AP® CALCULUS AB
2004 SCORING GUIDELINES (Form B)

Question 2

For $0 \le t \le 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

(a) Show that the number of mosquitoes is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \le t \le 31$? Show the analysis that leads to your conclusion.

(a) Since $R(6) = 4.438 > 0$, the number of mosquitoes is increasing at $t = 6$.

1 : shows that $R(6) > 0$

(b) $R'(6) = -1.913$

Since $R'(6) < 0$, the number of mosquitoes is increasing at a decreasing rate at $t = 6$.

2 : { 1 : considers $R'(6)$; 1 : answer with reason }

(c) $1000 + \int_0^{31} R(t)\,dt = 964.335$

To the nearest whole number, there are 964 mosquitoes.

2 : { 1 : integral; 1 : answer }

7.854 23.562

(d) $R(t) = 0$ when $t = 0$, $t = 2.5\pi$, or $t = 7.5\pi$

$R(t) > 0$ on $0 < t < 2.5\pi$

$R(t) < 0$ on $2.5\pi < t < 7.5\pi$

$R(t) > 0$ on $7.5\pi < t < 31$

The absolute maximum number of mosquitoes occurs at $t = 2.5\pi$ or at $t = 31$.

$1000 + \int_0^{2.5\pi} R(t)\,dt = 1039.357,$

There are 964 mosquitoes at $t = 31$, so the maximum number of mosquitoes is 1039, to the nearest whole number.

4 : {
- 2 : absolute maximum value
  - 1 : integral
  - 1 : answer
- 2 : analysis
  - 1 : computes interior critical points
  - 1 : completes analysis

}

AP® CALCULUS AB
2010 SCORING GUIDELINES

Question 1

There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

(a) $\int_0^6 f(t)\,dt = 142.274$ or 142.275 cubic feet	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$
(b) Rate of change is $f(8) - g(8) = -59.582$ or -59.583 cubic feet per hour.	1 : answer
(c) $h(0) = 0$ For $0 < t \le 6$, $h(t) = h(0) + \int_0^t g(s)\,ds = 0 + \int_0^t 0\,ds = 0.$ For $6 < t \le 7$, $h(t) = h(6) + \int_6^t g(s)\,ds = 0 + \int_6^t 125\,ds = 125(t-6).$ For $7 < t \le 9$, $h(t) = h(7) + \int_7^t g(s)\,ds = 125 + \int_7^t 108\,ds = 125 + 108(t-7).$ Thus, $h(t) = \begin{cases} 0 & \text{for } 0 \le t \le 6 \\ 125(t-6) & \text{for } 6 < t \le 7 \\ 125 + 108(t-7) & \text{for } 7 < t \le 9 \end{cases}$	$3: \begin{cases} 1: h(t) \text{ for } 0 \le t \le 6 \\ 1: h(t) \text{ for } 6 < t \le 7 \\ 1: h(t) \text{ for } 7 < t \le 9 \end{cases}$
(d) Amount of snow is $\int_0^9 f(t)\,dt - h(9) = 26.334$ or 26.335 cubic feet.	$3: \begin{cases} 1: \text{integral} \\ 1: h(9) \\ 1: \text{answer} \end{cases}$

AP® CALCULUS AB
2010 SCORING GUIDELINES

Question 2

t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon $(t = 0)$ and 8 P.M. $(t = 8)$. The number of entries in the box t hours after noon is modeled by a differentiable function E for $0 \le t \le 8$. Values of $E(t)$, in hundreds of entries, at various times t are shown in the table above.

(a) Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t = 6$. Show the computations that lead to your answer.

(b) Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)\,dt$. Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)\,dt$ in terms of the number of entries.

(c) At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function P, where $P(t) = t^3 - 30t^2 + 298t - 976$ hundreds of entries per hour for $8 \le t \le 12$. According to the model, how many entries had not yet been processed by midnight $(t = 12)$?

(d) According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

(a) $E'(6) \approx \dfrac{E(7) - E(5)}{7 - 5} = 4$ hundred entries per hour

1 : answer

(b) $\frac{1}{8}\int_0^8 E(t)\,dt \approx$

$\frac{1}{8}\left(2\cdot\frac{E(0)+E(2)}{2} + 3\cdot\frac{E(2)+E(5)}{2} + 2\cdot\frac{E(5)+E(7)}{2} + 1\cdot\frac{E(7)+E(8)}{2}\right)$

$= 10.687$ or 10.688

$\frac{1}{8}\int_0^8 E(t)\,dt$ is the average number of hundreds of entries in the box between noon and 8 P.M.

3 : { 1 : trapezoidal sum; 1 : approximation; 1 : meaning }

(c) $23 - \int_8^{12} P(t)\,dt = 23 - 16 = 7$ hundred entries

2 : { 1 : integral; 1 : answer }

(d) $P'(t) = 0$ when $t = 9.183503$ and $t = 10.816497$.

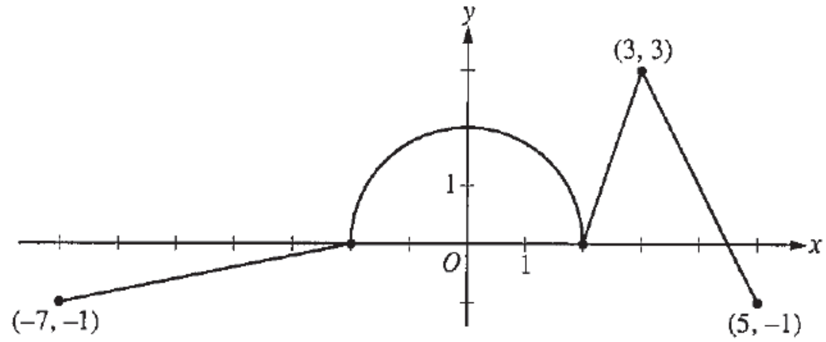
t	$P(t)$
8	0
9.183503	5.088662
10.816497	2.911338
12	8

Entries are being processed most quickly at time $t = 12$.

3 : { 1 : considers $P'(t) = 0$; 1 : identifies candidates; 1 : answer with justification }

AP® CALCULUS AB
2010 SCORING GUIDELINES

Question 5



Graph of g'

The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find $g(3)$ and $g(-2)$.

(b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

(a) $g(3) = 5 + \int_0^3 g'(x)\,dx = 5 + \frac{\pi \cdot 2^2}{4} + \frac{3}{2} = \frac{13}{2} + \pi$

$g(-2) = 5 + \int_0^{-2} g'(x)\,dx = 5 - \pi$

$3 : \begin{cases} 1 : \text{uses } g(0) = 5 \\ 1 : g(3) \\ 1 : g(-2) \end{cases}$

(b) The graph of $y = g(x)$ has points of inflection at $x = 0$, $x = 2$, and $x = 3$ because g' changes from increasing to decreasing at $x = 0$ and $x = 3$, and g' changes from decreasing to increasing at $x = 2$.

$2 : \begin{cases} 1 : \text{identifies } x = 0, 2, 3 \\ 1 : \text{explanation} \end{cases}$

(c) $h'(x) = g'(x) - x = 0 \Rightarrow g'(x) = x$

On the interval $-2 \le x \le 2$, $g'(x) = \sqrt{4 - x^2}$.

On this interval, $g'(x) = x$ when $x = \sqrt{2}$.

The only other solution to $g'(x) = x$ is $x = 3$.

$h'(x) = g'(x) - x > 0$ for $0 \le x < \sqrt{2}$

$h'(x) = g'(x) - x \le 0$ for $\sqrt{2} < x \le 5$

Therefore h has a relative maximum at $x = \sqrt{2}$, and h has neither a minimum nor a maximum at $x = 3$.

$4 : \begin{cases} 1 : h'(x) \\ 1 : \text{identifies } x = \sqrt{2}, 3 \\ 1 : \text{answer for } \sqrt{2} \text{ with analysis} \\ 1 : \text{answer for 3 with analysis} \end{cases}$

AP® CALCULUS AB
2011 SCORING GUIDELINES

Question 2

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

(a) $H'(3.5) \approx \dfrac{H(5) - H(2)}{5 - 2}$

$= \dfrac{52 - 60}{3} = -2.666$ or -2.667 degrees Celsius per minute

1 : answer

(b) $\frac{1}{10}\int_0^{10} H(t)\,dt$ is the average temperature of the tea, in degrees Celsius, over the 10 minutes.

$$\frac{1}{10}\int_0^{10} H(t)\,dt \approx \frac{1}{10}\left(2\cdot\frac{66+60}{2} + 3\cdot\frac{60+52}{2} + 4\cdot\frac{52+44}{2} + 1\cdot\frac{44+43}{2}\right)$$
$$= 52.95$$

3 : { 1 : meaning of expression; 1 : trapezoidal sum; 1 : estimate }

(c) $\int_0^{10} H'(t)\,dt = H(10) - H(0) = 43 - 66 = -23$

The temperature of the tea drops 23 degrees Celsius from time $t = 0$ to time $t = 10$ minutes.

2 : { 1 : value of integral; 1 : meaning of expression }

(d) $B(10) = 100 + \int_0^{10} B'(t)\,dt = 34.18275$; $\quad H(10) - B(10) = 8.817$

The biscuits are 8.817 degrees Celsius cooler than the tea.

3 : { 1 : integrand; 1 : uses $B(0) = 100$; 1 : answer }