Section 4 Projectile Motion: Launching Things into the Air

What Do You See?

Learning Outcomes

In this section, you will

- **Apply** the terms free fall, projectile, trajectory, and range.
- **Provide** evidence concerning projectiles launched horizontally from the same height at different launch speeds (including zero launch speed).
- **Explain** the relationship between the vertical and horizontal components of a projectile's motion.
- **Recognize** the factors that affect the range of a projectile.
- **Infer** the shape of a projectile's trajectory.

What Do You Think?

Some track and field events involve launching things into the air, such as a shot put, a javelin, or even one's body in the case of the long jump. In golf, football, tennis, and baseball, balls move through the air as well.

- **What determines how far an object thrown into the air travels before landing?**

Record your ideas about this question in your *Active Physics* log. Be prepared to discuss your response with your small group and the class.

Investigate

Part A: Observe Two Coins Dropping

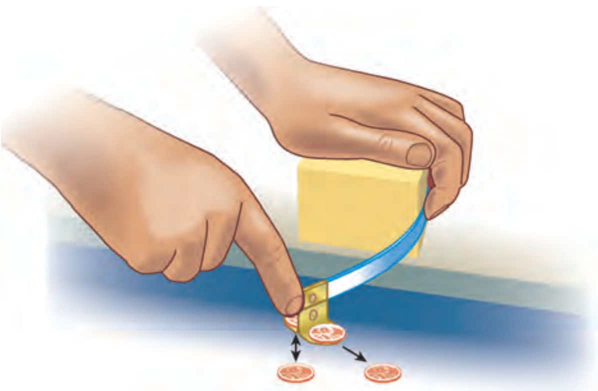
In this part of the *Investigate*, you will observe two coins as they fall from a table. One coin will be dropped from the table, and the other will be projected from the table.

1. Hold two coins the same distance above the floor. Drop them at the same time. Listen to the sound they make as they strike the floor.

   a) Do they hit the floor at the same time?

2. Place one coin at the edge of a table with about half of the coin hanging over the edge. Place another coin flat on the table. Use your fingers to "flick" this coin across the tabletop to strike the first coin. Aim "off center" so that the coin at the edge of the table drops straight down and the projected coin leaves the edge of the table with some horizontal speed.

Your teacher may also decide to use a "coin launcher" for this experiment as shown in the diagram below.



Repeat the event as many times as needed to record your answer to the following question in your log.

a) Do the coins hit the floor at the same or different times? (Hearing is the key to observation here, although you may wish to rely on sight as well.)

3. Vary the speed of the projected coin.

a) Does the speed of the projected coin affect whether the two coins hit the floor at the same time? Explain your answer.

b) Does the speed of the projected coin change how far it lands horizontally from the coin that fell straight down? Explain your answer.

c) Draw a single sketch that includes both the path of the coin that is falling and the coin that is projected. Imagine where each coin is at four identical points in time and note these predicted locations on your sketch. Label these times A, B, C, D.

4. Use a box, chair, or a stack of books to change the height from which you project the coins.

a) Do the coins hit the floor at the same or different times?

b) How does changing the height affect how far the projected coin travels horizontally as it falls?

Part B: Vertical and Horizontal Motion of a Projectile

In this part of the *Investigate*, the class will observe a student throwing a ball into the air while sitting in a moving chair.

1. To illustrate an object that has both vertical and horizontal motion at the same time, your teacher will supervise a class activity in which one student sits on a chair that is moving at constant speed. While the chair is moving, the student on the chair will throw a ball straight up into the air and try to catch it when it comes down. The class will stand in a line beside the path of the chair to observe the event, prepared to mark the vertical position of the ball as it passes them.

   a) In your log, write your prediction of what you think will happen.

2. This activity can be done in several ways. It is ideal if the chair's path can be parallel to the chalkboard. Another option is to take a large roll of paper and tape the paper to a wall parallel to the path of the chair on the opposite side from the observing students. The bottom horizontal side of the board or paper should be at the height the student in the chair launches and catches the ball. Each student observing the event draws a vertical line on the board or paper marking their position beside the path. As the event takes place, each observing student keeps track of the height of the ball as it passes the line representing their position. After the event, each student puts a mark on the board or paper corresponding to the point where the ball passed the line.

   a) Write in your log what you observed about the ball's trajectory (shape of the ball's path) and the ball's approximate range (horizontal distance) for trials in which you varied the speed of the chair and the launching speed of the ball. Remember, to see the effect of both the ball's trajectory and approximate range, be sure to only change one variable at a time during your trials.

   b) According to your observations, what factors affect the range of the ball?

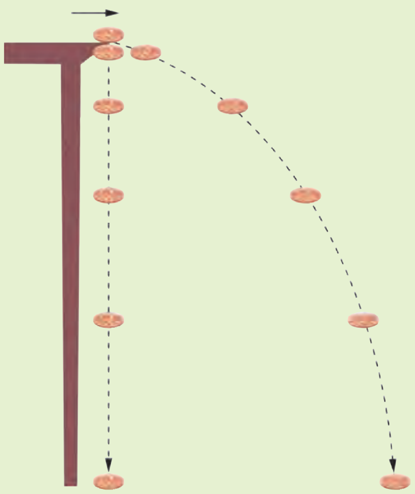
Physics Talk

PROJECTILES AND TRAJECTORIES

By observing two falling coins and by tossing a ball in a moving chair, you gained evidence of two very important aspects of how thrown objects move. Since the javelin, the baseball, the football, and even a high jumper are objects thrown in the air, the two observations of **projectile** motions are crucial to your voice overdub of sporting events.

The horizontally thrown coin and the dropped coin hit the ground at the same time when there is little or no air resistance. (This does not work for a falling feather.) Under careful observations, you find that this is always true — the horizontal motion of the coin does not affect its downward motion. If you were to take a picture of the coin every tenth of a second, you would observe the two coins as shown in the diagram.

Both coins fall the same amount in each tenth of a second. The vertical motions of the coin falling directly down to the floor near the table and the coin landing further away from the table are identical. The projected coin kept moving to the right, but its vertical motion was identical to the dropped coin.



"Believing is Seeing"

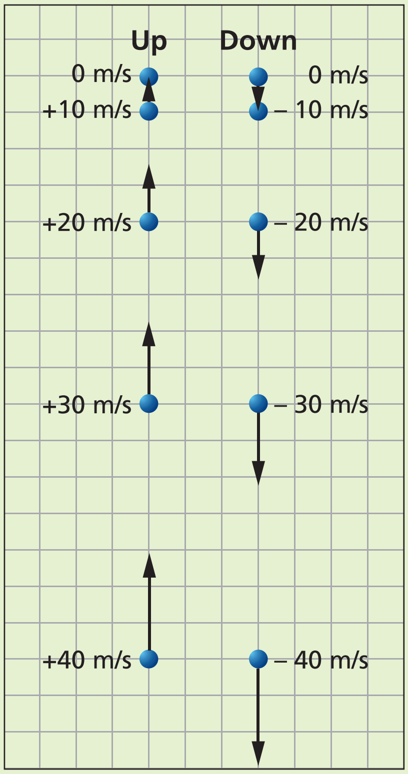
The *Investigate* you completed may not have convinced you that the two coins hit the ground at the same time. Intuition tells you that the dropped coin should hit first. In this case, intuition is wrong. The two coins do hit at the same time. If you believe strongly that the dropped coin hits first, you fool yourself into seeing that. The phrase "seeing is believing" should actually be "believing is seeing." If you believe that the dropped coin hits first, you will see it hit first even though it hits at the same time. To defend your intuition, you may even state that the dropped coin hit a tiny, tiny bit before the coin landing further away. There have been high-speed photos taken that show they hit at the same time. There are computer simulations that you can find on the Internet that also try to help people accept this "hard to accept" truth about motion.

The projected coin has a constant speed to the right, when there is no air resistance. The vertical motion does not affect this constant horizontal speed. The falling coin has no speed to the right, in a perfect fall.

Physics Words

projectile: an object traveling through the air or other medium.

Any hit or thrown ball travels horizontally and vertically. The horizontal velocity remains the same (if there is no air resistance). The vertical velocity is constantly changing. As it rises, the ball slows down. As it falls, the ball speeds up. The change in velocity of the ball is always 9.8 m/s every second or 9.8 m/s². For ease of discussion and problem solving, it is sometimes convenient to round this number to be 10 m/s every second or 10 m/s². Since the acceleration is always down to Earth, use –10 m/s² as the value. Think of any velocity in the "up" direction as + and any velocity in the down direction as –.

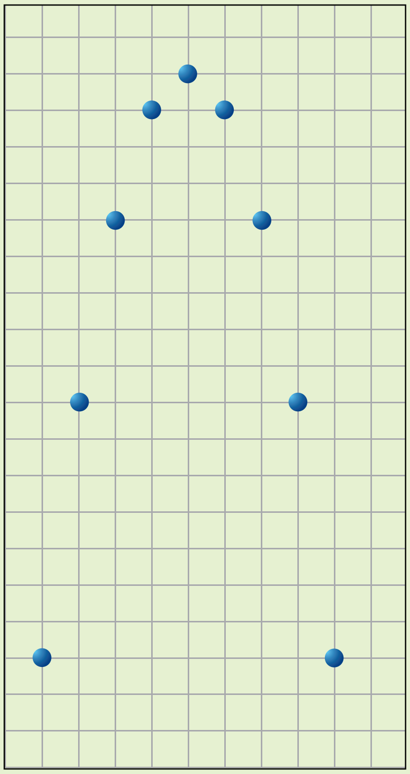


If an object is thrown straight up at 40 m/s, its velocity decreases by 10 m/s every second. Its speed at the end of each second is shown in the top diagram.

It comes to rest at the top of the path because its velocity is 0 m/s. Its acceleration will still be –10 m/s² because its speed is still changing by –10 m/s every second. Its new speed is –10 m/s one second after it begins its fall.

The horizontal speed of the object will remain constant since no force acts on the ball horizontally.

These two motions can be combined to allow you to mathematically predict the motion of a thrown object. If you space the horizontal position of the ball at equal distances as it rises and falls, you can represent the motion of the ball.



Checking Up

1. If a pen and a ruler are dropped together from the same height, will they reach the ground at the same time? Explain your answer.
2. When an object falls vertically down, does its velocity remain the same? Explain your answer.
3. If a ball is thrown upward, what is the ball's velocity at its point of highest rise? What is the ball's acceleration?

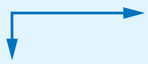
Active Physics Plus

+Math	+Depth	+Concepts	+Exploration
◆			

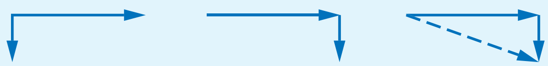
Vector Components

In the investigation you just completed, the projected coin left the table horizontally. At any point in its motion, the projected coin is moving down and to the right. You can draw its velocity at any time. This velocity has two parts. One part describes the horizontal motion and the second part describes the vertical motion.

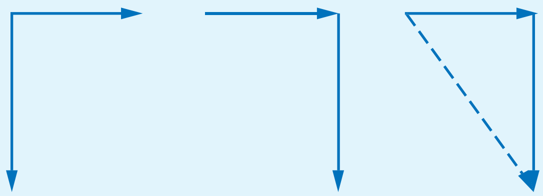
A short time after leaving the table, the projected coin has a small vertical speed and a constant horizontal speed.



You can add these parts as vectors. To add these two velocity vectors, use the "tip-to-tail" method. By sliding one vector over (maintaining its length and direction), the resultant is then drawn from the tail of the first vector to the tip of the second vector.

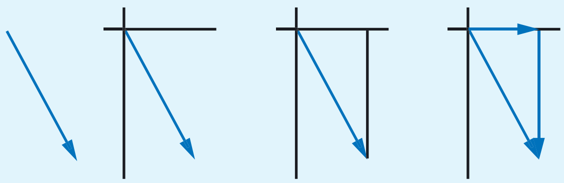


When you look at the coin's velocity some time later, you notice that the coin is moving faster in the vertical direction but continues horizontally at the same speed. If you have the values for the speeds of the vertical and horizontal motions of the coin, you can add the vectors to determine what happens to the total (or resultant) vector.



The resultant velocity or total velocity (often simply called the coin's velocity) has become larger and its direction has changed. The coin's resultant or total velocity is pointing in a more vertical direction.

If you measure the velocity at any one point in the path, you could also use that resultant or total velocity vector to find its horizontal and vertical "components." The components of a vector are themselves vectors, namely, the vectors along two perpendicular axes that add up to the vector. First, you draw the total or resultant velocity vector to the correct size and pointing in the correct direction. Second, you draw horizontal and vertical axes from the tail of the vector. Third, you draw lines from the tip of the vector to each axis, making sure the lines are parallel to the other axis. By doing this, you can obtain the horizontal and vertical components of the velocity (the vectors that add together to produce the total or resultant velocity vector).



If you were to construct numerous velocity vectors along the path of the object, you would notice two things. First, the horizontal velocity components are always equal. Second, the vertical velocity components increase as time goes on.

Sample Problem

a) A football is thrown at 20.0 m/s at an angle of 30° with respect to the horizontal. What is its horizontal velocity (often called the x-component of its velocity)?

b) If the football were thrown at 20.0 m/s at an angle of 60°, what is its horizontal velocity?

c) How far does each football travel in the horizontal direction in 3.0 s?

Strategy: You can solve the first two parts by drawing vector diagrams to scale and finding the x-components. In *c)*, you can find how far each football traveled by using the relationship for steady motion:

Distance = (velocity) × (time)

$$d = vt$$

Solution:

a) The first vector must be 20 units long at an angle of 30°. (The scale is 1 unit = 1 m/s.) Use a protractor to draw the angle accurately.

Measuring the x-component and using the scale, you find the x-component is 17.3 m/s.

b) The second vector is also 20 units long at an angle of 60°.

Measuring the x-component and using the scale, you find the x-component is 10 m/s.

c) The first trajectory has a horizontal velocity component of 17.3 m/s for 3.0 s. Its distance is:

$$\begin{aligned} d &= vt \\ &= (17.3 \text{ m/s})(3.0 \text{ s}) \\ &= (17.3 \times 3.0)\left(\frac{\text{m}}{\cancel{\text{s}}} \times \cancel{\text{s}}\right) \\ &= 51.9 \text{ m or } 52 \text{ m} \end{aligned}$$

The second trajectory has a horizontal velocity of 10 m/s. Its distance is:

$$\begin{aligned} d &= vt \\ &= (10 \text{ m/s})(3.0 \text{ s}) \\ &= 10 \times 3.0\left(\frac{\text{m}}{\cancel{\text{s}}} \times \cancel{\text{s}}\right) \\ &= 30 \text{ m} \end{aligned}$$

1. A football is kicked with a vertical velocity of 30 m/s and a horizontal velocity of 10 m/s.

   a) Calculate the vertical and horizontal velocities for each second that the football is in the air.

   b) Draw a diagram showing the vertical and horizontal positions of the ball after each second.

2. A batted baseball leaves the bat with a velocity of 50 m/s at an angle of 30° from the horizontal.

   a) If the ball leaves the bat at time equal to zero, what are the horizontal and vertical components of the velocity at time equal to zero?

   b) What are the horizontal and vertical components of the velocity at time equal to 1 s?

   c) What are the horizontal and vertical components of the velocity at time equal to 5 s?

   d) What is the magnitude and direction of the velocity at time equal to 5 s?

What Do You Think Now?

At the beginning of the section, you were asked the following:

- **What determines how far an object thrown into the air travels before landing?**

From the observations you made in the *Investigate* section, what do you now think determines how far an object travels after it is thrown?

Physics Essential Questions

What does it mean?

A very important principle of physics is that motion in the horizontal direction and motion in the vertical direction are independent of each other. What does it mean to say that the horizontal motion of a projectile is independent of the vertical motion of the same projectile?

How do you know?

What evidence do you have to convince yourself that the horizontal and vertical motions of a projectile are independent of each other?

Why do you believe?

Connects with Other Physics Content	Fits with Big Ideas in Science	Meets Physics Requirements
Force and motion	Models	* Experimental evidence is consistent with models and theories

* Physics attempts to explain as much as possible with a single concept. Both the motion of a projectile and a swimmer crossing a river can be understood if their horizontal and vertical motions are considered independently. Give a reason why you believe that all motion can be examined in this way.

Why should you care?

Many sports involve projectiles. Think of a sport where a projectile is involved and describe how the independence of its horizontal and vertical motions explains its trajectory.

Reflecting on the Section and the Challenge

In *Part A* of this *Investigate* (two falling coins), you observed that the time required for a coin to fall is independent of its horizontal speed. If two long jumpers rise to the same height, they will then remain in the air for identical times.

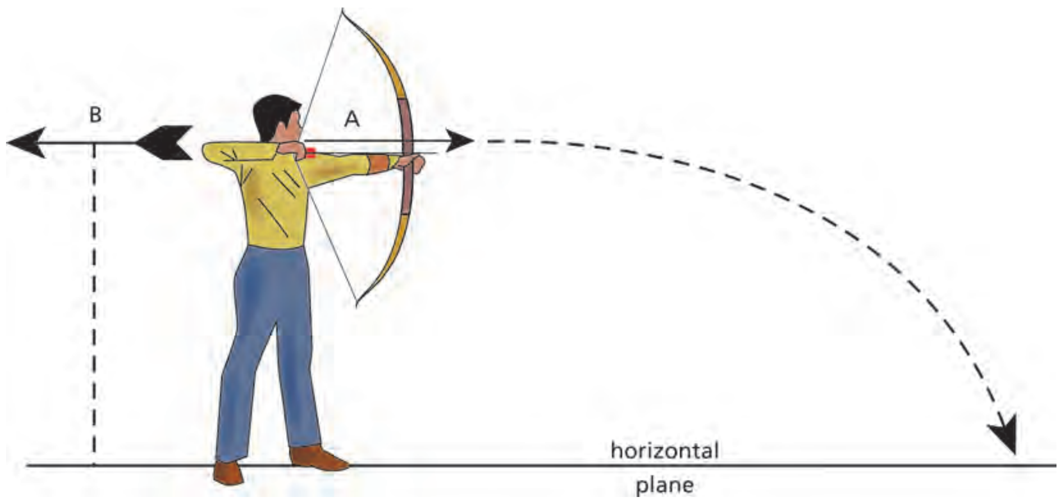
In *Part B* of the *Investigate* (the rolling chair), you saw that the faster the chair is moving, the farther the ball will travel horizontally. If a long jumper is able to increase horizontal speed, then the jumper will travel farther.

Most sports have objects or people "flying through the air." You can describe how projectile motion relates to a sport you might choose for the challenge.

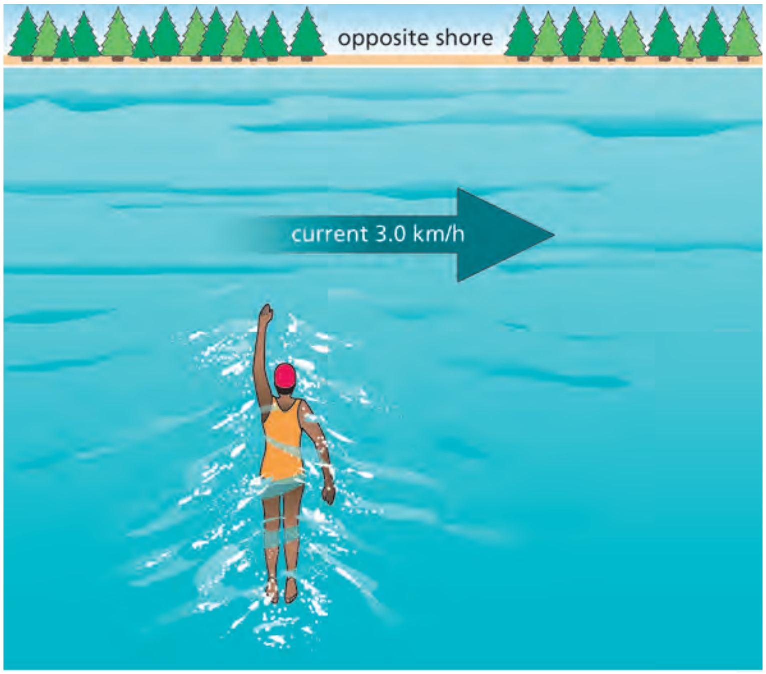
Physics to Go

1. Draw a sketch of two coins leaving the table. Show where each coin is at the end of each tenth of a second. Remember to emphasize that they both hit the ground at the same time.
2. Repeat the sketch of the two coins leaving the table, but this time have one of the coins moving at a very high speed.
3. It is said that a bullet shot horizontally and a bullet dropped will both hit the ground at the same time if air resistance is neglected. Draw sketches of this (the bullet is like a very, very fast-moving coin).
4. Survey your friends and family members to find out which they think will hit the ground first, a bullet that is dropped, or a horizontally-shot bullet (neglecting air resistance).

   Explain why you think people may believe that the two coins hit the ground at the same time, but that they have a more difficult time believing the same fact about bullets.
5. Use evidence from your observations of the two coins in this section to prove that a 100 mi/h pitch thrown horizontally by a major league player will hit the ground in the same amount of time as a 10 mi/h pitch thrown horizontally from the same height by a child.
6. Use evidence from your observations of the ball and chair in this section to show the truth of the statement, "A projectile's horizontal motion has no effect on its vertical motion, and vice versa."
7. Look at the diagram of an arrow being shot horizontally from a bow and another arrow dropped from the same height. Arrow A is shot horizontally at a speed of 50 m/s. A second arrow, B is dropped from the same height and at the same instant as arrow A is released. Neglecting air friction, how does the time A takes to strike the horizontal plane (ground) compare to the time B takes to strike the horizontal plane?



Use a protractor for *Questions 8 – 10.*



8. A swimmer jumps into a river and swims directly for the opposite shore at 2.0 km/h as shown in the diagram. The current in the river is 3.0 km/h and flows from left to right in the diagram. What is the swimmer's velocity relative to the shore?

9. Active Physics *Plus* A football is thrown at 15 m/s at an angle of 37° in the horizontal direction.

   a) What is its velocity in the horizontal direction?

   b) How far in the horizontal direction has the football traveled in 2.0 s?

10. Active Physics *Plus* A shot put is released at 12 m/s at an angle of 45° in a horizontal direction.

    a) What is its velocity in the horizontal direction?

    b) How far in the horizontal direction has the shot put traveled in 0.5 s?

11. ***Preparing for the Chapter Challenge***
    Write a script for a sports telecast that describes the motion of a baseball while it is pitched and then hit into the outfield.

Inquiring Further

Investigating more *x*- and *y*-components of motion

Ask another student to roll a marble slowly across the table in front of you. As the marble rolls by, apply a momentary force to it with a small object such as a block of wood. Make sure the force you apply is directly away from you and perpendicular to the initial velocity of the marble. If you define the *x*-direction to be along the initial velocity of the marble, then you can define the *y*-direction as the direction of the force you are applying. Investigate whether you can change the *x*-component of the marble's velocity by applying a force in the *y*-direction. Relate your observation to what you have learned about projectile motion.