Name Answer Key Date Class

Day #43 Homework

A particle moves along the x axis such that its position, for $t > 0$, is given by the function $p(t) = e^{2t} - 5t$. Use this information to complete exercises 1 – 4.

1. What are the values of $p'(2)$ and $p''(2)$? Explain what each value represents.

$p'(t) = 2e^{2t} - 5$ $\quad p''(t) = 4e^{2t}$

$p'(2) = 2e^4 - 5$ $\quad p''(2) = 4e^4$

$p'(2)$ represents the velocity and $p''(2)$ represents the acceleration of the particle at $t=2$.

2. Based on the values found in part (a), what can be concluded about the speed of the particle at $t = 2$? Give a reason for your answer.

Since $p'(2) > 0$ and $p''(2) > 0$, the speed of the particle is increasing at $t=2$.

3. On what interval(s) of t is the particle moving to the left? To the right? Justify your answers.

$p'(t) = 0$

$2e^{2t} - 5 = 0$

$e^{2t} = 5/2$

$2t = \ln(5/2)$

$t = 0.458$

$p'(t)$: 0 Neg 0.458 Pos

Since $p'(t) < 0$, the particle is moving left on $0 < t < 0.458$. Since $p'(t) > 0$, the particle is moving right on $t > 0.458$.

4. Does the particle ever change directions? Justify your answer.

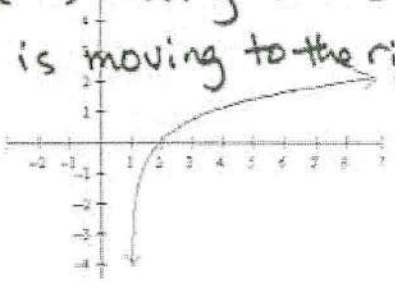
Since $p'(t) = v(t)$ changes signs at $t = 0.458$, the particle changes directions.

5. The graph of $v(t)$, the velocity of a moving particle, is given below. What conclusions can be made about the movement of the particle along the x – axis and the acceleration, $a(t)$, of the particle for $t > 0$? Give reasons for your answers.

Since $v(t) < 0$ on $0 < t < 2$, the particle is moving to the left.

Since $v(t) > 0$ for $t > 2$, the particle is moving to the right.

Since $v(t)$ is always increasing, then $a(t)$ is always positive for $t > 0$.



6. If the position of a particle is defined by the function $x(t) = t^3 - 9t^2 + 24$ for $t > 0$, is the speed of the particle increasing or decreasing when $t = 2.5$? Justify your answer.

$x'(t) = v(t) = 3t^2 - 18t + 24$ $\quad x''(t) = a(t) = 6t - 18$

$v(2.5) = 3(2.5)^2 - 18(2.5) + 24$ $\quad a(2.5) = 6(2.5) - 18$

$v(2.5) = -2.25$ $\quad = -3$

Since both $v(2.5)$ and $a(2.5) < 0$, the speed of the particle is increasing at $t = 2.5$.

The position of a particle is given by the function $p(t) = (2t - 3)e^{2-t}$ for $t > 0$. Answer questions 7 – 9.

7. What is the average velocity from $t = 1$ to $t = 3$?

Average velocity $= \dfrac{p(1) - p(3)}{1 - 3} = 1.911$

8. Find an equation for $v(t)$, the velocity of the particle.

$p'(t) = v(t) = 2e^{2-t} + -1(2t-3)e^{2-t}$

$v(t) = 2e^{2-t} - (2t-3)e^{2-t}$

$v(t) = e^{2-t}(2 - 2t + 3)$

$\boxed{v(t) = (5 - 2t)e^{2-t}}$

9. For what value(s) of t will the $v(t) = 0$?

$v(t) = 0$

$5 - 2t = 0$ $\quad e^{2-t} = 0$

$-2t = -5$ $\quad 2 - t = \ln 0$

$\boxed{t = 5/2}$ $\quad t =$ undefined

2003 AP Calculus AB #2 (Partial)

A particle moves along the $x-$ axis so that its velocity at time t is given by $v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$.

10. Find the acceleration of the particle at $t = 2$. Is the speed of the particle increasing at $t = 2$? Explain why or why not.

$v'(2) = a(2) = 1.588$

$v(2) = -2.728$

Since $v(2)$ and $a(2)$ are different signs, then the speed is decreasing at $t=2$.

11. Find all times in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

On the interval $0 < t < 3$, $v(t)$ changes from negative to positive, hence changing directions, when $t = 2.507$.