# CHINO VALLEY UNIFIED SCHOOL DISTRICT
## INSTRUCTIONAL GUIDE
## CALCULUS BC ADVANCED PLACEMENT

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| Course Number | 5125 |
| Department | Mathematics |
| Prerequisites | Successful completion of Honors Pre-Calculus or Trigonometry and teacher recommendation |
| Length of Course | Two (2) semesters/One (1) year |
| Grade Level | 11-12 |
| Credit | 5 units per semester/10 total credits of math |
| UC/CSU | Meets “c” math requirement |
| Board Approved | November 6, 2008 |

**Description of Course -** Calculus BC will cover topics in differential and integral calculus. This course in mathematics consists of a full and intensive academic year of work in the calculus of functions of a single variable. The courses emphasize a multirepresentational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations also are important. Technology should be used regularly by students and teachers to reinforce the relationships among the multiple representations of functions, to confirm written work, to implement experimentation, and to assist in interpreting results. Through the use of the unifying themes of derivatives, integrals, limits, approximation, and applications and modeling, the course becomes a cohesive whole rather than a collection of unrelated topics. These themes are developed using all the functions listed in the prerequisites.

Calculus BC is an extension of Calculus AB rather than an enhancement; common topics require a similar depth of understanding. Both courses are intended to be challenging and demanding. The topic outline for Calculus BC includes all Calculus AB topics. Additional topics involve the concept of series, series of constants and Taylor series.

**Rationale for Course -** When taught in high school, calculus should be presented with the same level of depth and rigor as entry level college and university calculus courses. It is expected that students who take this course in calculus will seek college credit or placement, or both, from institutions of higher learning. The content of Calculus BC is designed to qualify the student for placement and credit in a course that is one course beyond that granted for Calculus AB.

**Student Selection** - Admission to an AP course should depend on the student interest in the subject as well as on such formal credentials as an outstanding record of academic performance. Many highly motivated students with less-than-outstanding records have successfully completed AP courses and have obtained college credit, advanced placement, or both, through an AP Examination.

**Standard 1 –** Students understand functions, graphs, and limits.

1.1 Objective: Analyze graphs. With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function.

1.2 Objective: Understand limits of functions (including one-sided limits).

1.2.1 Performance Indicator: Students will have an intuitive understanding of the limiting process.

1.2.2 Performance Indicator: Students can calculate limits using algebra.

1.2.3 Performance Indicator: Students can estimate limits from graphs or tables of data.

1.3 Objective: Understand asymptotic and unbounded behavior.

1.3.1 Performance Indicator: Students understand asymptotes in terms of graphical behavior.

1.3.2 Performance Indicator: Students can describe asymptotic behavior in terms of limits involving infinity.

1.3.3 Performance Indicator: Students can compare relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth).

1.4 Objective: Understand continuity as a property of functions.

1.4.1 Performance Indicator: Students have an intuitive understanding of continuity. (The function values can be made as close as desired by taking sufficiently close values of the domain.)

1.4.2 Performance Indicator: Students understand continuity in terms of limits.

1.4.3 Performance Indicator: Students have a geometric understanding of graphs of continuous functions (Intermediate Value Theorem and Extreme Value Theorem).

1.4.4 Performance Indicator: Students understand parametric, polar, and vector functions.

**Standard 2** – Students understand the derivative.

2.1 Objective: Students understand the concept of the derivative.

2.1.1 Performance Indicator: Students understand the derivative presented graphically, numerically, and analytically.

2.1.2 Performance Indicator: Students understand the derivative interpreted as an instantaneous rate of change.

2.1.3 Performance Indicator: Students understand the derivative defined as the limit of the difference quotient.

2.1.4 Performance Indicator: Students understand the relationship between differentiability and continuity.

2.2 Objective: Understand the derivative at a point.

2.2.1 Performance Indicator: Students understand the slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.

2.2.2 Performance Indicator: Students understand the connection of the tangent line to a curve at a point and local linear approximation.

2.2.3 Performance Indicator: Students understand instantaneous rate of change as the limit of average rate of change.

2.2.4 Performance Indicator: Students can approximate rate of change from graphs and tables of values.

2.3 Objective: Understand the derivative as a function.

2.3.1 Performance Indicator: Students can connect characteristics of graphs of $f$ and $f'$.

2.3.2 Performance Indicator: Students understand the relationship between the increasing and decreasing behavior of $f$ and the sign of $f'$.

2.3.3 Performance Indicator: Students understand the Mean Value Theorem and its geometric interpretation.

2.3.4 Performance Indicator: Students understand equations involving derivatives. Verbal descriptions are translated into equations involving derivatives and vice versa.

2.4 Objective: Understand second derivatives.

2.4.1 Performance Indicator: Students understand the corresponding characteristics of the graphs of $f$ , $f'$ and $f''$ .

2.4.2 Performance Indicator: Students understand the relationship between the concavity of $f$ and the sign of $f''$ .

2.4.3 Performance Indicator: Students will be able to demonstrate an understanding of the points of inflection as places where concavity changes.

2.5 Objective: Understand the applications of derivatives.

2.5.1 Performance Indicator: Students will be able to demonstrate an understanding of the analysis of curves, including the notions of monotonicity and concavity.

2.5.2 Performance Indicator: Students can analyze planar curves given in parametric form, polar form, and vector form, including velocity and acceleration.

2.5.3 Performance Indicator: Students will be able to demonstrate an understanding of the optimization, both absolute (global) and relative (local) extrema.

2.5.4 Performance Indicator: Students will be able to demonstrate modeling rates of change, including related rates problems.

2.5.5 Performance Indicator: Students will be able to use of implicit differentiation to find the derivative of an inverse function.

2.5.6 Performance Indicator: Students understand the interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.

2.5.7 Performance Indicator: Students will be able to demonstrate an understanding of the geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.

2.5.8 Performance Indicator: Students can find the numerical solution of differential equations using Euler's method.

2.5.9 Performance Indicator: Students understand L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series.

2.6 Objective: Understand the computation of derivatives.

2.6.1 Performance Indicator: Students will be able to demonstrate the knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.

2.6.2 Performance Indicator: Students will be able to demonstrate an understanding of the derivative rules for sums, products, and quotients of functions.

2.6.3 Performance Indicator: Students will be able to demonstrate an understanding of the chain rule and implicit differentiation.

2.6.4 Performance Indicator: Students understand how to compute derivatives of parametric, polar, and vector functions.

**Standard 3** – Students understand the interpretations and properties of definite integrals.

3.1 Objective: Understand the interpretations and properties of definite integrals.

3.1.1 Performance Indicator: Students will be able to demonstrate an understanding of the definite integral as a limit of Riemann sums.

3.1.2 Performance Indicator: Students will be able to demonstrate an understanding of the definite integral of the rate of change of a quantity over an interval.

3.1.3 Performance Indicator: Students will be able to demonstrate an understanding of the basic properties of definite integrals (examples include additivity and linearity).

3.2 Objective: Understand the applications of integrals.

3.2.1 Performance Indicator: Students can find the area of a region (including a region bounded by polar curves).

3.2.2 Performance Indicator: Students can find the volume of a solid with known cross sections.

3.2.3 Performance Indicator: Students understand and can compute the average value of a function.

3.2.4 Performance Indicator: Students understand how to compute the distance traveled by a particle along a line.

3.2.5 Performance Indicator: Students understand how to compute the length of a curve (including a curve given in parametric form).

3.2.6 Performance Indicator: Students understand how to use integration as an accumulation function in a variety of applications.

3.3 Objective: Understand the Fundamental Theorem to evaluate definite integrals.

3.3.1 Performance Indicator: Students will be able to demonstrate an understanding of the use of the Fundamental Theorem to evaluate definite integrals.

3.3.2 Performance Indicator: Students will be able to demonstrate an understanding of the Fundamental Theorem to represent a particular anti-derivative, and the analytical and graphical analysis of functions so defined techniques of anti-differentiation.

3.3.3 Performance Indicator: Students will be able to demonstrate an understanding of anti-derivatives following directly from derivatives of basic functions.

3.3.4 Performance Indicator: Students will be able to demonstrate an understanding of anti-derivatives by substitution of variables (including change of limits for definite integrals).

3.3.5 Performance Indicator: Students will be able to demonstrate an understanding of anti-derivatives by parts, and simple partial fractions (non-repeating linear factors only).

3.3.6 Performance Indicator: Students will be able to demonstrate an understanding of improper integrals (as limits of definite integrals).

3.4 Objective: Understand the applications of anti-differentiation.

3.4.1 Performance Indicator: Students will be able to find specific anti-derivatives using initial conditions, including applications to motion along a line.

3.4.2 Performance Indicator: Students will be able to solve separable differential equations and use them in modeling, and numerical approximations to definite integrals.

3.4.3 Performance Indicator: Students will be able to solve logistic differential equations and use them in modeling.

3.5 Objective: Understand how to use Reimann sums.

3.5.1 Performance Indicator: Students know how to use Riemann sums (using left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

**Standard 4** – Students understand polynomial approximations and series.

4.1 Objective: Understand the concept of a series.

4.1.1 Performance Indicator: Students understand a series is defined as a sequence of partial sums, and convergence is defined in terms of the limit of the sequence of partial sums.

4.1.2 Performance Indicator: Students understand how to use technology to explore convergence and divergence.

4.2 Objective: Understand series of constants.

4.2.1 Performance Indicator: Students understand how to use decimal expansion.

4.2.2 Performance Indicator: Students understand geometric series with applications and test the series for convergence or divergence.

4.2.3 Performance Indicator: Students understand the harmonic series.

4.2.4 Performance Indicator: Students understand the alternating series test with error bound to establish the convergence or divergence of a series.

4.2.5 Performance Indicator: Students understand the terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of *p*-series.

4.2.6 Performance Indicator: Students understand the ratio and root test to establish the convergence and divergence of a series.

4.2.7 Performance Indicator: Students understand the comparison test to establish the convergence or divergence of a series.

4.2.8 Performance Indicator: Students understand functions defined by power series.

4.2.9 Performance Indicator: Students understand radius and interval of convergence of power series.

4.3 Objective: Have the knowledge of Taylor series.

4.3.1 Performance Indicator: Students understand Taylor polynomial approximation with graphical demonstration of convergence (for example, viewing graphs of various Taylor polynomials of the sine function approximating the sine curve).

4.3.2 Performance Indicator: Students understand Maclaurin series and the general Taylor series centered at *x* = *a.*

4.3.3 Performance Indicator: Students understand Maclaurin and Taylor series for the functions $e^x$, $\sin x$, $\cos x$, and $\frac{1}{1-x}$.

4.3.4 Performance Indicator: Students understand formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, anti-differentiation, and the formation of new series from known series.

4.3.5 Performance Indicator: Students understand Lagrange error bound for Taylor polynomials.