



## **AP<sup>®</sup> Calculus AB 2011 Free-Response Questions**

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**2011 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

**CALCULUS AB  
SECTION II, Part A**

**Time—30 minutes**

**Number of problems—2**

**A graphing calculator is required for these problems.**

1. For  $0 \leq t \leq 6$ , a particle is moving along the  $x$ -axis. The particle's position,  $x(t)$ , is not explicitly given. The velocity of the particle is given by  $v(t) = 2 \sin(e^{t/4}) + 1$ . The acceleration of the particle is given by  $a(t) = \frac{1}{2} e^{t/4} \cos(e^{t/4})$  and  $x(0) = 2$ .
- (a) Is the speed of the particle increasing or decreasing at time  $t = 5.5$ ? Give a reason for your answer.
  - (b) Find the average velocity of the particle for the time period  $0 \leq t \leq 6$ .
  - (c) Find the total distance traveled by the particle from time  $t = 0$  to  $t = 6$ .
  - (d) For  $0 \leq t \leq 6$ , the particle changes direction exactly once. Find the position of the particle at that time.
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$t$ (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function  $H$  for  $0 \leq t \leq 10$ , where time  $t$  is measured in minutes and temperature  $H(t)$  is measured in degrees Celsius. Values of  $H(t)$  at selected values of time  $t$  are shown in the table above.
- (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time  $t = 3.5$ . Show the computations that lead to your answer.
- (b) Using correct units, explain the meaning of  $\frac{1}{10} \int_0^{10} H(t) dt$  in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate  $\frac{1}{10} \int_0^{10} H(t) dt$ .
- (c) Evaluate  $\int_0^{10} H'(t) dt$ . Using correct units, explain the meaning of the expression in the context of this problem.
- (d) At time  $t = 0$ , biscuits with temperature  $100^\circ\text{C}$  were removed from an oven. The temperature of the biscuits at time  $t$  is modeled by a differentiable function  $B$  for which it is known that  $B'(t) = -13.84e^{-0.173t}$ . Using the given models, at time  $t = 10$ , how much cooler are the biscuits than the tea?

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**END OF PART A OF SECTION II**

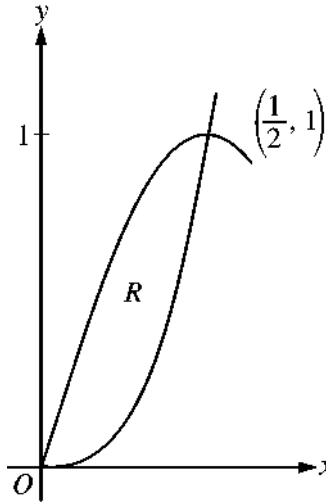
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CALCULUS AB  
SECTION II, Part B

Time—60 minutes

Number of problems—4

No calculator is allowed for these problems.

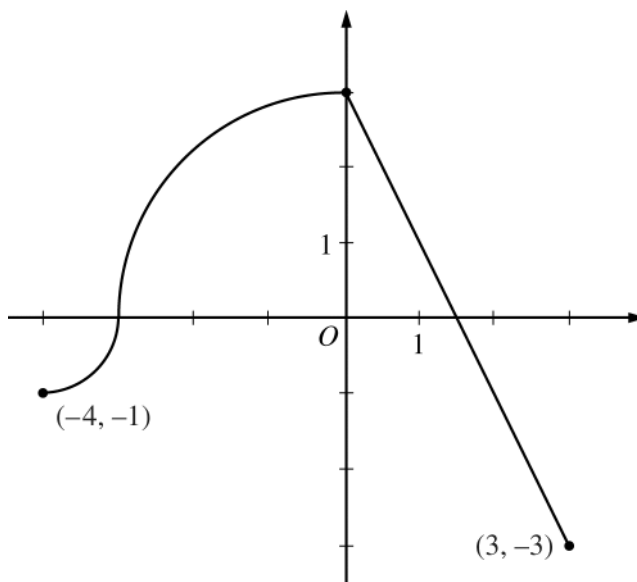


3. Let  $R$  be the region in the first quadrant enclosed by the graphs of  $f(x) = 8x^3$  and  $g(x) = \sin(\pi x)$ , as shown in the figure above.
- (a) Write an equation for the line tangent to the graph of  $f$  at  $x = \frac{1}{2}$ .
- (b) Find the area of  $R$ .
- (c) Write, but do not evaluate, an integral expression for the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = 1$ .

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Graph of  $f$

4. The continuous function  $f$  is defined on the interval  $-4 \leq x \leq 3$ . The graph of  $f$  consists of two quarter circles and one line segment, as shown in the figure above. Let  $g(x) = 2x + \int_0^x f(t) dt$ .
- Find  $g(-3)$ . Find  $g'(x)$  and evaluate  $g'(-3)$ .
  - Determine the  $x$ -coordinate of the point at which  $g$  has an absolute maximum on the interval  $-4 \leq x \leq 3$ . Justify your answer.
  - Find all values of  $x$  on the interval  $-4 < x < 3$  for which the graph of  $g$  has a point of inflection. Give a reason for your answer.
  - Find the average rate of change of  $f$  on the interval  $-4 \leq x \leq 3$ . There is no point  $c$ ,  $-4 < c < 3$ , for which  $f'(c)$  is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

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5. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function  $W$  models the total amount of solid waste stored at the landfill. Planners estimate that  $W$  will satisfy the differential equation  $\frac{dW}{dt} = \frac{1}{25}(W - 300)$  for the next 20 years.  $W$  is measured in tons, and  $t$  is measured in years from the start of 2010.
- (a) Use the line tangent to the graph of  $W$  at  $t = 0$  to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time  $t = \frac{1}{4}$ ).
- (b) Find  $\frac{d^2W}{dt^2}$  in terms of  $W$ . Use  $\frac{d^2W}{dt^2}$  to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time  $t = \frac{1}{4}$ .
- (c) Find the particular solution  $W = W(t)$  to the differential equation  $\frac{dW}{dt} = \frac{1}{25}(W - 300)$  with initial condition  $W(0) = 1400$ .
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6. Let  $f$  be a function defined by  $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

- (a) Show that  $f$  is continuous at  $x = 0$ .
- (b) For  $x \neq 0$ , express  $f'(x)$  as a piecewise-defined function. Find the value of  $x$  for which  $f'(x) = -3$ .
- (c) Find the average value of  $f$  on the interval  $[-1, 1]$ .
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**END OF EXAM**