AP Calculus AB Y Assignments

it #5—L'Hôpital, Curve Sketching

You are responsible for doing all of the homework and checking your work. If you get stuck, the solutions are worked out at the end of the unit and the odd numbered exercises are also available online through the textbook publisher. If you still have questions on the homework problems after going over the solutions, then come in at lunch by appointment, afterschool, or during intervention as class time will not be devoted to going over the homework.

Assignment #1:	**L'Hopital** **Page 246: 1, 6, 7, 11, 12, 13, 14, 18, 25, 29, 39, 51**
Assignment #2:	**Curve Sketching Polynomials** **Worksheet**
Assignment #3:	**Curve Sketching Radical Functions-Fractional Powers** **Page 255: 22, 25, 27, 28, 35**
Assignment #4:	**Curve Sketching Rational Functions** **Worksheet**
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Test

Homework Heading

Assignment Number

Name

Period

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Topic: L'Hôpital **Lesson 1**

Goal: To use L'Hopital to find the limits when the limit is indeterminate.

What is indeterminate?

$\frac{0}{0}, \frac{\infty}{\infty}, \frac{-\infty}{\infty}, \frac{\infty}{-\infty}$, or $\frac{-\infty}{-\infty}$

L'Hôpital

If a limit is ***indeterminate*** (see above) ***then*** you can do the following:

$$\lim_{x\to c}\frac{f(x)}{g(x)}=\lim_{x\to c}\frac{f'(x)}{g'(x)}$$

What is this limit?

removable discontinuity

$\lim_{x\to 2}\frac{3x-6}{x^2-4}=$ $\frac{3\cdot 2-6}{2^2-4}=\frac{0}{0}$ $\lim_{x\to 2}\frac{3\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}=\frac{3}{4}$

Now again using ***L'Hôpital.*** Take deriv of num and denom

$\lim_{x\to 2}\frac{3x-6}{x^2-4}=$ $\lim_{x\to 2}\frac{3}{2x}=\frac{3}{4}$

$\lim_{x\to\infty}\left(\frac{5x^3}{2x^3+7x^2}\right)=$ $\frac{\infty}{\infty}$ $\lim_{x\to\infty}\frac{15x^2}{6x^2+14x}=\frac{\infty}{\infty}$

$\lim_{x\to\infty}\frac{30x}{12x+14}=\frac{\infty}{\infty}$ $\lim_{x\to\infty}\frac{30}{12}=\frac{5}{2}$

deg num = deg denom

1. $\lim_{x\to 0}\frac{\sin x}{x}=$ $\frac{\sin 0}{0}=\frac{0}{0}$

$\lim_{x\to 0}\frac{\cos x}{1}=\cos 0=1$

2. $\lim_{x\to 0}\frac{1-\cos x}{x}=$ $\frac{1-\cos 0}{0}=\frac{0}{0}$

$\lim_{x\to 0}\frac{\sin x}{1}=\sin 0=0$

3. $\lim_{x\to 1}\frac{x+3}{x^2+1}=$ $\frac{1+3}{1^2+1}=2$

Not Indeterminant

4. $\lim_{x\to 1}\frac{x+3}{x^2-1}=$ $\frac{1+3}{1^2-1}=\frac{4}{0}$

Not Indeterminant

$f(x)=\frac{x+3}{x^2-1}=\frac{x+3}{(x-1)(x+1)}$

There is a vert asymptote at $x=1,-1$

5. $\lim\limits_{x\to 0}\frac{\sin^2 x}{x\cos x} = \frac{\sin^2 0}{0\cos 0} = \frac{0}{0}$

$$= \lim_{x\to 0}\frac{2\sin x\cos x}{1\cdot\cos x + x(-\sin x)} = \frac{2\sin 0\cos 0}{\cos 0 + 0} = \frac{0}{1} = 0$$

6. $\lim\limits_{x\to\infty} e^{-x}\sqrt{x} = \lim\limits_{x\to\infty}\frac{x^{1/2}}{e^x} = \frac{\infty}{\infty}$

$$\lim_{x\to\infty}\frac{\frac{1}{2}x^{-1/2}}{e^x} = \frac{1}{2e^x x^{1/2}} = \frac{1}{\infty} = 0$$

~~7. $\lim\limits_{x\to 1^+}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right) = \frac{1}{\ln 1} - \frac{1}{1-1}$ = und~~

8. $\lim\limits_{x\to 0^+} x\ln x = 0\cdot\overset{?}{\ln 0}$

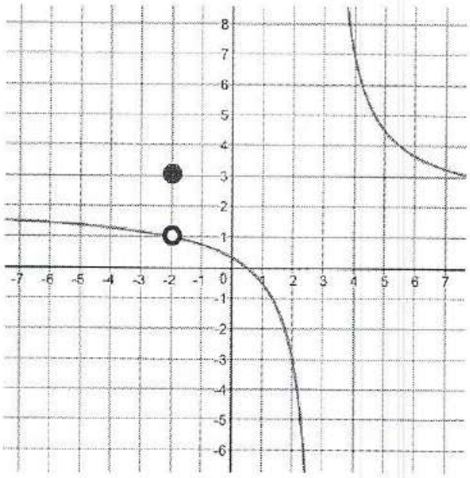
$$= \lim_{x\to 0^+} 1\cdot\ln x + x\cdot\frac{1}{x} = \overset{?}{\ln 0} + 1$$

$$= \lim_{x\to 0^+}\frac{1}{x}$$

9. $\lim\limits_{x\to 1}\frac{e^x - e}{\ln x} = \frac{e^1 - e}{\ln 1} = \frac{0}{0}$

$$\lim_{x\to 1}\frac{e^x}{\frac{1}{x}} = \lim_{x\to 1} e^x x = e^1\cdot 1 = e$$

Find each limit using the graph of *f* shown at the right.



$\lim_{3^-} f(x) = -\infty$ / DNE $\qquad \lim_{x\to 3^+} f(x) = \infty$ / DNE

$\lim_{x\to -2} f(x) = 1 \qquad f(-2) = 3$

$\lim_{x\to -\infty} f(x) = 2 \qquad \lim_{x\to +\infty} f(x) = 2$

$\lim_{x\to 4} f(x) = 7 \qquad f(4) = 7$

Now find the limits algebraically using the function $f(x) = \frac{2x^2+3x-2}{x^2-x-6}$ compare your answers to the ones above.

$\lim_{x\to 3^-} f(x) =$ DNE / $-\infty$ $\qquad \lim_{x\to 3^+} f(x) =$ DNE / ∞

$$= \frac{2\cdot 3^2 + 3(-2) - 2}{3^2 - 3 - 6} = \frac{25}{0}$$

x	y
3.5	12
2.5	-8

$$f(x) = \frac{(2x-1)(x\cancel{+}2)}{(x-3)(\cancel{x}+2)}$$

VA x = 3

HA y = 2

Not indeterminant. This is a vertical asymptote.

$\lim_{-2} f(x) = 1$

$$= \frac{2(-2)^2 + 3(-2) - 2}{(-2)^2 + 2 - 6} = \frac{0}{0} \quad \text{L'H} \lim_{x\to -2} \frac{4x+3}{2x-1} = \frac{4(-2)+3}{2(-2)-1} = \frac{-5}{-5} = 1$$

$\lim_{x\to -\infty} f(x) = 2 \qquad \lim_{x\to +\infty} f(x) =$ Same for $\pm\infty$: 2

$$\lim_{x\to -\infty} \frac{4x+3}{2x-1} = \frac{\infty}{\infty} \quad \text{L'H} \lim_{x\to -\infty} \frac{4}{2} = 2$$

$\lim_{x\to 4} f(x) = 7$

$$= \frac{2(4)^2 + 3(4) - 2}{4^2 - 4 - 6} = \frac{42}{6} = 7$$

Assignment #1: Page 246: 1, 6, 7, 11, 12, 13, 14, 18, 25, 29, 39, 51

Warm Up

Lesson 2

Give the vertical and horizontal asymptotes of $f(x) = \frac{2x^2-3x-2}{x^2+2x-3}$

Remember, vertical asymptotes are when the denominator equals 0 and the numerator does not equal 0 and horizontal asymptotes are given by the $\lim_{x\to\pm\infty} f(x)$.

HA $\lim_{x\to\pm\infty} \frac{2x^2-3x-2}{x^2+2x-3} = 2$

VA $x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3 \quad x=1$

Fill in This page so Time is not needed in class.

First Derivative Test

Fill In

Increasing: $f'(x) > 0$ for all x in (a, b), then f is increasing on [a, b]

Decreasing: $f'(x) < 0$ for all x in (a, b), then f is decreasing on [a, b]

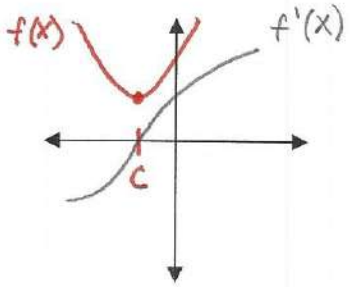
Constant: $f'(x) = 0$ for all x in (a, b), then f is constant on [a, b]

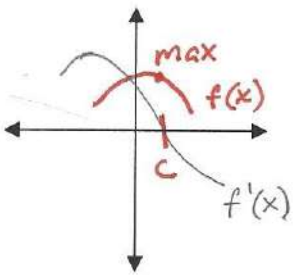
If $f'(x)$ changes from negative to positive at c, **then** $f(c)$ is a minimum.

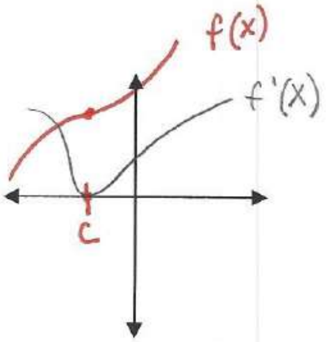
If $f'(x)$ changes from positive to negative at c, **then** $f(c)$ is a maximum.

If $f'(x)$ has an x-intercept at c but does not change signs at c, **then** $f(c)$ is neither a relative maximum nor a relative minimum.

Draw a picture to illustrate each.







Second Derivative Test

Concavity: Let f be differentiable on an open interval.

Method I

If f' is increasing on the interval **then** the graph of f is concave up on the interval.

If f' is decreasing on the interval **then** the graph of f is concave down on the interval.

Method II

If $f''(x) > 0$ for all x on an interval, **then** the graph of f is concave up on the interval.

If $f''(x) < 0$ for all x on an interval, **then** the graph of f is concave down on the interval.

Points of Inflection: The point at which a graph changes concavity. If $(c, f(c))$ is a point of inflection of the graph of f, then either $f''(c)=0$ or f'' DNE at x = c and the graph changed concavity.

Note: If $f'(c) = 0$ and…

If $f''(c) > 0$, **then** f has a relative minimum at (c, f(c)) or

If $f''(c) < 0$, **then** f has a relative maximum at (c, f(c)).

If $f''(c) = 0$, **then** the test fails and you have to use the first derivative test.

It can have a max, min, or neither.

Goal: To sketch the derivative of a function and correlate where the derivative is positive and negative to where the function is increasing or decreasing.

$\ldots) = x^3 - \frac{3}{2}x^2$

Without Calculus, what can you tell about the graph?

y-int $(0,0)$

x-int $0 = x^3 - \frac{3}{2}x^2$

$0 = x^2(x - \frac{3}{2})$

$x = 0$ mult of 2 $\quad x = \frac{3}{2}$

Domain $x \in \mathbb{R}$ $\quad$ Range $y \in \mathbb{R}$

Right side ↗ Pos lead term

Left side ↙ degree 3

First Derivative

What information about the function do we get from the first derivative?

Step 1: Compute the Derivative

Step 2: Find all critical numbers

Step 3: Set up intervals and test.

Step 4: Identify intervals of increasing and decreasing and maximum and minimum points.

$f'(x) = 3x^2 - 3x$

Critical $<$ f' und none / f' = 0

$0 = 3x(x-1)$

$x = 0, 1$

	$x=-1$		$x=.5$		$x=2$
$f'(x)$	Pos		Neg		Pos
		0		1	
$f(x)$	inc	Max	dec	Min	inc

$f(x)$ is inc on $(-\infty, 0)$ and $(1, \infty)$ b/c $f' > 0$

$f(x)$ is dec on $(0,1)$ b/c $f' < 0$

max $x = 0$

$f(0) = 0 \quad (0,0)$

f has a max at $x = 0$ b/c f' went from pos to neg ⇒ f went from inc to dec.

min $x = 1$

$f(1) = 1^3 - \frac{3}{2} \cdot 1^2$

$= -\frac{1}{2}$

$(1, -\frac{1}{2})$

f has a min at $x = 1$ b/c f' went from neg to pos ⇒ f went from dec to inc.

Second Derivative

What information about the function do we get from the second derivative?

Step 1: Second Derivative

Step 2: Find all critical numbers.

Step 3: Set up intervals and test.

Step 4: Identify intervals of concavity and inflection points.

$f'(x) = 3x^2 - 3x$

Critical # < f'' und None; $f'' = 0$

$f''(x) = 6x - 3$

$0 = 6x - 3$

$3 = 6x \quad x = \frac{1}{2}$

f'' x = 0 x = 1

Neg | Pos

f cc dn $\frac{1}{2}$ cc up

POI

POI $x = \frac{1}{2}$

$f(\frac{1}{2}) = (\frac{1}{2})^3 - \frac{3}{2}(\frac{1}{2})^2$

$= \frac{1}{8} - \frac{3}{8} = -\frac{1}{4}$

$(\frac{1}{2}, -\frac{1}{4})$

f has a POI at $x = \frac{1}{2}$ b/c f'' goes from neg to pos.

Organize your data on the number lines. Then sketch *f(x)*.

$f'(x)$ POS | Neg | POS

$f(x)$ inc 0 max dec 1 min inc

$f''(x)$ Neg | Pos

$f(x)$ cc dn $\frac{1}{2}$ POI cc up

x-int $(\frac{3}{2}, 0)$ $(0,0)$

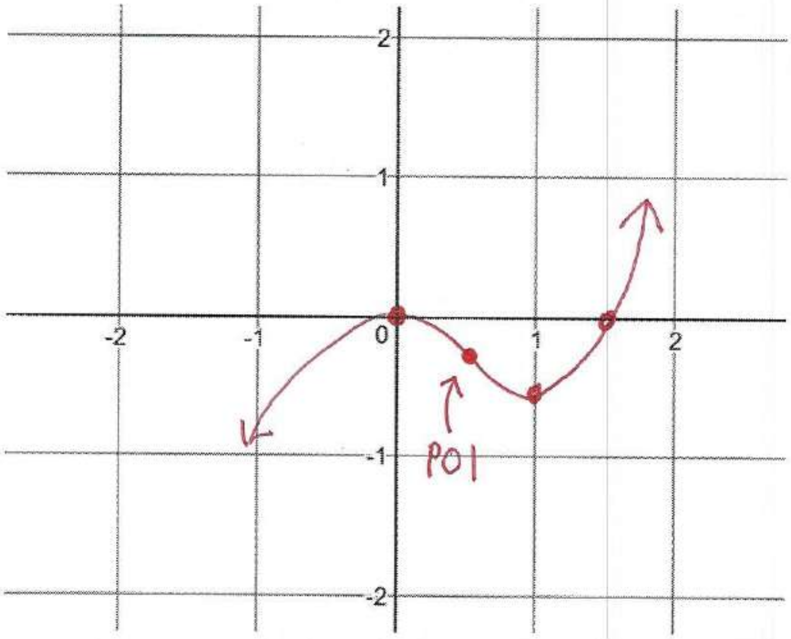
y-int $(0,0)$ max

$f(1) = -\frac{1}{2}$ min

$f(\frac{1}{2}) = -\frac{1}{4}$ POI

RT: up

Lt: dn



Assignment #2: Worksheet

Lesson 3

Topic: Curve Sketching Radical Functions–Fractional Powers – SKIPPED

***Goal:** To sketch a function accurately using the information from the first and second derivative of the function and everything else known about the function.*

Find all critical numbers from f' and f''. Then determine on which intervals the function is decreasing, increasing, concaved up, concaved down, any maximums, minimums, inflection points, vertical asymptotes, horizontal asymptotes, and the end behavior (what happens to the function's y-values as x goes to $-\infty$ and ∞). **Then graph $f(x)$.**

$f(x) = x^{1/3} + 1$

y-int $(0,1)$

x-int $0 = x^{1/3}+1$

$-1 = x^{1/3}$

$(-1)^3 = (x^{1/3})^3$

$x = -1$

$(-1,0)$

Domain

$x \in \mathbb{R}$

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to -\infty} f(x) = -\infty$

X	Y
10	3.1
1000	11
1,000,000	101
−10	−1.2
−1,000	−9
−1,000,000	−99

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{2/3}}$

critical # < und $x=0$; $f'=0$ None

$0 = \frac{1}{3x^{2/3}}$

$0 \neq 1$

f' $x=-1$ Pos | $x=1$ Pos

0

f' inc inc

$f''(x) = -\frac{2}{9}x^{-5/3} = \frac{-2}{9x^{5/3}}$

Critical # < f'' und $x=0$ POI $(0,1)$; $f''=0$ None

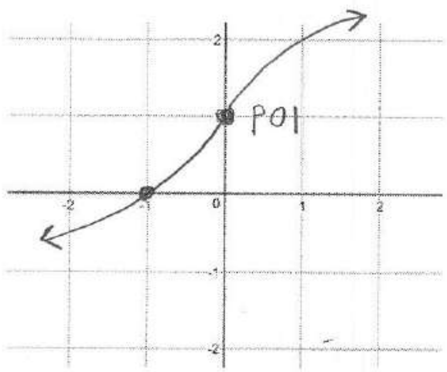
$0 = \frac{-2}{9x^{5/3}}$

$0 \neq -2$

f'' $x=-1$ Pos | $x=1$ Neg

f cc up 0 POI cc dn

f'	Pos		Pos
f	Inc	0	Inc
f''	Pos		Neg
f	cc up	0 POI	cc dn



Assignment #3: Page 255: ~~27~~, ~~28~~, ~~35~~

25, 22,

Lesson 4

Warm Up

1. The table represents the values of a function *f*. The function is continuous on the interval [0, 15] and differentiable on the interval (0, 15).

x	0	1	3	6	7	10	12	14	15
f(*x*)	12	25	34	30	−32	−8	30	−4	6

A. Does *f*(*x*) ever equal 27? Explain.

Yes. Since f is diff on $(0,15)$ with $f(1) = 25$ and $f(3) = 34$, Then There exists a c in $(1,3)$ such that $f(c) = 27$. IVT

Also on $(6,7)$ $(10,12)$ $(12,14)$

B. What is the minimum number of times *f* (*x*) = 0. Explain.

4 Times. Since f is diff on $(0,15)$ and f goes from pos to neg or neg to pos on $(6,7)$, $(10,12)$, $(12,14)$, and $(14,15)$, Then There exists a c in each interval such That $f(c) = 0$. IVT

C. Is there an interval when *f* ′(*c*) = 0, for some *c* on that interval? If yes, give the interval and a justification.

Yes. On the interval $(6,12)$, $\dfrac{f(12) - f(6)}{12-6} = \dfrac{0}{6}$

Since f is diff on $(0,15)$ then there exists a $c \in (6,12)$ such that $f'(c) = 0$.

MVT

Topic: Curve Sketching Rational Functions

Goal: *To sketch a function accurately using the information from the first and second derivative of the function and everything else known about the function.*

Find all critical numbers from f' and f''. Then determine on which intervals the function is decreasing, increasing, concaved up, concaved down, any maximums, minimums, inflection points, vertical asymptotes, horizontal asymptotes, and the end behavior (what happens to the function's y-values as x goes to $-\infty$ and ∞). **Then graph $f(x)$.**

$f(x) = \frac{2(x^2-9)}{x^2-4}$

y-int $(0, \frac{9}{2})$

x-int

$0 = \frac{2(x^2-9)}{x^2-4}$

$0 = 2(x^2-9)$

$0 = x^2-9$

$9 = x^2$

$x = \pm 3 \quad (-3,0) \ (3,0)$

$\lim_{x \to \pm\infty} f(x) = 2$

vert Asymp

$f(x) = \frac{2(x+3)(x-3)}{(x+2)(x-2)}$

$x = -2, \ x = 2$

$f(x) = \frac{2x^2-18}{x^2-4}$

$f'(x) = \frac{4x(x^2-4) - (2x^2-18)2x}{(x^2-4)^2}$

$= \frac{4x^3-16x-4x^3+36x}{(x^2-4)^2}$

$= \frac{20x}{(x^2-4)^2}$ undef $x = \pm 2$ 0 at $x = 0$

$f''(x) = \frac{20(x^2-4)^2 - 20x(2)(x^2-4)2x}{[(x^2-4)^2]^2}$

$= \frac{20(x^2-4) - 80x^2}{(x^2-4)^3} = \frac{-60x^2-80}{(x^2-4)^3}$

$0 = 20(x^2-4) - 80x^2$

$0 = 20x^2 - 80 - 80x^2$

$80 = -60x^2$

$\frac{80}{-60} = x^2$ no sol

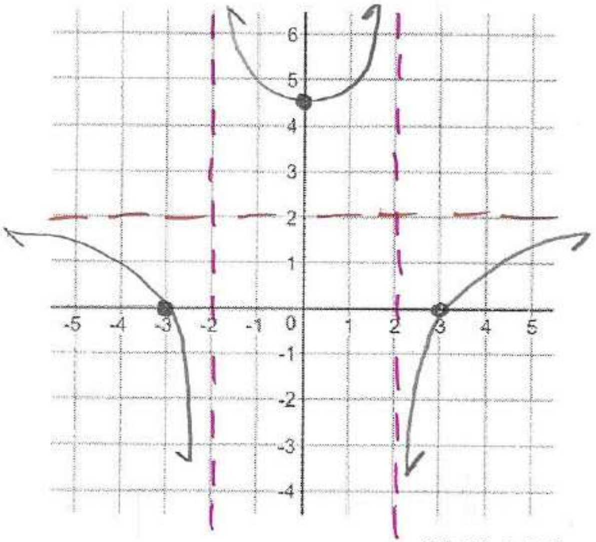
$x=-3 \quad x=-1 \quad x=1 \quad x=3$

$f'(x)$: Neg | -2 | Neg | 0 | Pos | 2 | Pos

$f(x)$: dec | dec | inc | inc

$f''(x)$: Neg | -2 | Pos | 2 | Neg

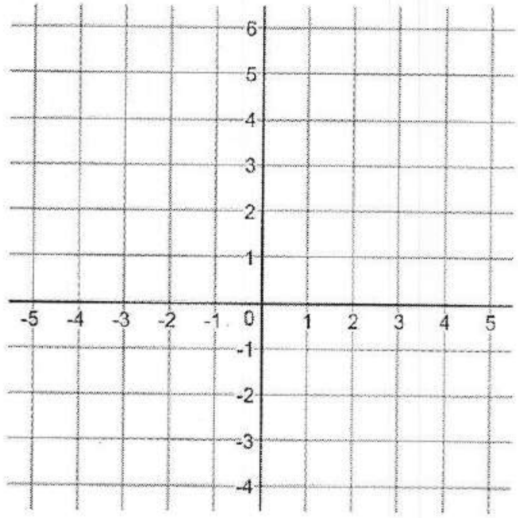
$f(x)$: cc dn | cc up | cc dn



Assignment #4: Worksheet

Find all critical numbers from f' and f''. Then determine on which intervals the function is decreasing, increasing, concaved up, concaved down, any maximums, minimums, inflection points, vertical asymptotes, horizontal asymptotes, and the end behavior (what happens to the function's *y*-values as *x* goes to $-\infty$ and ∞). **Then graph** (x).

$$f(x) = \frac{2x^2}{x-4}$$



Lesson 5

Warm Up

Find the equation of the tangent line to $f(x) = x^3 e^{4x-4}$ at $x = 1$.

Point

$(1,1)$

$f(1) = 1^3 e^{4 \cdot 1 - 4}$

$f(1) = 1$

Slope $f'(x) = 3x^2 e^{4x-4} + x^3 e^{4x-4} \cdot 4$

$f'(1) = 3 \cdot e^0 + 1 \cdot e^0 \cdot 4$

$= 7$

$y - 1 = 7(x-1)$

Use the tangent line to approximate $f(1.1)$.

$y - 1 = 7(1.1 - 1)$

$y = 7(0.1) + 1$

$y = 1.7$

Topic: Curve Sketching Other Functions

Goal: *To sketch a function accurately using the information from the first and second derivative of the function and everything else known about the function.*

Find all critical numbers from f' and f''. Then determine on which intervals the function is decreasing, increasing, concaved up, concaved down, any maximums, minimums, inflection points, vertical asymptotes, horizontal asymptotes, and the end behavior (what happens to the function's y-values as x goes to $-\infty$ and ∞). **Then graph $f(x)$.**

$f(x) = xe^x + 2$

$\lim_{x \to \infty} f(x) = \infty$ $\lim_{x \to -\infty} f(x) = 2$

X	Y
-10	1.9995
-1,000	2
-10,000	2

CALC

y-int $(0, 2)$

x-int $0 = xe^x + 2$

$-2 = xe^x$ no sol - can't ln

critical # $<$ und, $= 0$

$f'(x) = e^x + xe^x$

$0 = e^x + xe^x$

$0 = e^x(1 + x)$

$e^x = 0$ NO sol $x = -1$

$f''(x) = e^x + e^x + xe^x$

$0 = 2e^x + xe^x$

$0 = e^x(2 + x)$

no sol $x = -2$

Min $(-1, 1.632)$

POI $(-2, 1.729)$

$x = -3$ $x = 0$

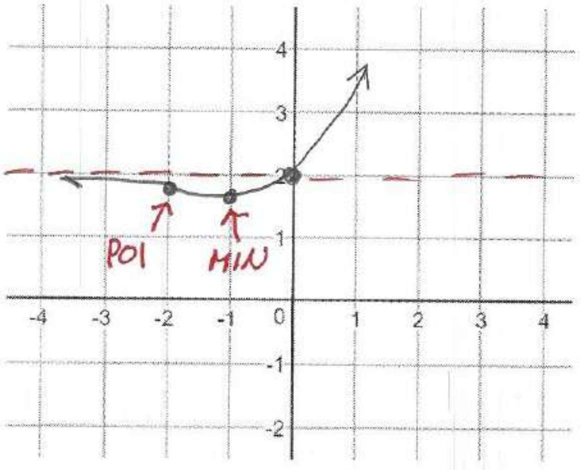
$f'(x)$ Neg | Pos

$f(x)$ dec -1 inc

min

$f''(x)$ Neg POI Pos

$f(x)$ CC DN -2 CC UP



Assignment #5:

Assignment #6: Review Handout