

AP CALCULUS SUMMER ASSIGNMENT

Have this assignment completed and the material that it covers understood by the first day of school. You will be tested on its contents when AP calculus starts.

1. Are the following statements true? If not, explain in words why not.

a) $\frac{2k}{2x+h} = \frac{k}{x+h}$

b) $\frac{1}{p+q} = \frac{1}{p} + \frac{1}{q}$

c) $\frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$

d) $3\left(\frac{a}{b}\right) = \frac{3a}{3b}$

e) $3\left(\frac{a}{b}\right) = \frac{3a}{b}$

f) $3\left(\frac{a+b}{c}\right) = \frac{3a+b}{c}$

2. Simplify.

a) $\frac{\frac{x}{2}}{\frac{x}{4}}$

b) $h + \frac{x+h}{h}$

c) $\frac{\sqrt{x-2} + \frac{5}{\sqrt{x-2}}}{x-2}$

3. Solve for y' .

a) $xy' + y = 1 + y'$

b) $3y^2y' + 2yy' = 5y' + 2x$

c) $3x^2yy' + 2xy^2 = 2yy'$

4. Solve the quadratic equation.

a) $4x^2 - 21x - 18 = 0$

b) $2x^2 - 3x + 3 = 0$

c) $x^4 - 9x^2 + 8 = 0$

5. Write as a single fraction with the denominator in factored form.

a) $\frac{7x^2+5x}{x^2+1} - \frac{5x}{x^2-6}$

b) $20\left(\frac{2}{x+1} - \frac{3}{x}\right)$

6. Graph the equation $y = x^3 - x$ and answer the following questions.

- Is the point (3, 2) on the graph?
- Is the point (2, 6) on the graph?
- Is the function even, odd, or neither?
- Find the x-intercepts.

7. Determine algebraically if the function is even, odd, or neither.

a) $f(x) = 2x^2 - 7$

b) $f(x) = -4x^3 - 2x$

c) $f(x) = 4x^2 - 4x + 4$

8. Find the equation of the line that passes through the point (2, 4) and is parallel to the line $2x + 3y - 8 = 0$.

9. Find the equation of the line that is perpendicular to the line $2x + 3y - 8 = 0$ at the point $(1, 2)$.

10. The line with slope 5 that passes through the point $(-1, 3)$ intersects the x-axis at a point. What are the coordinates of this point?

11. What are the coordinates of the point at which the line passing through the points $(1, -3)$ and $(-2, 4)$ intersects the y-axis?

12. Find $f(1) - f(5)$ given $f(x) = |x - 3| - 5$.

13. Find $f(x+2) - f(2)$ given $f(x) = x^2 - 3x + 4$.

14. Use interval notation to identify the domain for each of the following functions.

a) $h(x) = \frac{1}{4x^2 - 21x - 18}$ b) $k(x) = \sqrt{x^2 - 5x - 14}$ c) $p(x) = \frac{\sqrt[3]{x-6}}{\sqrt{x^2 - x - 30}}$

d) $d(x) = \ln(2x - 12)$

15. Find $f(x + \Delta x)$ for $f(x) = x^2 - 2x - 3$.

16. Find $\frac{f(x + \Delta x) - f(x)}{\Delta x}$ if $f(x) = 8x^2 + 1$.

17. Find $\frac{f(x + h) - f(x)}{h}$ if $f(x) = \frac{1}{x}$.

18. Graph the function.

a) $f(x) = \begin{cases} 1 & x \leq 0 \\ -1 & x > 0 \end{cases}$ b) $f(x) = \begin{cases} 2x & (-\infty, -1) \\ 2x^2 & [-1, 2) \\ -x + 3 & (2, \infty) \end{cases}$ c) $f(x) = \sqrt{16 - x^2}$

19. Given $f(x) = x - 3$ and $g(x) = \sqrt{x}$ complete the following.

a) $f(g(x)) =$ b) $g(f(x)) =$ c) $f(f(x)) =$

20. Given $f(x) = \frac{1}{x-5}$ and $g(x) = x^2 - 5$ complete the following.

a) $f(g(7)) =$ b) $g(f(v)) =$ c) $g(g(x)) =$

21. Let $f(x) = 2x - 2$. Complete the following.

- a) Graph f .
- b) Determine whether f has an inverse function.
- c) Graph f^{-1} .
- d) Give the equation for f^{-1} .

22. Simplify using only positive exponents. Do not rationalize the denominator.

a) $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$ b) $\left(\frac{1}{x^{-2}} + \frac{4}{x^{-1}y^{-1}} + \frac{1}{y^{-2}}\right)^{-\frac{1}{2}}$ c) $\left(\frac{x^{-2}}{y^{-1}} - x\right)^{-3}$

23. If $f(x) = x^2 - 1$, describe in words what the following would do to the graph of $f(x)$.

- a) $f(x) - 4$ b) $f(x - 4)$ c) $-f(x + 2)$
- d) $5f(x) + 3$ e) $f(2x)$ f) $|f(x)|$

24. Find the surface area of a box of height h whose base dimensions are p and q , and satisfies the following condition.

- a) The box is closed.
- b) The box has an open top.
- c) The box has an open top and a square base with side length p .

25. A seven foot ladder, leaning against a wall, touches the wall x feet above the ground. Write an expression in terms of x for the distance from the foot of the ladder to the base of the wall.

26. A piece of wire 5 inches long is to be cut into two pieces. One piece is x inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of x .

27. Evaluate. Be sure to answer in radians.

a) $\cos 0$ b) $\sin 0$ c) $\tan \frac{\pi}{2}$

d) $\cos \frac{\pi}{4}$

e) $\sin \frac{\pi}{2}$

f) $\sin \pi$

g) $\arcsin \frac{\sqrt{3}}{2}$

h) $\arctan(-1)$

i) $\arccos\left(-\frac{1}{2}\right)$

28. Find the solution of the equation for $\theta \in [0, 2\pi)$.

a) $2 \sin^2 \theta = 1 - \sin \theta$

b) $2 \tan \theta - \sec^2 \theta = 0$

c) $\sin 2\theta + \sin \theta = 0$

29. Which of the following expressions are identical?

a) $\cos^2 x$

b) $(\cos x)^2$

c) $\cos x^2$

30. Which of the following expressions are identical?

a) $(\sin x)^{-1}$

b) $\arcsin x$

c) $\sin x^{-1}$

d) $\frac{1}{\sin x}$

31. Solve for x .

a) $\ln e^3 = x$

b) $\ln e^x = 4$

c) $\ln x + \ln x = 0$

d) $e^{\ln 5} = x$

e) $\ln 1 - \ln e = x$

f) $\ln 6 + \ln x - \ln 2 = 3$

g) $\ln(x+5) = \ln(x-1) - \ln(x+1)$