

Advance Placement Calculus BC

AP CALCULUS - BC - 3460

Grade 12

Full year - 1 credit

Prerequisite: Introduction to BC Calculus Honors or faculty approval – Honors Policy Applies

AP Overview

Our AP Calculus BC class meets for a 42 minutes period 5 times a week. Students entering the class have taken a calculus honors class in their junior year which covered many pre-calculus prerequisites as well as an introduction to Calculus 1 topics.

AP Calculus is the continuation of our Introduction to AP Calculus BC Honors course. The course is intended for students who have a thorough knowledge of analytic geometry and elementary functions in addition to college preparatory mathematics. All students in this course prepare to take the BC Calculus Advanced Placement examination in May. Calculus BC curriculum content is considerably more extensive than Calculus AB including the units of parametric and polar functions and infinite sequences and series. The use of a graphics calculator is an integral part of this course and is required for the AP examination.

General Department Philosophy

The Garden City Mathematics Department presents courses that align with either the New York State Regents curriculum or the College Board's Advanced Placement curriculum. In either case, the course material prepared by the Department (Grades 6 – 12) is fully consistent with these standards. In particular, our Advanced Placement syllabi have been approved by the College Board. Our Regents courses address the five NYS content strands as well as the five process strands. Our instructional activities are created to promote written and verbal mathematical communication and critical thinking skills that employ accurate mathematical ideas. The Department develops student assessments that are also consistent with the New York State and/or College Board assessment in both style and content. The scoring rubrics employed by the Department are modeled after the particular associated scoring guides. Additional information about the NYS Mathematics program can be found at <https://www.engageny.org/resource/grades-9-12-mathematics-curriculum-map> and Advanced Placement program at <http://apcentral.collegeboard.com>.

Members of the Department encourage their students to explore, discover and question the many fundamental concepts developed within each courses. The primary objective is to engage our students in lessons that are meaningful, inspiring and enjoyable and promote a greater interest in mathematics – at the post secondary level and beyond. Technology applications, such as calculator usage, are incorporated as developmentally appropriate and as specified by College Board curriculum. The department wants each student to realize that they can make a contribution to their class and that others can benefit from their input. The department wants all students to maximize their mathematical potential as we move through the challenging curriculum and prepare to master all course requirements.

Specific AP Teaching Strategies/Philosophy

In class, we promote a comfortable yet challenging learning atmosphere. We encourage students to question, explore and discover concepts as they are learning. The objective is to engage students in enjoyable lessons to promote a greater interest in mathematics. We want each student to realize they have something to contribute to the class and that others will benefit from their input as well as from my lessons. We want students to feel that sense of accomplishment after moving through a challenging topic and looking forward to what lies ahead of them. Students need to have a balanced approach to their solutions: Analytic, algebraic, numerical, graphic, and verbal methods of representing problems. Students must understand the proper use of technology with this course. Students must have a strong mathematics foundation and make the important connections between the concepts learned. The graphing calculator is a powerful tool to help develop a visual understanding of the material as well as an excellent computational resource to work on an application possibly in the area of science, business, or engineering.

Graphing Calculator Applications and Integration

We are fortunate in our school in that students enter BC Calculus with an extensive knowledge base in graphing calculator applications. Starting in the seventh grade, our students are presented with ways to integrate graphing calculator technology into their mathematical discovery and skill development. Functions such as window, trace, max/min, degree/radian, table, plot format, regression models, roots and intersections are fully developed and implemented. The calculator technology is further enhanced through the BC Calculus instruction.

Students learn how to visualize graphs (in rectangular, polar and parametric modes) as a way to interpret data and reach conclusions. While viewing the graph of $f'(x)$, students clearly understand ways to extrapolate details about $f(x)$ and $f''(x)$. Through slope field and other differential functions calculator applications, students are able to visualize significant results based upon the output. Students are encouraged to explore alternate graphing calculator methods to find solutions and to develop a deeper understanding of the calculus. Through instructional models, students quickly learn how to represent functions graphically, numerically, analytically and verbally. They also learn ways to integrate these representations into a meaningful cohesive entity.

Functions from Multiple Representations

As we all know, functions can be represented a variety of ways and methods. Certainly, these include graphical, numerical, analytical and verbal. As a regular and consistent component of the instructional model, students develop an appreciation for the different function representations and, more importantly, learn how to integrate each of these forms to promote greater understanding of the calculus.

Each representation format is often associated with various advantages and disadvantages. For example, when a function is described in graphical form, students learn ways to draw broad conclusions about the functions behavior and also how to reach specific details. Graphing calculator technology is very helpful in providing students with visual opportunities to connect multiple representations for a more meaningful learning experience. This type of instruction also allows students to practice better mathematical communication as they share ideas, discoveries and skills with their classmates.

Student Evaluation

Students are given homework on a regular basis. They work on questions assigned from the textbook and from supplemental sources. Towards the end of a topic, the students are assigned practice AP questions reflecting the different ideas learned. When reviewing homework assignments, students share their methodology, calculator applications and problem-solving strategies with the class.

In addition to homework, problem sets are assigned every two weeks. Students are encouraged to work cooperatively to discuss their ideas with their classmates but they are to turn in their own solutions. Students are assigned released multiple-choice from past AP exams on a monthly basis which is followed by a discussion in class regarding the examples from each test. Students are given quizzes to cover small pieces of material, usually work from the prior few days. Tests cover large amounts of material and are always cumulative. Tests are divided into calculator/non calculator sections and contain multiple-choice and free-response questions.

Course Outline

I Limits & Continuity

- a. Concept of a Limit
 - i. A graphic approach to limits
 - ii. Properties of limits
 - iii. Algebraic techniques to evaluating limits
 - iv. Limits involving infinity
- b. Continuity
 - i. Definition of Continuity
 - ii. Geometric understanding of the graphs of continuous functions
 - iii. Differentiability and Continuity

II The Derivative

- a. Definition of the derivative – the tangent line problem
- b. Differentiation Rules and Procedures
- c. Derivatives of Trigonometric Functions
- d. Derivatives of Exponentials and Logarithmic Functions
- e. Derivatives of Inverses Trigonometric Functions
- f. Implicit Differentiation

III Applications of the Derivatives

- a. Related Rates
- b. Intermediate Value Theorem and Extreme Value Theorem
- c. Mean Value Theorem and Rolle's Theorem
- d. Critical values, relative(local) and absolute (global)
- e. The first and second derivative tests
- f. Concavity and points of inflection
- g. Comparing the graphs of f , f' , and f''
- h. Modeling Maximum and Minimum Problems

- i. Local linearity & Newton's Approximation Method
- j. Particle motion; position, velocity, and other rates of change including rectilinear motion

IV The Definite Integral

- a. Concept of Riemann Sums
 - a. Includes left-hand, right-hand and midpoint applications
- b. Estimation of change in a function using Riemann sums
- c. Definition & Interpretation of the Definite Integral
- d. Definite Integral & Antidifferentiation
- e. The Fundamental Theorem of Calculus
- f. Average Value of a Function
- g. Trapezoidal Rule
 - a. Discovery exercise
- h. Exercises to explore connection between concavity and over/under approximations
- i. Definite integral as an accumulator

V Techniques of Integration

- a. Integration by u-substitution
- b. Integration by parts
- c. Integration by partial fractions

VI Applications of Definite Integrals

- a. Area between two functions
 - i. Discovery/Exploration activity (calculator active)
- b. Volume of solids of revolution
 - i. Disk method
 - ii. Washer method
 - iii. Shell method (optional)
 - iv. Volume of solid with known parallel cross sectional areas
- c. Arc length and surface of revolution (optional)

VII Differential Equations

- a. Definition of a differential equation
- b. Exponential growth & decay
- c. Solving differential equations by separation of variables
- d. Solving differential equations with initial conditions
 - i. Logistic Models
- e. Slope Fields
- f. Euler's Method

VIII Parametric Functions

- a. Introduction to Parametric Equations
- b. Derivatives of Parametric Equations & Length of Curve
 - i. Derivative at a point
 - ii. $\frac{d^2y}{dx^2}$
- c. Vectors in the Plane

- d. Vector Valued Functions
 - i. Velocity, Speed, Acceleration, Direction, Distance

IX Polar Curves

- a. Polar coordinates and polar graphs
 - i. Exploration activity
 - ii. Polar graph lab experience (calculator active)
- b. Derivatives of polar curves
- c. Area in polar coordinates
- d. Arc length of polar curves

X Improper Integrals

- a. L'Hopital's rule revisited
- b. Related Rates of Growth
- c. Improper Integrals
 - i. Introduction to convergence tests
 - ii. Discovery activity

XI Sequences & Series

- a. Sequences
 - i. Review of properties of sequences
 - ii. Convergence of n^{th} term of a sequence
- b. Series
 - i. Introduction to Infinite Series and their properties
 - ii. Convergent and Divergent Series (partial sums, telescoping, etc.)
 - iii. Geometric Series
 - a. Convergence of a Geometric Series
 - b. Sum of geometric series
 - iv. p-series
 - a. Convergence of a p-series (illustrated by integral test)
 - b. Harmonic Series and Alternating Harmonic Series
 - v. Alternating Series
 - a. Alternating series test
 - b. Alternating series remainder/error
 - c. Absolute and conditional convergence
- d. Convergence Tests
 - i. n^{th} term test for divergence, integral test, direct comparison test, limit comparison test, alternating series test, ratio test, root test
- e. Power Series
 - i. Radius and Interval of Convergence
 - ii. Testing Convergence at Endpoints
 - iii. Term-by-term Differentiation and Integration
 - iv. Representation of Functions by a Power Series
- f. Taylor Series

- i. Taylor polynomials as approximations for functions
 - 1. Graphic exploration activity
- ii. Maclaurin polynomials
- iii. Taylor's Theorem with the Lagrange Error Form of the Remainder (Lagrange Error Bound)
- iv. Find a Taylor series for a function including $\ln x$.
- v. Maclaurin series for $\sin x$, $\cos x$, e^x , $\frac{1}{1-x}$
- vi. Forming a new series from a known Taylor series

Review for the AP Calculus BC Test

During this period, students work on free-response questions from the released exams. Students are given two days to complete six questions. The questions are collected and graded according to the scoring guidelines and then the questions and scoring are discussed the next day in class. Students are given the opportunity to experience rubric scoring and the AP scoring process. Over the course of the year, students learn to communicate in a mathematical fashion and to provide all necessary justifications. Similarly, students make extensive use of graphing calculator technology (TI83+ or TI84+) as it is incorporated into every unit of study.

Students are also completing multiple choice questions on a weekly basis. These questions are addressed in class as well. Students are encouraged to work cooperatively on in-class assignments and extended problem sets. Several assessments will be administered during class meeting times that are similar in design to the questions being studied by the students. Special consideration is given to assessment items that represent multiple topics. This ensures mastery of all the necessary concepts reflected in the released AP examinations and further develops an appreciation for the integrated nature of mathematics.

Primary Textbook

Finney, Ross L., Franklin Demana, Bert Waits, and Daniel Kennedy.
Calculus: Graphical, Numerical, Algebraic. Media Update 3rd Edition Pearson Education Inc., 2010.

Other Resources

Hughes-Hallet, Deborah, et al. *Calculus: Single Variable*. New York: John Wiley & Sons, 2005.

Larson, Ron, Bruce H. Edwards, and Robert P. Hostetler.
Calculus of a Single Variable. Boston: Houghton Mifflin, 2002.

Leithold, Louis. *The Calculus with Analytic Geometry*. New York: Harper & Row, Publishers, 1990.

Stewart, James

Calculus, Belmont, CA: Thomson Learning, Inc. 2003.

http://apcentral.collegeboard.com/apc/public/courses/teachers_corner/2118.html

<http://www.sosmath.com/>

<http://archives.math.utk.edu/visual.calculus/>

http://mathforum.org/library/drmath/sets/high_calculus.html

<http://www.calculus.org/>

<http://www.calculus-help.com/funstuff/phobe.html>

<http://www.math.ucdavis.edu/~kouba/ProblemsList.html>

<http://www.plu.edu/~heathdj/java/>

<http://www.ma.iup.edu/projects/CalcDEMma/Summary.html>

<http://education.ti.com/educationportal/sites/US/homePage/index.html>