AP® CALCULUS AB
2011 SCORING GUIDELINES (Form B)

Question 1

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S, where $S(t)$ is measured in millimeters and t is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

(a) According to the model, what is the height of the water in the can at the end of the 60-day period?

(b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.

(c) Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time $t = 7$? Indicate units of measure.

(d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M, where $M(t) = \frac{1}{400}\left(3t^3 - 30t^2 + 330t\right)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \le t \le 60$ to justify that there exists a time t, $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

(a) $S(60) = \int_0^{60} S'(t)\,dt = 171.813$ mm	$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(b) $\dfrac{S(60) - S(0)}{60} = 2.863$ or 2.864 mm/day	1 : answer
(c) $V(t) = 100\pi S(t)$ $V'(7) = 100\pi S'(7) = 602.218$ The volume of water in the can is increasing at a rate of 602.218 mm^3/day.	$2: \begin{cases} 1: \text{relationship between } V \text{ and } S \\ 1: \text{answer} \end{cases}$
(d) $D(0) = -0.675 < 0$ and $D(60) = 69.37730 > 0$ Because D is continuous, the Intermediate Value Theorem implies that there is a time t, $0 < t < 60$, at which $D(t) = 0$. At this time, the heights of water in the two cans are changing at the same rate.	$2: \begin{cases} 1: \text{considers } D(0) \text{ and } D(60) \\ 1: \text{justification} \end{cases}$
	1 : units in (b) or (c)

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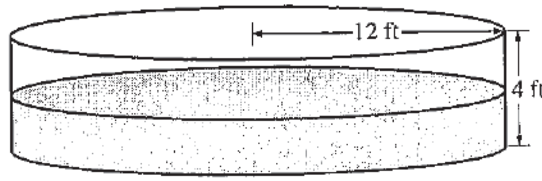
Question 3

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t. During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

(b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours.

(c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

(d) Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

(a) $\int_0^{12} P(t)\,dt \approx 46 \cdot 4 + 57 \cdot 4 + 62 \cdot 4 = 660 \text{ ft}^3$

$2 : \begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{answer} \end{cases}$

(b) $\int_0^{12} R(t)\,dt = 225.594 \text{ ft}^3$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $1000 + \int_0^{12} P(t)\,dt - \int_0^{12} R(t)\,dt = 1434.406$

At time $t = 12$ hours, the volume of water in the pool is approximately 1434 ft^3.

1 : answer

(d) $V'(t) = P(t) - R(t)$

$V'(8) = P(8) - R(8) = 60 - 25e^{-0.4} = 43.241 \text{ or } 43.242 \text{ ft}^3/\text{hr}$

$V = \pi(12)^2 h$

$\frac{dV}{dt} = 144\pi \frac{dh}{dt}$

$\left.\frac{dh}{dt}\right|_{t=8} = \frac{1}{144\pi} \cdot \left.\frac{dV}{dt}\right|_{t=8} = 0.095 \text{ or } 0.096 \text{ ft/hr}$

$4 : \begin{cases} 1 : V'(8) \\ 1 : \text{equation relating } \frac{dV}{dt} \text{ and } \frac{dh}{dt} \\ 1 : \left.\frac{dh}{dt}\right|_{t=8} \\ 1 : \text{units of ft}^3/\text{hr and ft/hr} \end{cases}$

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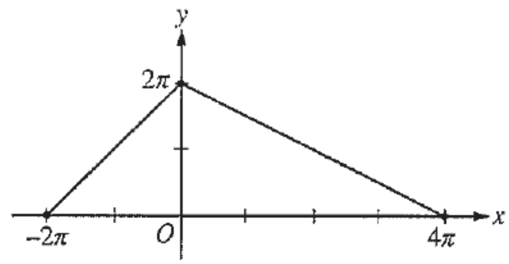
Question 6

Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.



Graph of g

(a) Find $\int_{-2\pi}^{4\pi} f(x)\,dx$. Show the computations that lead to your answer.

(b) Find all x-values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.

(c) Let $h(x) = \int_0^{3x} g(t)\,dt$. Find $h'\left(-\frac{\pi}{3}\right)$.

(a) $\int_{-2\pi}^{4\pi} f(x)\,dx = \int_{-2\pi}^{4\pi} \left(g(x) - \cos\left(\frac{x}{2}\right)\right) dx$

$= 6\pi^2 - \left[2\sin\left(\frac{x}{2}\right)\right]_{x=-2\pi}^{x=4\pi}$

$= 6\pi^2$

$2 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) $f'(x) = g'(x) + \frac{1}{2}\sin\left(\frac{x}{2}\right) = \begin{cases} 1 + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } -2\pi < x < 0 \\ -\frac{1}{2} + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } 0 < x < 4\pi \end{cases}$

$f'(x)$ does not exist at $x = 0$.
For $-2\pi < x < 0$, $f'(x) \neq 0$.
For $0 < x < 4\pi$, $f'(x) = 0$ when $x = \pi$.

f has critical points at $x = 0$ and $x = \pi$.

$4 : \begin{cases} 1 : \frac{d}{dx}\left(\cos\left(\frac{x}{2}\right)\right) \\ 1 : g'(x) \\ 1 : x = 0 \\ 1 : x = \pi \end{cases}$

(c) $h'(x) = g(3x) \cdot 3$

$h'\left(-\frac{\pi}{3}\right) = 3g(-\pi) = 3\pi$

$3 : \begin{cases} 2 : h'(x) \\ 1 : \text{answer} \end{cases}$