Unit #8—Integration (Antiderivatives)

You are responsible for doing all of the homework and checking your work. If you get stuck, the solutions are worked out at the end of the unit and the odd numbered exercises are also available online through the textbook publisher. If you still have questions on the homework problems after going over the solutions, then come in at lunch by appointment, afterschool, or during intervention as class time will not be devoted to going over the homework.

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Homework Heading

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Warm-Up **Lesson 1**

Give the derivative of each.

1. $y = (3x+2)^5$

$y' = 5(3x+2)^4 \cdot 3$

2. $y = (2x^3 + 9)^4$

$y' = 4(2x^3+9)^3 \cdot 6x^2$

3. $y = \sqrt{5x+8}$

$y = (5x+8)^{1/2}$

$y' = \frac{1}{2}(5x+8)^{-1/2} \cdot 5$

4. $y = (e^x + 9)^{10}$

$y' = 10(e^x+9)^9 \cdot e^x$

5. $y = sin(5x)$

$y' = \cos(5x) \cdot 5$

6. $y = tan(x^2 + 1)$

$y' = \sec^2(x^2+1) \cdot 2x$

7. $y = \csc(x^2+1)$

$y' = -\csc(x^2+1)\cot(x^2+1) \cdot 2x$

Topic: *U-Substitutions*

Goal: ***Be able to use u-substitutions to find antiderivatives.***

U-Substitutions

$$\int f(g(x))\,g'(x)dx = \int f(u)du$$

$u = g(x) \qquad du = g'(x)\,dx$

If $y = (7x^3 - 2)^3$,

then $y' = 3(7x^3-2)^2 \cdot 21x^2$

So $\int 21x^2(7x^3 - 2)^2\,dx =$

$\dfrac{(7x^3-2)^3}{3} + C$

$\int (x^2+1)^2(2x)\,dx$

Sub from right

$\int u^2 \cancel{(2x)} \dfrac{du}{\cancel{2x}}$

$\int u^2\,du$

$\dfrac{u^3}{3} + C = \dfrac{(x^2+1)^3}{3} + C$

let $u = x^2+1$

$\frac{d}{dx}(u = x^2+1)$

$\dfrac{du}{dx} = 2x$

$du = 2x\,dx$

$\dfrac{du}{2x} = dx$

$\int (3x^5+1)^5(15x^4)\,dx$

$\int u^5 \cancel{(15x^4)} \dfrac{du}{\cancel{15x^4}}$

$\int u^5\,du$

$\dfrac{u^6}{6} + C = \dfrac{(3x^5+1)^6}{6} + C$

let $u = 3x^5+1$

$\dfrac{du}{dx} = 15x^4$

$du = 15x^4\,dx$

$\dfrac{du}{15x^4} = dx$

$\int 3(3x-1)^4\,dx$

$u = 3x-1$

$\dfrac{du}{dx} = 3$

$du = 3\,dx$

$\dfrac{du}{3} = dx$

$\int \cancel{3}(u)^4 \dfrac{du}{\cancel{3}}$

$\dfrac{u^5}{5} + C$

$\dfrac{(3x-1)^5}{5} + C$

deriv of

$\int (2x+1)(x^2+x)\,dx$

$u = x^2+x$

$du = 2x+1\,dx$

$\dfrac{du}{2x+1} = dx$

$\int (2x+1)\,u\,\dfrac{du}{2x+1}$

$\int u\,du$

$\dfrac{u^2}{2} + C$

$\dfrac{(x^2+x)}{2} + C$

$\int 3x^2\sqrt{x^3-2}\,dx$

$u = x^3-2$

$du = 3x^2\,dx$

$\dfrac{du}{3x^2} = dx$

$\int 3x^2 \cdot u^{1/2} \dfrac{du}{3x^2}$

$\int u^{1/2}\,du$

$\dfrac{2}{3}u^{3/2} + C$

$\dfrac{2}{3}(x^3-2)^{3/2} + C$

$\int \dfrac{-4x}{(1-x^2)^2}\,dx$

$\int -4x(1-x^2)^{-2}dx$

$u = 1-x^2$

$du = -2x\,dx$

$\dfrac{du}{-2x} = dx$

$\int -4x \cdot u^{-2} \dfrac{du}{-2x}$

$\int 2u^{-2}\,du$

$-2u^{-1} + C$

$-2(1-x^2) + C$

$\int \sqrt{2x-1}\,dx =$

$\int (2x-1)^{1/2}\,dx$

$u = 2x-1$

$du = 2\,dx$

$\frac{du}{2} = dx$

$\int u^{1/2}\,\frac{du}{2}$

$\frac{2}{3}u^{3/2}\cdot\frac{1}{2} + C$

$\frac{1}{3}(2x-1)^{3/2} + C$

$\int \frac{x^2+2}{x^4}\,dx =$

$\int \frac{x^2}{x^4} + \frac{2}{x^4}\,dx$

$\int x^{-2} + 2x^{-4}\,dx$

$-1x^{-1} + \frac{2x^{-3}}{-3} + C$

$\int (x^2-9)^3(2x)\,dx =$

deriv of x^2-9

build in reverse

$\frac{(x^2-9)^4}{4} + C$

$\int x^2(x^3+5)^4\,dx =$

deriv of x^3+5

$\frac{(x^3+5)^5}{5} + C$

$\int t^3\sqrt{t^4+5}\,dt =$

$\int t^3(t^4+5)^{1/2}\,dt$

deriv of t^4+5 is $4t^3$

$\frac{1}{4}\,\frac{2}{3}(t^4+5)^{3/2} + C$

$\int x\sqrt{2x-1}\,dx$

$\int x(2x-1)^{1/2}\,dx$

$\int x\cdot u^{1/2}\,\frac{du}{2}$

$u = 2x-1$

$du = 2\,dx$

$\frac{du}{2} = dx$

Need to sub for x

$u = 2x-1$

$u+1 = 2x$

$\frac{u+1}{2} = x$

$\int\left(\frac{u+1}{2}\right)\cdot u^{1/2}\,\frac{du}{2}$

$\int \frac{u^{3/2}}{4} + \frac{u^{1/2}}{4}\,du$

$\frac{2}{5}\cdot\frac{u^{5/2}}{4} + \frac{2}{3}\frac{u^{3/2}}{4} + C$

$\frac{2(2x-1)^{5/2}}{20} + \frac{2(2x-1)^{3/2}}{12} + C$

Assignment #1 Page 334: 27, 31, 33, 37, 39, 43, 65

Warm-Up **Lesson 2**

Give the derivative of each.

1. $y = \sin(x)$

$y' = \cos x$

2. $y = \cos(x)$

$y' = -\sin x$

3. $y = \tan(x)$

$y' = \sec^2 x$

4. $y = \csc(x)$

$y' = -\csc x \cot x$

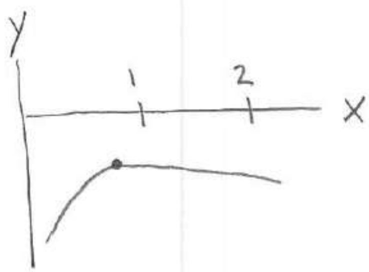
5. $y = \sec(x)$

$y' = \sec x \tan x$

6. $y = \cot(x)$

$y' = -\csc^2 x$

7. Determine on what interval $y = ln(x) - x^2$ is increasing on the interval $0 < x \leq 2$. Use your calculator.



y is inc on $(0, 0.707)$

By Hand

$y' = \frac{1}{x} - 2x$ $\quad \underset{\text{und}}{x = 0}$ but not in $(0, 2)$

$0 = \frac{1}{x} - 2x$

$2x = \frac{1}{x}$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \pm \frac{1}{\sqrt{2}}$

$x = \frac{1}{\sqrt{2}}$ in $(0, 2)$

$x = \frac{1}{2}$ $\quad x = 1$

y' Pos Neg

$0 \qquad \frac{1}{\sqrt{2}} \qquad 2$

y inc dec

$\therefore x = \frac{1}{\sqrt{2}}$ is a max

Topic: *Antiderivatives and Indefinite Integration*

Goal: ***Be able to find the antiderivative of simple trigonometric functions and trigonometric functions using u-substitution.***

Trigonometric Antiderivatives

$\int \sin x\,dx =$

$\int \cos x\,dx =$

$\int \sec^2 x\,dx =$

$\int \csc^2 x\,dx =$

$\int \sec x \tan x\,dx =$

$\int \csc x \cot x\,dx =$

If $y = \sin 8x$

then $y' =$ $\cos(8x)\,8$

So, $\int \cos 8x\,dx =$ $\frac{\sin 8x}{8} + C$

If $y = \tan 6x$

then $y' =$ $\sec^2(6x)\,6$

So, $\int \sec^2 6x\,dx =$ $\frac{\tan 6x}{6} + C$

If $y = \sec 13x$

then $y' =$ $13 \sec 13x \tan 13x$

So, $\int \sec 13x \tan 13x\,dx =$ $\frac{\sec 13x}{13} + C$

If $y = \sin x^2$

then $y' =$ $\cos(x^2)\,2x$

So, $\int 2x \cos x^2\,dx =$ $\sin x^2 + C$

If $y = \tan x^3$

then $y' =$ $\sec^2(x^3)\,3x^2$

So, $\int x^2 \sec^2 x^3\,dx =$ $\frac{\tan x^3}{3} + C$

If $y = \cos x^4$

then $y' =$ $-\sin(x^4)\,4x^3$

So, $\int \sin x^4\,dx =$ $-\cos x^4$?

can't do b/c the deriv of x^4 is missing in the integral.

$\int 5\cos 5x\, dx$

$\sin 5x + C$

$\int \frac{\sin x}{\cos^2 x} dx$

$u = \cos x$

$du = -\sin x\, dx$

$\frac{du}{-\sin x} = dx$

$\int \frac{\sin x}{u^2} \frac{du}{-\sin x}$

$\int \frac{-1}{u^2} du$

$\int -1u^{-2} du$

$u^{-1} + C$

$(\cos x)^{-1} + C$

$\frac{1}{\cos x} + C$

deriv of $\sin 3x$

$\int 3\sin^2 3x \cos 3x\, dx$

$u = \sin 3x$

$du = \cos 3x \cdot 3\, dx$

$\frac{du}{3\cos 3x} = dx$

$\int 3u^2 \cos 3x \cdot \frac{du}{3\cos 3x}$

$\int u^2 du$

$\frac{u^3}{3} + C$

$\frac{\sin^3 3x}{3} + C$

$\int \cos^2 11x \sin 11x\, dx$

$u = \cos 11x$

$du = -\sin(11x)\, 11\, dx$

$\frac{du}{-11\sin(11x)} = dx$

$\int u^2 \sin 11x \cdot \frac{du}{-11\sin 11x}$

$\int \frac{u^2}{-11} du$

$\frac{u^3}{-11 \cdot 3} + C$

$\frac{\cos^3 11x}{-33} + C$

Assignment #2: Page 334: 47, 48, 50, 52, 55

Assignment #2A: Worksheet

Topic: Antiderivatives of Natural Logarithms—Integration Lesson 3

Goal: To integrate with natural logarithms.

Find the derivative of each.

$y = ln(x^2 + 1)$

$y' = \frac{2x}{x^2+1}$

$y = ln(x^3 + x - 2)$

$y' = \frac{3x^2+1}{x^3+x-2}$

$y = (x^2 + 1)^{-2}$

$y' = -2(x^2+1)^{-3} \cdot 2x$

The general rule for integrating is below:

$\int \frac{1}{x}\, dx = ln|x| + c$

$\int \frac{2}{x}\, dx = 2\ln|x| + C$

$\int \frac{1}{x-1}\, dx = \ln|x-1| + C$

$\int_1^4 \frac{1}{5x-1}\, dx = \frac{\ln|5x-1|}{5}\Big|_1^4 = \frac{\ln 19}{5} - \frac{\ln 4}{5}$

$= \frac{1}{5}(\ln 19 - \ln 4) = \frac{1}{5}\ln\frac{19}{4}$ or $\ln\left(\frac{19}{4}\right)^{1/5}$

deriv of x^2-1

$\int_2^4 \frac{2x}{x^2-1}\, dx = \ln|x^2-1|\Big|_2^4$

$= \ln 15 - \ln 3$

$= \ln\frac{15}{3} = \ln 5$

$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{sec^2 x}{tan\, x}\, dx = \ln|\tan x|\Big|_{\pi/4}^{\pi/3}$

$= \ln|\tan\frac{\pi}{3}| - \ln|\tan\frac{\pi}{4}|$

$= \ln\sqrt{3} - \ln 1$ $\ln 1 = 0$

$= \ln\sqrt{3}$

$\int \frac{1}{3x^2-1}\, dx \neq \ln|3x^2-1| + C$

doesn't work because there is no x in the numerator for deriv.

$\int \frac{x}{3x^2-1}\, dx = \frac{\ln|3x^2-1|}{6} + C$

$\int \frac{x+1}{x^2+x}\,dx = \frac{\ln|x^2+x|}{2} + C$

$\int \frac{x^2+x+1}{x}\,dx = \int x + 1 + \frac{1}{x}\,dx$

$= \frac{x^2}{2} + x + \ln|x| + C$

$\int_0^2 \frac{1}{3x+2}\,dx = \frac{\ln|3x+2|}{3}\Big|_0^2$

$= \frac{\ln 8}{3} - \frac{\ln 2}{3}$

$= \frac{1}{3}(\ln 8 - \ln 2) = \frac{1}{3}\ln 4$

$\int \frac{x-1}{x}\,dx = \int \frac{x}{x} - \frac{1}{x}\,dx$

$= \int 1 - \frac{1}{x}\,dx$

$= x - \ln|x| + C$

$\int \frac{(\ln x)^5}{x}\,dx =$

let $u = \ln x$

$du = \frac{1}{x}\,dx$

$x\,du = dx$

$\int \frac{u^5}{x}\cdot x\,du$

$\frac{u^6}{6} + C$

$\frac{(\ln|x|)^6}{6} + C$

$\int \frac{1}{x(\ln x)^3}\,dx = \int \frac{(\ln x)^{-3}}{x}\,dx$

$= \frac{(\ln|x|)^{-2}}{-2} + C$

Let u = x + 3, then rewrite the integral $\int_{x=0}^{2} \frac{x^2+1}{x+3}\,dx$ and integrate with respect to u.

$u = x + 3 \rightarrow u - 3 = x$

$du = 1\,dx$

$du = dx$

$(u-3)^2 + 1 = x^2 + 1$

$u^2 - 6u + 9 + 1 =$

$u^2 - 6u + 10 = x^2 + 1$

limits

$x = 0 \quad u = 3$

$x = 2 \quad u = 5$

$\int_3^5 \frac{u^2 - 6u + 10}{u}\,du$

$\int_3^5 u - 6 + \frac{10}{u}\,du$

$\frac{u^2}{2} - 6u + 10\ln|u|\Big|_3^5$

$\frac{5^2}{2} - 6\cdot 5 + 10\ln 5 - \left(\frac{3^2}{2} - 6\cdot 3 + 10\ln 3\right)$

Assignment #3: Page: 339: 1–5, 8, 59, 60

Topic: Antiderivatives of Natural Logarithms—Integration Lesson 4

Goal: To integrate with natural logarithms involving trigonometric functions.

$\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx$ or let $u = \cos x$

$= -\ln|\cos x| + C$

Flash Cards

Trigonometric Identities
You need to memorize each one of these.

$\int \sin u \, du = -\cos u + c$

$\int \cos u \, du = \sin u + c$

$\int \tan u \, du = \int \frac{\sin u}{\cos u} du = -\ln|\cos u| + c$

$\int \cot u \, du = \int \frac{\cos u}{\sin u} du = \ln|\sin u| + c$

$\int \sec u \, du = \ln|\sec u + \tan u| + c$

$\int \csc u \, du = -\ln|\csc u + \cot u| + c$

$\int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$

$\int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + c$

$\int \cos 8x \, dx = \frac{\sin 8x}{8} + C$

$\int \sin 11x \, dx = -\frac{\cos 11x}{11} + C$

$\int \sec^2 4x \, dx = \frac{\tan 4x}{4} + C$

$\int \csc^2 \frac{1}{2}x \, dx = -\cot\left(\frac{1}{2}x\right) \cdot 2 + C$

$\int \sec 2x \tan 2x \, dx = \frac{\sec 2x}{2} + C$

$\int tan\, 3\,\theta\; d\theta$

$\frac{-\ln|\cos 3\theta|}{3} + C$

$\int sec\frac{\theta}{8}\; d\theta$

$8 \cdot \ln\left|\sec\frac{\theta}{8} + \tan\frac{\theta}{8}\right| + C$

deriv of θ^3

$\int 3\theta^2 \cot\theta^3\; d\theta$

$\int 3\theta^2 \frac{\cos\theta^3}{\sin\theta^3}\, d\theta$

$\ln|\sin\theta^3| + C$

$\int csc\,\theta^2\, sin\,\theta^2\; d\theta$

$\int \frac{1}{\sin\theta^2} \cdot \sin\theta^2\, d\theta$

$\int 1\, d\theta$

$\theta + C$

$\int \frac{cos^3\,\theta}{1-sin^2\,\theta}\; d\theta$

$\int \frac{\cos^3\theta}{\cos^2\theta}\, d\theta$

$\int \cos\theta\, d\theta$

$\sin\theta + C$

deriv of $\tan\theta$

$\int \frac{sec^2\,\theta}{1+tan\,\theta}\; d\theta$

$\ln|1+\tan\theta| + C$

Assignment #4: Handout

Topic: Integrating e^x Lesson 5

Goal: To find the antiderivative of e^x.

Find the derivative of each.

1. $y = e^{5x}$
$y' = 5e^{5x}$

2. $y = e^{x^3}$
$y' = 3x^2 e^{x^3}$

3. $y = e^{\sin x}$
$y' = \cos x \cdot e^{\sin x}$

General Rule for integrating e^x: $\int e^u\, du = e^u + c$

$\int 9e^{9x}\, dx = e^{9x} + C$

$\int e^{3x+1}\, dx = \frac{1}{3} e^{3x+1} + C$

let $u = 3x + 1$
$du = 3\, dx$
$\frac{du}{3} = dx$

$\int e^u \frac{du}{3}$
$\frac{1}{3} e^u + C$
$\frac{1}{3} e^{3x+1} + C$

$\int 5xe^{-x^2}\, dx = \frac{5e^{-x^2}}{-2} + C$

let $u = -x^2$
$du = -2x\, dx$
$\frac{du}{-2x} = dx$

$\int 5x e^u \frac{du}{-2x}$
$\int \frac{5}{-2} e^u\, du$
$\frac{5}{-2} e^u + C$
$-\frac{5}{2} e^{-x^2} + C$

$\int \frac{e^{\frac{1}{x}}}{x^2}\, dx =$

let $u = \frac{1}{x} = x^{-1}$
$du = -1x^{-2}\, dx$
$du = \frac{-1}{x^2}\, dx$

$-x^2 du = dx$

$\int \frac{e^u}{x^2} \cdot -x^2 du$

$\int -e^u\, du$ $\quad -e^u + C \quad -e^{1/x} + C$

$\int (\sin x) e^{\cos x}\, dx =$

$-e^{\cos x} + C$

$\int_0^1 e^{-2x}dx =$ $\dfrac{e^{-2x}}{-2}\Big|_0^1 = -\frac{1}{2}e^{-2} - -\frac{1}{2}e^{0} = -\dfrac{1}{2e^2} + \dfrac{1}{2}$

$\int_0^1 xe^{-x^2}dx =$ $\dfrac{e^{-x^2}}{-2}\Big|_0^1 = -\frac{1}{2}e^{-1} - -\frac{1}{2}e^{0} = -\dfrac{1}{2e} + \dfrac{1}{2}$

$\int_1^3 \frac{e^{\frac{3}{x}}}{x^2}dx =$

$u = \frac{3}{x} = 3x^{-1}$

$du = -3x^{-2}dx$

$du = -\frac{3}{x^2}dx$

$\frac{x^2}{-3}du = dx$

$\int_1^3 \frac{e^u}{x^2}\cdot\frac{x^2}{-3}du$

$\int_1^3 -\frac{1}{3}e^u du$

$-\frac{1}{3}e^u \Big|_{x=1}^3$

$-\frac{1}{3}e^{\frac{3}{x}}\Big|_1^3$

$-\frac{1}{3}e^1 - -\frac{1}{3}e^3$

Mixed Integration

$\int \frac{e^{\ln x}}{x}dx =$

$e^{\ln x} + C = x + C$

$\int e^{\ln x}\cdot\frac{1}{x}dx$

deriv of $\ln x$

$\int_0^{\sqrt{2}} xe^{-\left(\frac{x^2}{2}\right)}dx =$

$\int_{u=0}^{-1} x e^u \frac{du}{-x} = \int_{u=0}^{-1} -e^u du = -e^u\Big|_0^{-1}$

$= -e^{-1} - -e^0$

$= -\frac{1}{e} + 1$

$u = -\frac{x^2}{2}$

$du = -x\,dx$

$\frac{du}{-x} = dx$

limits $x = 0$ $u = -\frac{0^2}{2} = 0$

$x = \sqrt{2}$ $u = -\frac{\sqrt{2}^2}{2} = -1$

$\int_0^1 \frac{e^x}{1+e^x}\,dx =$ — deriv of denom

$\ln|1+e^x| \Big|_0^1 = \ln|1+e| - \ln|1+e^0|$

$= \ln|1+e| - \ln 2$

$= \ln \frac{1+e}{2}$

$\int_{-1}^0 e^x \cos e^x\,dx =$ $\sin e^x \Big|_{-1}^0$

$= \sin e^0 - \sin e^{-1}$

$= \sin 1 - \sin \frac{1}{e}$

$\int_{-1}^1 \frac{1}{x+2}\,dx =$ $\ln|x+2| \Big|_{-1}^1$

$= \ln 3 - \ln 1$

$= \ln \frac{3}{1}$

$= \ln 3$

$\int_e^{e^2} \frac{1}{x\ln x}\,dx =$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$x\,du = dx$

$\int_e^{e^2} \frac{1}{x}\cdot\frac{1}{u}\,x\,du$

$\int_e^{e^2} \frac{1}{u}\,du$

$\ln|u| \Big|_{x=e}^{e^2}$

$\ln|\ln x| \Big|_e^{e^2}$

$\ln|\ln e^2| - \ln|\ln e|$

$\ln 2 - \ln 1$

$\ln 2$

Assignment #5: Page: 334: 63, 64, 65, 66 Page 339: 33, 38, 47, 48, 49, 57

Topic: Exponential Functions in General—Derivatives Lesson 6

Goal: To differentiate functions of the form y = a^x.
General Rule: $\frac{d}{dx}a^u = (\ln a)a^u u'$
Find y′ if …

$y = 3^x$

$y' = 3^x \ln 3$

$y = 12^x$

$y' = 12^x \ln 12$

$y = 5^{\tan x}$

$y' = 5^{\tan x} \ln 5 \sec^2 x$

$y = 10^x$

$y' = 10^x \ln 10$

$y = 23^{9x^7}$

$y' = 23^{9x^7} \ln 23 \cdot 63x^6$

$y = 4^{\sec x}$

$y' = 4^{\sec x} \ln 4 \sec x \tan x$

Topic: Exponential Functions in General--Integrating

Goal: To integrate functions of the form ∫ a^{x}dx.

General Formula: $\int a^u du = \frac{1}{\ln a}a^u + C$

$\int 8^x dx = \frac{1}{\ln 8} 8^x + C$

$\int 3^{\sin 2x} \cos 2x\, dx =$

deriv of exp

$\frac{1}{2 \cdot \ln 3} 3^{\sin 2x} + C$

$\int_0^1 \frac{1}{2^x} dx = \int_0^1 2^{-x} dx$

$\frac{-1}{\ln 2} 2^{-x} \Big|_0^1$

$\frac{-1}{\ln 2} \cdot 2^{-1} - \frac{-1}{\ln 2} \cdot 2^0$

$\frac{-1}{2 \ln 2} + \frac{1}{\ln 2}$

$\int_1^2 5^x dx = \frac{1}{\ln 5} \cdot 5^x \Big|_1^2$

$= \frac{1}{\ln 5} \cdot 5^2 - \frac{1}{\ln 5} \cdot 5$

$= \frac{25}{\ln 5} - \frac{5}{\ln 5}$

$= \frac{20}{\ln 5}$

$\int x5^{x^2}dx=$ $\frac{1}{2\ln 5}\cdot 5^{x^2}+C$

$\int(2x+1)7^{x^2+x}\,dx=$ $\frac{1}{\ln 7}\cdot 7^{x^2+x}+C$ $u=x^2+x$

deriv of exponent

$\int(\sec^2 x)12^{\tan x}\,dx=$ $\frac{1}{\ln 12}\cdot 12^{\tan x}+C$

deriv of exp

$\int 2^x(2^x+1)^3\,dx=$

$u=2^x+1$

$du=\ln 2\cdot 2^x\,dx$

$\frac{du}{\ln 2\cdot 2^x}=dx$

$\int \cancel{2^x}(u)^3\frac{du}{\ln 2\cdot\cancel{2^x}}$

$\int u^3\frac{du}{\ln 2}$

$\frac{1}{\ln 2}\cdot\frac{u^4}{4}+C$

$\frac{1}{4\ln 2}(2^x+1)^4+C$

$\int\frac{9^x}{9^x+4}\,dx=$ $\frac{\ln|9^x+4|}{\ln 9}+C$ $u=9^x+4$

Assignment #6: Page: 339: 27, 28, 29, 30, 31, 37, 65, 70

Lesson 7

Topic: Integrals Involving Inverse Trigonometric Functions

***Goal:** To be able to integrate using the inverse trigonometric functions.*

The formulas for the inverse trigonometric functions are:

$$\int \frac{1}{\sqrt{1-u^2}}\,du = sin^{-1}u + c \qquad \int \frac{-1}{\sqrt{1-u^2}}\,du = cos^{-1}u + c \qquad \int \frac{1}{1+u^2}\,du = tan^{-1}u + c$$

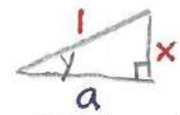
Play Time

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$

$y = \sin^{-1} x$

$\sin y = \sin \sin^{-1} x$

$\sin y = x$



$\sqrt{1-x^2}$

$1^2 = x^2 + a^2$

$1 - x^2 = a^2$

$\sqrt{1-x^2} = a$

$\frac{d}{dx}(\sin y = x)$

$\cos y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\cos y}$ see Δ

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

So if $y = \sin^{-1} x$

$y' = \frac{1}{\sqrt{1-x^2}}$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$$

$$\int \frac{x^2}{\sqrt{1-x^6}}\,dx = \int \frac{x^2}{\sqrt{1-(x^3)^2}}\,dx = \frac{\sin^{-1} x^3}{3} + C$$

$\int \frac{1}{1+9x^2}\,dx = \int \frac{1}{1+(3x)^2}\,dx = \frac{\tan^{-1} 3x}{3} + C$

$\int \frac{-1}{\sqrt{1-4u^2}}\,du = \int \frac{-1}{\sqrt{1-(2u)^2}}\,du = \frac{\cos^{-1} 2u}{2} + C$

Assignment 7: Page 338: 9, 13, 15, 17, 19, 21, 23, 35, 44, 45, 54, 68

Assignment 8: Review Handout