

### About the Advanced Placement Program<sup>®</sup> (AP<sup>®</sup>)

The Advanced Placement Program<sup>®</sup> enables willing and academically prepared students to pursue college-level studies — with the opportunity to earn college credit, advanced placement, or both — while still in high school. AP Exams are given each year in May. Students who earn a qualifying score on an AP Exam are typically eligible to receive college credit and/or placement into advanced courses in college. Every aspect of AP course and exam development is the result of collaboration between AP teachers and college faculty. They work together to develop AP courses and exams, set scoring standards, and score the exams. College faculty review every AP teacher's course syllabus.

### AP Calculus Program

AP Calculus AB and AP Calculus BC focus on students' understanding of calculus concepts and provide experience with methods and applications. Although computational competence is an important outcome, the main emphasis is on a multirepresentational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations are important.

Teachers and students should regularly use technology to reinforce relationships among functions, to confirm written work, to implement experimentation, and to assist in interpreting results. Through the use of the unifying themes of calculus (e.g., derivatives, integrals, limits, approximation, and applications and modeling) the courses become cohesive rather than a collection of unrelated topics.

### AP Calculus BC Course Overview

AP Calculus BC is roughly equivalent to both first and second semester college calculus courses and extends the content learned in AB to different types of equations and introduces the topic of sequences and series. The AP course covers topics in differential and integral calculus, including concepts and skills of limits, derivatives, definite integrals, the Fundamental Theorem of Calculus, and series. The course teaches students to approach calculus concepts and problems when they are represented graphically, numerically, analytically, and verbally, and to make connections amongst these representations.

Students learn how to use technology to help solve problems, experiment, interpret results, and support conclusions.

#### PREREQUISITE

Before studying calculus, all students should complete four years of secondary mathematics designed for college-bound students: courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise-defined functions. In particular, before studying calculus, students must be familiar with the properties of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and so on) and know the values of the trigonometric functions at the numbers  $0$ ,  $\pi/6$ ,  $\pi/4$ ,  $\pi/3$ ,  $\pi/2$ , and their multiples.

### Use of Graphing Calculators

Professional mathematics organizations have strongly endorsed the use of calculators in mathematics instruction and testing. The use of a graphing calculator in AP Calculus BC is considered an integral part of the course.

### Goals of AP Calculus BC

Students who are enrolled in AP Calculus BC are expected to

- Work with functions represented in multiple ways: graphical, numerical, analytical, or verbal. They should understand the connections among these representations.
- Understand the meaning of the derivative in terms of a rate of change and local linear approximation and use derivatives to solve problems.
- Understand the meaning of the definite integral as a limit of Riemann sums and as the net accumulation of change and use integrals to solve problems.
- Understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- Communicate mathematics and explain solutions to problems verbally and in writing.
- Model a written description of a physical situation with a function, a differential equation, or an integral.
- Use technology to solve problems, experiment, interpret results, and support conclusions.
- Determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
- Develop an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

### Topic Outline for AP Calculus BC

#### I. Functions, Graphs, and Limits

- Analysis of Graphs
- Limits of Functions (including one-sided limits)
- Asymptotic and Unbounded Behavior
- Continuity as a Property of Functions
- Parametric, Polar, and Vector Functions

#### II. Derivatives

- Concept of the Derivative
- Derivative at a Point
- Derivative as a Function
- Second Derivatives
- Applications and Computation of Derivatives

#### III. Integrals

- Interpretations and Properties of Definite Integrals
- Applications of Integrals
- Fundamental Theorem of Calculus
- Techniques and Applications of Antidifferentiation
- Numerical Approximations to Definite Integrals

#### IV. Polynomial Approximations and Series

- Concept of Series
- Series of constants
- Taylor Series

## AP Calculus BC Exam Structure

### AP BC CALCULUS EXAM: 3 HOURS 15 MINUTES

#### Assessment Overview

The AP Calculus BC Exam questions measure students' understanding of the concepts of calculus, their ability to apply these concepts, and their ability to make connections among graphical, numerical, analytical, and verbal representations of mathematics. Adequate preparation for the exam also includes a strong foundation in algebra, geometry, trigonometry, and elementary functions, though the course necessarily focuses on differential and integral calculus. Students may not take both the Calculus AB and Calculus BC Exams within the same year. A Calculus AB sub-score is reported based on performance on the portion of the Calculus BC Exam devoted to Calculus AB topics.

The free-response section tests students' ability to solve problems using an extended chain of reasoning. During the second timed portion of the free-response section (Part B), students are permitted to continue work on problems in Part A, but they are not permitted to use a calculator during this time..

#### Format of Assessment

**Section I:** Multiple Choice | 45 Questions | 105 minutes | 50% of Exam Score

- **Part A:** 28 questions; 55 minutes (no calculator permitted)
- **Part B:** 17 questions; 50 minutes (graphing calculator permitted)

**Section II:** Free Response | 6 Questions | 90 minutes | 50% of Exam Score

- **Part A:** 2 problems; 30 minutes (graphing calculator permitted)
- **Part B:** 4 problems; 1 hour (no calculator permitted)

## AP CALCULUS BC SAMPLE QUESTIONS

### Sample Multiple-Choice Question

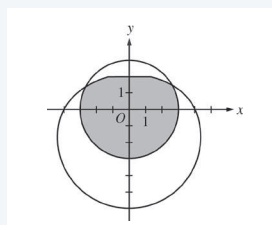
$x$	2	5	7	8
$f(x)$	10	30	40	20

The function  $f$  is continuous on the closed interval  $[2,8]$  and has values that are given in the table above. Using the subintervals  $[2,5]$ ,  $[5,7]$ , and  $[7,8]$ , what is the trapezoidal approximation of  $\int_2^8 f(x)dx$ ?

- (A) 110      (B) 130      (C) 160      (D) 190      (E) 210

Correct Answer: C

### Sample Free-Response Question: Experimental Design



The graphs of the polar curves  $r = 3$  and  $r = 4 - 2\sin \theta$  are shown in the figure above. The curves intersect when  $\theta = \frac{\pi}{6}$  and  $\theta = \frac{5\pi}{6}$ .

- Let  $S$  be the shaded region that is inside the graph of  $r = 3$  and also inside the graph of  $r = 2\sin \theta$ . Find the area of  $S$ .
- A particle moves along the polar curve  $r = 4 - 2\sin \theta$  so that at time  $t$  seconds,  $\theta = t^2$ . Find the time  $t$  in the interval  $1 \leq t \leq 2$  for which the  $x$ -coordinate of the particle's position is  $-1$ .
- For the particle described in part (b), find the position vector in terms of  $t$ . Find the velocity vector at time  $t = 1.5$ .