x) = 7x - 3x^3$ and $g(x) = e^{x-\frac{1}{2}}$ in the first quadrant. You may use a calculator for all parts.

   (a) Find the area of R.

   (a) ______________

   (b) The region R is rotated around the line $y = 5$. Find the volume of the solid that is formed.

   (b) ______________

   (c) The line $x = k$ equally divides the area of R. Using one or more integrals, write, but do not evaluate, an expression that could be used to find a value for k.

   (c) ______________

Answers

1. **Question #1**
   (a) 4
   (b) 16
   (c) 0
   (d) DNE
   (e) -5
   (f) 6

2. **Question #2**
   (a) $\frac{1}{5}$
   (b) 4

3. **Question #3**
   (a) 6
   (b) 8
   (c) 25
   (d) -5

4. **Question #4**
   (a) ∞
   (b) 0
   (c) $\frac{21}{40}$

5. **Question #5**
   (a) ∞
   (b) $-\infty$
   (c) ∞
   (d) $-\infty$

6. **Question #6**
   Show that $\lim_{x\to 0} \frac{\tan(x)}{x} = 1$.

7. **Question #7**
   Show that $\lim_{x\to 0} \frac{1-\cos(x)}{x} = 0$.

8. **Question #8**
   (a) $3x^2 + 4x - \frac{1}{2}$
   (b) $-\frac{4}{3x^{\frac{7}{3}}} - \frac{5}{2x^{\frac{3}{2}}} + \frac{1}{x}$
   (c) $4\sec^2(4x)$
   (d) $\cos 2x$
   (e) $\sin 2x$
   (f) $\frac{2}{2x+1}$
   (g) $2^x \ln 2$
   (h) $2\mathrm{e}^{2x}$
   (i) $2\mathrm{e}^{2x}$
   (j) $\frac{5}{1+25x^2}$
   (k) $-\csc^2 x$

9. **Question #9**
   (a) $\frac{1}{4}$
   (b) $\frac{1}{10+8\sqrt{5}}, \frac{1}{10-8\sqrt{5}}, \frac{1}{11}$
   (c) $(x^{10} - 7x^8)(\sin^{-1} x)$
   (d) $2x\sqrt{1-x^8}$
   (e) $15x^2\mathrm{e}^{\cos(5x^3)} - \mathrm{e}^{\cos x}$

10. **Question #10**
    (a) $-\frac{3}{20\pi}\,\frac{\text{ft}}{\text{min}}$

11. **Question #11**
    (a) 3.232, overestimate
    (b) 2.881
    (c) 2.640, underestimate

12. **Question #12**
    (a) $0.265\,\frac{\text{m}}{\text{s}^3}$
    (b) $12.503\,\text{m}$
    (c) $16.68\,\frac{\text{m}}{\text{s}}$
    (d) $-6.256\,\frac{\text{m}}{\text{s}}$
    (e) Yes, there is at least one time t for which the ball's acceleration is the same as the drag force acceleration on the interval $5 \leq t \leq 10$.
    (f) $23.954\,\frac{\text{m}}{\text{s}}$
    (g) $Q(t) = \ln|t^2+9|^{\frac{3}{2}} + 2t + \frac{5}{2} - \ln 27$

13. **Question #13**
    (a) $g(2) = -7, g'(2) = 2$
    (b) g has a relative minimum at $x = 1$.
    (c) g does not have a point of inflection at $x = 6$.
    (d) g has an absolute minimum at $(1, -8)$ on $-2 < x < 8$.
    (e) g has an absolute maximum at $(6, 6)$ on $-2 \leq x \leq 8$.

14. **Question #14**
    (a) $y = \frac{10}{11}(x-2) + 1$
    (b) $\frac{d^2y}{dx^2} = \frac{(y+9)(1-6x)}{(3x^2-1)^2}$
    (c) $g(x) = 7|3x^2+1|^{\frac{1}{6}} - 9$

15. **Question #15**
    (a) $\frac{e^x(\sin x + \cos x)}{2} + C$
    (b) $-x^2 \cos x + 2x \sin x + 2\cos x + C$
    (c) $-\mathrm{e}^{\cos x} + C$
    (d) 2π
    (e) $\frac{1}{\ln|1-4x^2|^{\frac{1}{4}}} + C$
    (f) $\ln|(3x-2)^{\frac{8}{15}}(x+1)^{\frac{4}{5}}| + C$
    (g) $\frac{1}{2}x - \frac{\sin(2x)}{4} + C$
    (h) $\frac{8}{7}\ln\left|\frac{x-2}{x+5}\right| + C$

16. **Question #16**
    (a) 2.187
    (b) 35.667
    (c) Two possible solutions to find k using one or more integral expressions.

