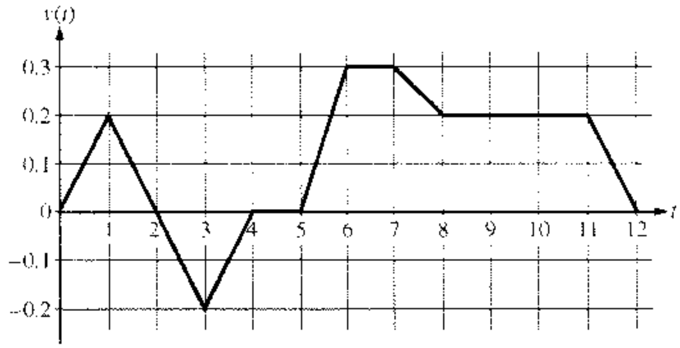
AVD (HW)

AP® CALCULUS AB
2009 SCORING GUIDELINES

Question 1



Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

(a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\, dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|\, dt$.

(c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

(d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

(a) $a(7.5) = v'(7.5) = \frac{v(8) - v(7)}{8 - 7} = -0.1$ miles/minute^2

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{units} \end{cases}$

(b) $\int_0^{12} |v(t)|\, dt$ is the total distance, in miles, that Caren rode during the 12 minutes from $t = 0$ to $t = 12$.

$$\int_0^{12} |v(t)|\, dt = \int_0^{2} v(t)\, dt - \int_2^{4} v(t)\, dt + \int_4^{12} v(t)\, dt$$
$$= 0.2 + 0.2 + 1.4 = 1.8 \text{ miles}$$

$2 : \begin{cases} 1 : \text{meaning of integral} \\ 1 : \text{value of integral} \end{cases}$

(c) Caren turns around to go back home at time $t = 2$ minutes. This is the time at which her velocity changes from positive to negative.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

(d) $\int_0^{12} w(t)\, dt = 1.6$; Larry lives 1.6 miles from school.

$\int_0^{12} v(t)\, dt = 1.4$; Caren lives 1.4 miles from school.

Therefore, Caren lives closer to school.

$3 : \begin{cases} 2 : \text{Larry's distance from school} \\ \quad 1 : \text{integral} \\ \quad 1 : \text{value} \\ 1 : \text{Caren's distance from school and conclusion} \end{cases}$

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Question 6

t (seconds)	0	8	20	25	32	40
$v(t)$ (meters per second)	3	5	−10	−8	−4	7

The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters, and time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

(a) Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_{20}^{40} v(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)\,dt$.

(c) For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in the table? If so, identify the subintervals and explain your reasoning. If not, explain why not.

(d) Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.

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1 : units in (a) and (b)

(a) $a(36) = v'(36) \approx \dfrac{v(40) - v(32)}{40 - 32} = \dfrac{11}{8}$ meters/sec^2

1 : answer

(b) $\int_{20}^{40} v(t)\,dt$ is the particle's change in position in meters from time $t = 20$ seconds to time $t = 40$ seconds.

$$\int_{20}^{40} v(t)\,dt \approx \frac{v(20) + v(25)}{2} \cdot 5 + \frac{v(25) + v(32)}{2} \cdot 7 + \frac{v(32) + v(40)}{2} \cdot 8$$
$$= -75 \text{ meters}$$

3 : { 1 : meaning of $\int_{20}^{40} v(t)\,dt$; 2 : trapezoidal approximation }

(c) $v(8) > 0$ and $v(20) < 0$
$v(32) < 0$ and $v(40) > 0$
Therefore, the particle changes direction in the intervals $8 < t < 20$ and $32 < t < 40$.

2 : { 1 : answer; 1 : explanation }

(d) Since $v'(t) = a(t) > 0$ for $0 < t < 8$, $v(t) \ge 3$ on this interval.

Therefore, $x(8) = x(0) + \int_0^8 v(t)\,dt \ge 7 + 8 \cdot 3 > 30$.

2 : { 1 : $v'(t) = a(t)$; 1 : explanation of $x(8) > 30$ }

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Question 2

For time $t \geq 0$ hours, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by $g(x) = 0.05x\left(1 - e^{-x/2}\right)$.

(a) How many kilometers does the car travel during the first 2 hours?

(b) Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t = 2$ hours. Indicate units of measure.

(c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

(a) $\int_0^2 r(t)\,dt = 206.370$ kilometers

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\dfrac{dg}{dt} = \dfrac{dg}{dx} \cdot \dfrac{dx}{dt}; \quad \dfrac{dx}{dt} = r(t)$

$\left.\dfrac{dg}{dt}\right|_{t=2} = \left.\dfrac{dg}{dx}\right|_{x=206.370} \cdot r(2)$

$= (0.050)(120) = 6$ liters/hour

$3 : \begin{cases} 2 : \text{uses chain rule} \\ 1 : \text{answer with units} \end{cases}$

(c) Let T be the time at which the car's speed reaches 80 kilometers per hour.

Then, $r(T) = 80$ or $T = 0.331453$ hours.

At time T, the car has gone

$x(T) = \int_0^T r(t)\,dt = 10.794097$ kilometers

and has consumed $g(x(T)) = 0.537$ liters of gasoline.

$4 : \begin{cases} 1 : \text{equation } r(t) = 80 \\ 2 : \text{distance integral} \\ 1 : \text{answer} \end{cases}$