Honors Pre-Calculus Summer Preparation

Welcome to Honors Pre-Calculus! We look forward to working with you during the 2021 – 2022 school year. While you are enjoying your summer, please take time to continue to practice and make sure you have mastered the prerequisite skills necessary for your success in the course, and that you maintain the skill so that you can start the year strong.

This packet and the Canvas course (which you should have received an invitation for) are for your use and reference. **None of the work done this summer will be graded at any time.** It is for you to use to make sure you are prepared for the year**. We will have an Entrance Test on the second day of school that covers these concepts, so that you can show your readiness for the course.**

Within this Canvas course you will find 6 modules. These modules contain prerequisite skills that are crucial to your success in Honors Pre-Calculus. You should recognize these concepts from Algebra 2. Each module has videos, paper practice, and a practice quiz. Here is our suggestion for how you should approach each module:

1. Take the quiz to see if you already understand the pre-requisite skills. If you earn a 100% on the quiz, you have shown mastery of the skills and do not need to complete the notes/practice *(unless you really want to…and if so, go for it!)*.
2. If you do NOT earn a 100%, read the notes, complete the practice on a separate sheet of paper using the key and videos as needed to help you review the concepts and fine-tune your skills before attempting the quiz again.
3. Once you feel more confident, try the quiz again! You have unlimited attempts to complete the quiz to earn a 100%.
4. Once you have earned a 100% on the quiz, move on to the next module and repeat!

If you have questions or concerns, you can reach out to Mrs. Luksic (luksicde@delawarecityschools.net) by email.

We look forward to meeting you in August! Go Pacers! - Hayes Math Department

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Algebra Review for Pre-Calculus & Calculus

You will find that most calculus concepts are not difficult to grasp; however, solving calculus problems requires *strong algebra and trigonometry skills*. If you are weak in these areas, you will likely struggle, despite having a solid understanding of calculus. It is therefore crucial that you brush up on these skills before entering pre-calculus! Completing this packet (and holding on to it!!) will certainly help!

The following pages contain various algebra topics that are used in the study of both pre-calculus and calculus. These pages were designed so that students can refresh their knowledge of topics to help better prepare themselves for upper-level math courses. Each section contains practice problems for you to try, but please do not feel you need to do all of the problems.

Adapted from <https://www.swarthmore.edu/math-stat-academic-support/review-math-15-math-25

1. Exponents

Laws of Exponents Assume *a* and *b* are real numbers and *m* and *n* are integers	**Examples using the Laws of Exponents**
1. $a^m \cdot a^n = a^{m+n}$	**1.** a. $x^7 \cdot x^5 = x^{12}$ b. $2^3 \cdot 2^2 = 2^5 = 32$
2. $(ab)^n = a^n b^n$	**2.** a. $(xy)^4 = x^4 y^4$ b. $(2 \cdot 5)^3 = 2^3 \cdot 5^3 = 8 \cdot 125 = 1000$
3. $\frac{a^m}{a^n} = a^{m-n}$	**3.** a. $\frac{x^{10}}{x^7} = x^3$ b. $\frac{4^5}{4^7} = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
4. $(a^m)^n = a^{mn}$	**4.** a. $(z^3)^2 = z^6$ b. $(5^2)^3 = 5^6 = 15625$
5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad (b \neq 0)$	**5.** a. $\left(\frac{x}{y}\right)^4 = \frac{x^4}{y^4}$ b. $\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3} = \frac{8}{125}$
Zero Exponent: If a is a nonzero number, then **6.** $a^0 = 1$	**6.** a. $x^0 = 1$ b. $12^0 = 1$
Negative Exponent: If a is a nonzero number and n is a nonzero integer, then **7.** $a^{-n} = \frac{1}{a^n}$	**7.** a. $x^{-3} = \frac{1}{x^3}$ b. $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

Watch out for the following common exponent mistakes!

1. Exponents applied to polynomials: remember, *these need to be multiplied out!*
   $(a+b)^2 \neq a^2 + b^2$. The correct way is $(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$

2. Parenthesis and negative signs:
   a. $(-2)^4 = 16$ b. $-2^4 = 16$ c. $(-2)^3 = -8$

   In part (a), the exponent is applied to the number -2; notice the *even* exponent makes the outcome positive.
   In part (b), the exponent is applied ONLY to the number 2, so the outcome is negative (since the negative gets applied after the exponent is applied to the 2.
   In part (c), the exponent is applied to the number -2, but since the exponent is *odd*, the outcome is negative.
3. Negative exponents: remember, a negative exponent requires a reciprocal to make it positive!
   Note: $3^{-2} \neq -3^2$, The correct way is $3^{-2} = \frac{1}{3^2}$.

Practice Problems:

Use the properties of exponents to simplify each exponential expression. Write all expressions in the numerator.

1. $(-4)^3\cdot(-4)^2$	2. $(-5)^2\cdot(-5)^6$	3. 2^0	4. -2^0
5. $(5m)^0$	6. $(2^2)^5$	7. $(2x^5y^4)^3$	8. $(-4m^3n^9)^2$
9. $-\left(\frac{p^4}{q}\right)^2$	10. $\left(\frac{r^8}{s^2}\right)^3$	11. $\left(\frac{x^8}{x^4}\right)^2$	12. $(x^4\cdot x^3)^3$

Simplify each of the following so that no negative exponents remain.

13. $(-4)^{-3}$	14. $(-5)^{-2}$	15. $\left(\frac{1}{5}\right)^{-2}$	16. $2^0\cdot5^{-3}$
17. $3x^{-2}$	18. $(5y)^{-2}$	19. $(x^{-2}y^3)(x^4y^{-4})$	20. xy^{-3}
21. $5m^2n^{-4}$	22. $\frac{2x^{-3}z^0}{y^{-4}}$	23. $\left(\frac{x^3}{y^{-5}}\right)^{-2}$	24. $5^0\cdot y^{-3}$

Perform the indicated operations. Write answers using only positive exponents.

25. $2^{-3}\cdot2^{-4}$	26. $5^{-2}\cdot5^{-6}$	27. $9^{-4}\cdot9^{-1}$	28. $\frac{4^{-2}\cdot4^{-1}}{4^{-3}}$
29. $\frac{3^{-1}\cdot3^{-4}}{3^2\cdot3^{-2}}$	30. $\frac{7^3 7r^{-3}}{7^2r^{-2}}$	31. $\left(\frac{r^{-2}}{s^{-5}}\right)^{-3}$	32. $\frac{-4a^{-1}\cdot a^4}{a^{-2}}$
33. $\frac{x^2\cdot x^{-7}}{4y^{-3}}$	34. $\left(\frac{5x^7}{x^7y^6}\right)^0$	35. $\left(\frac{x^2\cdot x^4}{y^{-1}\cdot y^{-3}}\right)^{-2}$	36. $\frac{(6z)^{-2}z^3}{y^2\cdot3y^{-2}}$

Solutions:

1. $(-4)^5$	2. $(-5)^8$	3. 1	4. -1	5. 1	6. 2^{10}
7. $8x^{15}y^{12}$	8. $16m^6n^{18}$	9. $-p^8q^{-2}$	10. $r^{24}s^{-6}$	11. x^8	12. x^{21}
13. $-\frac{1}{4^3}$	14. $\frac{1}{25}$	15. 25	16. $\frac{1}{125}$	17. $\frac{3}{x^2}$	18. $\frac{1}{25y^2}$
19. $\frac{x^2}{y}$	20. $\frac{x}{y^3}$	21. $\frac{5m^2}{n^4}$	22. $\frac{2y^4}{x^3}$	23. $\frac{1}{x^6y^{10}}$	24. $\frac{1}{y^3}$
25. $\frac{1}{2^7}$	26. $\frac{1}{5^8}$	27. $\frac{1}{9^5}$	28. 1	29. $\frac{1}{3^5}$	30. $\frac{7^2}{r}$
31. $\frac{r^6}{s^{15}}$	32. $-4a^5$	33. $\frac{y^3}{4x^5}$	34. 1	35. $\frac{1}{x^{12}y^8}$	36. $\frac{z}{108}$

2. Radicals & Rational Exponents

Definitions of $a^{\frac{1}{n}}$ and $\sqrt[n]{a}$:

For any positive integer n, $a^{\frac{1}{n}} = \sqrt[n]{a}$. *Note that $\sqrt[n]{a}$ is not a real number if $a < 0$ and n is even.*

Rational Exponents:

Assume m and n are integers with $n > 0$: $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(a^m\right)^{\frac{1}{n}}$ or equivalently: $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \left(\sqrt[n]{a^m}\right)$

Properties of Radicals Assume $a, b, \sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers	**Examples Using the Properties of Radicals**
1. $\left(\sqrt[n]{a}\right)^n = \sqrt[n]{a^n} = a$ *This is a special case of* $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$	**1.** a. $\left(\sqrt{4}\right)^2 = (2)^2 = 4$ b. $\sqrt[3]{8} = \sqrt[3]{2^3} = 2$
2. $\sqrt[n]{a} \cdot \sqrt[n]{b} = a^{\frac{1}{n}} \cdot b^{\frac{1}{n}} = (a \cdot b)^{\frac{1}{n}} = \sqrt[n]{a \cdot b}$ *This is a special case from the rule* $(ab)^n = a^n b^n$ *from the exponent rules section (pg. 3)*	**2.** $\sqrt[4]{8} \cdot \sqrt[4]{2} = \sqrt[4]{8 \cdot 2} = \sqrt[4]{16} = 2$
3. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \dfrac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} = \left(\dfrac{a}{b}\right)^{\frac{1}{n}} = \sqrt[n]{\dfrac{a}{b}}$ where $b \neq 0$	**3.** $\dfrac{\sqrt[3]{54}}{\sqrt[3]{2}} = \sqrt[3]{\dfrac{54}{2}} = \sqrt[3]{27} = 3$

Watch out for the following!

1. $\sqrt{8} = \sqrt[2]{8} = 8^{\frac{1}{2}}$; Remember: if there is no index given, it means the square root!

2. $\sqrt[4]{x^3} = x^{\frac{3}{4}}$; When switching from radical notation to exponential notation, remember that the root goes on the bottom of the fractional exponent. Think of it like a tree; *the roots are always at the bottom!*

Practice Problems:

Simplify each of the following *(show how to rewrite it as a radical – do NOT just use your calculator!!!)*:

1. $25^{\frac{1}{2}}$
2. $(-8)^{\frac{2}{3}}$
3. $(16x^4)^{\frac{1}{2}}$
4. $\left(\frac{1}{8}\right)^{\frac{-5}{3}}$
5. $\left(\frac{121}{100}\right)^{\frac{-3}{2}}$
6. $\left(\frac{4}{9}\right)^{\frac{-3}{2}}$
7. $-81^{\frac{3}{4}}$
8. $(27x^6)^{\frac{2}{3}}$
9. $(-32)^{\frac{-4}{5}}$
10. $(36r^6)^{\frac{1}{2}}$
11. $(64a^{12})^{\frac{5}{6}}$
12. $(16)^{\frac{1}{4}}$

Write all radicals with fractional exponents.

13. $\sqrt[3]{x}$
14. $\sqrt[5]{x}$
15. $\sqrt[3]{x^2}$
16. $\left(\sqrt[5]{x}\right)^2$
17. $\sqrt{x}$
18. $\sqrt[4]{x}$
19. $\sqrt{x^3}$
20. $\left(\sqrt[4]{x}\right)^3$

Perform the indicated operations. Write answers using only positive exponents.

21. $\left(m^{\frac{2}{3}}\right)\left(m^{\frac{5}{3}}\right)$
22. $\left(2y^{\frac{3}{4}}z\right)\left(3y^{-2}z^{\frac{-1}{3}}\right)$
23. $\left(4a^{-1}b^{\frac{2}{3}}\right)\left(a^{\frac{3}{2}}b^{-3}\right)$
24. $\left(\frac{x^4y^3z}{16x^{-16}yz^5}\right)^{\frac{1}{2}}$
25. $\left(\frac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}}\right)^2$
26. $\left(\frac{x^3y^{\frac{1}{3}}}{x^5y^{\frac{4}{3}}}\right)^4$

Solutions:

1. 5
2. 4
3. $4x^2$
4. 32
5. $\frac{10^3}{11^3}$
6. $\frac{27}{8}$
7. - 27
8. $9x^4$
9. $\frac{1}{16}$
10. $6r^3$
11. $32a^{10}$
12. 2
13. $x^{\frac{1}{3}}$
14. $x^{\frac{1}{5}}$
15. $x^{\frac{2}{3}}$
16. $x^{\frac{2}{5}}$
17. $x^{\frac{1}{2}}$
18. $x^{\frac{1}{4}}$
19. $x^{\frac{3}{2}}$
20. $x^{\frac{3}{4}}$
21. $m^{\frac{7}{3}}$
22. $\frac{6z^{\frac{2}{3}}}{y^{\frac{5}{4}}}$
23. $\frac{4a^{\frac{1}{2}}}{b^{\frac{7}{3}}}$
24. $\frac{x^{10}y}{4z^2}$
25. x^2
26. $\frac{1}{x^8y^4}$

3. Factoring

Being able to factor is an ***essential*** skill needed in calculus. Below are some techniques used to factor polynomials.

Factoring Technique & General Forms	Examples
1. Greatest Common Factor (GCF) The greatest common factor is the largest factor that divides into *every* term in a given polynomial.	**1.** a. $4x+12=4(x+3)$ b. $6x^2y+9xy^2+3xy=3xy(2x+3y+1)$
2. Difference of Squares $x^2-y^2=(x+y)(x-y)$	**2.** a. $x^2-25=(x+5)(x-5)$ b. $4x^2-9y^2=(2x+3y)(2x-3y)$
3. Trinomials (when leading coefficient is 1) $x^2+(a+b)x+ab=(x+a)(x+b)$	**3.** a. $x^2+7x+12=(x+3)(x+4)$ b. $x^2-5x+6=(x-2)(x-3)$ c. $x^2-4x-21=(x-7)(x+3)$
4. Trinomials with leading coefficient greater than 1 **use the split the middle method!!!** →	**4.** $6x^2+23x+7$ $(6x^2+21x)+(2x+7)$ $3x(2x+7)+1(2x+7)$ $(3x+1)(2x+7)$
5. Sum or Difference of Two Perfect Cubes $x^3+y^3=(x+y)(x^2-xy+y^2)$ $x^3-y^3=(x-y)(x^2+xy+y^2)$ SOAP: **S**ame, **O**pposite, **A**lways **P**ositive ☺	**5.** a. $x^3+27y^3=(x+3y)(x^2-3xy+9y^2)$ b. $8x^3-y^3=(2x-y)(4x^2+2xy+y^2)$ $64x^3-125y^3=(4x-5y)(16x^2+20xy+25y^2)$
6. Grouping (only applies to polynomials of 4 terms)	**6.** $5x^3-20x^2+3x-12$ $5x^2(x-4)+3(x-4)$ $(5x^2+3)(x-4)$

Remember!

Always try to factor out the greatest common factor (GCF) first! A polynomial may look like it is not factorable, but after taking out a common factor, you may be able to factor it with ease.

- 2 Terms: Try GCF, Difference of Squares or Sum/Difference of Cubes
- 3 Terms: Try GCF and/or Split the Middle
- 4 Terms: Try GCF and/or Grouping
- More than 4 Terms: GCF is your only possibility!

Practice Problems:

Factor each trinomial.

1. $x^2 + 12x + 20$
2. $x^2 + x - 12$
3. $2x^2 + 9x + 10$
4. $x^3 + 10x^2 + 21x$
5. $6a^2 - 48a - 120$
6. $6y^2 - 13y - 5$
7. $x^2 - 7x + 6$
8. $6x^2 - 19x + 10$
9. $x^2 + 7x - 44$
10. $x^2 - 5x + 6$
11. $3y^3 + 12y^2 + 9y$
12. $2x^2 - x - 3$

Factor each binomial.

13. $144 - 49x^2$
14. $125x^3 - 1$
15. $8x^2 - 12x$
16. $27x^2 + 3$
17. $y^2 - 36$
18. $64y^2 - 16$
19. $x^3 + 9$
20. $49x^2 - 4$
21. $6xy^2 - 4z$
22. $9x^2 + 25$
23. $x^3 + 8y^3$
24. $6y^2 - 14y$

Solutions:

1. $(x+10)(x+2)$
2. $(x+4)(x-3)$
3. $(x+2)(2x+5)$
4. $x(x+7)(x+3)$
5. $6(a-10)(a+2)$
6. $(3y+1)(2y-5)$
7. $(x-1)(x-6)$
8. $(3x-2)(2x-5)$
9. $(x+11)(x-4)$
10. $(x-3)(x-2)$
11. $3y(y+3)(y+1)$
12. $(x+1)(2x-3)$
13. $(12-7x)(12+7x)$
14. $(5x-1)(25x^2+5x+1)$
15. $4x(2x-3)$
16. $3(9x^2+1)$
17. $(y-6)(y+6)$
18. $16(2y-1)(2y+1)$
19. Not Factorable
20. $(7x-2)(7x+2)$
21. $2(3xy^2-2z)$
22. Not Factorable
23. $(x+2y)(x^2-2xy+4y^2)$
24. $2y(3y-7)$

4. Solving Quadratic Equations (and other polynomials)

Given a quadratic equation, $ax^2+bx+c=0$, there are two basic methods that one can use to solve for the value of x: factoring or using the quadratic formula *(Note: completing the square and graphing are also methods you could use, but we are going to focus on factoring and the quadratic formula in this review, as they are the most commonly used)*

Factoring Example:

$x^2-3x-10=0$

$(x-5)(x+2)=0$ *factor as much as possible

$x-5=0 \ \ or \ \ x+2=0$ *set each factor equal to zero

$x=5 \ \ or \ \ x=-2$ *solve each equation

Thus, the solutions are $x=5 \ \ or \ \ x=-2$.

Quadratic Formula Example:

$x^2-3x-10=0$

Recall the Quadratic Formula:

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$, so with this example…

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-10)}}{2(1)}=\frac{3\pm\sqrt{9+40}}{2}$$

$x=\dfrac{3\pm\sqrt{49}}{2}=\dfrac{3\pm7}{2}$ So, $x=\dfrac{3+7}{2}=5 \ \ or \ \ x=\dfrac{3-7}{2}=-2$

Thus, the solutions are $x=5 \ \ or \ \ x=-2$.

Note that the quadratic formula is typically used to solve an equation that cannot be solved by the method of factoring. When a factorable quadratic is given (as in the example above), the quadratic formula is not necessary, but the solutions do end up the same whichever method is used. The quadratic formula allows for the finding of EXACT solutions, specifically when the solutions are irrational (or contain radicals!).

*The quadratic formula can only be used on **quadratic** equations $ax^2+bx+c=0$ -- if the polynomial is not a quadratic, try factoring to solve!

Practice Problems:

Solve for x.

1. $2x^2+5x-7=0$
2. $5x^2-15x-10=0$
3. $8x^3-32x=0$
4. $x^2-7x+12=0$
5. $6x^2+18x+2=0$
6. $x^3+3x^2+2x=0$
7. $x^2-5x+4=0$
8. $5x^2-3x-6=0$
9. $9x^2-6x^3+x^4=0$
10. $5x^3-20x=0$
11. $\dfrac{3}{2}x^3-6x=0$
12. $-3x^2+9x+10=0$

Solutions:

1. $x=1,-\dfrac{7}{2}$
2. $x=\dfrac{3\pm\sqrt{17}}{2}$
3. $x=0,2,-2$
4. $x=3,4$
5. $x=\dfrac{-9\pm\sqrt{69}}{6}$
6. $x=0,-1,-2$
7. $x=1,4$
8. $x=\dfrac{3\pm\sqrt{129}}{10}$
9. $x=0,3$

10 & 11. $x=0,2,-2$

12. $x=\dfrac{9\pm\sqrt{201}}{6}$

5. Operations with Fractions

Adding & Subtracting Fractions

The sum or difference of fractions with the same denominator is given by the sum or difference of the numerators divided by the common denominator. If fractions do not have a common denominator, one must be obtained before addition or subtraction can take place.

Simple Example: Find the sum.

$$\frac{1}{2}+\frac{2}{3}=\frac{1}{2}\cdot\left(\frac{3}{3}\right)+\frac{2}{3}\cdot\left(\frac{2}{2}\right)$$ **get denominator of 6 for both fractions*

$$=\frac{3}{6}+\frac{4}{6}=\frac{7}{6}$$ **add numerators of fractions together*

Example with algebraic fractions (fractions with variables):

When we add or subtract algebraic fractions, the method is exactly the same! First a common denominator must be obtained. To find the common denominator:

1. Factor each denominator completely.
2. The least common denominator is the product of all of the different factors – with each factor raised to the highest power to which it appears in any one factorization

Find the difference: $\frac{x}{x+3}-\frac{5}{x}$ *the common denominator is $x(x+3)$ *(neither denominator needed factored in this instance)*

1. $\frac{x}{x+3}\cdot\left(\frac{x}{x}\right)-\frac{5}{x}\cdot\left(\frac{x+3}{x+3}\right)$ *get denominator of $x(x+3)$ for both fractions
2. $\frac{x(x)-5(x+3)}{x(x+3)}$ *subtract the numerators
3. $\frac{x^2-5x-15}{x(x+3)}$ *simplify and distribute the negative sign

Multiplying Fractions

The product of fractions is given by the product of the numerators divided by the product of the denominators. *NO COMMON DENOMINATOR IS NEEDED TO MULTIPLY!!!*

Simple Example: Find the product.

$\frac{1}{2}\cdot\frac{2}{3}=\frac{2}{6}$ *multiply the numerators, multiply the denominators – and in this case, reduce the product to $\frac{1}{3}$

Example with algebraic fractions:

In this example, instead of distributing each numerator and denominator, you will need to factor each numerator and denominator separately. Then, look to cancel common factors and simplify the result.

Find the product: $$\frac{2x^2+12x}{4x+8}\cdot\frac{x^2-4x-12}{x^2-36}=\frac{2x(x+6)}{4(x+2)}\cdot\frac{(x-6)(x+2)}{(x+6)(x-6)}$$

Cancel common factors and simplify: $$\frac{2x\cancel{(x+6)}}{4\cancel{(x+2)}}\cdot\frac{\cancel{(x-6)}\cancel{(x+2)}}{\cancel{(x+6)}\cancel{(x-6)}}=\frac{2x}{4}=\frac{x}{2}$$

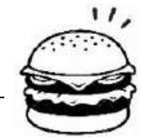
Dividing Fractions

The quotient of fractions is found by taking the reciprocal of the second fraction and then multiplying the two fractions together (using the same steps as outlined in the previous section) *NO COMMON DENOMINATOR IS NEEDED!!!*

KEEP *(the first fraction the same)*, **CHANGE** *(the division to multiplication), and* **FLIP** *(the second fraction)*

Simple Example: Find the quotient: $\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$

The same process applies when your quotient looks like this: $\dfrac{\frac{1}{2}}{\frac{2}{3}}$



You can rewrite it as the division of 2 fractions and follow the same process as above, or you could use the "burger" shortcut. Multiply the numerator of the top fraction with the denominator of the bottom fraction (think of these as the buns of your burger) – this product becomes the numerator of your final answer. Then, multiply the denominator of the top fraction with the numerator of the bottom fraction (think of these are your patties) – this product becomes the denominator of your final answer: $\frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$. It seems silly, but it's a helpful trick that can save you time later on.

Practice Problems:

Find the sum or difference.

1. $\frac{1}{4x} + \frac{1}{3x}$
2. $\frac{3x+4}{x+2} - \frac{2x+5}{x+2}$
3. $z + \frac{1}{z}$
4. $\frac{5}{8y} - \frac{2}{12y}$
5. $\frac{2-y}{9y+6} + \frac{y-2}{6y+4}$
6. $\frac{7}{n^2} - \frac{5n-2}{n}$
7. $\frac{3}{a-3} - \frac{3}{a}$
8. $\frac{2x+3}{2x^3-4x^2} - \frac{1}{x-2}$

Find the product or quotient.

9. $\frac{1}{4x} \cdot \frac{1}{3x}$
10. $\dfrac{\frac{x^2-36}{12x}}{\frac{x-6}{3x}}$
11. $\frac{4x-16}{7x^3} \div \frac{8x-32}{21x}$
12. $\frac{3x^2-2x}{5x^2-5x} \cdot \frac{x^2-1}{3x^2+x-2}$

Solutions:

1. $\frac{7}{12x}$
2. $\frac{x-1}{x+2}$
3. $\frac{z^2+1}{z}$
4. $\frac{11}{24y}$
5. $\frac{y-2}{6(3y+2)}$
6. $\frac{-5n^2+2n+7}{n^2}$
7. $\frac{9}{a(a-3)}$
8. $\frac{3+2x-2x^2}{2x^2(x-2)}$
9. $\frac{1}{12x^2}$
10. $\frac{x+6}{4}$
11. $\frac{3}{2x^2}$
12. $\frac{1}{5}$

6. Right Triangle Trigonometry

Your Tools for Solving Right Triangles:

1. Pythagorean Theorem: $a^2 + b^2 = c^2$

Use to solve for any side! C is the hypotenuse, always!!

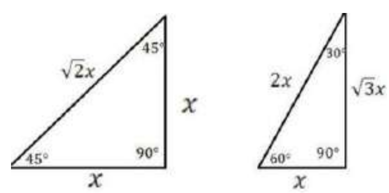
2. Trigonometry

$$\sin(q) = \frac{opposite}{hypotenuse} = \frac{o}{h} \qquad \cos(q) = \frac{adjacent}{hypotenuse} = \frac{a}{h}$$

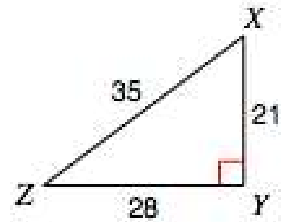
$$\tan(A) = \frac{opposite}{adjacent} = \frac{o}{a}$$

3. General Relationships

- The largest angle is always across from the largest side, and the smallest angle is across from the smallest side.
- The sum of the angles is always 180 degrees.
- Pythagorean triples: **3-4-5**, 5-12-13, 7-24-25, 8-15-17…
- Special Right Triangles:

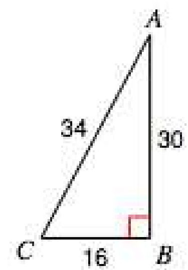


Example 1: What is $\sin(Z)$?



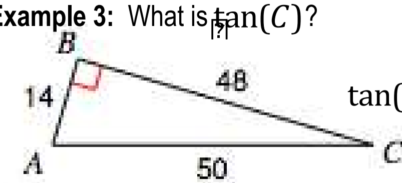
$$\sin(Z) = \frac{21}{35} = \frac{3}{5}$$

Example 2: Find $\cos(A)$.



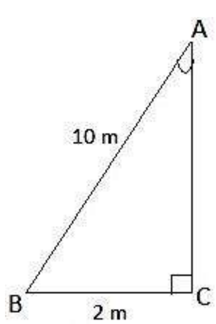
$$\cos(A) = \frac{30}{34} = \frac{15}{17}$$

Example 3: What is $\tan(C)$?



$$\tan(C) = \frac{14}{48} = \frac{7}{24}$$

Example 4: Find the missing side length of the triangle below. Then, find ALL three trigonometric ratios for the triangle.



Step 1: Use $a^2 + b^2 = c^2$ to solve for the missing side.

$$2^2 + b^2 = 10^2$$
$$b^2 = 100 - 4$$
$$b^2 = 96$$
$$b = \sqrt{96} = \sqrt{16 \cdot 6} = 4\sqrt{6}$$

Step 2: Fill in the trig ratios with the values. Simplify the fractions. Rationalize denominators.

Trigonometric Ratios for Angle A:

$$\sin(A) = \frac{2}{10} = \frac{1}{5} \qquad \cos(A) = \frac{4\sqrt{6}}{10} = \frac{2\sqrt{6}}{5}$$

$$\tan(A) = \frac{2}{4\sqrt{6}} = \frac{1}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{1\sqrt{6}}{12}$$

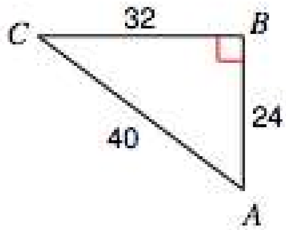
Trigonometric Ratios for Angle B:

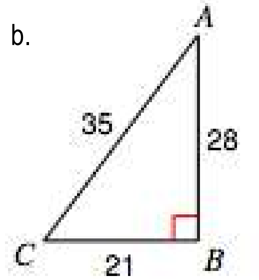
$$\sin(B) = \frac{4\sqrt{6}}{10} = \frac{2\sqrt{6}}{5} \qquad \cos(B) = \frac{2}{10} = \frac{1}{5}$$

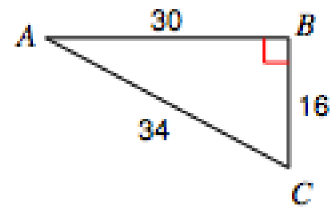
$$\tan(B) = \frac{4\sqrt{6}}{2} = 2\sqrt{6}$$

Practice Problems:

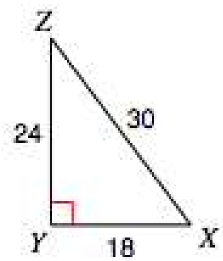
1. Find $\sin(C)$ in each of the following triangles. Simplify your answer fully.

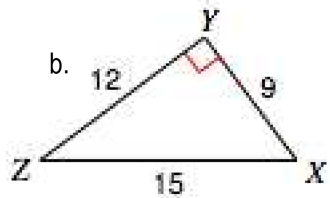
a. 

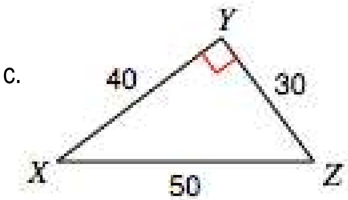
b. 

c. 

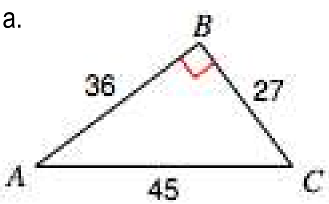
2. Find $\cos(Z)$ in each of the following triangles. Simplify your answer fully.

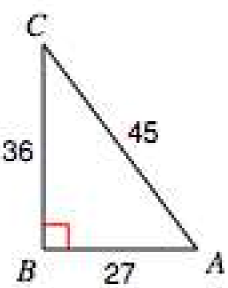
a. 

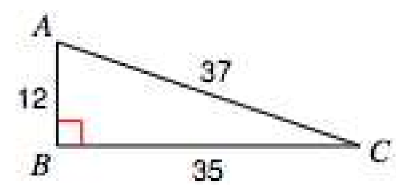
b. 

c. 

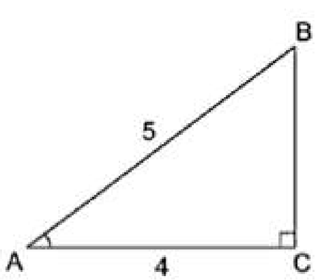
3. Find $\tan(A)$ in each of the following triangles. Simplify your answer fully.

a. 

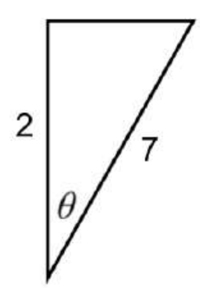
b. 

c. 

4. Using the triangle below, find $\sin A$.

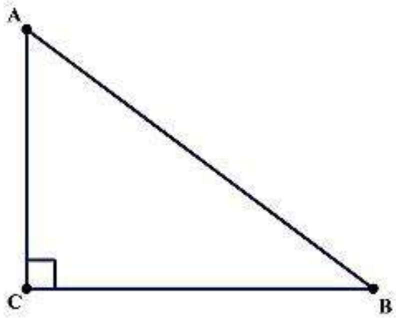


5. Using the triangle below, find $\tan q$.



6. In the triangle below, $AC = 7$ and $BC = 24$. Find the missing side length, *AB,* then find all the trig ratios for angles A and B.

Hint – write the side lengths down first that are given to you!



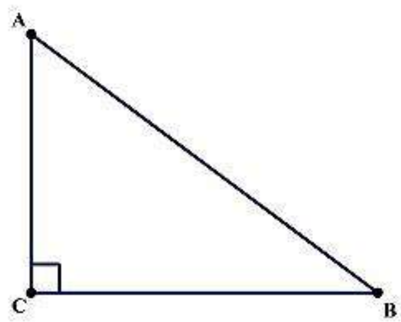
Trigonometric Ratios for Angle A:

$\sin(A) =$ $\cos(A) =$ $\tan(A) =$

Trigonometric Ratios for Angle B:

$\sin(B) =$ $\cos(B) =$ $\tan(B) =$

7. In the triangle below, $AB = 22$ and $BC = 14$. Find the missing side length, *AC,* then find all the trig ratios for angles A and B.



Trigonometric Ratios for Angle A:

$\sin(A) =$ $\cos(A) =$ $\tan(A) =$

Trigonometric Ratios for Angle B:

$\sin(B) =$ $\cos(B) =$ $\tan(B) =$

Solutions:

1a. $\frac{3}{5}$ 1b. $\frac{4}{5}$ 1c. $\frac{15}{17}$ 2a. $\frac{4}{5}$ 2b. $\frac{4}{5}$ 2c. $\frac{3}{5}$ 3a. $\frac{3}{4}$ 3b. $\frac{4}{3}$

3c. $\frac{35}{12}$ 4. Missing side is 3, $\sin A = \frac{3}{5}$ 5. Missing side is $3\sqrt{5}$, $\tan q = \frac{3\sqrt{5}}{2}$

6. Missing side is 25, $\sin A = \frac{24}{25}$, $\cos A = \frac{7}{25}$, $\tan A = \frac{24}{7}$, $\sin B = \frac{7}{25}$, $\cos B = \frac{24}{25}$, $\tan B = \frac{7}{24}$

7. Missing side is $12\sqrt{2}$, $\sin A = \frac{7}{11}$, $\cos A = \frac{6\sqrt{2}}{11}$, $\tan A = \frac{7\sqrt{2}}{12}$, $\sin B = \frac{6\sqrt{2}}{11}$, $\cos B = \frac{7}{11}$, $\tan B = \frac{6\sqrt{2}}{7}$