Section 5 The Range of Projectiles: The Shot Put

TECH LAB

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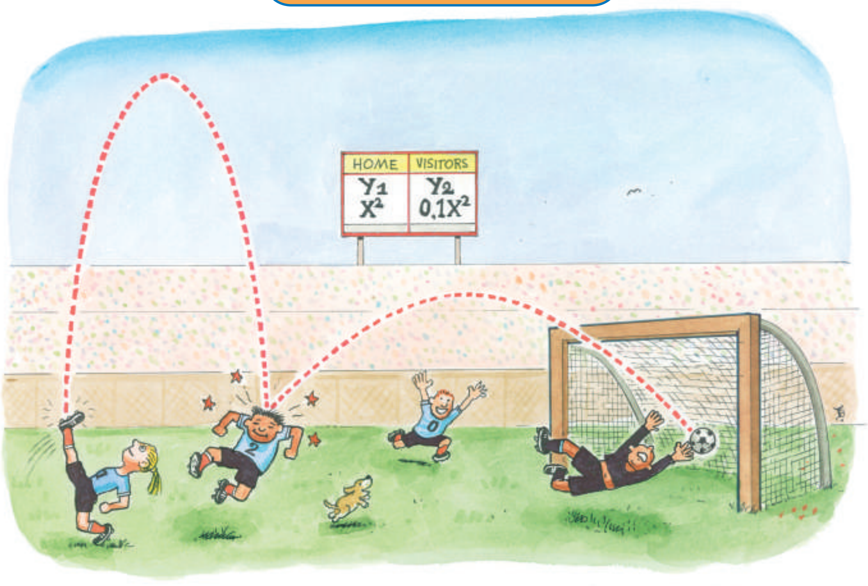
Learning Outcomes

In this section, you will

- **Measure** the acceleration due to gravity.
- **Calculate** the speed attained by an object that has fallen freely from rest.
- **Identify** the relationship between the average speed of an object that has fallen freely from rest and the final speed attained by the object.
- **Calculate** the distance traveled by an object that has fallen freely from rest.
- **Use** mathematical models of free fall and uniform speed to construct a physical model of the trajectory of a projectile.
- **Use** the motion of a real projectile to test a physical model of projectile motion.
- **Use** a physical model of projectile motion to infer the effects of launch speed and launch angle on the range of a projectile.

What Do You See?



What Do You Think?

A world record in the men's shot put of 23.12 m was set by Randy Barnes of the United States in 1990. In the women's javelin throw, Osleidys Menendez of Cuba broke the world record at 71.70 m in 2005.

- **Describe the trajectories of projectiles launched from the ground at various angles.**
- **Describe how a greater launch speed of a projectile might change the range when the launch angle is the same.**

Record your ideas in your *Active Physics* log. Be prepared to discuss your responses with your small group and the class.

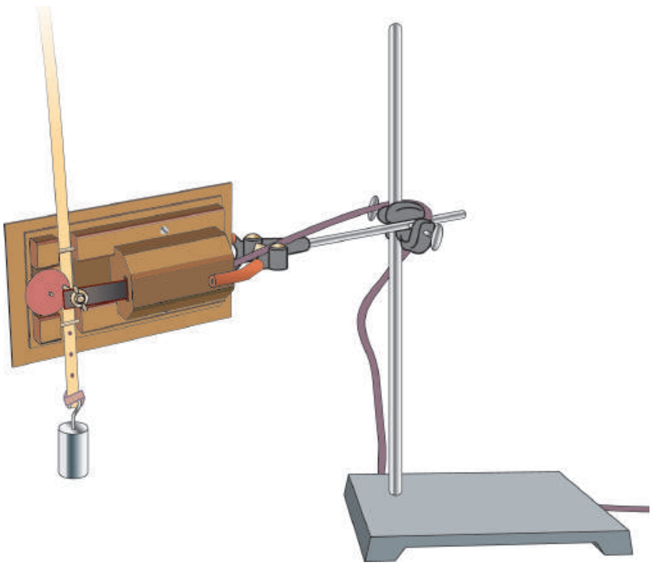
Investigate

In this *Investigate*, you will measure the acceleration due to gravity. You will then use a mathematical model to construct a physical model of the trajectory of a projectile. Finally, you will use a real projectile to test the physical model.

1. Your teacher will provide you with a method of measuring the acceleration caused by Earth's gravity for objects in a condition of free fall.

One simple recommended method uses a "picket fence" and a photogate timer attached to a computer. The picket fence is dropped and the computer measures the time between black slats of the fence. The computer then displays the acceleration due to gravity.

A second method uses a ticker-tape timer and a mass. The mass is attached to the ticker tape then dropped and the ticker tape is analyzed. One pair of successive dots allows you to calculate the velocity at the time those two dots were made. Another pair of successive dots allows you to calculate the velocity at the time those two dots were made. The acceleration can then be calculated by finding the change in velocity during the time between the first pair of dots and the second pair of dots. To increase the precision of the calculation, many pairs of dots can be used and an average acceleration can be found.



a) In your log, describe the procedure, data, calculations, and the value of the acceleration of gravity obtained. As you have learned, the acceleration due to gravity comes up often and has its own symbol, g.

2. After calculating the acceleration due to gravity (or using the value of $g = 10$ m/s^2), you can use this knowledge to analyze the path of a projectile.

a) In your log, make a table similar to the following:

Time of fall (s)	Final speed (m/s)	Average speed (m/s)	Distance (m)
0.0	0	0.0	
0.1	1	0.5	
0.2	2		
0.3			
0.4			
0.5			

(Some data for a falling object has already been calculated and entered in the table to help you get started.)

b) In the table, calculate and record the speed of a falling object at the end of each 0.10 s of its fall for a total of 0.5 s. To simplify the calculations, use a rounded off value for g of 10 m/s^2. The first three values are provided in the second column. Complete the table using the example below as a guide.

Example:

What you know: $g = 10$ m/s^2

Speed = acceleration × time

Speed at the end of 0.2 s = (10 m/s^2) × (0.2 s)

Speed = 2 m/s

c) When speeds are changing at a constant rate, then the average speed during a time interval is the average of the speeds at the beginning and the end of the time interval. Calculate and record the average speed for each time interval in the table. The falling object's speed has increased uniformly from zero to the final speed. In each time interval, the average speed will be the average of zero and the final speed reached at the end of each 0.10 s of falling. This average speed will come out to one half of the final speed.

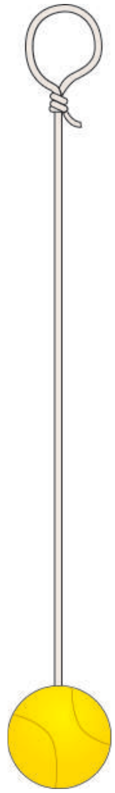
> Example:
>
> Average speed =
>
> $$\frac{\text{zero + speed at the end of time interval}}{2}$$
>
> Average speed during 0.2 s of fall =
>
> $$\frac{(0 \text{ m/s} + 2 \text{ m/s})}{2}$$
>
> $$= 1 \text{ m/s}$$

Complete the third column of the chart.

d) Calculate and record the distance the object has fallen at the end of each 0.10 s of its fall. To do this, use the familiar equation:

Distance = average speed × time.

> Example:
>
> The average speed during 0.2 s of falling is 1 m/s.
>
> Distance = average speed × time
>
> $= (1 \text{ m/s}) \times (0.2 \text{ s})$
>
> $= 0.2 \text{ m}$



3. The table you have completed is a mathematical model of an object falling freely from rest. Now you will change the mathematical model into a physical model. Your teacher will assign your group a particular row in the data table providing information about the falling object.

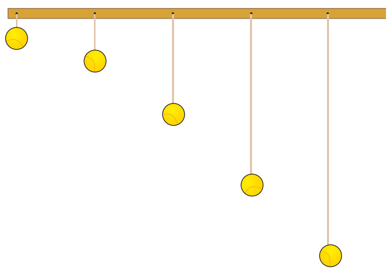
   Assemble two identical string and mass assemblies, as shown in the diagram, with an assembly length equal to the distance of fall assigned to your group.

4. Label the mass showing your group's name and the time of fall.

5. Your teacher will place a horizontal row of pins or tape labeled 0.0 s, 0.1 s, 0.2 s, and so on, along the top edge of a chalkboard in your classroom. The times noted on the labels correspond to the instants for which you calculated distances of fall in the table. The horizontal spacing of the pins is a model of the positions an object would have every 0.10 s if it traveled along the horizontal row of pins at a constant speed.

   a) Calculate the horizontal speed by dividing the distance traveled during each 0.1-s time interval by 0.1 s. (Dividing a number by 0.1 is equivalent to multiplying the number by 10.) Show your calculation and the result in your log.

6. Hang one of your string and mass assemblies from the pin corresponding to the time assigned to your group. Place a small mark on the chalkboard at the bottom end of the string and mass assembly.



7. A volunteer from the class should draw a smooth curve connecting the marks on the chalkboard. This curve corresponds to the path of an object thrown horizontally. Another volunteer should try to match the path, the trajectory, by throwing a tennis ball horizontally from your starting point (time = 0.0 s). To match the trajectory, the ball will need to be thrown horizontally at the speed calculated in *Step 5.a*). This may require a few practice tries.

   a) Write your observations in your log.

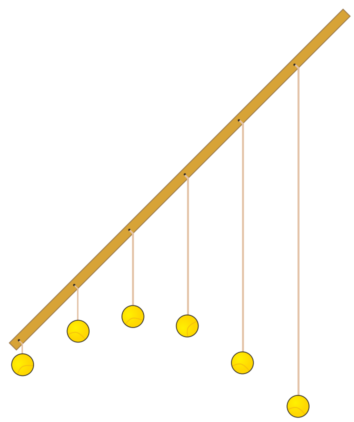
8. Create the other half of the trajectory by hanging your other mass assembly at the corresponding position to the left of the 0.0 pin. Hang the string and mass assemblies, mark the chalkboard, and connect the points to create the other half of an "arch-shaped" model of a trajectory. The goal is to put the two halves together to produce a single trajectory for an object thrown into the air.

9. If this curve represents the path of a ball, then you should be able to get a thrown ball's path to match this curve. A volunteer should try to throw a ball to match this trajectory. Have another person prepared to catch the ball.

   a) What conditions seem to be necessary to match the trajectory? Write your observations in your log.

   b) When a volunteer is able to match the trajectory, the class should agree upon and give the volunteer instructions to test, one at a time, the effects of launch speed and launch angle on the range of the projectile. Write your observations in your log.

10. Your teacher will show you a "portable" version of the row of pins used in *Step 5*.



11. Rest the end of the stick corresponding to 0.0 s on the tray at the bottom of the chalkboard while inclining the stick at an angle of 30°.

   a) Is the path indicated by the bottom ends of the string and mass assemblies a "true" trajectory? Have a volunteer try to match it. Record your observations.

   b) Repeat for angles of 45°, 60°, and other angles of interest. Record your observations (it may be necessary to rest the lower end of the model on the floor to prevent the upper end from hitting the ceiling of the room).

   c) What was your observation?

   d) Incline the stick to 90° (straight up). Do this outdoors if the ceiling is not high enough. What is being modeled in this case? Record your thoughts.

Physics Talk

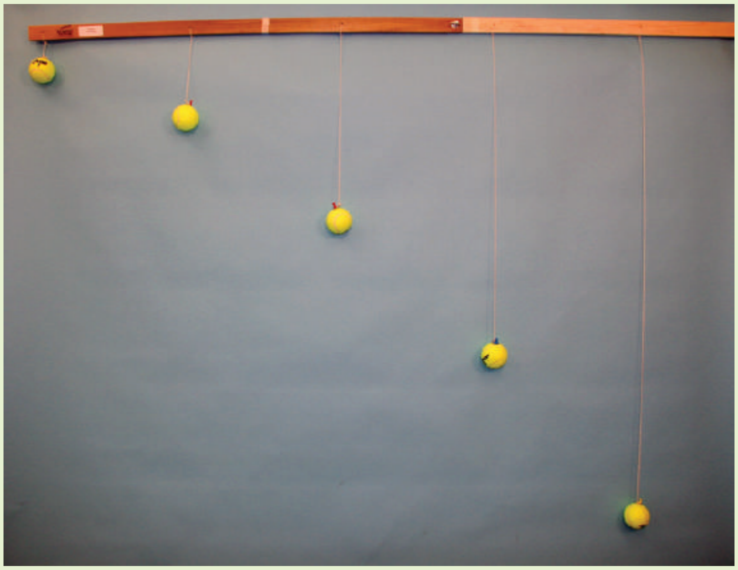
MODELING PROJECTILE MOTION

The *Investigate* you completed in this section and the last section demonstrate that a projectile has two motions that act at the same time and do not affect one another. One of the motions is constant speed along a straight line, corresponding to the amount of launch speed and its direction. The second motion is downward acceleration at 9.8 m/s^2 caused by Earth's gravitational force, which takes effect immediately upon launch. The trajectory of a projectile becomes simple to understand when these two simultaneous motions are kept in mind.

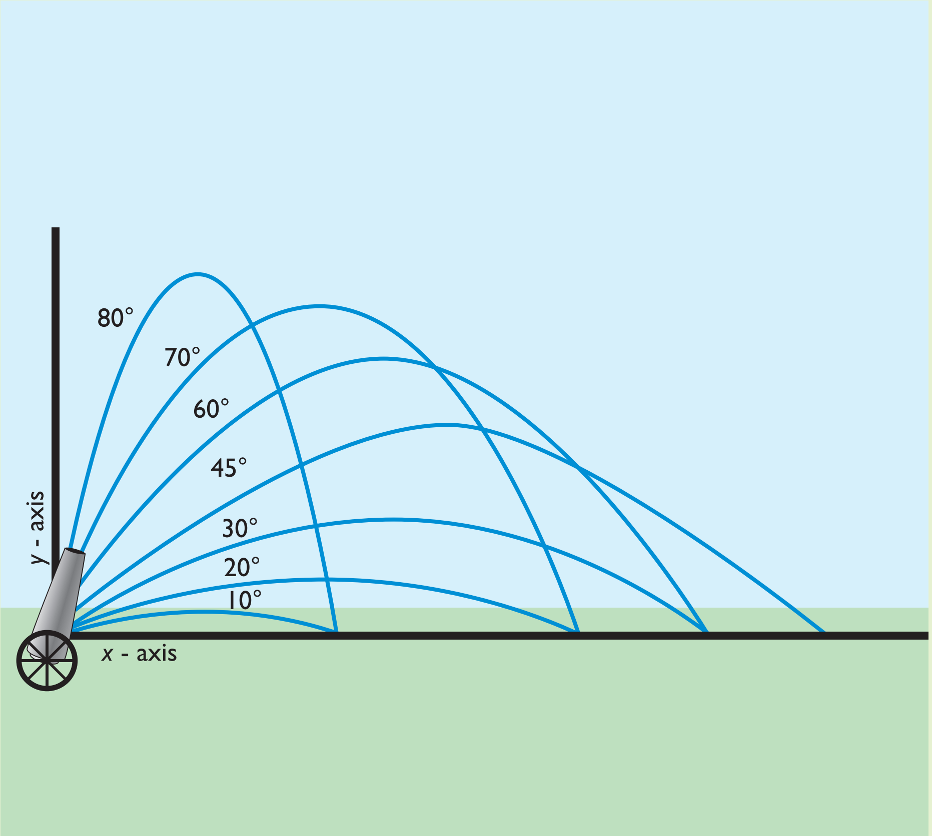
This section also demonstrates the main thing that scientists do: create models to help understand how things in nature work. In this section, you saw how two kinds of models, a mathematical model (the table of times, speeds, and distances during falling) and a physical model (the evenly spaced strings of calculated lengths) correspond to reality when a ball is thrown. For a scientific model to be accepted, the model must match reality in nature. By that requirement, the models used in this section were good ones.

Trajectories of projectiles can be modeled using a computer or graphing calculator. These tools allow you to manipulate variables such as launch angle, launch speed, launch height, and range to enhance your ability to simulate, explore, and understand projectile motion. You can find projectile motion simulations on the Internet.

If you ignore air resistance, the path of all trajectories are parabolas (bowl-shaped curves). If you throw a ball, it follows a parabolic path. You demonstrated this as your ball toss matched the parabola that you calculated and modeled with the hanging masses.



The diagram below shows plots of trajectories launched at many different angles (10°, 20°, 30°, 45°, 60°, 70°, 80°), but always with the same initial speed.



Notice the following:

- All balls travel in parabolas.
- The 45° launch angle produces the greatest range (largest distance).
- The distance traveled at pairs of angles (30° and 60°, 20° and 70°, 10° and 80°) are identical.
- Small angles have greater horizontal velocities but are in the air a short time. Large angles have smaller horizontal velocities but are in the air a long time.

In the real world of sports, the air resistance makes trajectories more complex. Baseballs and golf balls do not follow true parabolic paths. Baseballs can curve if the pitcher puts a certain type of spin on the ball. The temperature of the air also affects the distance a ball will travel.

Checking Up

1. What are the two types of motion that help you understand the trajectory of a projectile?
2. What is the fundamental requirement a scientist must meet when proposing a model of some natural phenomenon?
3. For projectiles launched at various angles, summarize how the height and range of projectiles vary as the angle of launch is increased from 10° to 80°.

Active Physics Plus

+Math	+Depth	+Concepts	+Exploration
◆			

Analyzing Two-Dimensional Motion Mathematically

You now have a means to analyze two-dimensional motion mathematically. The analysis of two-dimensional motion begins with the recognition that the horizontal and vertical components are independent of one another, as you discovered in this and the previous section. The horizontal speed always remains the same. The vertical speed of a falling object always increases with time as the object descends.

During a long jump the athlete runs and then travels in a parabola. The faster she runs, the faster is her horizontal velocity. She must jump in the air to get height so she can stay in the air longer. She does this without slowing down the horizontal velocity.

If a jumper leaves the ground with the same total velocity but changes the angle, the longest jump occurs when the athlete leaves the ground at an angle of 45°.

Let's see if this makes sense. If the athlete jumps straight up, she maximizes her time in the air but has no horizontal velocity. She will be in the air a long time, but won't go anywhere horizontally. If the athlete jumps straight out at a very small angle, she has a large horizontal component, but is not in the air very long. If she leaves the ground at 45° she is in the air for quite some time and still has a large horizontal velocity. This angle of 45° gives the maximum range.

In physics, you can use mathematical equations to describe the world with accuracy and precision.

Here is a table that describes the horizontal and vertical motion of a trajectory.

	Horizontal Component	Vertical Component
Position	$x = v_x t$ where x is the horizontal displacement v_x is the horizontal component of the velocity t is the time	$y = \frac{1}{2}at^2$ where y is the vertical displacement traveled a is the acceleration due to gravity (a = 9.8 m/s^2 on Earth) t is the time
Velocity	The horizontal velocity is constant. There is no net force in the horizontal direction. With no force, there is no acceleration.	$v_y = at$ where v_y is the vertical velocity a is the acceleration due to gravity (a = 9.8 m/s^2 on Earth) t is the time
Acceleration	No acceleration in the x-direction.	Acceleration due to gravity in the y-direction = 9.8 m/s^2

Sample Problem

You can analyze a long jumper with the mathematics that you have practiced in this section. Suppose the height that the long jumper achieves is 1.6 m with a horizontal velocity of 6.0 m/s. How far does the jumper move horizontally?

Strategy: Begin by thinking about what will happen if the long jumper jumps horizontally from a ledge with a height of 1.6 m with a horizontal velocity of 6.0 m/s. Where will she land? Jumping from the ledge is identical to the second half of her jump from the maximum height of 1.6 m to the ground.

Solve for the vertical motion and then solve for the horizontal motion.

Step 1: Use the vertical-motion information to determine the time in the air for the second half of the trip. Her vertical fall is 1.6 m irrespective of the horizontal velocity. It is identical to her falling straight down.

If she fell straight down from 1.6 m or jumped horizontally from 1.6 m, her vertical motion would be identical.

You were able to find the vertical distance traveled by first finding the average speed and then multiplying that average speed by the time.

If the vertical speed at the start is zero, the vertical distance traveled can be found in one step by using the equation,

$$y = \frac{1}{2}at^2$$

where a is the acceleration due to gravity (9.8 m/s^2 on Earth).

The value of 9.8 m/s^2 is often rounded up to be 10 m/s^2.

Using the equation $y = \frac{1}{2}at^2$ you can find the time she is in the air.

Given:

$$y = 1.6 \text{ m}$$

Solution:

$$y = \frac{1}{2}at^2$$

You can use your calculator to find a value for t, such that:

$$1.6 \text{ m} = \frac{1}{2}at^2$$

or you can practice your algebra skills and rearrange the equation to solve for time.

$$t = \sqrt{\frac{2y}{a}}$$

$$= \sqrt{\frac{2(1.6\text{m})}{9.8 \text{ m/s}^2}}$$

$$= 0.57 \text{ s or } 0.6 \text{ s}$$

Strategy:

Step 2: If she has a horizontal velocity of 6.0 m/s and she is in the air for 0.6 s, where will she land? Her horizontal motion can be found by recognizing that distance equals velocity times time.

Solution:

$$x = v_x t$$
$$= (6.0 \text{ m/s})(0.6 \text{ s})$$
$$= 3.6 \text{ m}$$

The jumper moves horizontally 3.6 m on the way down for the second half of the trip.

Strategy:

Step 3: If a long jumper achieves a height of 1.6 m and has a horizontal velocity of 6.0 m/s for the second half of the trip, then her horizontal distance is twice the value of the distance for the entire trip, since she moves horizontally on the way up as well. (Remember modeling the other part of the motion in the *Investigate*.)

Solution: $x_{total} = 2(3.6 \text{ m})$
$$= 7.2 \text{ m}$$

1. Calculate how far horizontally a long jumper travels if she achieves a height of 1.7 m and a horizontal velocity of 7.0 m/s.

You can solve lots of problems by analyzing half of the motion like this. You can calculate the path of a football or baseball or golf ball. The calculations will not apply to real-life situations as well as you might expect because of the effects of air resistance.

The path of a golf ball should be a parabola. Air resistance changes the shape. A baseball should also travel in a parabola, but when the pitcher puts a certain type of spin on it, the air resistance allows it to curve and therefore change the calculated path of our model.

2. Calculate how far a ball will travel horizontally if the ball reaches a high of 1.5 m above the ground and is thrown at a horizontal velocity of 45.0 m/s. Assume that the ball is caught at the same height it is thrown and that there is no air resistance.

What Do You Think Now?

At the beginning of this section, you were asked the following:

- **Describe the trajectories of projectiles launched from the ground at various angles.**
- **Describe how a greater launch speed of a projectile might change the range when the launch angle is the same.**

You can use evidence from the mathematical model and the physical model of this section to describe the path of the object and to describe how the angle of the trajectory determines the distance the object travels.

Physics

Essential Questions

What does it mean?

It is said that any thrown object travels in a parabola. Describe three different paths and explain how they each can be a parabola.

How do you know?

What evidence do you have that the mathematics correctly predicted the path that a thrown object would take?

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Force and motion	* Models	Experimental evidence is consistent with models and theories

* The use of models is a physicist's way of making sense of the world. Did the mathematical model and the physical model in your investigation adequately describe the path of a trajectory?

Why should you care?

Many sports have objects moving in the air. Baseballs, footballs, and soccer balls all travel in parabolas. Divers and high jumpers also travel in parabolas. As a diver's body twists and turns in the air, how could a television broadcaster show that the path is a parabola?

Reflecting on the Section and the Challenge

The information learned about projectile motion in this section applies not only to the shot put, but to any sporting event that involves throwing things into the air (including the self-launching of a human body, as in the hurdles, long jump, or high jump). It has been reported that one Olympian who competed in the shot put increased his range in that event by nearly 4 m, based on suggestions made by a physicist. You are now a physicist specializing in projectile motion. Imagine what you might say in your voice-over when covering the long jump event or describing a home run ball or a punt in football. You may want to comment on how the vertical motion and horizontal motion are independent of one another. You may wish to mention that the angle will help determine the range of the ball, with 45° producing the longest range. You will certainly want to mention that the curved path of the ball is a parabola. In the real world of sports, the air resistance makes trajectories more complex. Baseballs and golf balls do not follow true parabolic paths. Baseballs can curve if the pitcher puts a certain type of spin on the ball. The temperature of the air affects the distance a ball will travel. Although the details of these are complex to analyze, you may wish to mention them in your voice-over.

Physics to Go

1. If the launching and landing heights for a projectile are equal, what angle produces the greatest range? Why?

2. Compared to a launch angle of 45°, what happens to the amount of time a projectile is in the air if the launch angle is

   a) greater than 45°?

   b) less than 45°?

3. For a constant launch speed, what angle produces the same range as a launch angle of

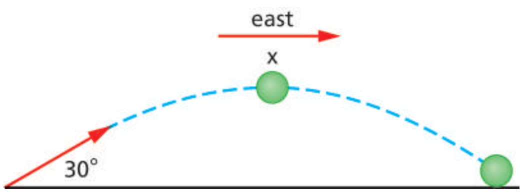
   a) 30°?

   b) 15°?

4. Analyses of performances of long jumpers has shown that the typical launch angle is about 18°, far less than the angle needed to produce maximum range. Why do you think this occurs?

5. You might be familiar with Carl Lewis as a medal-winning sprinter. But he is also an Olympic gold medalist in the long jump. Why do you think he was successful in both events?

6. The diagram below shows a ball thrown toward the east and upward at an angle of 30° to the horizontal. Point X represents the ball's highest point.



   a) What is the direction of the ball's acceleration at point X? (Ignore friction.)

   b) What is the direction of the ball's velocity at point X?

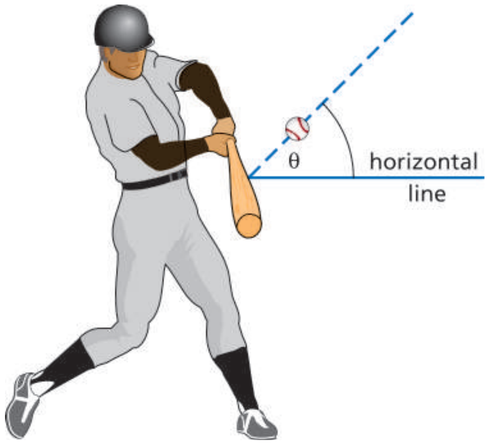
7. Active Physics Plus A diver jumps horizontally off a cliff with an initial velocity of 5.0 m/s. The diver strikes the water 3.0 s later.

   a) What is the vertical speed of the diver upon reaching the surface of the water?

   b) What is the horizontal speed of the diver 1.0 s after the diver jumps?

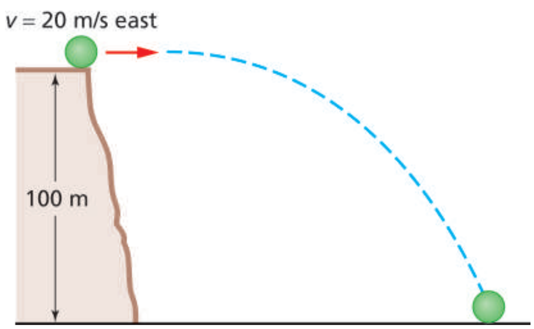
   c) How far from the base of the cliff will the diver strike the water?

8. The diagram of the baseball player shows a baseball being hit with a bat. Angle θ represents the angle between the horizontal and the ball's initial direction of motion. Which value of θ would result in the ball traveling the longest horizontal distance if air resistance is neglected?



9. Four balls, each with mass (m) and initial velocity (v), are thrown at different angles by a baseball player. Neglecting air friction, which angular direction produces the greatest projectile height?

10. Active Physics *Plus* The diagram below shows a ball projected horizontally with an initial velocity of 20.0 m/s east, off a cliff 100-m high.



a) During the flight of the ball, what is the direction of its acceleration?

b) How many seconds does the ball take to reach the ground?

c) How far from the base of the cliff does the ball land?