AP Calculus AB Y Assignments

Unit #7—Integration (Antiderivatives)

You are responsible for doing all of the homework and checking your work. If you get stuck, the solutions are worked out at the end of the unit and the odd numbered exercises are also available online through the textbook publisher. If you still have questions on the homework problems after going over the solutions, then come in at lunch by appointment, afterschool, or during intervention as class time will not be devoted to going over the homework.

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Assignment #3: **Riemann Sums–LRAM-RRAM-MRAM (RAM: Rectangular Approximation Method)**
Handout

Assignment #4: **Trapezoidal Rule**
Page 461: 1, 11 Ignore the directions and approximate the area bounded by the curve and the x–axis using Left Riemann Sums, Right Riemann Sums, Midpoint Sums, and the Trapezoid Method. Let the number of intervals be what is given in the

Assignment #5: **Infinite Riemann Sums**
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Assignment #6: **Evaluating Integrals Using Geometry**
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Assignment #8: **First Fundamental Theorem of Calculus (FTC1)**
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Assignment #9: **Second Fundamental Theorem of Calculus**
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Assignment #10: **Review**
Handout

Test

Open Math XL

Warm-Up **Lesson 1**

Give the derivative of each.

1. $y = x^5$

   $y' = 5x^4$

2. $y = x^{-3}$

   $y' = -3x^{-4}$

3. $y = \sqrt{x}$ $y = x^{1/2}$

   $y' = \frac{1}{2}x^{-\frac{1}{2}}$

4. $y = 3 + \ln x$

   $y' = \frac{1}{x}$

5. $y = e^{3x}$

   $y' = 3e^{3x}$

6. $y = 3^{5x}$

   $y' = 3^{5x}\ln(3)\cdot 5$

7. $y = 5x^2 + 3$

   $y' = 10x$

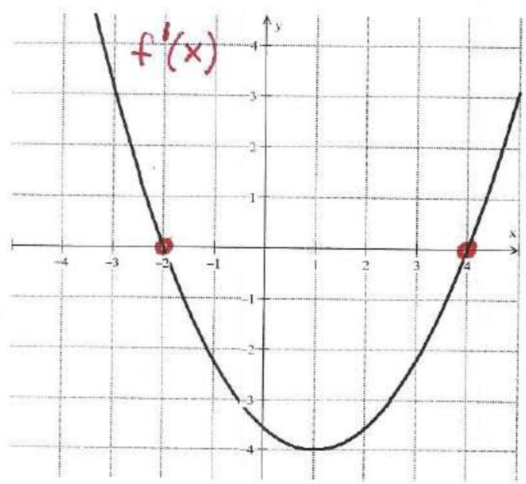
8. $y = 5x^2 - 9$

   $y' = 10x$

9. $y = 5x^2 + 143.2$

   $y' = 10x$

10. Given the graph of $f'(x)$ and $f(1) = 7$.



A) Identify the x–coordinates of any maximum values of $f(x)$. Justify your answer.

max at $x = -2$ b/c f' goes from pos to neg

B) Identify the x–coordinates of any minimum values of $f(x)$. Justify your answer.

min at $x = 4$ b/c f' goes from neg to pos

C) Identify the x–coordinates of any points of inflection of $f(x)$. Justify your answer.

f has a POI at $x = 1$ b/c f' goes from dec to inc

D) Give the equation of the tangent line of f at $x = 1$.

Point (1, 7) Slope -4

$y - 7 = -4(x - 1)$

Topic: Antiderivatives and Indefinite Integration

Goal: Be able to find the antiderivative of a simple function.

Definition Antiderviative:

A function F is an antiderivative of f on an open interval I if $F'(x) = f(x)\ \forall x \in I$.

Antidifferentiation (or indefinite integration) is an operation.

$$\int f(x)dx = F(x) + C$$

Integrand

Constant of Integration

Variable of Integration

Indefinite Integral is a synonym for antiderivative.

Every rule for derivatives has a companion rule for integrals.

How it Works

If $f(x) = 7x^3$, then $f'(x) =$ $21x^2$

If $f'(x) = 10x^4$, then $f(x) =$ $2x^5 + C$

some constant

Why it Works

$y' = 4x$ another way to write it is $\left[\frac{dy}{dx} = 4x\right] dx$

$dy = 4x\,dx$ — Separate dy and dx.

$\int dy = \int 4x\,dx$ — Integrate both sides (antidiferentiate). $\int$

$y = 2x^2 + C$

$\frac{d}{dx}\left[y = 2x^2 + C\right]$ taking the derivative

$\frac{dy}{dx} = 4x$

$\frac{d}{dx}\tan x = \sec^2 x \rightarrow \int \sec^2 x\, dx =$ $\tan x + C$

what functions derivative equals the integrand?

$\int 2x\, dx =$ $x^2 + C$

$\int x^2\, dx =$ $\frac{1}{3}x^3 + C$

$\int x^5\, dx =$ $\frac{1}{6}x^6 + C$

$\int 4x^3\, dx =$ $x^4 + C$

$\int x^{12}\, dx =$ $\frac{1}{13}x^{13} + C$

$\int 7x^{29}\, dx =$ $\frac{1}{30}\cdot 7x^{30} + C$

$\int x^3 + x^2 - 3x + 4 =$

$\frac{1}{4}x^4 + \frac{1}{3}x^3 - \frac{1}{2}\cdot 3x^2 + 4x + C$

$\int \frac{1}{x^5}dx =$ $\int x^{-5}dx$

$-\frac{1}{4}x^{-4} + C$

****To check each, take the derivative.**

$\int \frac{1}{x^7} dx = \int x^{-7} dx$

$-\frac{1}{6}x^{-6} + C$

$\int \sqrt{x}\, dx = \int x^{1/2} dx$

$\frac{2}{3}x^{3/2} + C$ reciprocals

$\int \frac{1}{x^{11}} dx = \int x^{-11} dx$

$-\frac{1}{10}x^{-10} + C$

$\int x^{\frac{3}{8}} dx =$

$\frac{8}{11}x^{11/8} + C$

$\int \frac{1}{x} dx = \int x^{-1} dx$ oops!

$= \ln|x| + C$

x cannot be neg

$\int \frac{x^4-2x}{x} dx = \int x^3 - 2\ dx$ Simplify 1st

$= \frac{1}{4}x^4 - 2x + C$

$\int \frac{x-3}{\sqrt{x}} dx = \int \frac{x}{x^{1/2}} - \frac{3}{x^{1/2}} dx$

$= \int x^{1/2} - 3x^{-1/2} dx$

$= \frac{2}{3}x^{3/2} - 2 \cdot 3x^{1/2} + C$

$\int \frac{x-3}{\sqrt[3]{x}} dx = \int \frac{x}{x^{1/3}} - \frac{3}{x^{1/3}} dx$

$= \int x^{2/3} - 3x^{-1/3} dx$

$= \frac{3}{5}x^{5/3} - \frac{3}{2} \cdot 3x^{2/3} + C$

$\int 5e^x\, dx = 5e^x + C$

$\int \frac{x^2+5x+6}{x+3} dx = \int \frac{(x+3)(x+2)}{x+3} dx$

$= \int x + 2\ dx$

$= \frac{1}{2}x^2 + 2x + C$

Trigonometric Antiderivatives

$\int \sin x\, dx = -\cos x + C$

$\int \cos x\, dx = \sin x + C$

$\int \sec^2 x\, dx = \tan x + C$

$\int \csc^2 x\, dx = -\cot x + C$

$\int \sec x \tan x\, dx = \sec x + C$

$\int \csc x \cot x\, dx = -\csc x + C$

Assignment #1 Page 280: 1, 2, 4–24, 26–29 all

Give the anti-derivative.

1. If $\frac{dy}{dx} = x^2 + 3$, then y = $\frac{1}{3}x^3 + 3x + C$

How would the answer change if I told you that when x = 1, y = 2?

$y = \frac{1}{3}x^3 + 3x + C$

$2 = \frac{1}{3}(1)^3 + 3(1) + C$

$2 = \frac{1}{3} + 3 + C$

$-1\frac{1}{3} = C$

$C = -\frac{4}{3}$

$y = \frac{1}{3}x^3 + 3x - \frac{4}{3}$

This is a specific or particular solution.

2. If $f''(x) < 0$ on the interval (1, 4), $f(2) = 7$, and $f'(2) = \frac{1}{3}$, then

   a) What is the equation of the tangent line at $x = 2$?

   Point (2,7)

   Slope = $\frac{1}{3}$

   $y - 7 = \frac{1}{3}(x-2)$

   b) Use the tangent line to approximate $f(2.3)$.

   $y - 7 = \frac{1}{3}(2.3 - 2)$

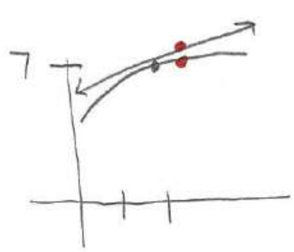
   $y - 7 = \frac{1}{3} \cdot \frac{3}{10}$

   $y - 7 = \frac{1}{10}$

   $y = 7\frac{1}{10}$

   c) Is the approximation greater than or less than the actual value? Explain.

   The approx is greater than the actual value b/c $f'' < 0 \Rightarrow f$ is cc dn



Topic: Initial Value Problems

Goal: Be able to solve initial value problems using integrals.

Initial Value Problems

Solving first order differential equations. You want to solve for y.

$y' = f(x)$

$\frac{dy}{dx} = f(x)$

$dy = f(x)\,dx$

Integrating both sides gives:

$\int dy = \int f(x)dx$

$y = F(x) + c$

antiderivative of $f(x)$

Solve the differential equation: $y' = 3x$

$\frac{dy}{dx} = 3X$

$dy = 3X\ dx$ separate variables

$\int dy = \int 3x\ dx$ Integrate both sides

$\int 1\ dy = \int 3x\ dx$

$y = \frac{1}{2} \cdot 3X^2 + C$

Solve: $y' = \frac{1}{x^2}$, $x > 0$ and $F(1) = 0$. **(Initial value problem)**

$\frac{dy}{dx} = X^{-2}$

$dy = X^{-2}\ dx$

$\int 1\ dy = \int X^{-2} dx$

$y = -1 \cdot X^{-1} + C$

$y = -\frac{1}{X} + C$

$0 = -\frac{1}{1} + C$

$1 = C$

$y = -\frac{1}{X} + 1$

Particular Solution

Solve: $\frac{dy}{dx} = 9x^2 - 4x + 5$ $\quad y(-1) = 0$

$dy = 9x^2 - 4x + 5 \; dx$

$\int dy = \int 9x^2 - 4x + 5 \; dx$

$y = 3x^3 - 2x^2 + 5x + C$

$0 = 3(-1)^3 - 2(-1)^2 + 5(-1) + C$

$0 = -3 - 2 - 5 + C$

$0 = -10 + C$

$C = 10$ $\quad y = 3x^3 - 2x^2 + 5x + 10$

Solve: $\frac{dy}{dx} = 3x^3 + 2x^2 + 7x$ $\quad y(1) = 1$

$dy = 3x^3 + 2x^2 + 7x \; dx$

$\int dy = \int 3x^3 + 2x^2 + 7x \; dx$

$y = \frac{1}{4} \cdot 3x^4 + \frac{1}{3} \cdot 2x^3 + \frac{1}{2} \cdot 7x^2 + C$

$\left[1 = \frac{1}{4} \cdot 3 + \frac{1}{3} \cdot 2 + \frac{1}{2} \cdot 7 + C \right] 12$

$12 = 9 + 8 + 42 + 12C$

$12 = 59 + 12C$

$-47 = 12C$

$C = -\frac{47}{12}$ $\quad y = \frac{3}{4}x^4 + \frac{2}{3}x^3 + \frac{7}{2}x^2 - \frac{47}{12}$

Assignment #2: Page 281: 47–52

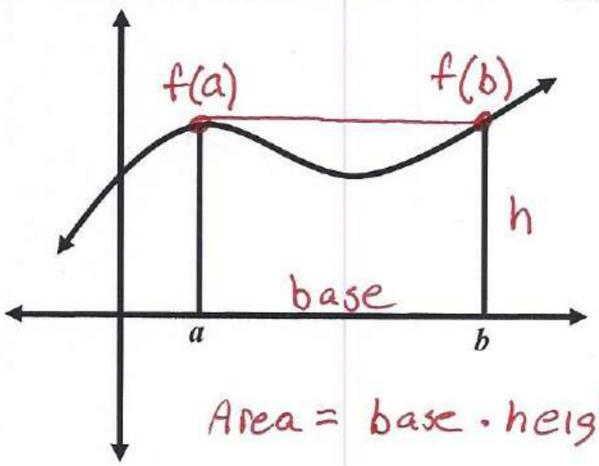
Lesson 3

Topic: Approximate the Area Under a Curve

Goal:* *Approximate the area under a curve using LRAM, MRAM, RRAM (Reimmann Sums).

Approximating the Area Under a Curve

Rectangular Approximation Methods RAM

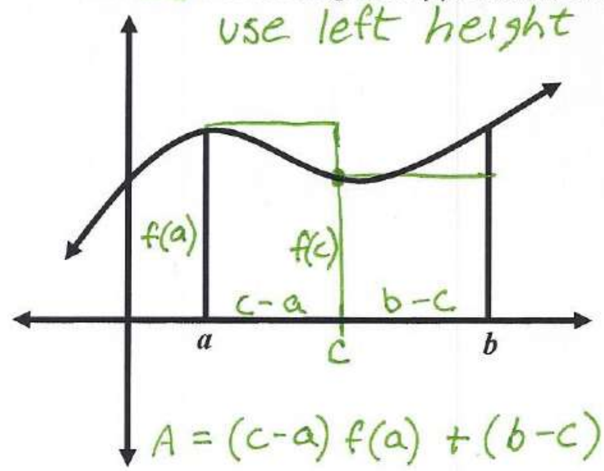


Area = base · height

$= (b-a) f(b)$

LRAM: Left Rectangular Approximation Method

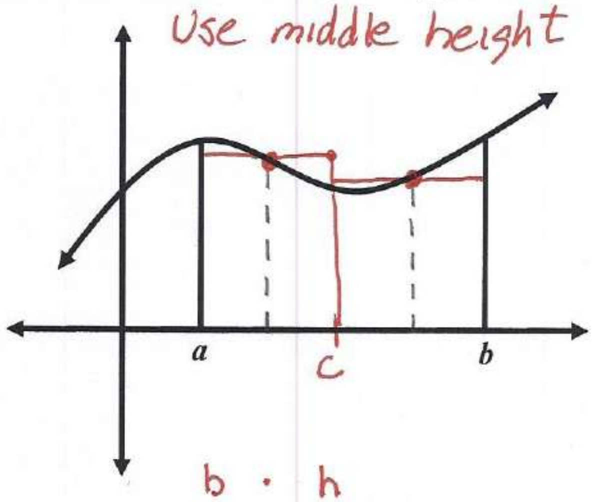
use left height



$A = (c-a) f(a) + (b-c) f(c)$

MRAM: Midpoint Rectangular Approximation

Use middle height

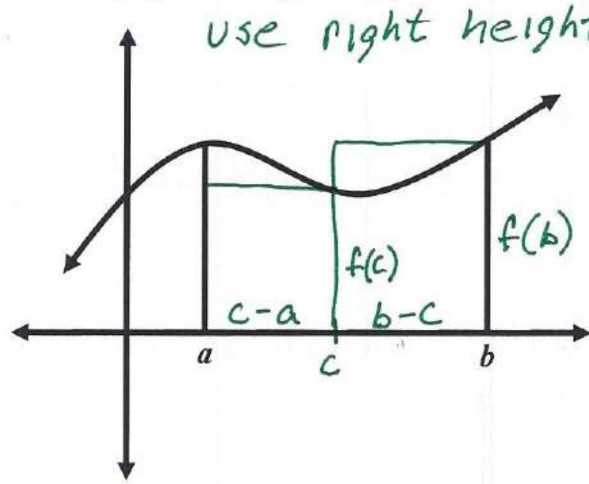


b · h

$A = (c-a) f\left(\frac{a+c}{2}\right) + (b-c) f\left(\frac{c+b}{2}\right)$

RRAM: Right Rectangular Approximation Method

use right height



$A = (c-a) f(c) + (b-c) f(b)$

All bases are the same

Use the table below to approximate the area under the curve using 3 equal intervals and LRAM, RRAM, and MRAM.

x	y
0	4
2	2
4	2
6	4
8	8
10	14
12	22

Left LRAM

$b \cdot h = A$

$[0,4]$ $4 \cdot 4 = 16$

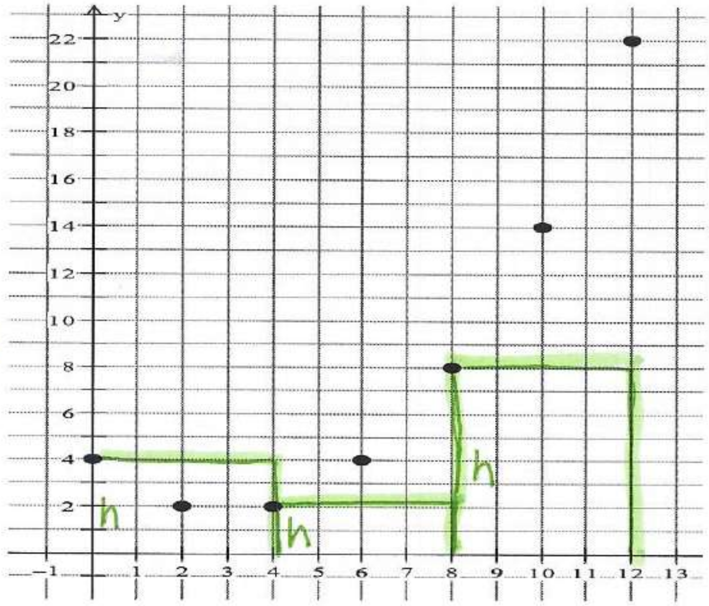
$[4,8]$ $4 \cdot 2 = 8$

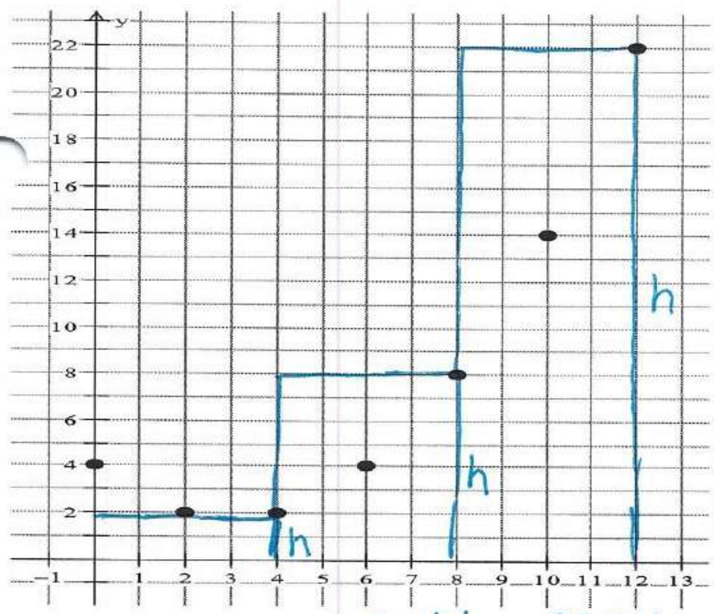
$[8,12]$ $4 \cdot 8 = 32$

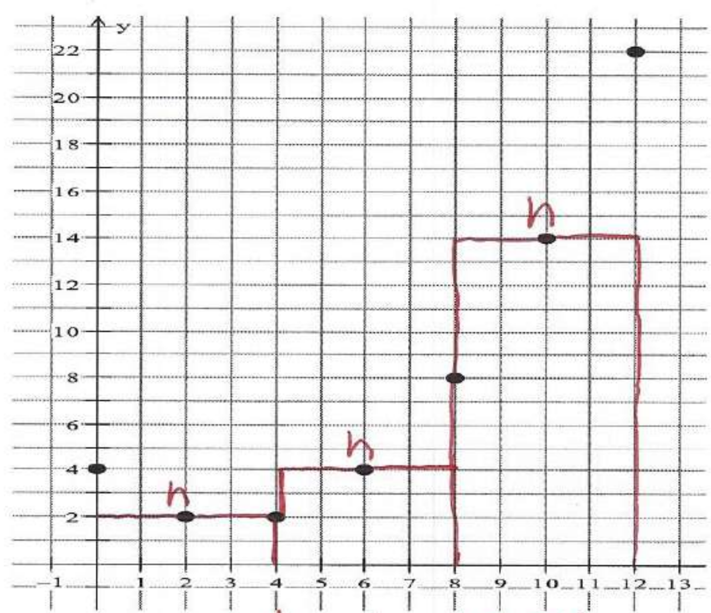
height of left side of rect.

Intervals $\frac{12-0}{3}$ 4 wide

$A = 56 \text{ un}^2$







Right RRAM

$b \cdot h = A$

x	y
0	4
2	2
4	2
6	4
8	8
10	14
12	22

$[0,4]$ $4 \cdot 2 = 8$

$[4,8]$ $4 \cdot 8 = 32$

$[8,12]$ $4 \cdot 22 = 88$

$A = 128 \text{ un}^2$

Midpoint MRAM

x	y
0	4
2	2
4	2
6	4
8	8
10	14
12	22

$[0,4]$ $4 \cdot 2 = 8$

$[4,8]$ $4 \cdot 4 = 16$

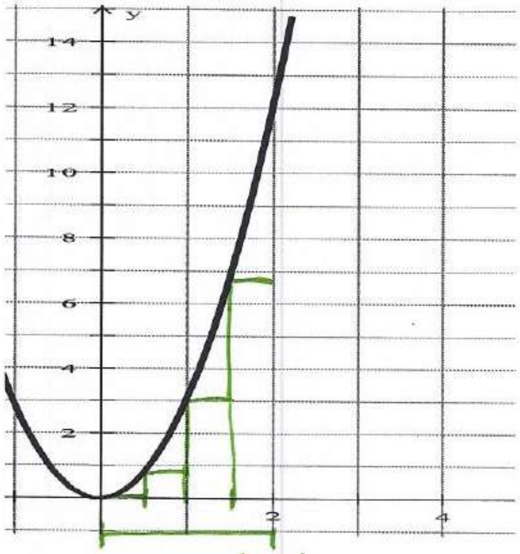
$[8,12]$ $4 \cdot 14 = 56$

$A = 80 \text{ un}^2$

LRAM---RRAM---MRAM

Find the area of the curve bounded by $y = 3x^2$ and the x-axis from $x = 0$ to $x = 2$ using $n = 4$ (note: this denotes the number of rectangles.)

LRAM

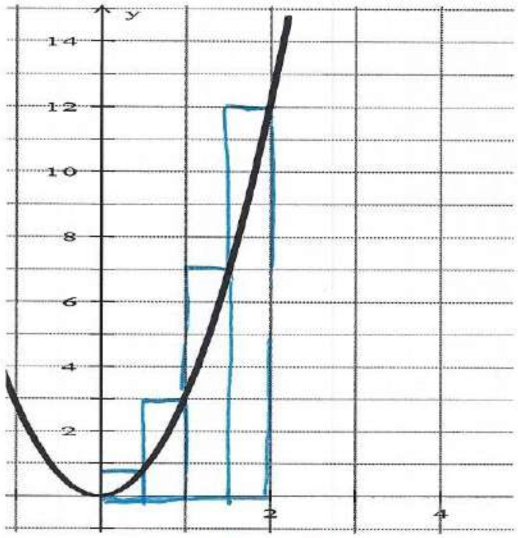


Intervals

$\frac{2-0}{4}$

$\frac{1}{2}$

RRAM

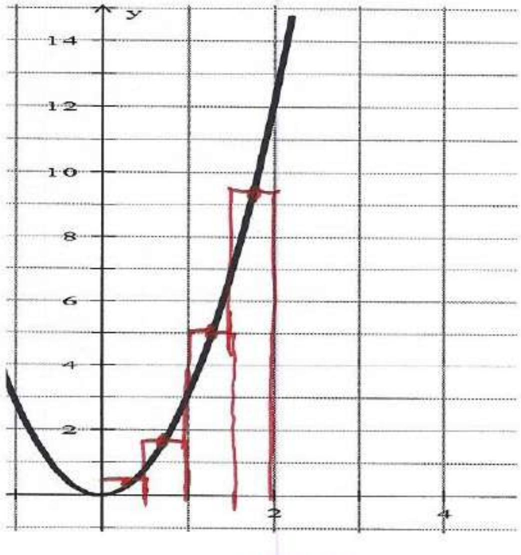


	b · h
$[0,\frac{1}{2}]$	$\frac{1}{2}\cdot f(0) = \frac{1}{2}\cdot 0 = 0$
$[\frac{1}{2},1]$	$\frac{1}{2}\cdot f(\frac{1}{2}) = \frac{1}{2}\cdot\frac{3}{4} = \frac{3}{8}$
$[1,\frac{3}{2}]$	$\frac{1}{2}\cdot f(1) = \frac{1}{2}\cdot 3 = \frac{3}{2}$
$[\frac{3}{2},2]$	$\frac{1}{2}\cdot f(\frac{3}{2}) = \frac{1}{2}\cdot\frac{27}{4} = \frac{27}{8}$

$A = \frac{42}{8}$

	b · h
$[0,\frac{1}{2}]$	$\frac{1}{2}\cdot f(\frac{1}{2}) = \frac{1}{2}\cdot\frac{3}{4} = \frac{3}{8}$
$[\frac{1}{2},1]$	$\frac{1}{2}\cdot f(1) = \frac{1}{2}\cdot 3 = \frac{3}{2}$
$[1,\frac{3}{2}]$	$\frac{1}{2}\cdot f(\frac{3}{2}) = \frac{1}{2}\cdot\frac{27}{4} = \frac{27}{8}$
$[\frac{3}{2},2]$	$\frac{1}{2}\cdot f(2) = \frac{1}{2}\cdot 12 = 6$

$A = \frac{90}{8}$



MRAM

	b · h
$[0,\frac{1}{2}]$	$\frac{1}{2}\cdot f(\frac{1}{4}) = \frac{1}{2}\cdot\frac{3}{16} = \frac{3}{32}$
$[\frac{1}{2},1]$	$\frac{1}{2}\cdot f(\frac{3}{4}) = \frac{1}{2}\cdot\frac{27}{16} = \frac{27}{32}$
$[1,\frac{3}{2}]$	$\frac{1}{2}\cdot f(\frac{5}{4}) = \frac{1}{2}\cdot\frac{75}{16} = \frac{75}{32}$
$[\frac{3}{2},2]$	$\frac{1}{2}\cdot f(\frac{7}{4}) = \frac{1}{2}\cdot\frac{147}{16} = \frac{147}{32}$

$A = \frac{252}{32}$

Midpoints

$\frac{0+\frac{1}{2}}{2} = \frac{1}{4}$ $\quad \frac{\frac{1}{2}+1}{2} = \frac{3}{4}$ $\quad \frac{1+\frac{3}{2}}{2} = \frac{5}{4}$

Approximate the area of the curve bounded by $y = x^3$ and the x-axis from $x = -2$ to $x = 2$ using $n = 4$ using Riemann Sums.

Interval	LRAM b	LRAM h	LRAM A	RRAM b	RRAM h	RRAM A	MRAM b	MRAM h	MRAM A
$[-2,-1]$	1	$f(-2) = -8$	8	1	$f(-1) = -1$	1	1	$f(-\frac{3}{2}) = \frac{-27}{8}$	$\frac{27}{8}$
$[-1,0]$	1	$f(-1) = -1$	1	1	$f(0) = 0$	0	1	$f(-\frac{1}{2}) = \frac{-1}{8}$	$\frac{1}{8}$
$[0,1]$	1	$f(0) = 0$	0	1	$f(1) = 1$	1	1	$f(\frac{1}{2}) = \frac{1}{8}$	$\frac{1}{8}$
$[1,2]$	1	$f(1) = 1$	1	1	$f(2) = 8$	8	1	$f(\frac{3}{2}) = \frac{27}{8}$	$\frac{27}{8}$
			$A = 10$			$A = 10$			$A = \frac{56}{8}$

Let the table of values be for a continuous function. Use the table below to approximate $\int_0^{42} f(x)\,dx$ using LRAM and RRAM using 6 intervals. Cannot split evenly b/c we don't know $f(x)$

x	y
0	7
5	6
12	15
14	10
21	16
38	8
42	19

Interval	LRAM b	LRAM h	LRAM A	RRAM b	RRAM h	RRAM A
$[0,5]$	5	7	35	5	6	30
$[5,12]$	7	6	42	7	15	105
$[12,14]$	2	15	30	2	10	20
$[14,21]$	7	10	70	7	16	112
$[21,38]$	17	16	272	17	8	136
$[38,42]$	6	8	48	6	19	76
			$A = 481$			$A = 479$

Why did we not do the table by MRAM? b/c we don't know midpoint values

Assignment #3: Handout

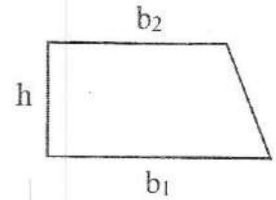
Lesson 4

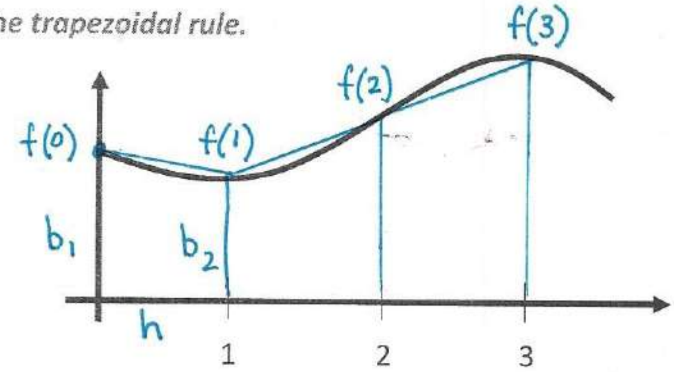
Topic: Trapezoidal Rule

***Goal:** Approximate the area under a curve using the trapezoidal rule.*

Formula for the area of a trapezoid is

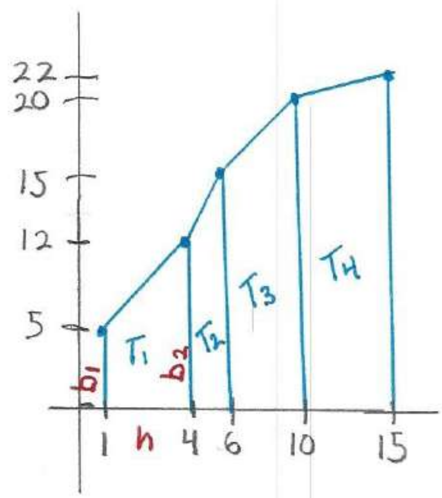
$$A = \frac{1}{2}h(b_1 + b_2)$$





Use the trapezoid rule with four trapezoids to approximate the area under the curve given by the points in the table.

x	1	4	6	10	15
$f(x)$	5	12	15	20	22



$T_1 = \frac{1}{2}(5+12)3 = \frac{51}{2}$

$T_2 = \frac{1}{2}(12+15)2 = 27$

$T_3 = \frac{1}{2}(15+20)4 = 50$

$T_4 = \frac{1}{2}(20+22)5 = 105$

$\int_1^{15} f(x)\,dx = \frac{51}{2} + 27 + 50 + 105$

approximation

Use the trapezoid rule with four trapezoids to approximate the area under the curve.

$\int_0^4 x^2 + 1\,dx$

x	y
0	1
1	2
2	5
3	10
4	17

$T_1 = \frac{1}{2}(1+2)\cdot 1 = \frac{3}{2}$

$T_2 = \frac{1}{2}(2+5)\cdot 1 = \frac{7}{2}$

$T_3 = \frac{1}{2}(5+10)\cdot 1 = \frac{15}{2}$

$T_4 = \frac{1}{2}(10+17)\cdot 1 = \frac{27}{2}$

$\int_0^4 x^2+1\,dx \approx \frac{52}{2}$

Use the data in the table to approximate the area under the curve on the interval [1, 16], using Left Riemann Sums and four subintervals.

x	1	3	8	10	16
$f(x)$	6	2	12	15	2

LRAM

Interval	b	h	A
[1, 3]	2	6	12
[3, 8]	5	2	10
[8, 10]	2	12	24
[10, 16]	6	15	90

$A = 136 \text{ un}^2$

Trap

$T_1 = \frac{1}{2}(6+2)2 = 8$

$T_2 = \frac{1}{2}(2+12)5 = 35$

$T_3 = \frac{1}{2}(12+15)2 = 27$

$T_4 = \frac{1}{2}(15+2)\cdot 6 = 51$

$A = 121 \text{ un}^2$

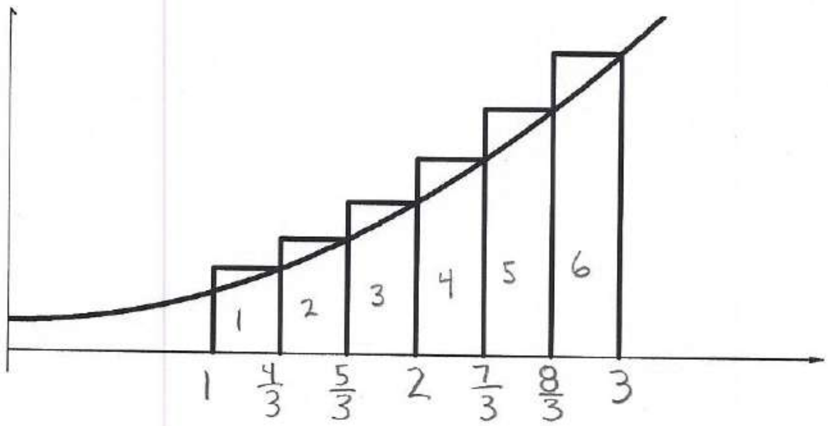
Assignment #2: Page 461: 1, 11 Ignore the directions and approximate the area bounded by the curve and the x–axis using Left Riemann Sums, Right Riemann Sums, Midpoint Sums, and the Trapezoid Method. Let the number of intervals be what is given in the book.

Lesson 5

Topic: Infinite Riemann Sums

***Goal:** To find the area under a curve using the limit of an infinite Riemann sum.*

Approximate the area of the curve bounded by $y = x^2 + 1$ and the x-axis from $x = 1$ to $x = 3$.



If right–endpoint approximations are used with $n = 6$, what are the values of x? Label them on the graph.

How would you find the height of each rectangle? Do it.

$r_1 = f(\frac{4}{3}) = \frac{16}{9} + 1 = \frac{25}{9}$

$r_2 = f(\frac{5}{3}) = \frac{25}{9} + 1 = \frac{34}{9}$

$r_3 = f(2) = 4 + 1 = 5$

$r_4 = f(\frac{7}{3}) = \frac{49}{9} + 1 = \frac{58}{9}$

$r_5 = f(\frac{8}{3}) = \frac{64}{9} + 1 = \frac{73}{9}$

$r_6 = f(3) = 9 + 1 = 10$

Since all bases are the same

height of each rectangle

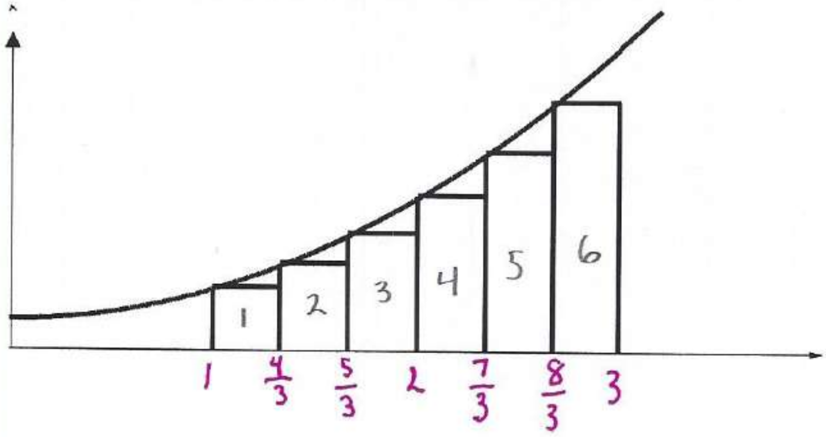
$A = \frac{1}{3}\left(\frac{25}{9} + \frac{34}{9} + 5 + \frac{58}{9} + \frac{73}{9} + 10\right)$

lenght of interval ÷ number of intervals

How would you estimate the total area under the function on this interval? Do it.

(SAME PROBLEM CONTINUED FROM PREVIOUS PAGE…)

Approximate the area of the curve bounded by $y = x^2 + 1$ and the x-axis from $x = 1$ to $x = 3$.



If left–endpoint approximations are used with $n = 6$, what are the values of x? How would you find the height of each rectangle? How would you find the area of each rectangle? Do it.

Heights

r_1 $f(1) = 2$

r_2 $f(\frac{4}{3}) = (\frac{4}{3})^2 + 1 = \frac{25}{9}$

r_3 $f(\frac{5}{3}) = (\frac{5}{3})^2 + 1 = \frac{34}{9}$

r_4 $f(3) = 3^2 + 1 = 10$

r_5 $f(\frac{7}{3}) = (\frac{7}{3})^2 + 1 = \frac{58}{9}$

r_6 $f(\frac{8}{3}) = (\frac{8}{3})^2 + 1 = \frac{73}{9}$

$$\underbrace{\tfrac{1}{3}}_{\text{base}}\left(2 + \frac{25}{9} + \frac{34}{9} + 10 + \frac{58}{9} + \frac{73}{9}\right) = 12.037$$

Now, what about right-endpoint approximations with $n = 50$? Write the first few two terms, the last two terms, and then the sigma notation for the total approximation.

$$\frac{\text{length of interval}}{\#\text{ of intervals } n} = \frac{2}{50} = \frac{1}{25} \quad \text{base of each rect}$$

r_1 $f(1 + \frac{1}{25})$ $A = \frac{1}{25} \cdot f(1 + \frac{1}{25})$

r_2 $f(1 + \frac{2}{25})$ $A = \frac{1}{25} f(1 + \frac{2}{25})$

r_3 $f(1 + \frac{3}{25})$ $A = \frac{1}{25} \cdot f(1 + \frac{3}{25})$

$\vdots$ $\vdots$

r_{49} $f(1 + \frac{49}{25})$ $A = \frac{1}{25} \cdot f(1 + \frac{49}{25})$

r_{50} $f(1 + \frac{50}{25})$ $A = \frac{1}{25} f(1 + \frac{50}{25})$

$A = \frac{1}{25} f(1 + \frac{n}{25})$

width of interval

beginning value of x

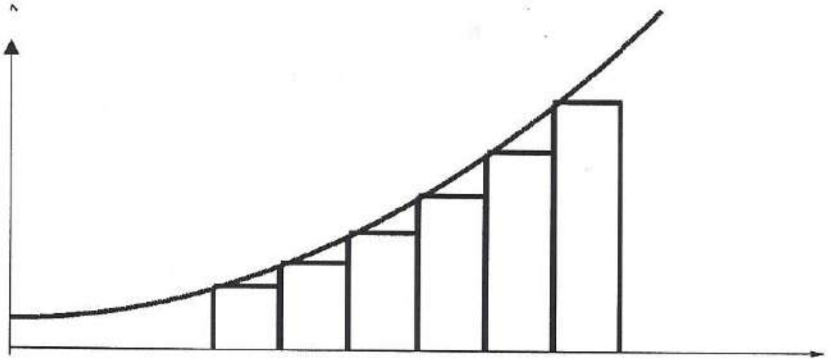
how far into interval

$$A = \sum_{n=1}^{50} \frac{1}{25}\, f\left(1 + \frac{n}{25}\right)$$

$$A = \sum_{n=1}^{50} \frac{1}{25}\left[\left(1 + \frac{n}{25}\right)^2 + 1\right] = 10.827$$

(SAME PROBLEM CONTINUED FROM PREVIOUS PAGE...)

Approximate the area of the curve bounded by $y = x^2 + 1$ and the x-axis from $x = 1$ to $x = 3$.



YOU TRY:

What about left-endpoint approximations with $n = 100$? Write the first few terms, the last two terms, and then the sigma notation for the total approximation.

$\frac{3-1}{100} = \frac{1}{50}$ length of interval (base of rect)

$$\sum_{n=1}^{100} \frac{1}{50}\left[\left(1+\frac{n-1}{50}\right)^2+1\right] \quad \text{or} \quad \sum_{n=0}^{99} \frac{1}{50}\left[\left(1+\frac{n}{50}\right)^2+1\right]$$

$= 10.587$

$\left(1+\frac{0}{50}\right)^2+1$ 1st rect height

$$\int_1^3 x^2+1\,dx = 10.667$$

If $f(x)$ is continuous on $[a, b]$, then the endpoint and midpoint approximations approach one and the same limit as $N \to \infty$. In other words, there is a value L, such that

Right Endpoint **Midpoint** **Left Endpoint**

$$\lim_{N\to\infty} R_N = \lim_{N\to\infty} L_N = \lim_{N\to\infty} M_N = \mathrm{L}$$

If $f(x) \geq 0$, we define the area under the graph over $[a, b]$ to be L.

1. Let A be the area under the graph of $f($ ... how you would set up an <u>infinite</u> Riemann sum to find A.

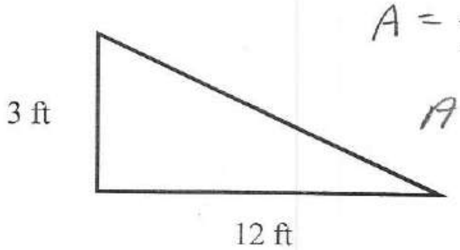
As the base of each rect gets smaller and smaller then

$$\lim_{\Delta x_i \to 0} \sum_{i=1}^{N} f(c_i)\,\Delta x_i = \int_a^b f(x)\,dx$$

Assignment #5: Handout

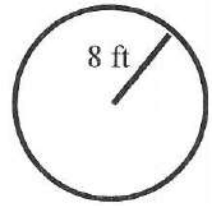
Warm Up **Lesson** 6

Find the area of each shape. No calculator.



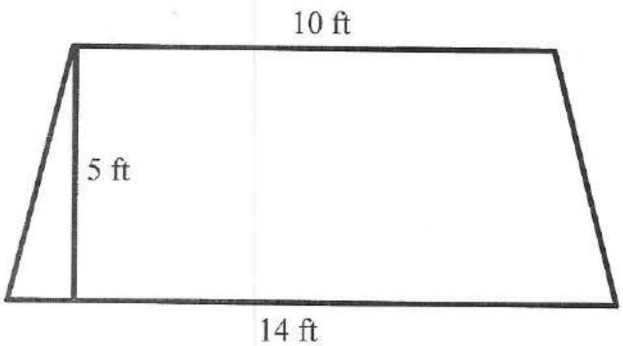
$A = \frac{1}{2}(12)\,3$

$A = 18\ ft^2$



$A = \pi \cdot 8^2$

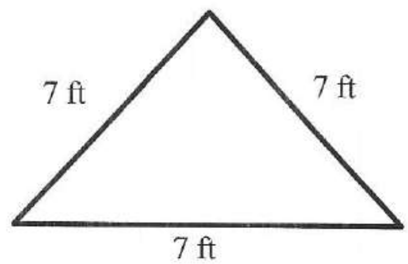
$A = 64\pi\ ft^2$



$A = \frac{1}{2}(b_1 + b_2)h$

$A = \frac{1}{2}(24)5$

$A = 60\ ft^2$



$A = \frac{\sqrt{3}}{4} s^2$

$A = \frac{\sqrt{3}}{4} \cdot 49$

$A = \frac{49\sqrt{3}}{4}\ ft^2$

Topic: *Evaluating Integrals using Geometry*

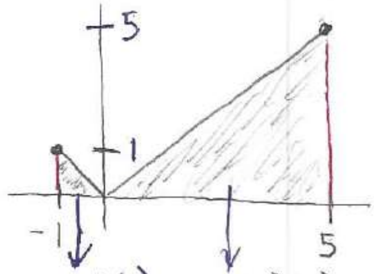
Goal: *Be able to find the area under a curve using geometry.*

An accurate graph will make it a lot easier to setup and evaluate the integrals needed.

$\int_{-1}^{5} |x|\, dx$

$|x| = -x \;,\; x < 0$

$\quad\;\; x \;,\; x > 0$



$\frac{1}{2}(1)(1) \quad \frac{1}{2}(5)(5)$

$\frac{1}{2} + \frac{25}{2} \qquad \frac{26}{2}$

$\frac{1}{2}(1)(1) + \frac{1}{2}(5)(5)$

$\frac{1}{2} + \frac{25}{2}$

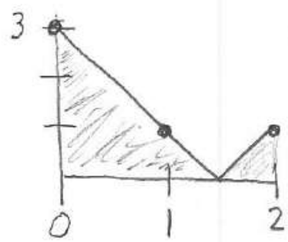
13

$\int_{0}^{2} |2x - 3|\, dx$

vertex

$2x - 3 = 0$

$x = \frac{3}{2}$

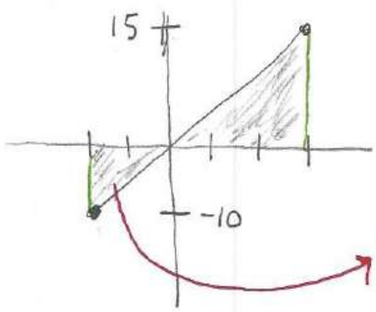


$\frac{1}{2}\left(\frac{3}{2}\right)(3) + \frac{1}{2}\left(\frac{1}{2}\right)(1)$

$\frac{9}{4} + \frac{1}{4}$

$\frac{10}{4}$

$\int_{-2}^{3} 5x\, dx$



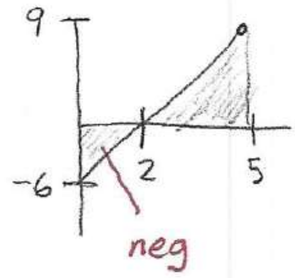
$-\frac{1}{2}(2)(10) + \frac{1}{2}(3)(15)$

$-10 + \frac{45}{2} \qquad -\frac{20}{2} + \frac{45}{2}$

$\frac{25}{2}$

Area under x-axis is Negative

$\int_0^5 3x - 6\,dx$



under x-axis above

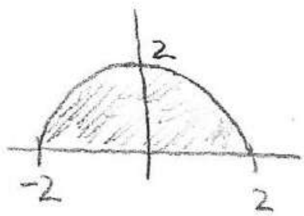
$-\frac{1}{2}(2)(6) + \frac{1}{2}(3)(9)$

$-6 + \frac{27}{2}$

$-\frac{12}{2} + \frac{27}{2}$ $\frac{15}{2}$

$\int_{-2}^{2} \sqrt{4 - x^2}\,dx$

↑
Top half of circle

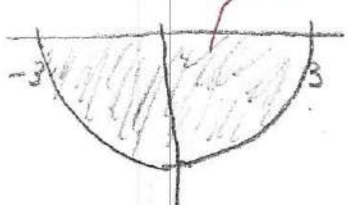


$\frac{1}{2}\pi(2)^2$

2π

$\int_{-3}^{3} -\sqrt{9 - x^2}\,dx$

neg



$-\frac{1}{2}\pi(3)^2$

$-\frac{9}{2}\pi$

Assignment #6: Page 307: 1–10, 13–16

Topic: Definite Integral Lesson 7

Goal: Evaluate an Integral using the first Fundamental Theorem of Calculus.

Indefinite Integral Equation of Anti-Derivative

$\int f(x)dx = F(x) + C$ → Only when no limits of Integration

General Antiderivative

Definite Integral Area between function and x-axis

$\int_a^b f(x)dx = F(x) \Big|_a^b = F(b) - F(a)$

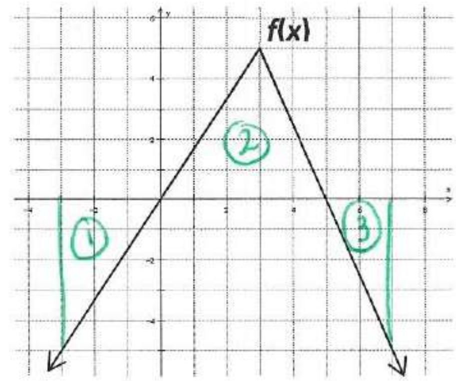
There are limits of integration: **a** and **b**.
a is the lower limit and **b** is the upper limit.

The graph at the right is *f(x)*.

A. Write a piecewise function for *f(x)*.

Rt side
Point (5,0)
Slope $-\frac{5}{2}$
$y - 0 = -\frac{5}{2}(x-5)$
$y = -\frac{5}{2}x + \frac{25}{2}$

$$f(x) = \begin{cases} 2x, & x < 3 \\ -\frac{5}{2}x + \frac{25}{2}, & x \geq 3 \end{cases}$$



B. There are three triangles formed by the graph and the x–axis on the interval [–3, 7].
Find the area of each triangle. assuming you a finding $\int f(x)$. Area below the x-axis is negative

① $-\frac{1}{2}\cdot 3\cdot 5$ = $-\frac{15}{2}$

② $\frac{1}{2}\cdot 5\cdot 5$ = $\frac{25}{2}$

③ $-\frac{1}{2}\cdot 2\cdot 5$ = -5

C. Use the areas from part B to evaluate:

1) $\int_0^5 f(x)dx = \frac{25}{2}$
△2

2) $\int_{-3}^0 f(x)dx = -\frac{15}{2}$
△1

3) $\int_5^7 f(x)dx = -5$
△3

4) $\int_{-3}^5 f(x)dx = -\frac{15}{2} + \frac{25}{2} = 5$
△1 + △2

5) $\int_{-3}^7 f(x)dx = -\frac{15}{2} + \frac{25}{2} - \frac{10}{2} = 0$

6) $\int_0^7 f(x)dx = \frac{25}{2} - 5 = \frac{10}{2}$

7) $\int_0^1 f(x)dx = \frac{1}{2}(1)\frac{5}{3} = \frac{5}{6}$
$\frac{1}{2}bh$

8) $\int_1^1 f(x)dx = \frac{1}{2}\cdot 0\cdot 0 = 0$
$\frac{1}{2}bh$ no base/height

First Fundamental Theorem of Calculus

If $f(x)$ is continuous on [a, b] and F(x) is the antiderivative of $f(x)$, then $\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$

$f(x) = \frac{5}{3}x \;,\; x < 3 \qquad f(x) = -\frac{5}{2}x + \frac{25}{2} \;,\; x \geq 3$

1) $\int_{-3}^{0} f(x)dx = \int_{-3}^{0} \frac{5}{3}x\,dx = \frac{1}{2}\cdot\frac{5}{3}x^2\Big|_{-3}^{0} = \frac{5}{6}\cdot 0^2 - \frac{5}{6}(-3)^2 = -\frac{45}{6} = -\frac{15}{2}$

see area of $\triangle 1$ — opp signs

Switch limits

$\int_{0}^{-3} f(x)dx = \int_{0}^{-3} \frac{5}{3}x\,dx = \frac{1}{2}\cdot\frac{5}{3}x^2\Big|_{0}^{-3} = \frac{5}{6}(-3)^2 - \frac{5}{6}\cdot 0^2 = \frac{45}{6} = \frac{15}{2}$

2) $\int_{5}^{7} f(x)dx = \int_{5}^{7} -\frac{5}{2}x + \frac{25}{2}\,dx = \frac{1}{2}\left(-\frac{5}{2}\right)x^2 + \frac{25}{2}x\Big|_{5}^{7}$

$= -\frac{5}{4}\cdot 7^2 + \frac{25}{2}\cdot 7 - \left(-\frac{5}{4}\cdot 5^2 + \frac{25}{2}\cdot 5\right)$ AP stop

$= -\frac{245}{4} + \frac{175}{2} - \left(-\frac{125}{4} + \frac{125}{2}\right) = -\frac{245}{4} + \frac{350}{4} + \frac{125}{4} - \frac{250}{4}$

$= -\frac{20}{4} = -5$ see $\triangle 3$ Easier to do geometrically

Switch Limits

$\int_{7}^{5} f(x)dx = -\int_{5}^{7} f(x)\,dx = 5$

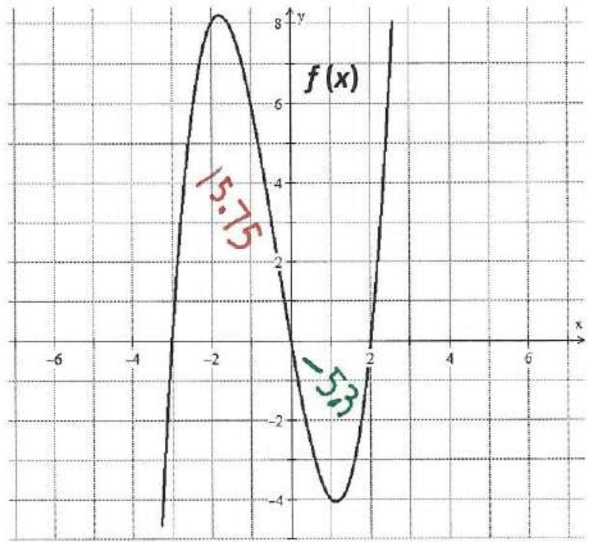
3) $\int_{0}^{1} f(x)dx = \int_{0}^{1} \frac{5}{3}x\,dx = \frac{1}{2}\cdot\frac{5}{3}x^2\Big|_{0}^{1} = \frac{5}{6}\cdot 1^2 - \frac{5}{6}\cdot 0 = \frac{5}{6}$

see #7 previous page

4) $\int_{1}^{1} f(x)dx = \int_{1}^{1} \frac{5}{3}x\,dx = \frac{1}{2}\cdot\frac{5}{3}x^2\Big|_{1}^{1} = \frac{5}{6}\cdot 1^2 - \frac{5}{6}\cdot 1^2 = 0$

The graph at the right is f(x). The area bounded by the curve and the x–axis on the interval [–3, 0] is 15.75 and the area bounded by the curve and the x–axis on the interval [0, 2] is 5.3.

Using the information above, evaluate each integral.



A. $\int_{-3}^{0} f(x)dx =$ 15.75

B. $\int_{0}^{-3} f(x)dx =$ −15.75

C. $\int_{0}^{2} f(x)dx =$ −5.3

D. $\int_{2}^{0} f(x)dx =$ 5.3

E. $\int_{-3}^{2} f(x)dx =$

15.75 − 5.3

10.45

F. $\int_{2}^{-3} f(x)dx =$ −10.45

G. If $\int_{-1}^{0} f(x)dx = 3$, then $\int_{-3}^{-1} f(x)dx =$

$$\int_{-3}^{0} f(x)\,dx = \int_{-3}^{-1} f(x)\,dx + \int_{-1}^{0} f(x)\,dx$$

$$15.75 = A + 3 \qquad A = 12.75$$

H. If $\int_{-1}^{0} f(x)dx = 3$ and $\int_{-2}^{-1} f(x)dx = 7.6$, then $\int_{-3}^{-2} f(x)dx =$

$$\int_{-3}^{0} f(x)\,dx = \int_{-3}^{-2} f(x)\,dx + \int_{-2}^{-1} f(x)\,dx + \int_{-1}^{0} f(x)\,dx$$

$$15.75 = A + 7.6 + 3 \qquad A = 5.15$$

I. If $\int_{1}^{2} f(x)dx = -2.9$, then $\int_{0}^{1} f(x)dx =$

$$\int_{0}^{2} f(x)\,dx = \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx$$

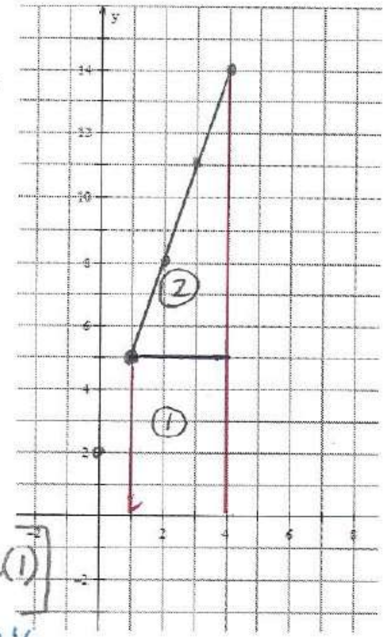
$$-5.3 = A + -2.9 \qquad A = -2.4$$

Let g(x) be a new function such that $\int_{-2}^{3} g(x)\,dx = 19.4$ and $\int_{1}^{3} g(x)\,dx = 3.8$, find:

A. $\int_{-2}^{1} g(x)dx$ $\quad \int_{-2}^{3} = \int_{-2}^{1} + \int_{1}^{3} \quad 19.4 = \int_{-2}^{1} + 3.8 \quad \int_{-2}^{1} = 15.6$

B. $\int_{-2}^{3} 5g(x)dx$ $= 5\int_{-2}^{3} g(x)\,dx = 5(19.4) = 97$

1) Evaluate $\int_1^4 3x+2\,dx$ by



A) Sketching the region bounded by the graph and the x–axis and evaluate using the area under the curve.

$$\underset{①}{3(5)} + \underset{②}{\tfrac{1}{2}(3)(9)} \qquad \frac{57}{2} = 28\tfrac{1}{2}$$

$$15 + \frac{27}{2}$$

B) Using the First Fundamental Theorem of Calculus to evaluate the integral.

$$\int_1^4 f(x)\,dx = F(4) - F(1)$$

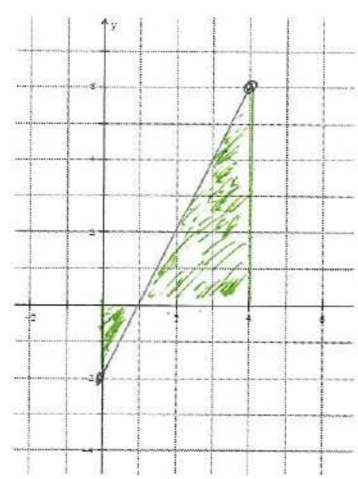
$$\int_1^4 3x+2\,dx = \frac{3x^2}{2} + 2x \Big|_1^4 = \left[\frac{3\cdot 4^2}{2} + 2(4)\right] - \left[\frac{3\cdot 1^2}{2} + 2(1)\right]$$

$$= 24 + 8 - \frac{3}{2} - 2 = 28\tfrac{1}{2}$$

2) Evaluate $\int_0^2 x^2 dx$ using the FTC.

$$\frac{x^3}{3}\Big|_0^2 = \frac{2^3}{3} - \frac{0^3}{3} = \frac{8}{3}$$

3) Evaluate $\int_0^4 2x - 2\,dx$ by



A) Sketching the region bounded by the graph and the x–axis and evaluate using the area under the curve.

$$-\tfrac{1}{2}(1)(2) + \tfrac{1}{2}(3)(6) \qquad 8$$

$$-1 + 9$$

B) Using the First Fundamental Theorem of Calculus to evaluate the integral.

$$\int_0^4 2x-2\,dx = x^2 - 2x \Big|_0^4$$

$$= \left[4^2 - 2(4)\right] - \left[0^2 - 2(0)\right] = 8$$

4) Evaluate $\int_{-2}^{2} x^2 dx = \frac{x^3}{3}\Big|_{-2}^{2} = \frac{2^3}{3} - \frac{(-2)^3}{3} = \frac{8}{3} + \frac{8}{3} = \frac{16}{3}$

Same as $2\int_0^2 x^2\,dx$

Assignment #7: Page 308: 43–46, 55–62,

Use the First Fundamental Theorem of Calculus to do problems 33–42

Topic: First Fundamental Theorem of Calculus

Goal: Evaluate an Integral using the area bounded by the graph and the x–axis and the first Fundamental Theorem of Calculus.

The First Fundamental Theorem of Calculus

If $f(x)$ is continuous on [a, b] and $F(x)$ is the antiderivative of f(x), then $\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$

Note: The result is a Area **number!!!!!!**

$\int_{-3}^{2}(6 - x - x^2)\,dx =$

$6x - \frac{1}{2}x^2 - \frac{1}{3}x^3 \Big|_{-3}^{2}$

$\left[6\cdot 2 - \frac{1}{2}\cdot 2^2 - \frac{1}{3}\cdot 2^3\right] - \left[6(-3) - \frac{1}{2}(-3)^2 - \frac{1}{3}(-3)^3\right]$

AP Test stop

$\int_0^{\frac{\pi}{2}} \sin x\,dx =$

$-\cos x \Big|_0^{\pi/2}$

$-\cos\frac{\pi}{2} - {}^-\cos 0$

$0 + 1$

1

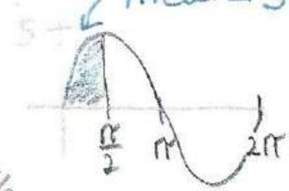
$\int_0^{\frac{\pi}{4}} \sec x \tan x\,dx =$

$\sec x \Big|_0^{\pi/4}$

$\sec\frac{\pi}{4} - \sec 0$

$\sqrt{2} - 1$

Area = 5



$\int_0^{\frac{\pi}{2}} 5\sin x\,dx =$

$-5\cos x \Big|_0^{\pi/2}$

$-5\cos\frac{\pi}{2} - {}^-5\cos 0$

$0 + 5(1)$

5

Same as

$5\int_0^{\pi/2} \sin x\,dx$

$\int_1^2 \left(3 - \frac{6}{x^2}\right) dx = \int_1^2 3 - 6x^{-2}\ dx$

$3x - \frac{6x^{-1}}{-1} \Big|_1^2$

$3x + \frac{6}{x} \Big|_1^2$

$\left(3\cdot 2 + \frac{6}{2}\right) - \left(3\cdot 1 + \frac{6}{1}\right)$ AP Stop

$9 - 9$

0

$\int_{-2}^2 (x^2 - 4)\, dx =$

$\frac{x^3}{3} - 4x \Big|_{-2}^2$

$\left[\frac{2^3}{3} - 4(2)\right] - \left[\frac{(-2)^3}{3} - 4(-2)\right]$ AP STOP

$\frac{8}{3} - 8 + \frac{8}{3} - 8$

$\frac{16}{3} - \frac{48}{3} \qquad -\frac{32}{3}$

$\int_0^1 \sqrt{x}\, dx = \int_0^1 x^{1/2}\ dx$

$\frac{2}{3}x^{3/2} \Big|_0^1$

$\frac{2}{3}\cdot 1^{3/2} - \frac{2}{3}\cdot 0^{3/2}$

$\frac{2}{3}$

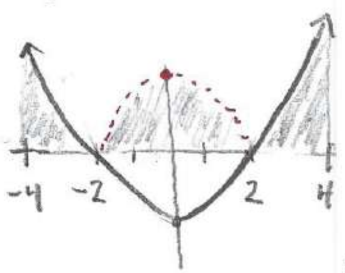
$\int_0^{\frac{\pi}{2}} \cos x\, dx =$

$\sin x \Big|_0^{\pi/2}$

$\sin\frac{\pi}{2} - \sin 0$

1

$\int_{-4}^4 |x^2 - 4|\ dx =$



Same

$\int_{-4}^{-2} x^2 - 4\ dx + \int_{-2}^{2} -x^2 + 4\ dx + \int_2^4 x^2 - 4\ dx$

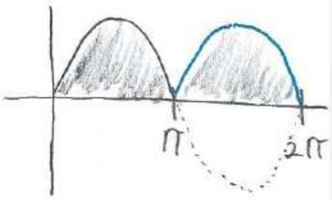
$\frac{x^3}{3} - 4x \Big|_{-4}^{-2} + -\frac{x^3}{3} + 4x \Big|_{-2}^{2} + \frac{-x^3}{3} - 4x \Big|_2^4$

$\left[\frac{(-2)^3}{3} - 4(-2)\right] - \left[\frac{(-4)^3}{3} - 4(-4)\right] + \left[-\frac{2^3}{3} + 4(2)\right] - \left[-\frac{(-2)^3}{3} + 4(-2)\right] +$

$\left[-\frac{4^3}{3} - 4(3)\right] - \left[-\frac{2^3}{3} - 4(2)\right]$

STOP

$\int_0^{2\pi} |\sin x|\, dx =$



$$\int_0^{\pi} \sin x\, dx + \int_{\pi}^{2\pi} -\sin x\, dx$$

$$-\cos x \Big|_0^{\pi} + \cos x \Big|_{\pi}^{2\pi}$$

$$-\cos \pi - \cos 0 + \cos 2\pi - \cos \pi$$

$$-(-1) - (-1) + 1 - (-1)$$

$$4$$

If $f(x) = 9x + \cos(x)$ and F is the antiderivative of f, with F(0) = –4, then find F(3).

$$\int_0^3 9x + \cos x\, dx = \frac{9x^2}{2} + \sin x \Big|_0^3$$

$$F(3) - F(0) = \frac{9}{2}x^2 + \sin x \Big|_0^3$$

$$F(3) - -4 = \frac{9}{2}\cdot 3^2 + \sin 3 - \left[\frac{9}{2}\cdot 0^2 + \sin 0\right]$$

$$F(3) + 4 = \frac{81}{2} + \sin 3$$

$$F(3) = \frac{81}{2} + \sin 3 - 4$$

could Also do as initial value

$y = \frac{9}{2}x^2 + \sin x + C$

$-4 = C$

$y = \frac{9}{2}x^2 + \sin x - 4$

Now plug in 3

$\int_1^9 f(x)\, dx = -5$ and $\int_4^9 f(x)\, dx = 6$, then $\int_1^4 f(x)\, dx = ?$ and $\int_1^9 3f(x) - 8\, dx = ?$

$$\int_1^9 f(x)\, dx = \int_1^4 f(x)\, dx + \int_4^9 f(x)\, dx$$

$$-5 = \int_1^4 f(x)\, dx + 6$$

$$-11 = \int_{-1}^4 f(x)\, dx$$

$$\int_1^9 3f(x) - 8\, dx$$

$$\int_1^9 3f(x)\, dx - \int_1^9 8\, dx$$

$$3\int_1^9 f(x)\, dx - 8x \Big|_1^9$$

$$3(-5) - [8\cdot 9 - 8\cdot 1]$$

$$-15 - 64$$

$$-79$$

Assignment #8: ~~Page 308: 43–46, 55–62~~,
P. 314: 5, 9, 12, 15, 17, 21, 23, 25, 33, 43, 46, 50, 55

Warm-Up **Lesson 9**

1. Determine on what interval $y = e^x - x^3$ is increasing, for $-1 \le x \le 2$. Use your calculator.

Lesson 9

Topic: Leibniz Rule (2nd Fundamental Theorem of Calculus) $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$

Goal: Apply the Second Fundamental Theorem of Calculus to find the derivative of functions defined in terms of an integral.

The 2nd Fundamental Theorem of Calculus: If f is continuous on [a, b] and u(x) and v(x) are differentiable functions of x whose value lie in [a, b], then:

$\frac{d}{dx}\left[\int_{u(x)}^{v(x)} f(t)dt\right] =$

sub upper limit — deriv of upper limit

$f(v(x))\frac{dv}{dx} - f(u(x))\frac{du}{dx}$

sub lower limit — deriv of lower limit

limit is a variable

F(x) = $\int_{\pi/2}^{x^3} \cos t\ dt$ What does this mean?

Find F '(x) =

Method I

Integrate then take the derivative.

$\int_{\pi/2}^{x^3} \cos t\ dt = \sin t \Big|_{\frac{\pi}{2}}^{x^3}$

$= \sin x^3 - \sin\frac{\pi}{2}$

$= \sin x^3 - 1$ Now take deriv

$\cos x^3 \cdot 3x^2$ Long way

Method II

Apply the 2nd Fundamental Theorem of Calculus.

$\frac{d}{dx}\int_{\pi/2}^{x^3} \cos t\ dt = \cos x^3 \cdot 3x^2 - \cos\frac{\pi}{2}\cdot 0 = 3x^2\cos x^3$

can't always integrate so Method 2 works better.

If $f(x) = \int_{1/x}^{x} \frac{1}{t}\, dt$, then find f '(x). $\frac{1}{x} = x^{-1}$

$f'(x) = \frac{1}{x}\cdot 1 - \frac{1}{\frac{1}{x}}(-1x^{-2})$

$= \frac{1}{x} + x\cdot\frac{1}{x^2} = \frac{1}{x} + \frac{1}{x} = \frac{2}{x}$

If $h(x) = \int_{\cos x}^{\sin x} \frac{1}{1-t^2}\, dt$, then find h '(x).

$h'(x) = \frac{1}{1-(\sin x)^2}\cdot\cos x - \frac{1}{1-(\cos x)^2}\cdot -\sin x$

If $g(y) = \int_{\sqrt{y}}^{2\sqrt{y}} \sin t^2\ dt$, then find g '(y). $2\sqrt{y} = 2y^{\frac{1}{2}}$

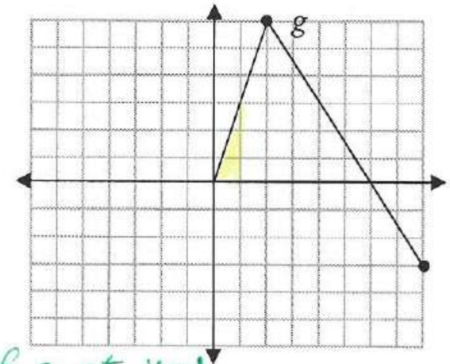
$g'(y) = \sin(2\sqrt{y})^2\, y^{-\frac{1}{2}} - \sin(\sqrt{y})^2\cdot\frac{1}{2}y^{-\frac{1}{2}}$

If F(x) = $\int_3^{2x} \sqrt{t^2+1}\, dt$, then find F '(x). $(t^2+1)^{\frac{1}{2}}$

$F'(x) = [(2x)^2+1]^{\frac{1}{2}}\cdot 2 - [3^2+1]^{\frac{1}{2}}\cdot 0$

$F'(x) = [(2x)^2+1]^{\frac{1}{2}}\cdot 2$

Let f be the function that is continuous and differentiable on the interval [0, 8] defined by f(x) = $\int_0^x g(t)dt$. The graph of g is the piecewise function shown at the right.



A. Find f(1), f '(1), f "(1).

$f(1) = \int_0^1 g(t)\,dt = \frac{1}{2}\cdot 1\cdot 3 = \frac{3}{2}$

$f'(x) = g(x)\cdot 1 - g(x)\cdot 0 \quad f'(1) = g(1) = 3$

$f''(x) = g'(x) \Rightarrow f''(1) = g'(1) = 3$

The slope of g at $x=1$

B. Find f(2), f '(2), f "(2).

$f(2) = \int_0^2 g(t)\,dt = \frac{1}{2}\cdot 2\cdot 6 = 6$

$f'(x) = g(x) \quad f'(2) = g(2) = 6$

$f''(2) = g'(2) = \text{DNE}$ b/c $\lim_{x\to 2^-} g'(x) \neq \lim_{x\to 2^+} g'(x)$ sharp turn is a clue

C. Is x = 6 a maximum or minimum of f? Explain your reasoning.

$x=6$ is a max b/c $f'(x) = g(x)$ and g goes from pos to neg at $x=6 \Rightarrow$ f goes from inc to dec.

D. How many points of inflection does f have? Explain your reasoning.

$g'(x) = f''(x)$ There is 1 POI b/c g goes from inc to dec at $x=2 \Rightarrow g'$ goes from pos to neg.

C. What is the equation of the tangent line of f at x = 1?

Point $(1, \frac{3}{2})$ from A

Slope $f'(1) = 3$

$y - \frac{3}{2} = 3(x-1)$

What is the value of x that maximizes the value of F(x) if F(x) = $\int_x^{x+3} t(5-t)\,dt$?

$F'(x) = (x+3)(5-(x+3))\cdot 1 - x(5-x)\cdot 1$

$F'(x) = (x+3)(-x+2) - 5x + x^2$

$F'(x) = -x^2 + 2x - 3x + 6 - 5x + x^2$

$0 = -6x + 6$

$6x = 6 \quad x = 1$

F': $x=0$ Pos | $x=2$ Neg

F: inc | dec

F has a max at $x=1$ b/c F' goes from pos To neg

Assignment #9: Page: 320: 21–24, 29–32 all, Handout
Assignment #10: Review Handout