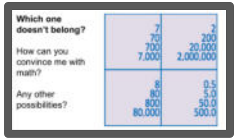
Table of Contents: Quick Links

This icon will take you back to this Table of Contents

Resources for Rich Thinking Tasks



Building Thinking Classrooms

Peter's Numeracy Tasks

Peter's Card Tricks

Peter's "Good" Problems

Week of Inspirational Math

Main Site: aliciaburdess.com (BTC resources/videos) LINK

2nd Grade 3rd Grade 8th Grade

BTC Book Companion Site: Link

Rich Tasks from Virginia Dept. of Education

Crowdsourced Spreadsheet **#1**

Crowdsourced Spreadsheet **#2**

Randomizing Cards

Pickerwheel Randomizer Spreadsheet Randomizer

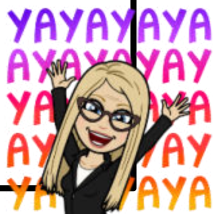
VRG at a Glance

Other Great Sites for Curricular Tasks

Making Math Moments that Matter (Problem-based Lessons)
Fostering Math Practices (Routines for Reasoning)
Illustrative Mathematics - Practices (Tasks by Standard)
2nd 3rd 4th 5th 6th 7th 8th

Emergent Math Problem-based Task Curr. Map
3rd 4th 5th 6th 7th 8th

Inside Mathematics
Desmos (Chelsea's Collections)
3-Acts: GFletchy TK-7
Robert Kaplinsky' (K-12) Kendra Lomax (K-3) Dan Meyer's (6-12)
Mike Wiernicki (K-8) Kristen Acosta (K-5) Andrew Stadel (6-12)
Kyle Pearce (3-12) Catherine Castillo(K-5) Dane Ehlert (6-12)
Geoff Krall (3-12) Jon Orr (6-12)
3-Act Training Deck by Van Lay for Pringle Ringle
YouCubed.org



Notes to Future Forgetful Self:

Worked Solution(s):	What I tried that didn't work: (...didn't work because…)
Assumptions I made: (We noticed...so we…)	What helped me get "unstuck?" (We got stuck on...so we…)

Non-Curricular Task Decks: By Grade Spans

K-3

4-8

Progression of Non-curricular Tasks: 4th-HS

- Frame the Cards (Concrete)
- Fewest Squares (Visual Representation)
- Tax Collector (Contextual/Abstract)
- Palindromes (Abstract)
- Hailstone Sequence (Abstract)
- People Puzzle (Movement/Concrete)

Sources for these tasks and where to find more:

Week of Inspirational Math

Teaching Through Problems Worth Solving (Grade 8, Grade 2, and Grade 3)

Nrich.maths.org

https://playwithyourmath.com/

Link back to this slide.

Frame the Cards

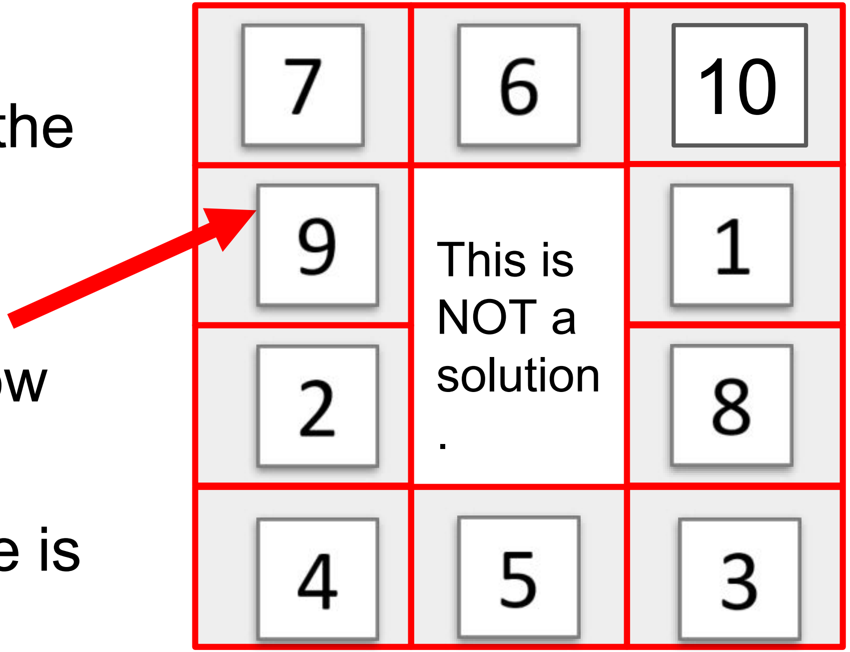
Frame the Cards

Arrange the cards from the ace to the ten into a picture frame so that the top, bottom, and sides add to the same total of spots (hearts/diamonds…) Right now the top row adds to 23, the bottom adds to 12, the left side is 22 and the right side is 22. These four numbers should be the same. Apparently there are many solutions to this problem.

Credit to Ian Stewart and Teaching Through Problems worth Solving 3.0

Frame the Cards

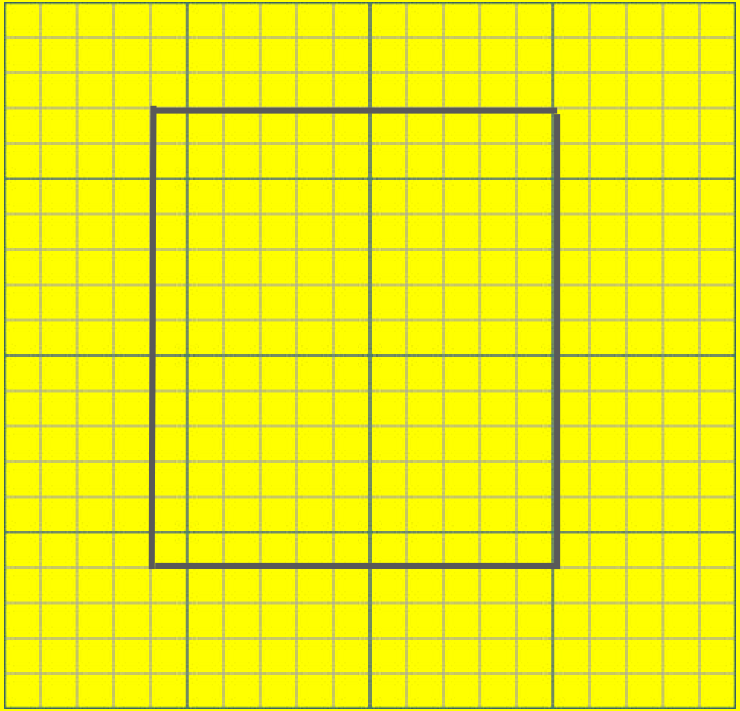
Arrange the cards from the one to the ten into a picture frame so that the top, bottom, and sides add to the same total …) Right now the top row adds to 23, the bottom adds to 12, the left side is 22 and the right side is 22. These four numbers should be the same. Apparently there are many solutions to this problem.



Credit to Ian Stewart and Teaching Through Problems worth Solving 3.0

Fewest Squares

What is the **FEWEST** number of squares you can draw inside the rectangle?



Tax Collector

Tax Collector Task:

I have 12 envelopes, numbered 1 to 12. Each contains a number of dollars equivalent to the number on it. The game starts with you taking one of the envelopes-the money inside of which is yours to keep. The tax collector will then take all of the remaining envelopes whose number is a factor of the envelope you took. The tax collector must be able to take at least one envelope every turn. Play continues until you can no longer take an envelope, at which point the tax collector will take any remaining envelopes. What is the most amount of money that you can get?

Tax Collector Extensions:

What if the envelopes had values from 1-20?

What if the envelopes had values from 1-24?

What if the envelopes had values from 1-32?

What if the envelopes had values from 1-36?

What if the envelopes had values from 1-50?

What if the envelopes had values from 1-60?

Palindromes

Mom

Hailstone Sequence

Pick any number. Write it down.

1. If the number is even, divide it by 2.

2. If the number is odd, multiply it by 3 and add 1.

Keep repeating step 1 and 2 each time you get an even or odd result.

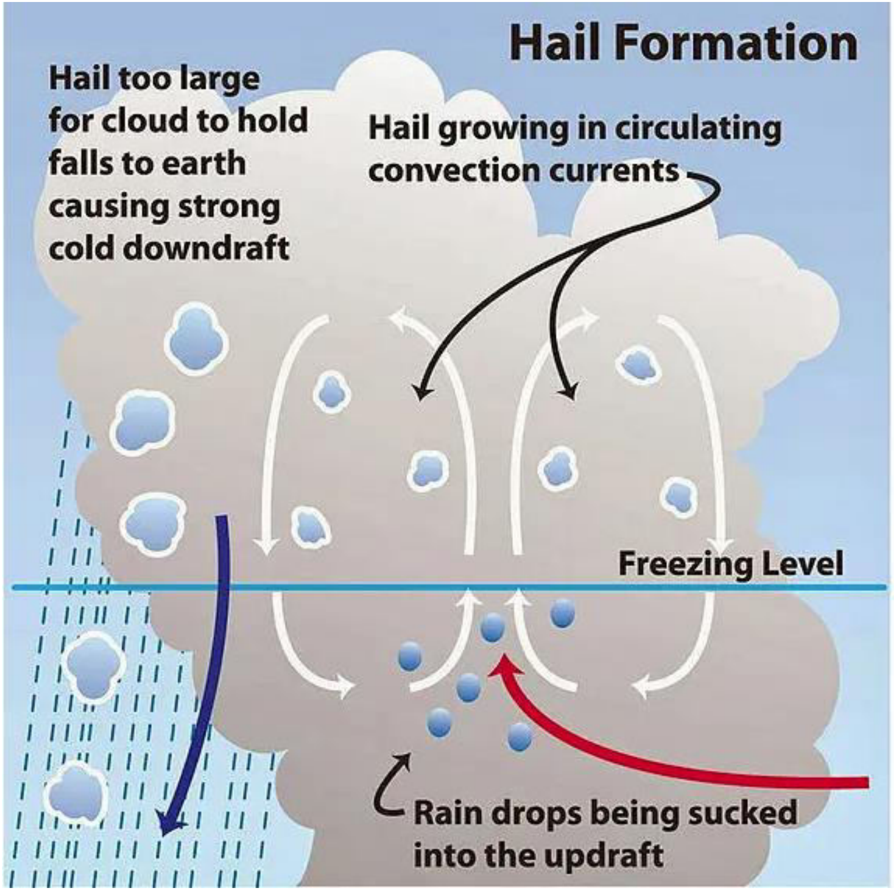
What happens each time?

Hailstone Sequence: an unsolved problem!

The problem you will work on today is one of the world's unsolved problems in mathematics, which is, in itself very cool. It involves a sequence of numbers called a Hailstone sequence. The sequence is called this because the numbers go up and down again, like this:

$$20 \rightarrow 10 \rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

Hailstones do this – they start in a cloud as drops of rainwater, then they are pushed higher in the atmosphere by wind where they freeze, sometimes several times, before eventually falling back to Earth. The number sequences are called hailstone sequences because they go up and down like hailstones.

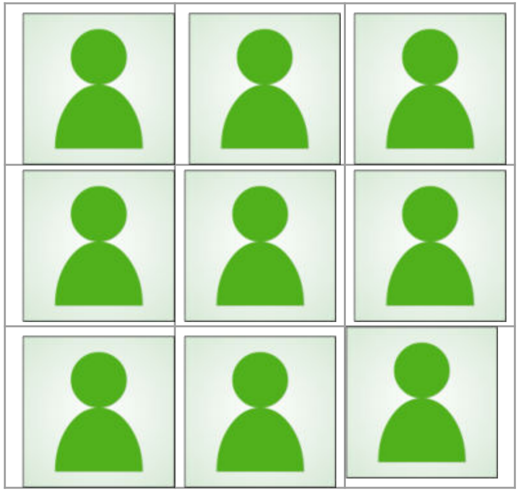

Hail Formation
Hail too large for cloud to hold falls to earth causing strong cold downdraft
Hail growing in circulating convection currents
Freezing Level
Rain drops being sucked into the updraft

Conjectures

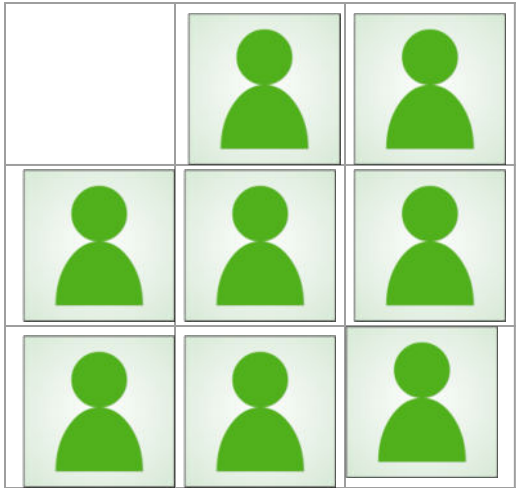
In mathematics people make conjectures - it is an idea that you think might be true but you do not know for sure. Conjectures are very important in mathematics, and making conjectures is something you can be doing as a math student.

Try working with some hailstone strings of numbers that have different starting numbers and make conjectures about what you find out.

People Puzzle



Launch:
T: Imagine there are 9 people spaced evenly in a 3x3 grid. The person in the top left corner is removed, leaving an empty space. (Show how you would remove a person - use sticky notes on the VNPS to demonstrate.)
T: How can you move the sticky note/person in the bottom right corner into this empty space?
S: Give ideas.
T: I forgot to tell you a rule. People/sticky notes can only move horizontally and vertically into empty spaces. Each time a sticky note/person moves, that is one move. How many moves are required?
Extension: Is this the minimum number of moves? How can you prove it?
What if the array was 4x4? 5x5? 6x6?



(Consider drawing a 3x3 array on the floor/concrete outside and having kids stand in the array shown on left. Then ask one student to step out of the array- top left kiddo. Have the students NOT inside the array discuss and direct the students inside the array how to move while keeping track of how many moves it took to complete the task.)

*You could give students colored tiles to work with at their tables.

Extra Non-Curricular Tasks

Fraction Army

1	1	1	1	2
2	2	3	3	3
4	5	5	6	6
7	7	8	9	10

$$\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}$$

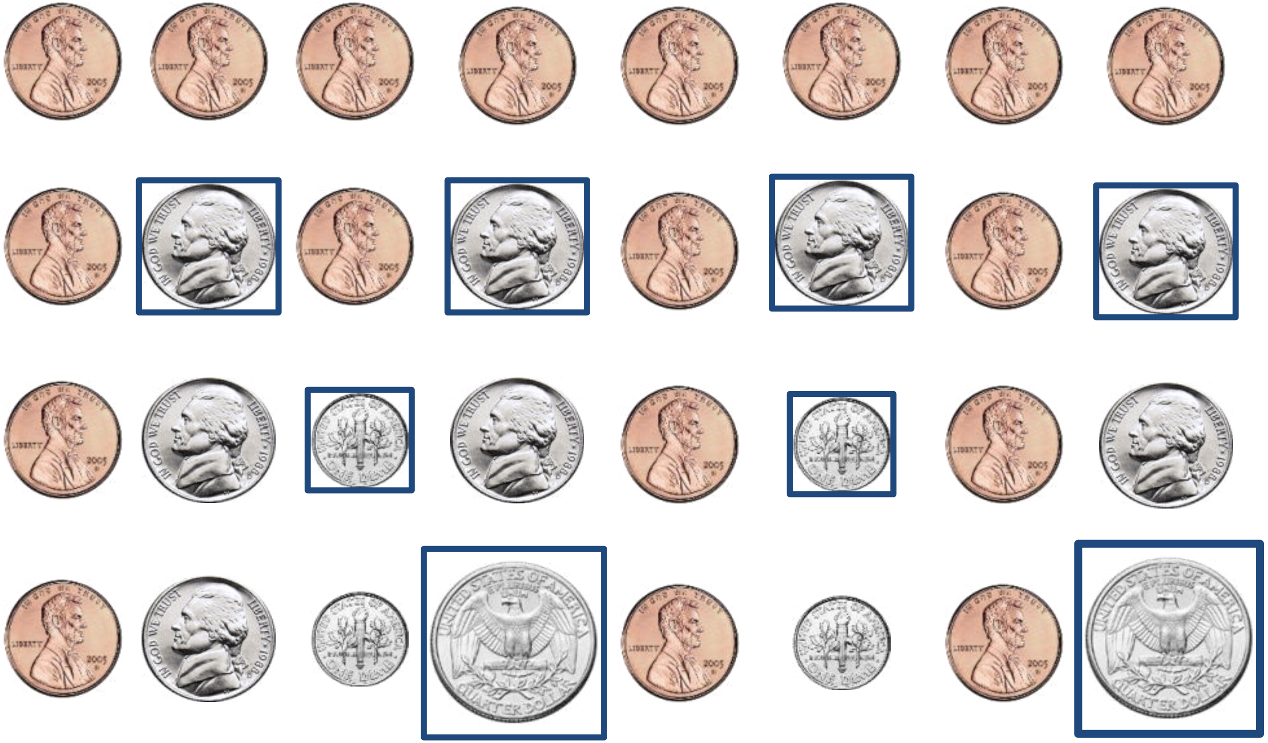
Coin Collection

Groups of	Pennies left over
2	1
3	1
5	1
6	1
7	0

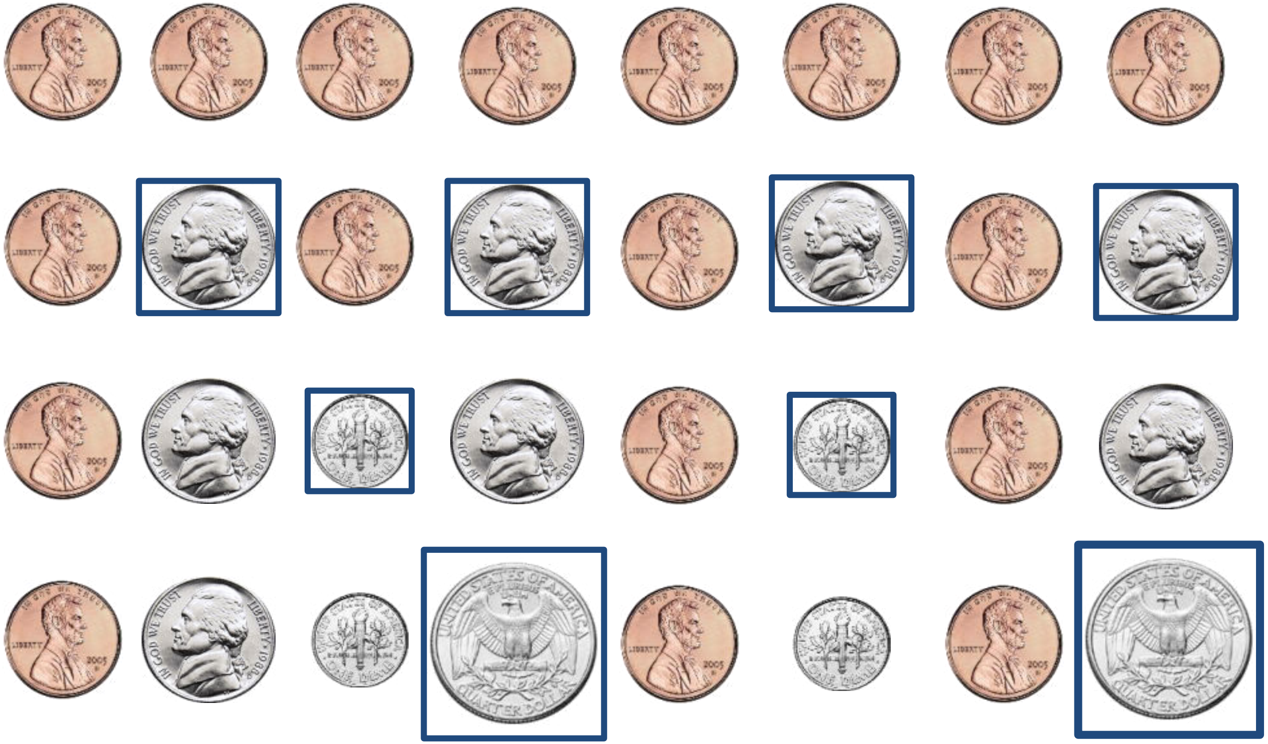
1001 Pennies


ONE CENT
UNITED STATES OF AMERICA
LIBERTY
IN GOD WE TRUST
QUARTER DOLLAR

P P P P P P P P



P P P P P P P P



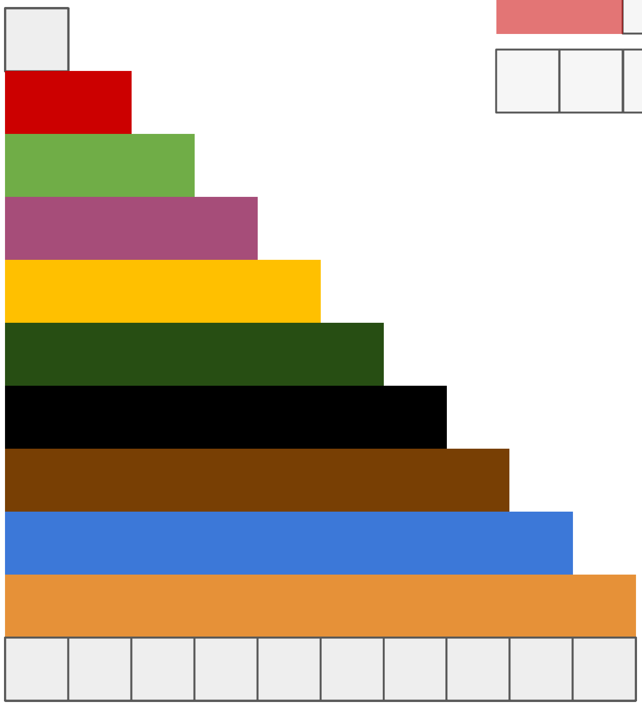
Do NOT write the words. Just say them as you draw letters or use images.

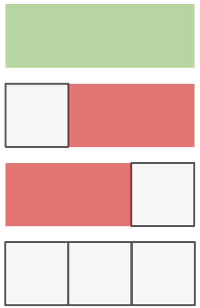
Every second coin is replaced with a nickel.

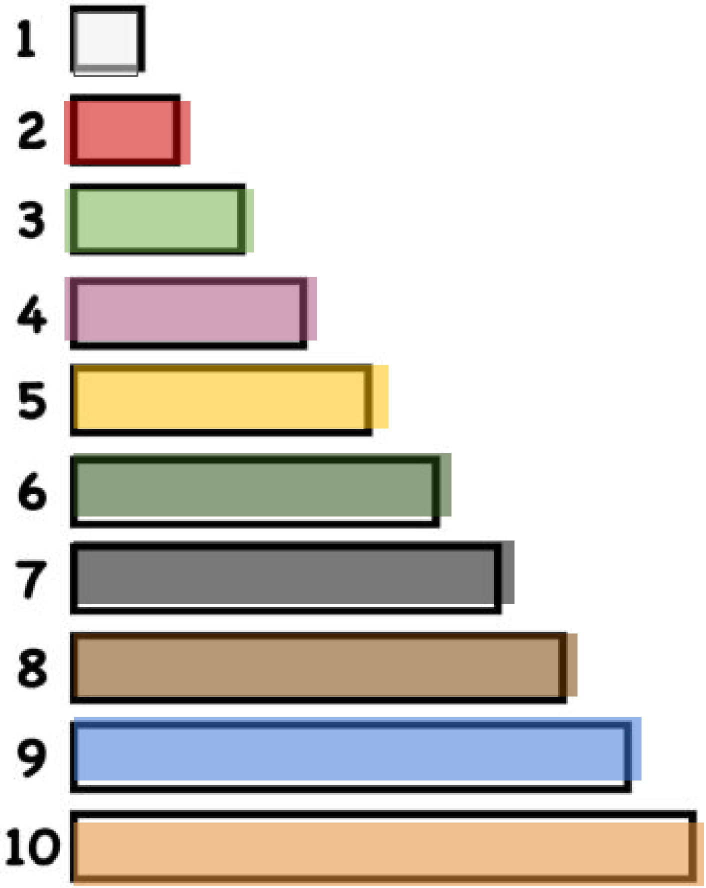
Every third coin is replaced with a dime.

Every fourth coin is replaced with a quarter.

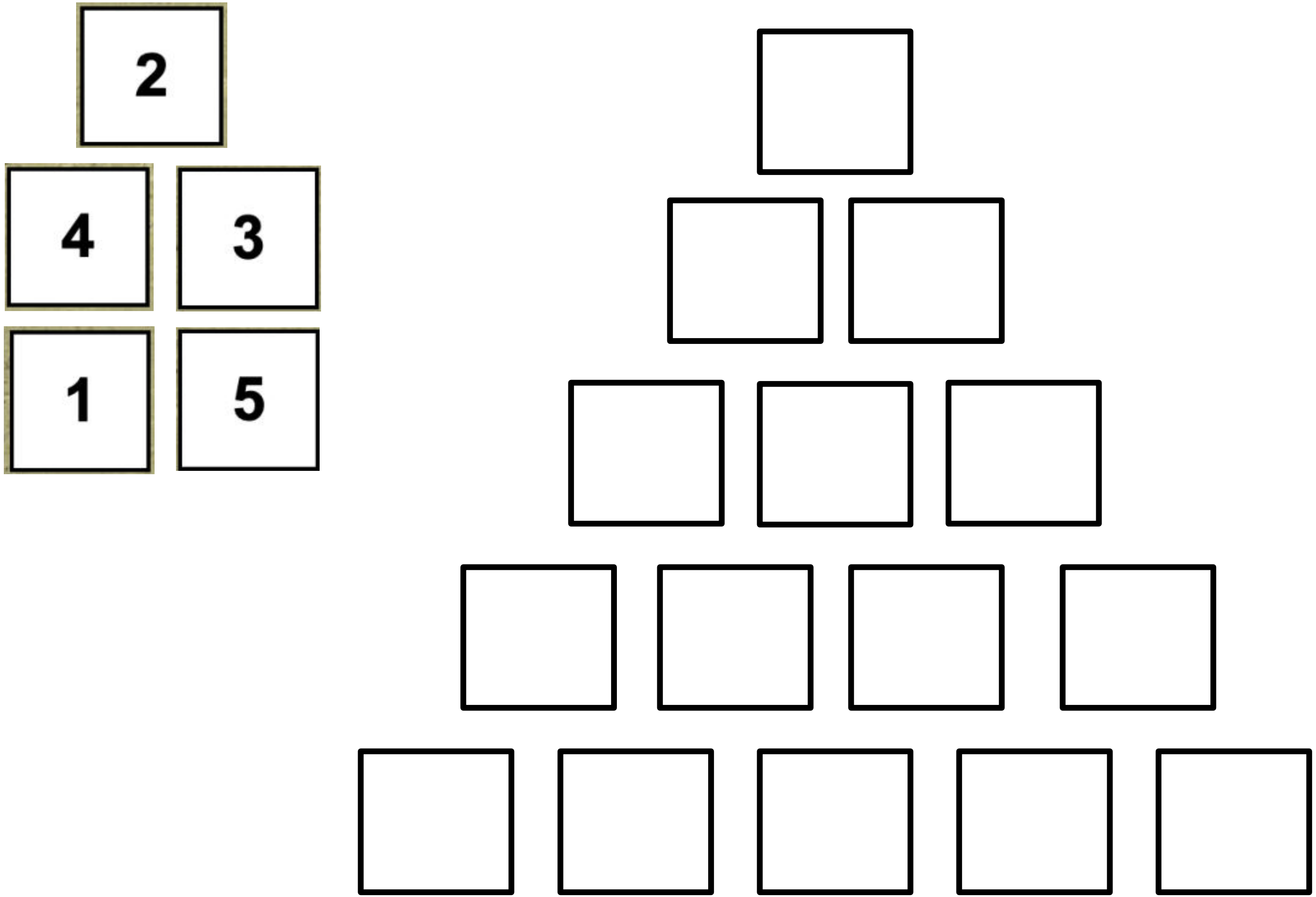
Rod Trains



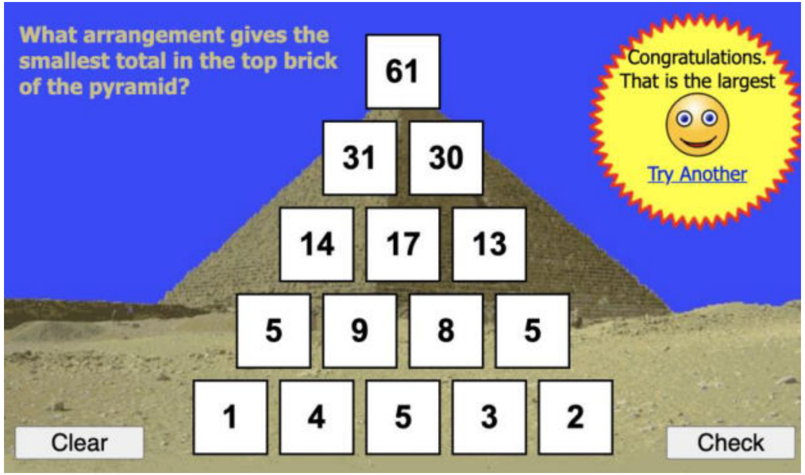


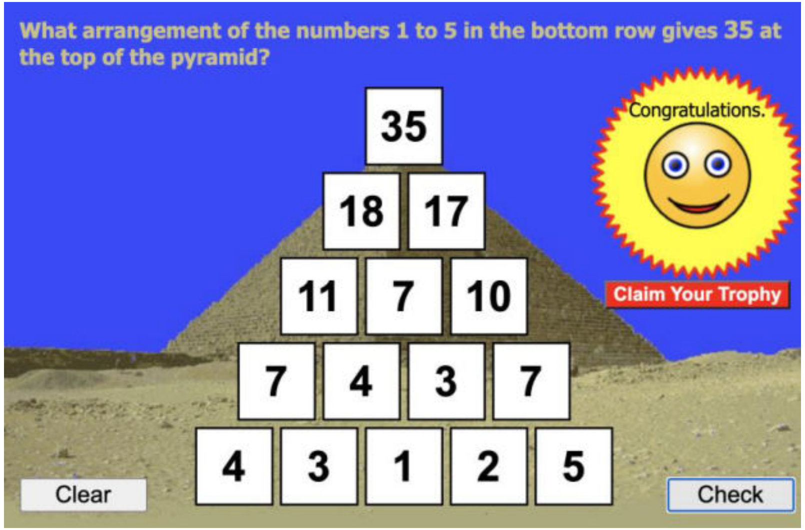

1
2
3
4
5
6
7
8
9
10

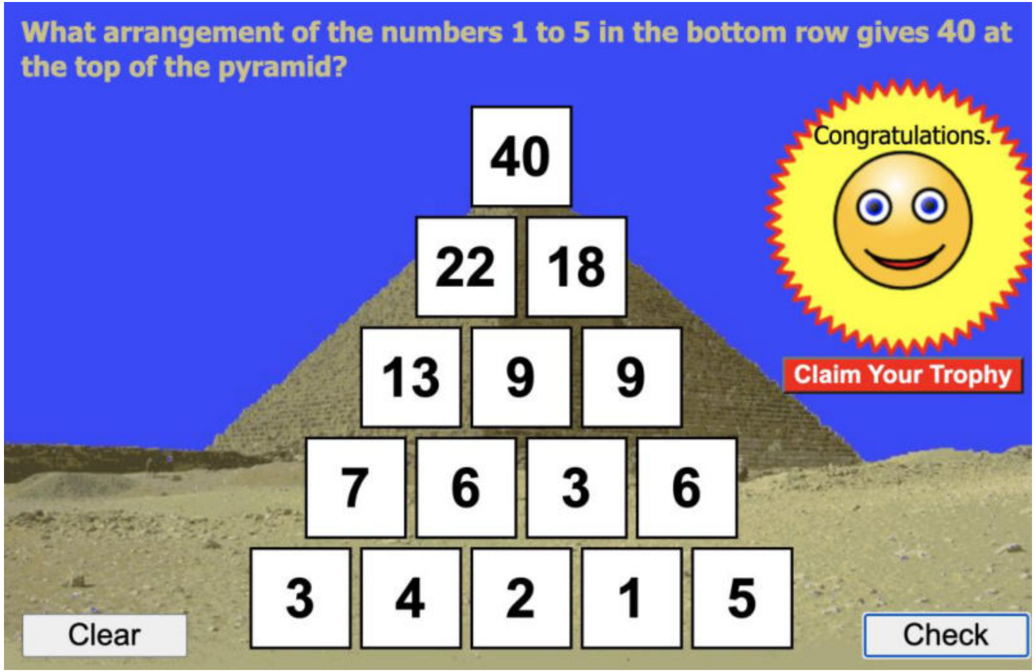
Pyramid Puzzle



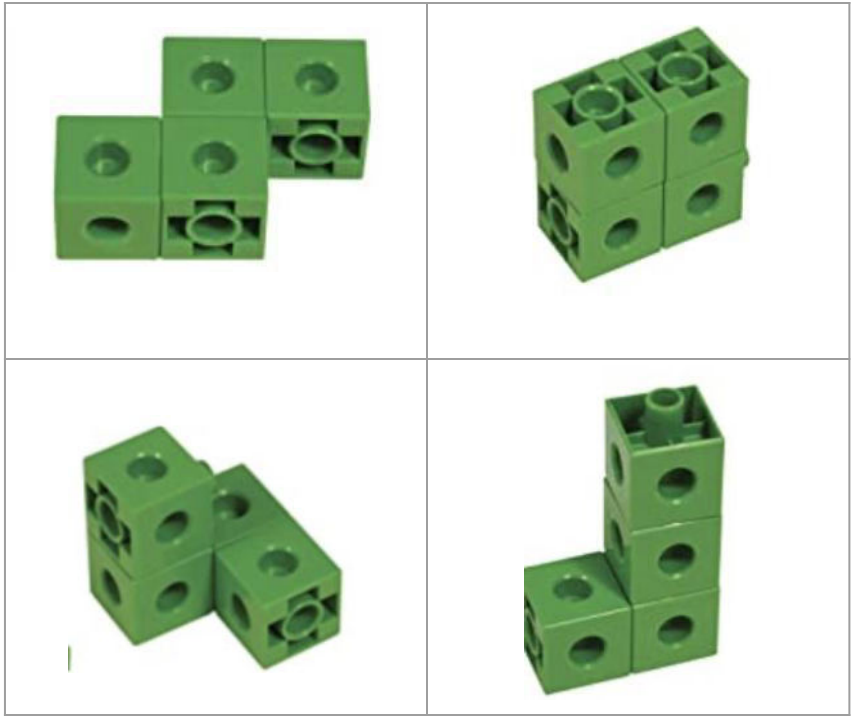
2
4
3
1
5


What arrangement gives the smallest total in the top brick of the pyramid?
61
31 30
14 17 13
5 9 8 5
1 4 5 3 2
Congratulations. That is the largest
Try Another
Clear
Check


What arrangement of the numbers 1 to 5 in the bottom row gives 35 at the top of the pyramid?
35
18 17
11 7 10
7 4 3 7
4 3 1 2 5
Congratulations.
Claim Your Trophy
Clear
Check

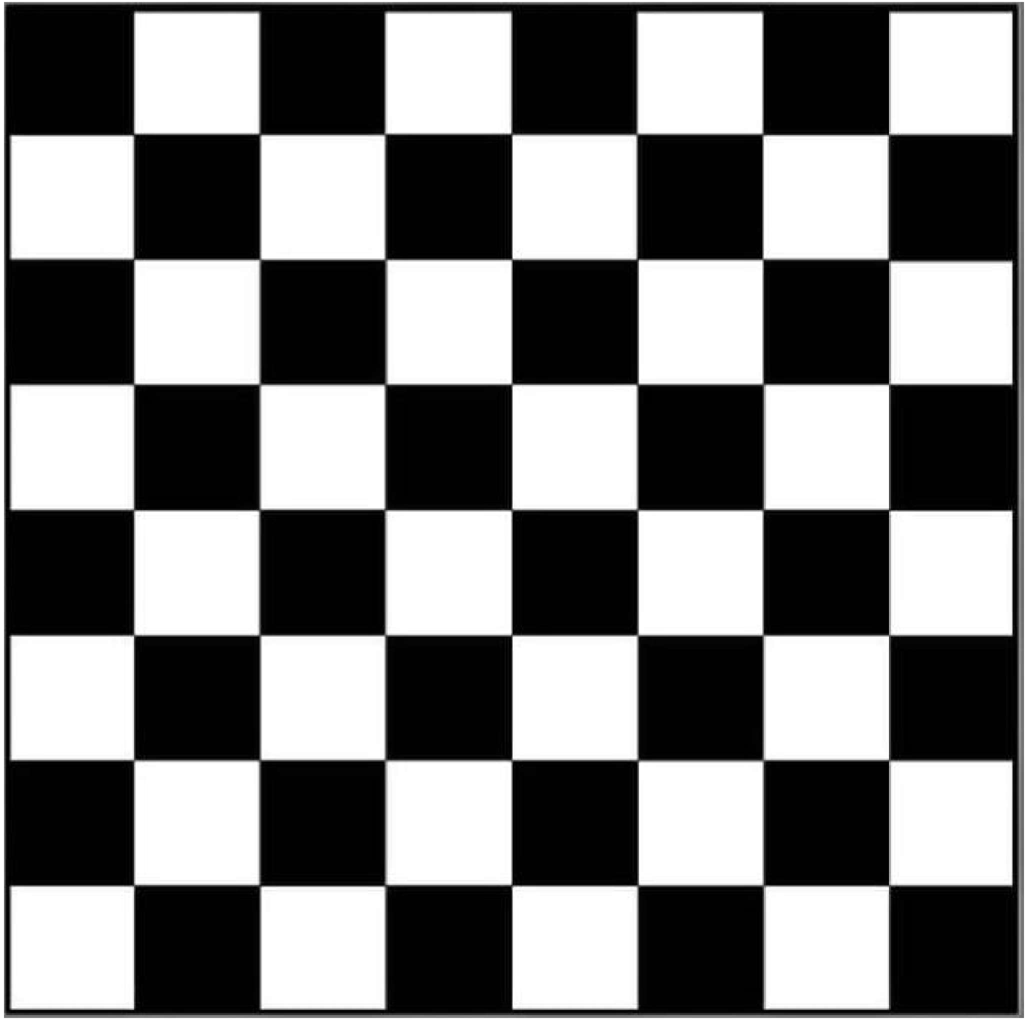

What arrangement of the numbers 1 to 5 in the bottom row gives 40 at the top of the pyramid?
40
22 18
13 9 9
7 6 3 6
3 4 2 1 5
Congratulations.
Claim Your Trophy
Clear
Check

Pentaminos



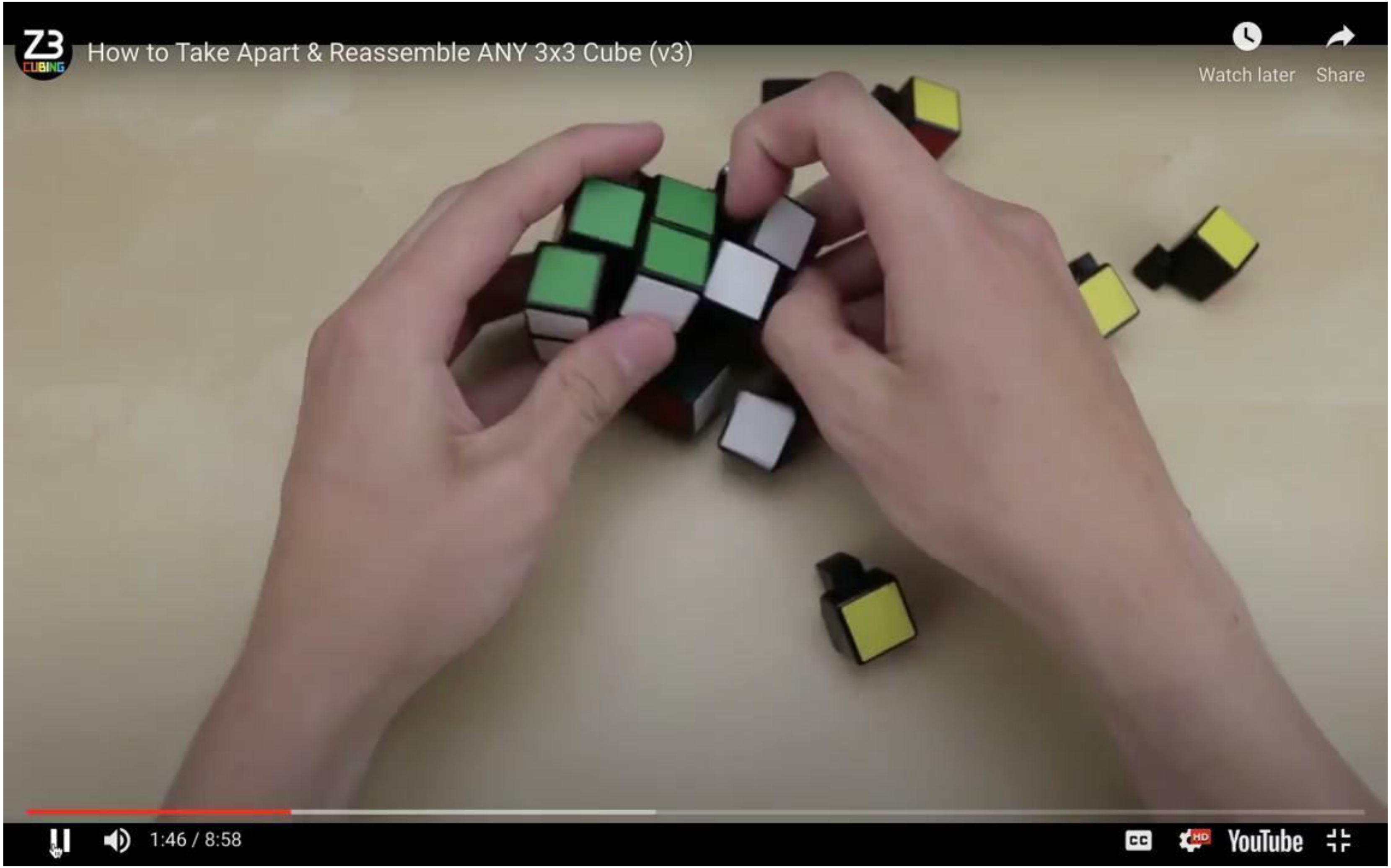
Adapted from Peter Liljedahl’s *Building Thinking Classrooms*, pg. 143 & maths300.com

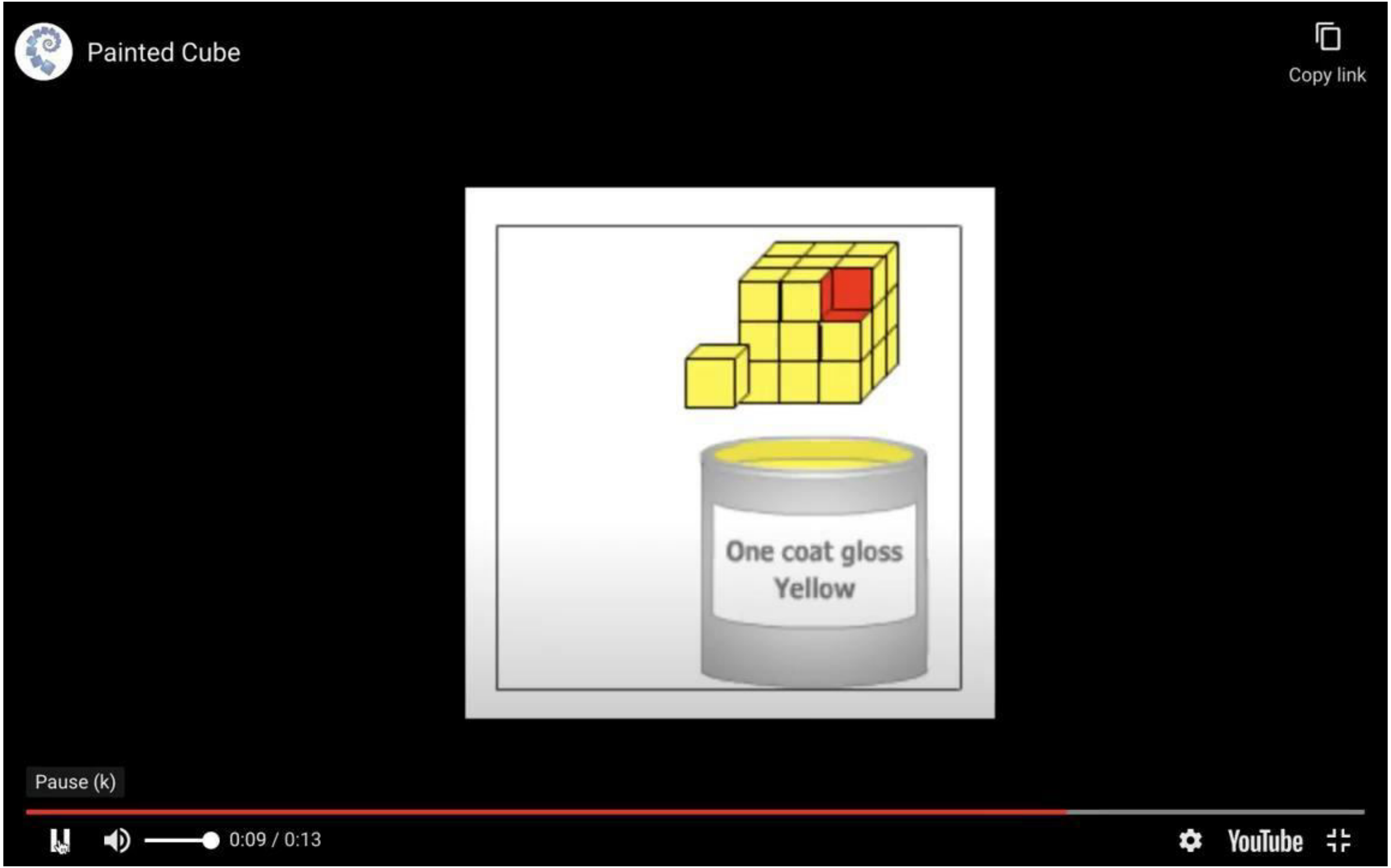
Checkerboard



Painted Cube

Lesson Plan by Abigail Bates


Z3
CUBING
How to Take Apart & Reassemble ANY 3x3 Cube (v3)
Watch later
Share
1:46 / 8:58
YouTube

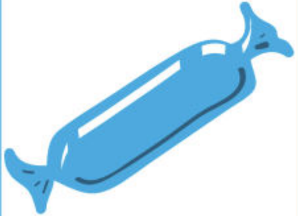
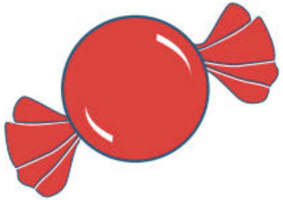

Painted Cube
Copy link
One coat gloss
Yellow
Pause (k)
0:09 / 0:13
YouTube

The Answers Are

1	2	3	4	5	6	7	8	9	10

+ − x ÷

How Many Sweets

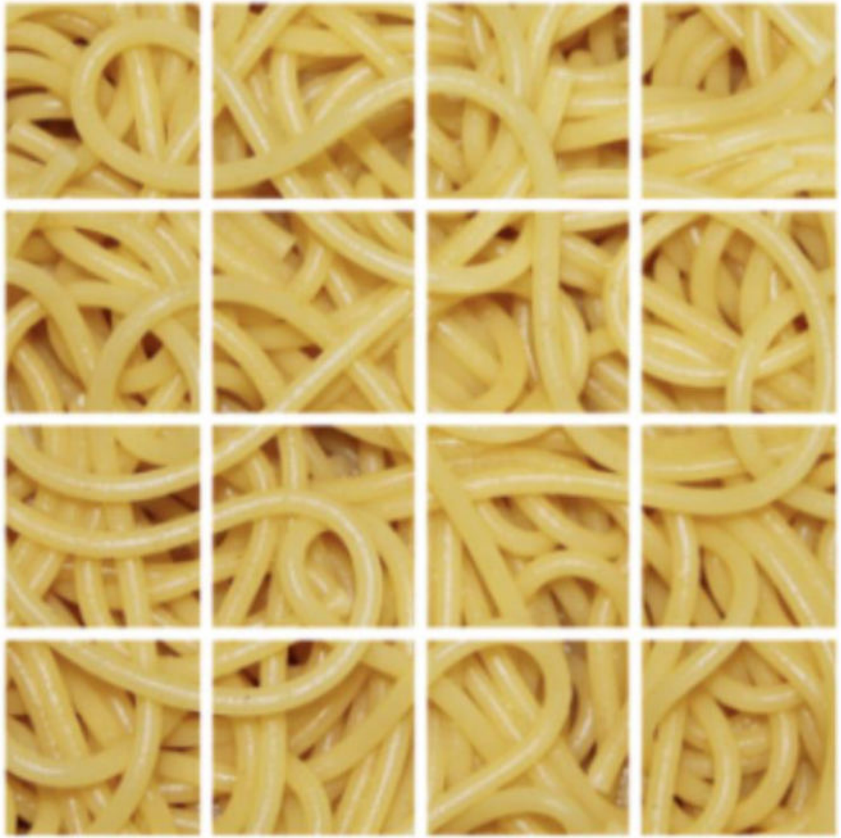
Sweet Type	 Jellies	 Minties	 Chockblocks	 Chewsies	Other
Pie Chart Angle	**90°**	**135°**	**60°**	**45°**	**?**

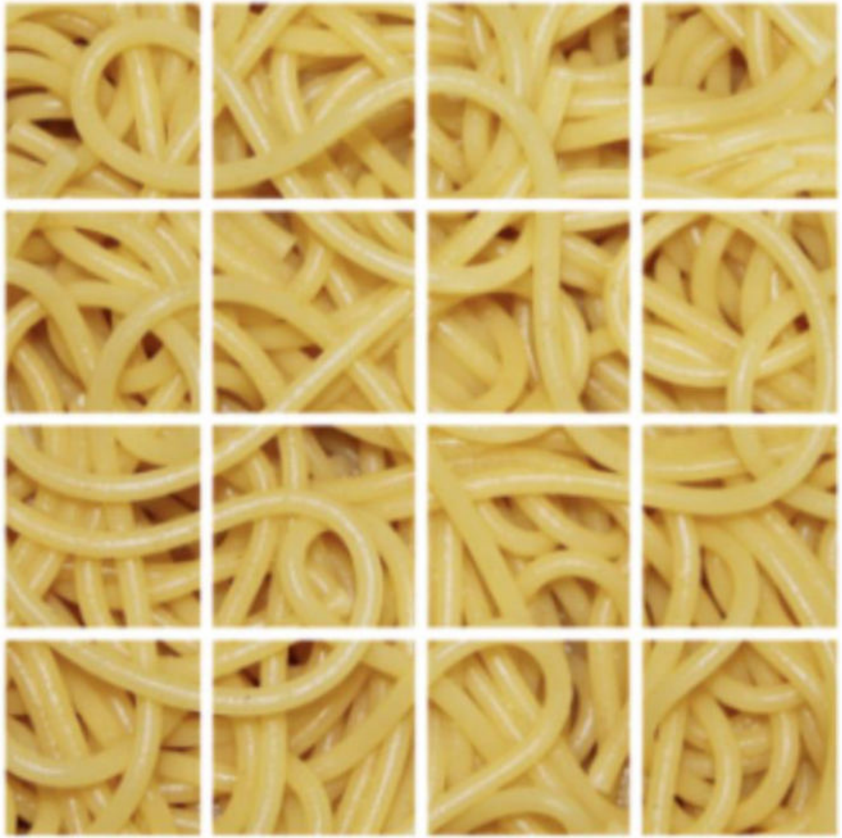
Wine Chest

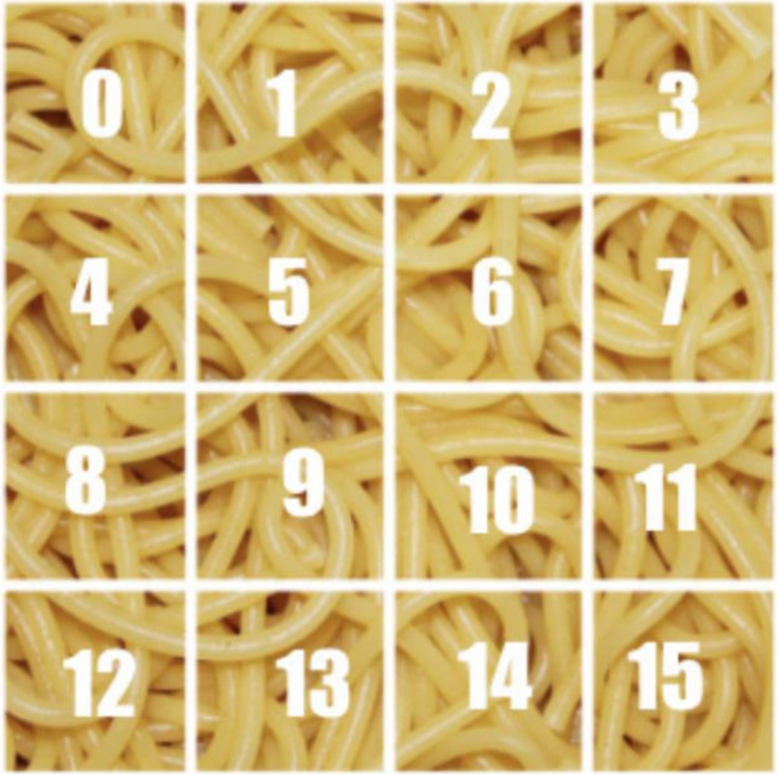
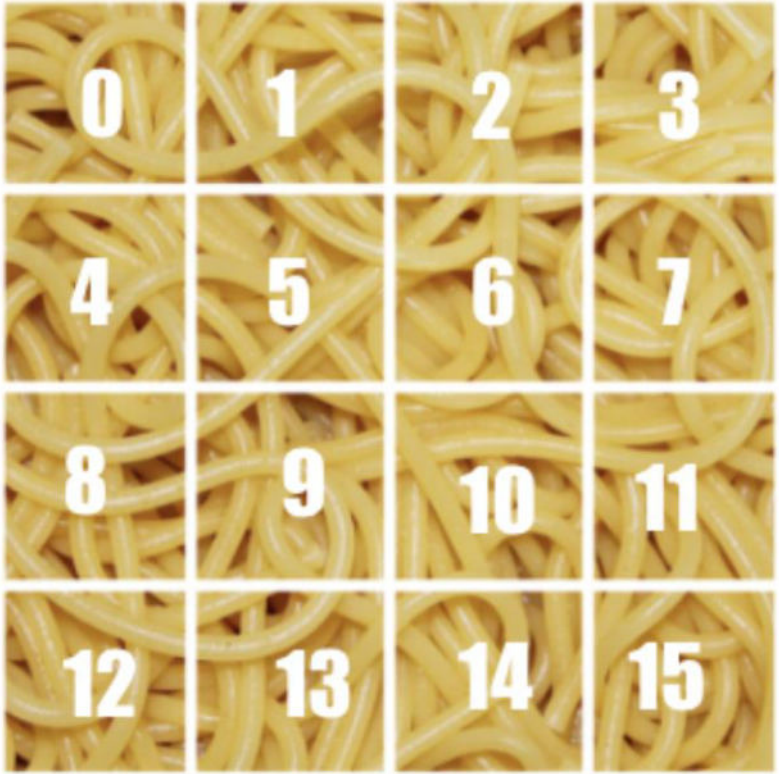
Mr. Snooty loves red wine. So much so that he drinks one bottle of wine a day. But he is very particular about his wine. First, it has to be the right type of wine. Second, it has to be the right temperature. An third, it cannot have been exposed to light more than five times. To make sure it is the right type of wine, Mr. Snooty goes to his favorite wine store, which is very far from home. To make sure that the wine is at the right temperature and not exposed to light, Mr. Snooty build two temperature-controlled wine chests in his house-one much bigger than the other. How often does Mr. Snooty have to go to his favorite wine store?

Mr. Patooty shops at the same store as Mr. Snooty, likes his wine at a certain temperature, but will not drink wine that has been exposed to light more than 10 times. How often does Mr. Patooty have to go to his favorite wine store?

Uncut Spaghetti

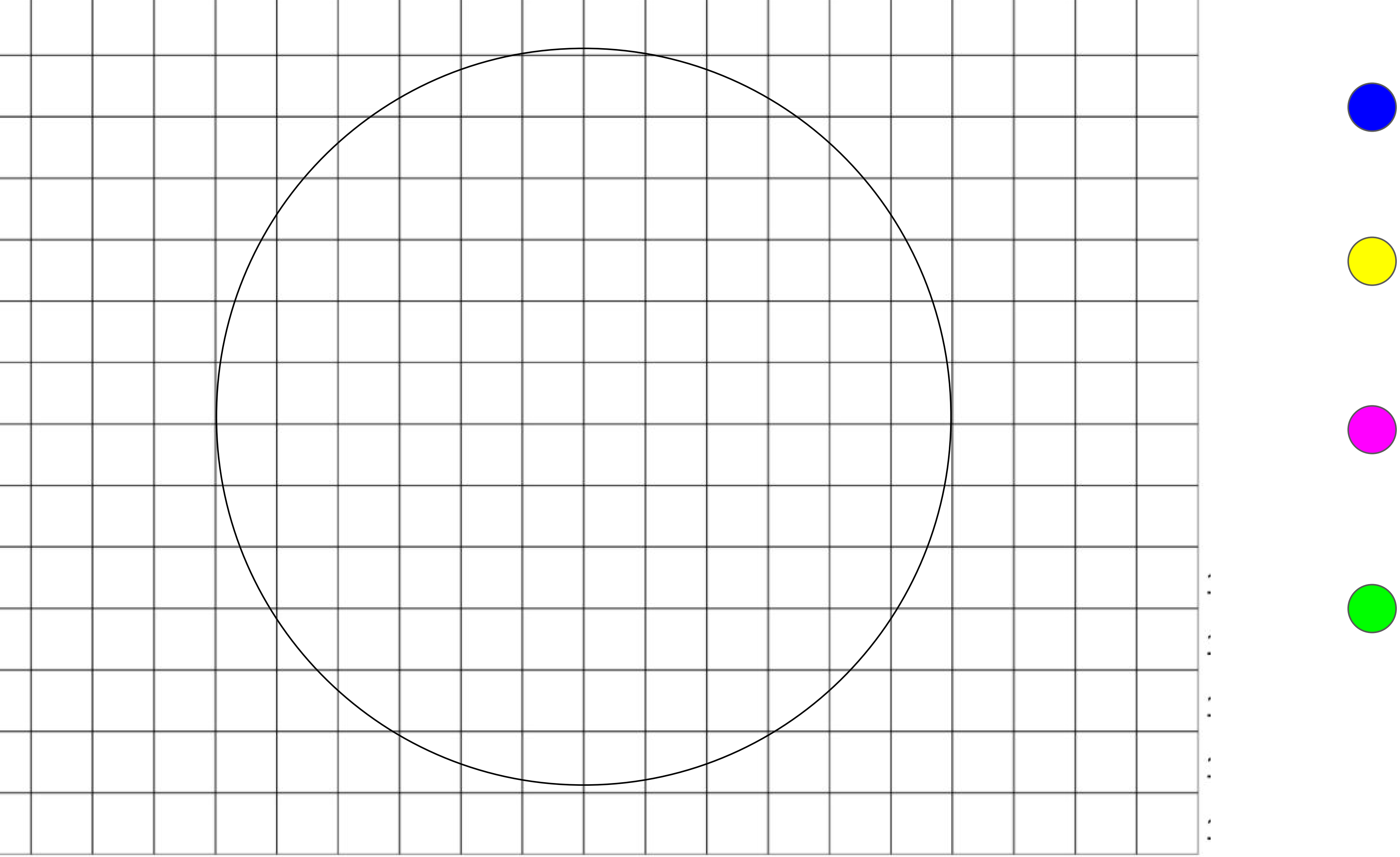





0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

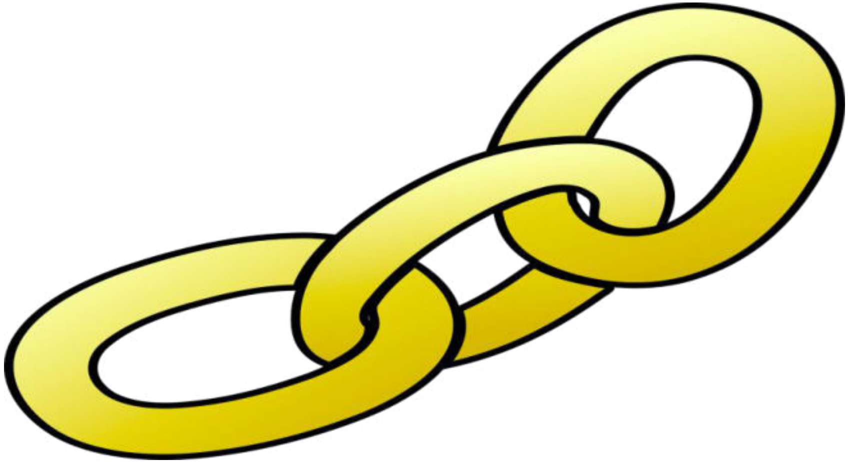
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

Birthday Cake

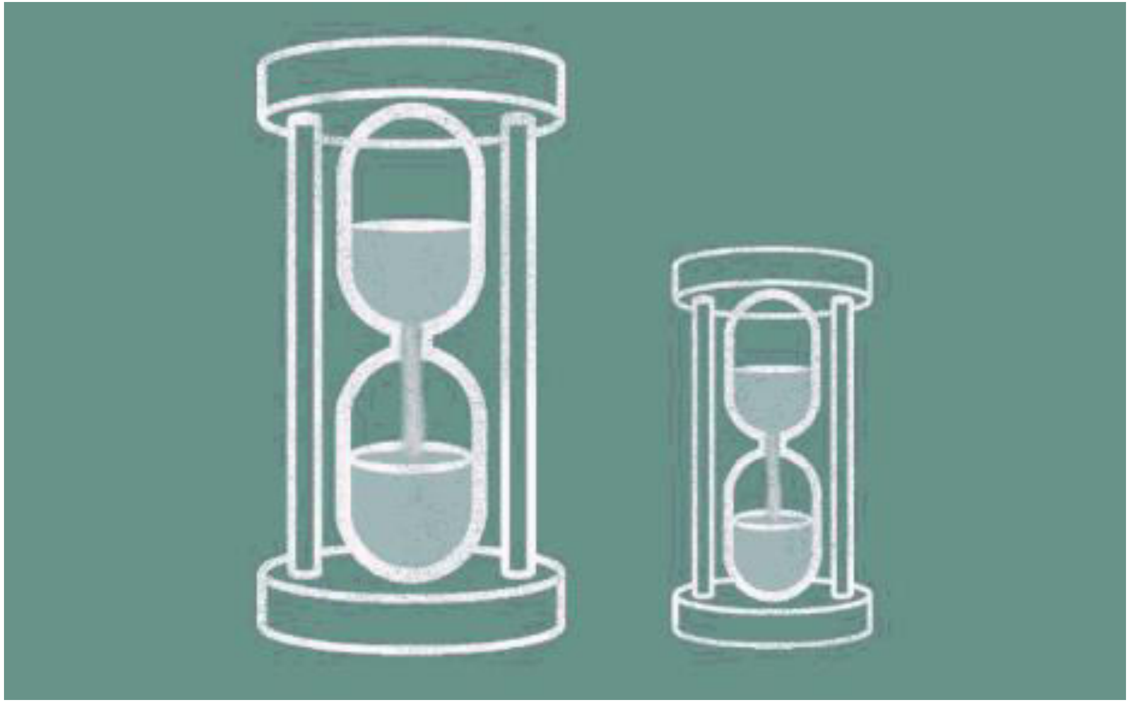


Gold Chain

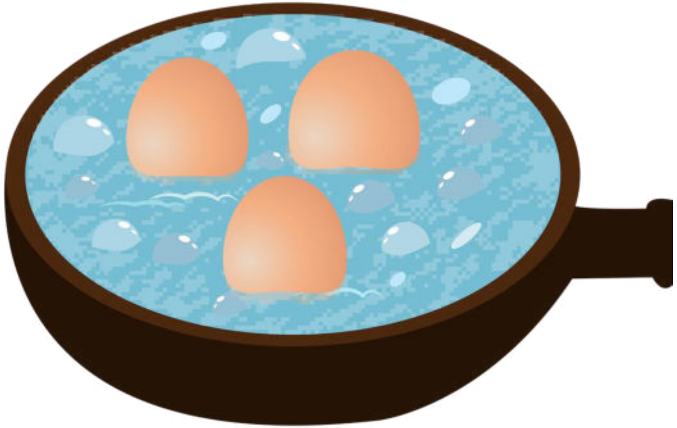
You are backpacking through Europe. You have one month left until your flight home, but you have run out of money. However, you have a 50 link gold chain. You have found a hotel that is willing to accept one link per night for payment of room and board. However, the manager wants payment every day. She is willing to help you out by cutting links for you. The problem is that she wants one gold link payment for every link she cuts. What is the fewest number of cuts you can make so that you will have the most number of links left when you fly home?



Egg Timer



7 minutes 4 minutes



Locker Problem

4th-5th Grade Non-curricular Tasks

4 more to choose from as you plan...

- Split 25 (playwithyourmath.com)
- Emoji Graph/Create a Graph (Week of Inspirational Math)
- Climbing Steps (Play with your math - found in Teaching through...link below)
- Wolf, Sheep, and Cabbage (nrich - found in Teaching through Probs...link below)

Sources for these tasks and where to find more:

Week of Inspirational Math

Teaching Through Problems Worth Solving (Grade 8, Grade 2, and Grade 3)

Nrich.maths.org

https://playwithyourmath.com/

Link to this slide

Split 25

$$25 = 2 + 23$$

2×23 is 46

$$25 = 10 + 10 + 5$$

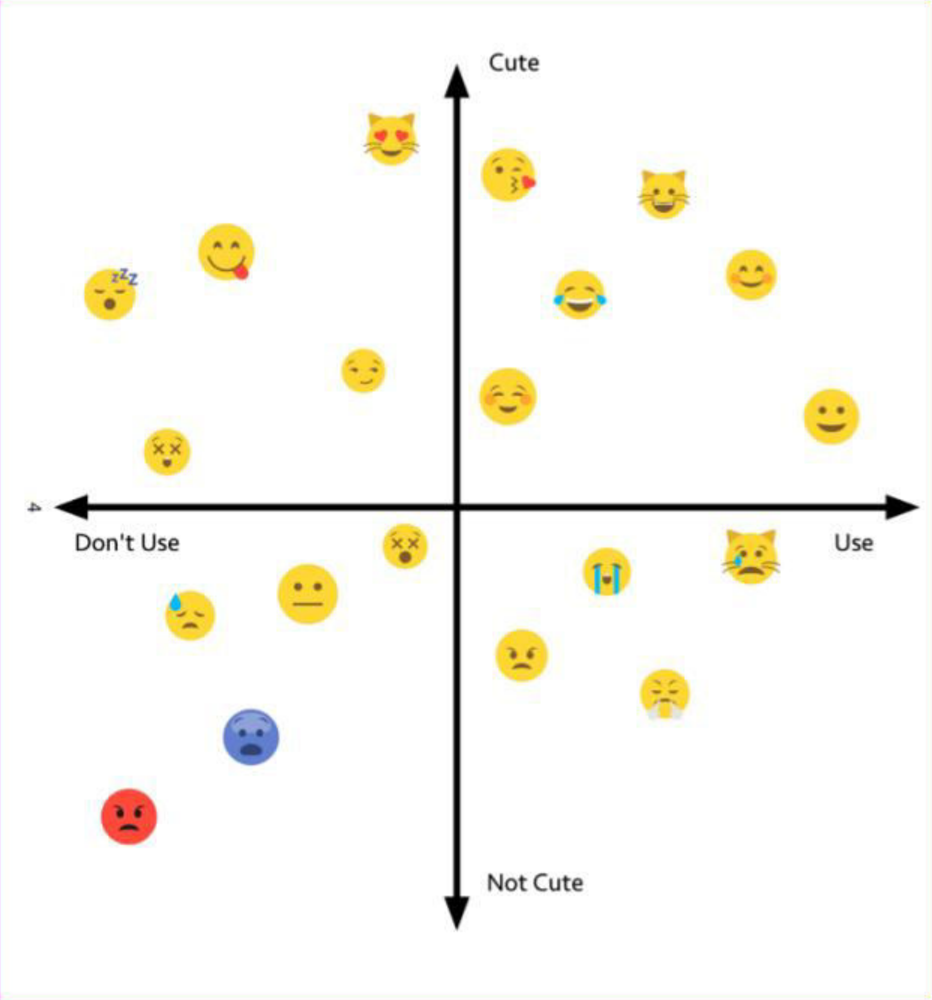
$10 \times 10 \times 5$ is 500

$$25 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$$

$$+ 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$$

$$+ 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$$

Source:
playwithyourmath.com



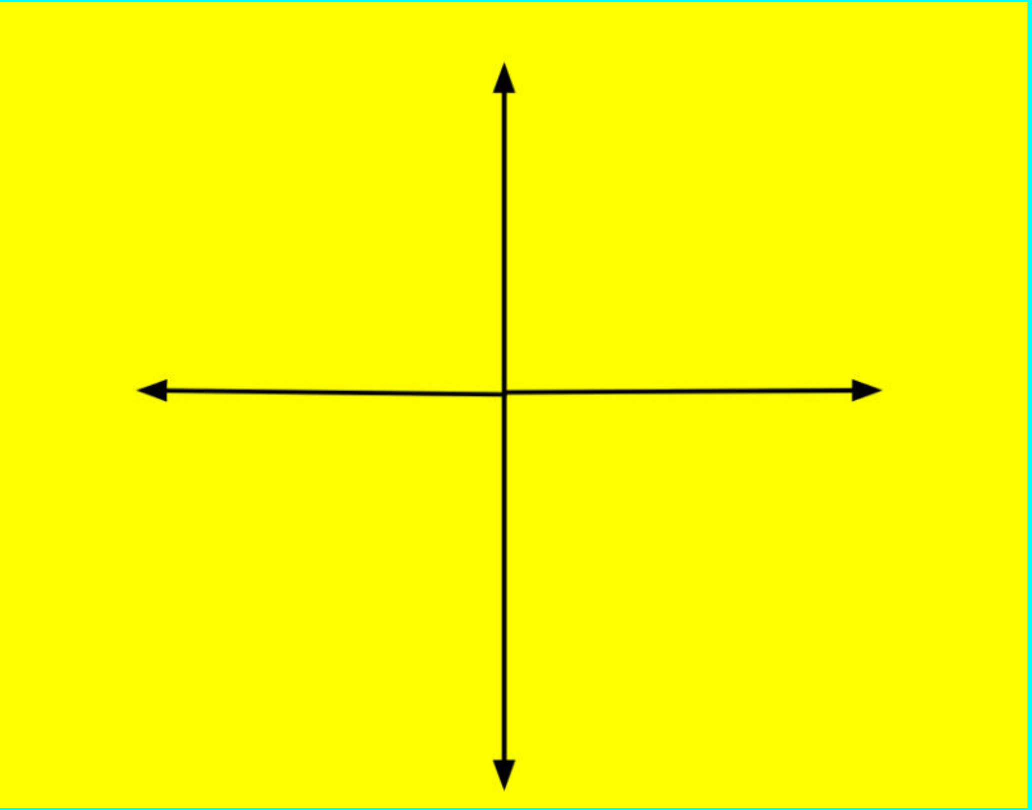
What do you notice?

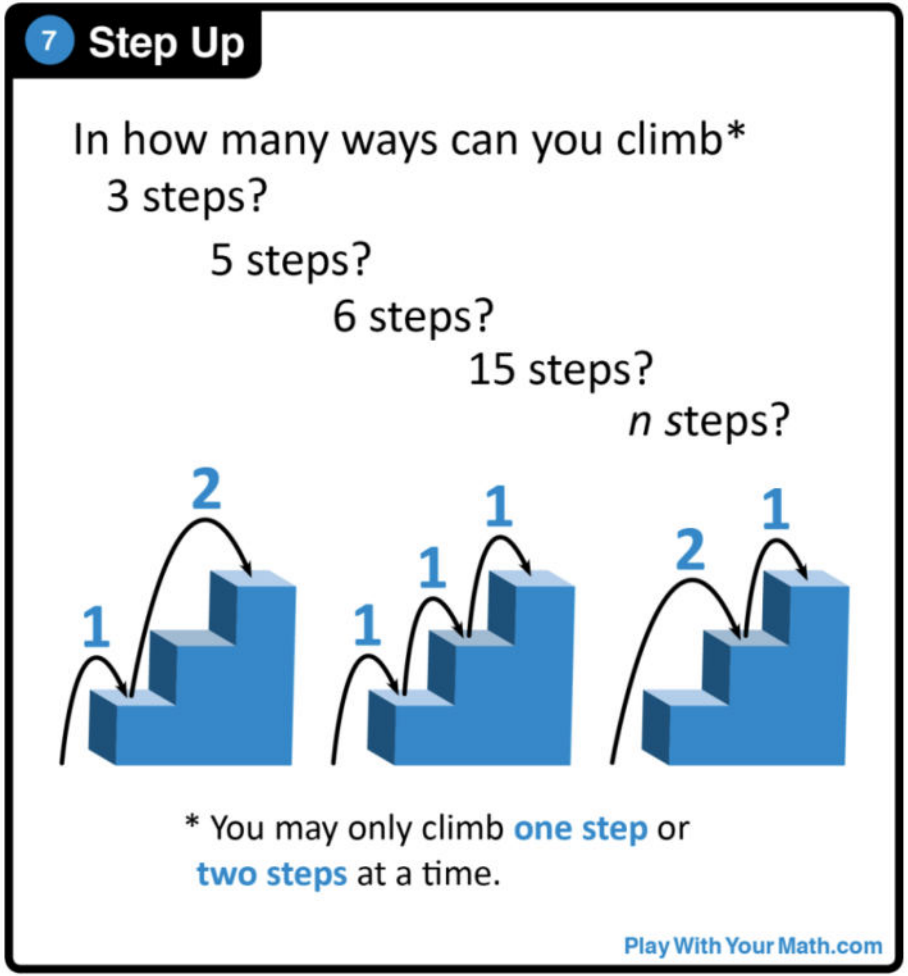
What do you wonder?

What questions do you have?

What information does this graph provide?

“Create a Graph”





*Credit to https://playwithyourmath.com/2017/07/27/7-step-up/

Climbing Steps

Extension:

What did you notice and why?

What if you also climb 3 steps at a time?

Credit also to Alicia Burdess (see link in notes)

Wolf, Sheep, and Cabbage

You need to move the wold, sheep, and cabbage to the opposite shore by rowing them over one at a time in a boat. It gets more difficult though because when you are not around, the wolf will eat the sheep, the sheep will also do the same when alone with the poor little cabbage. How do you do this?

Extension:
Is there another way to solve this problem?

5-4-3-2-1 Challenge

Use the digits 5, 4, 3, 2, and 1. The digits must remain in that order. Place the four arithmetic signs - plus, minus, times, and divided by - between the digits and as many parentheses as you like around the digits. How many numbers can you get from 1-40?

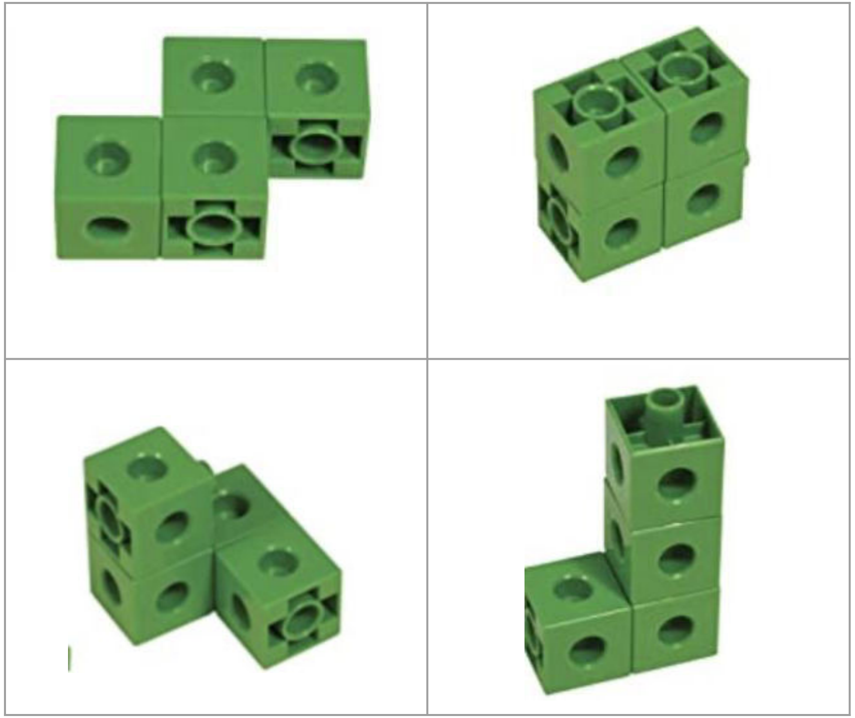
5 4 3 2 1

Credit: mathequalslove.net

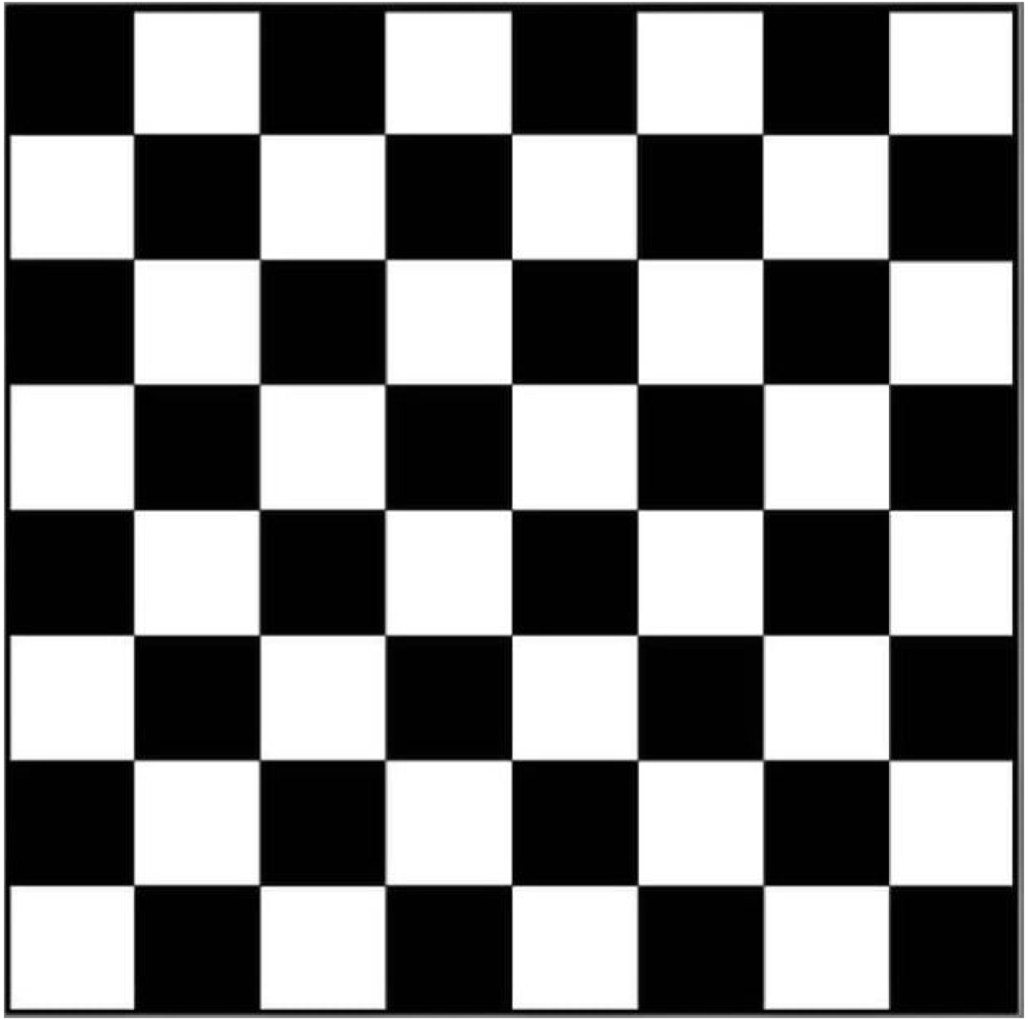

IN GOD WE TRUST
LIBERTY
1988 D

UNITED STATES OF AMERICA
E PLURIBUS UNUM
ONE DIME

UNITED STATES OF AMERICA
IN GOD WE TRUST
LIBERTY
QUARTER DOLLAR



Adapted from Peter Liljedahl's <u>*Building Thinking Classrooms*</u>, pg. 143 & maths300.com



106

11 62 2 76

4

65 29 93 79 54

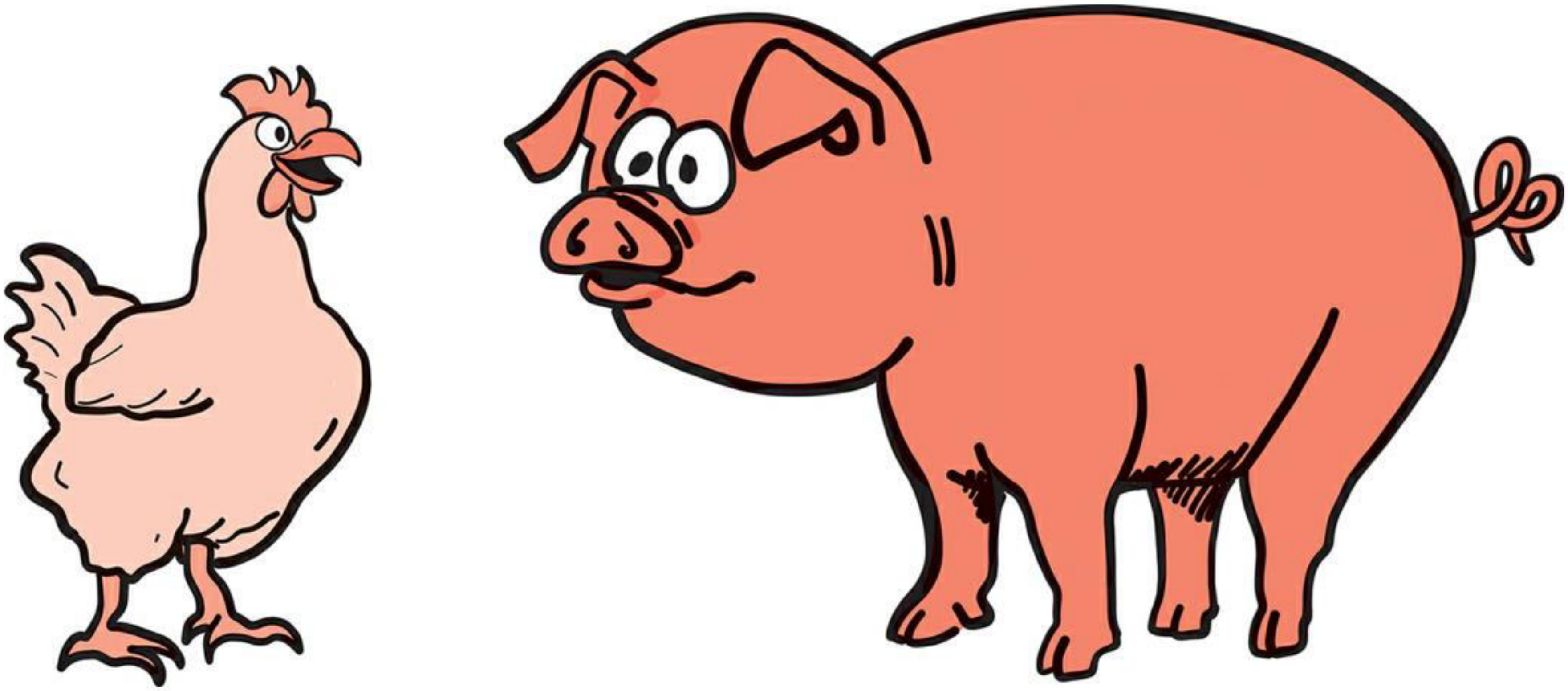
115

26 24 15 37

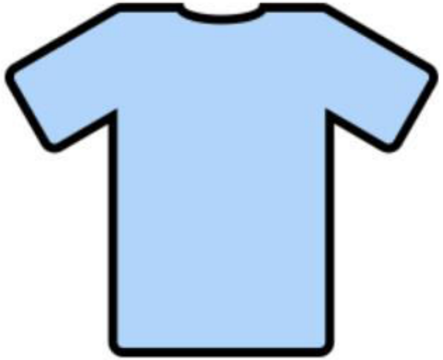
9

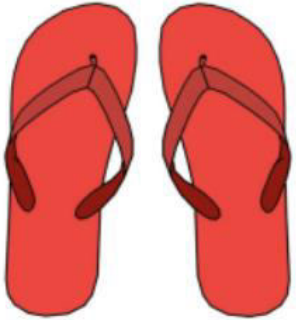
Credit to: [Play with your Math]

Farmer John



Wardrobe



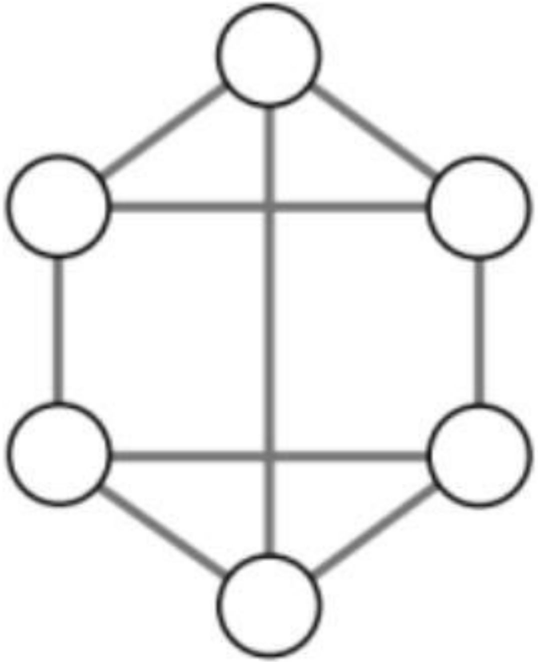


Cycling

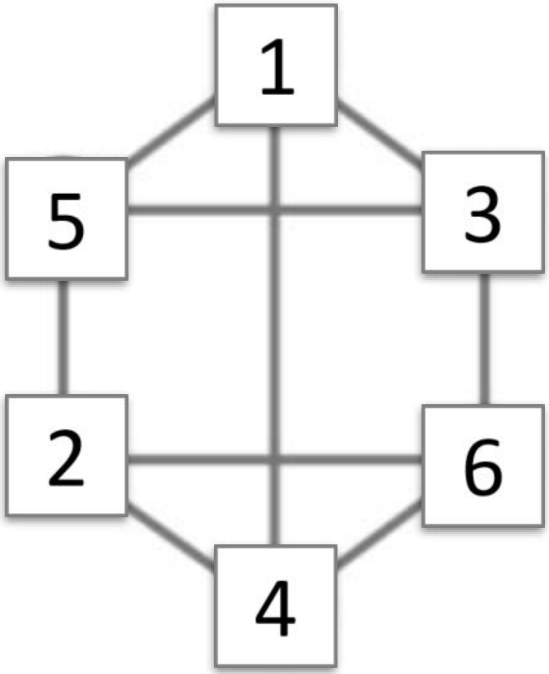


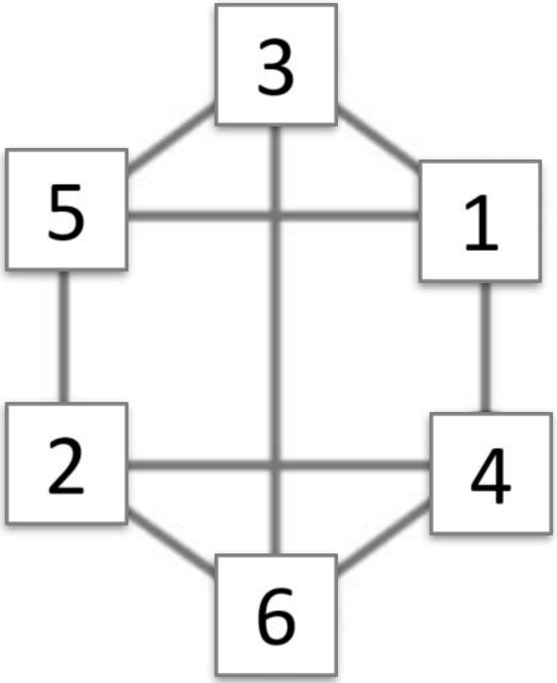
How Many 7’s

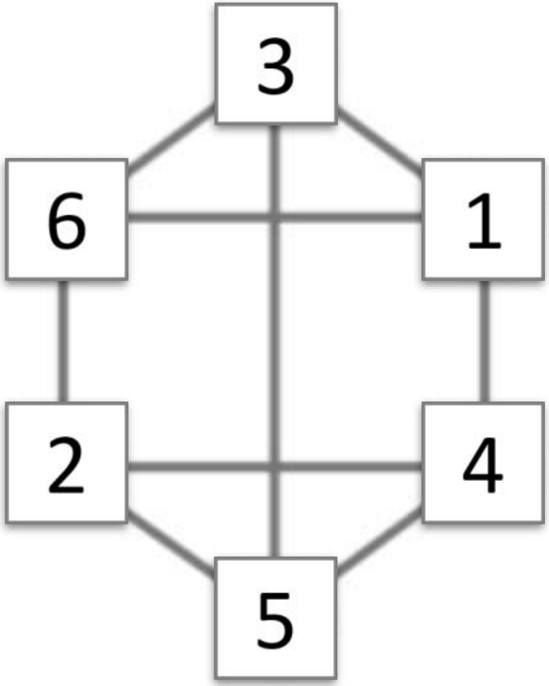
Neighbors

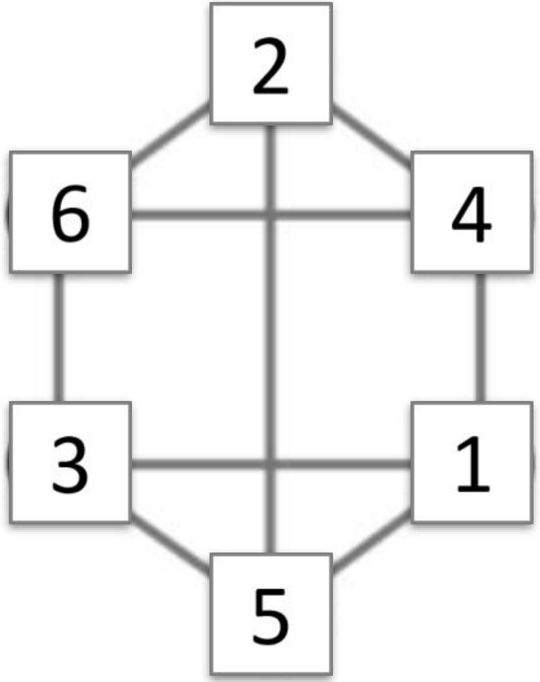


1 2 3 4 5 6


1
5
3
2
6
4


3
5
1
2
4
6


3
6
1
2
4
5


2
6
4
3
1
5

Country Road

Progression of Non-curricular Tasks (Gr. 2-3)

- Colored Shapes (Concrete)
- Waggies (Visual Representation)
- At the Fair (Abstract)

Sources for these tasks and where to find more:

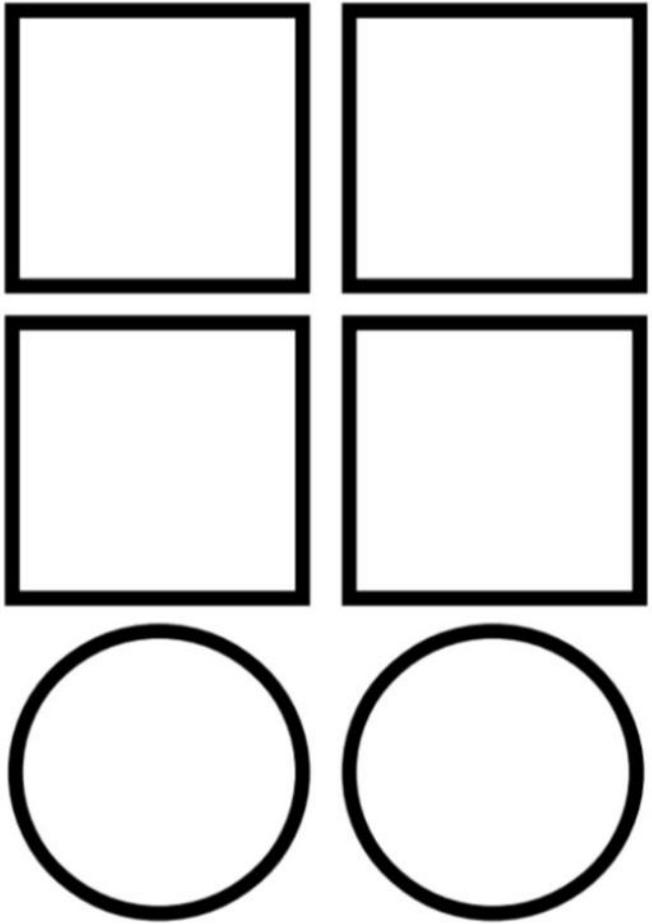
Week of Inspirational Math

Teaching Through Problems Worth Solving (Grade 8, Grade 2, and Grade 3)

Nrich.maths.org

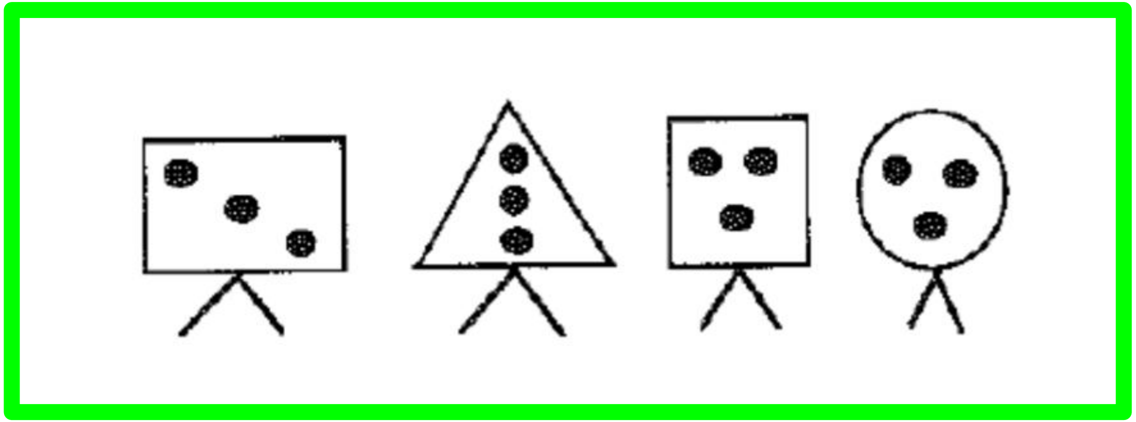
https://playwithyourmath.com/

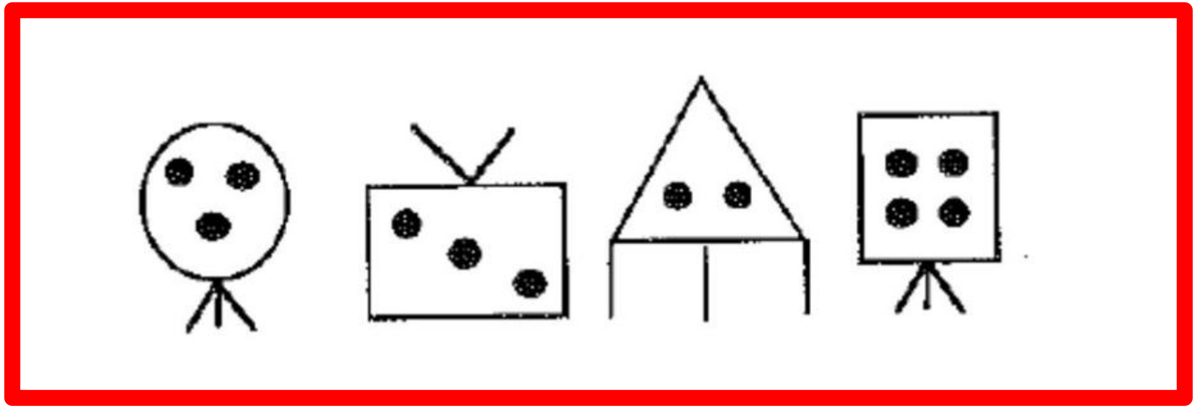
Link to this slide



Clues:

- Red is not next to grey.
- Blue is between white and grey.
- Green is not a square.
- Blue is on the right of pink.





Credit: Adapted from *Problem Solving Deck A Cards.pdf*. *Retrieved from http://maccss.ncdpi.wikispaces.net/file/view/Problem%20Solving%20Deck%20A.pdf/554108028/Problem%20Solving%20Deck%20A.pdf*

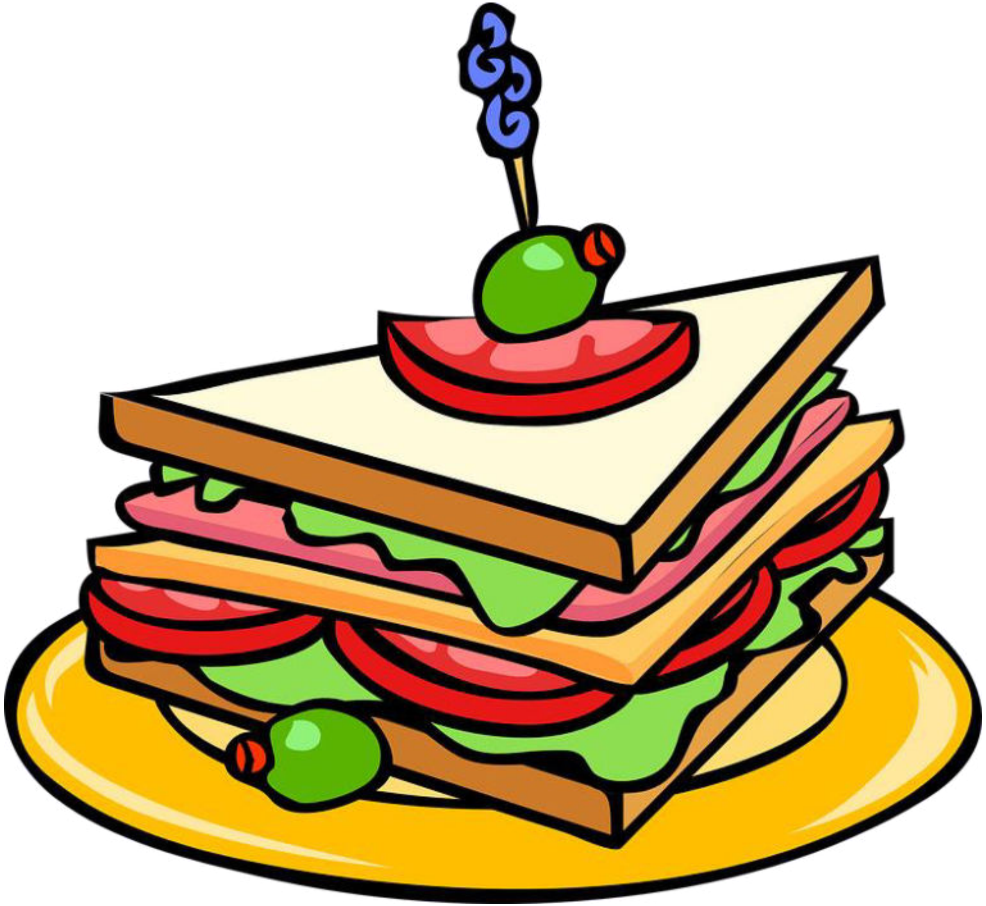
Credit: Aliciaburdess.com pg. 23

Rollercoaster - 6 minutes
Bumper Cars - 5 minutes
Tea Cups - 4 minutes

Ferris Wheel - 2 minutes
Swings - 1 minute
Merry-Go-Round - 3 minutes

Credit: Adapted from *Problem Solving Deck A Cards.pdf*. *Retrieved from http://maccss.ncdpi.wikispaces.net/file/view/Problem%20Solving%20Deck%20A.pdf/554108028/Problem%20Solving%20Deck%20A.pdf*

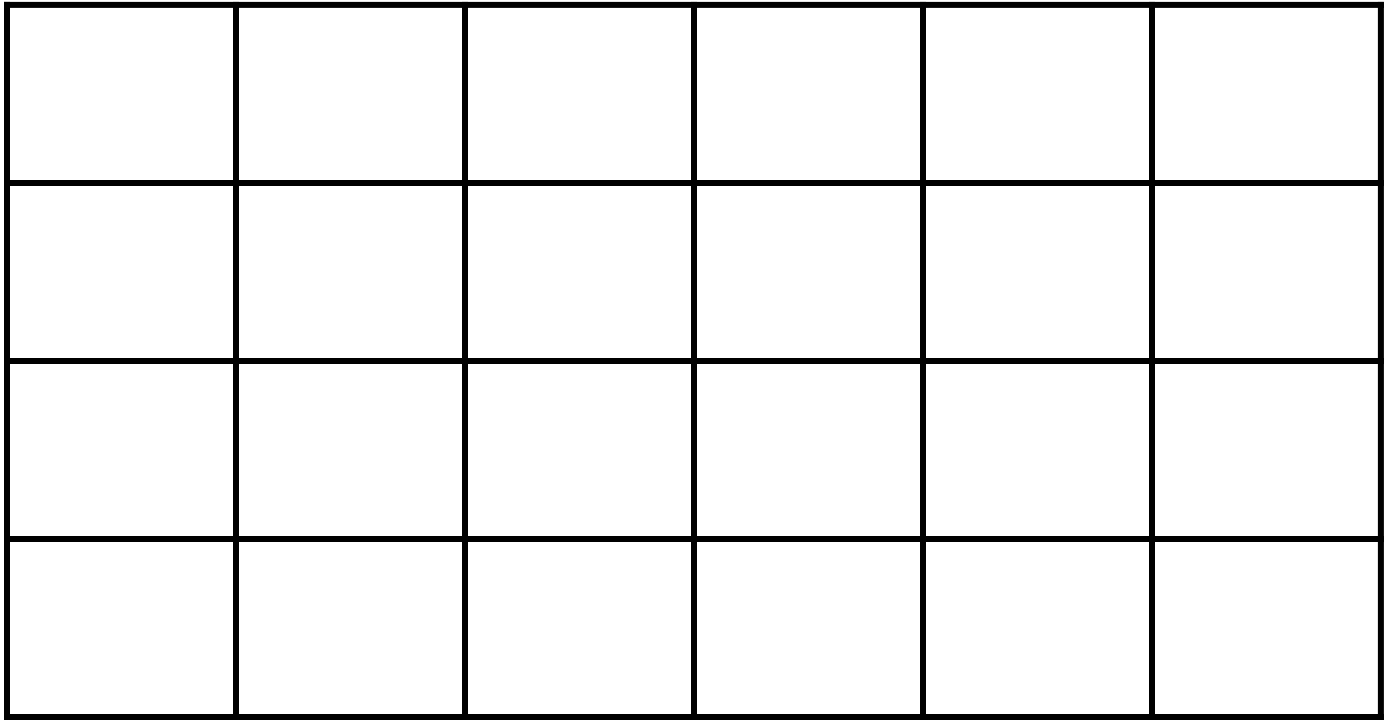
Credit: Aliciaburdess.com pg. 21



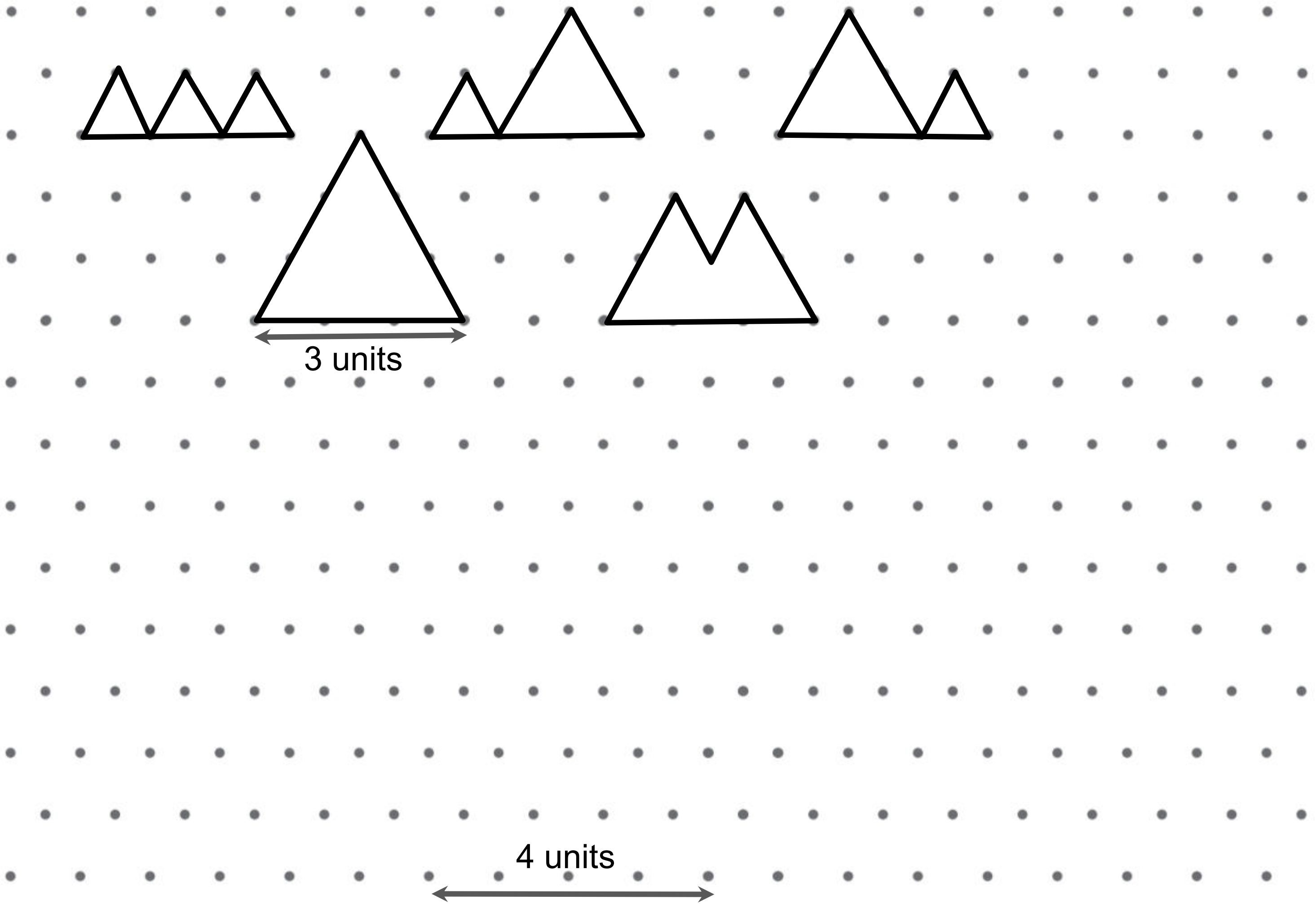
Adapted from the Hot Chocolate Task: Aliciaburdess.com pg. 16

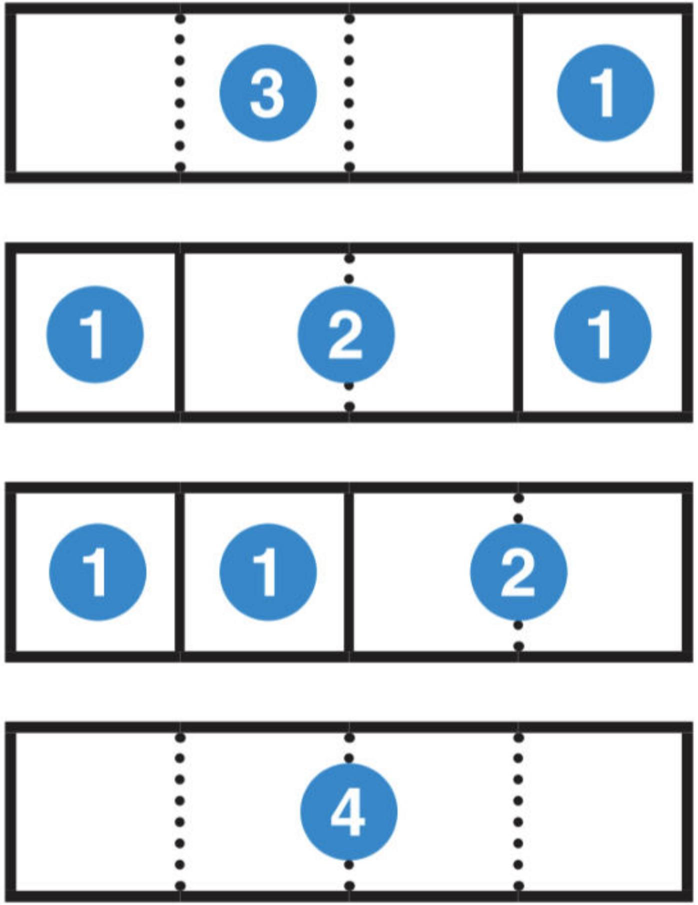
*Credit to John Mason – Thinking Mathematically

Milk Crate Extension



Credit to: http://www.primaryresources.co.uk/maths/problem.htm


3 units
4 units



Source: playwithyourmath.com

Progression of Non-curricular Tasks (Gr. 1 & 2)

- Snap Cubes (Concrete)
- Pentaminos (Concrete)
- Jellybeans - Candies (Visual Representation/Abstract)
- Next Door Numbers (Visual Representation)

Sources for these tasks and where to find more:

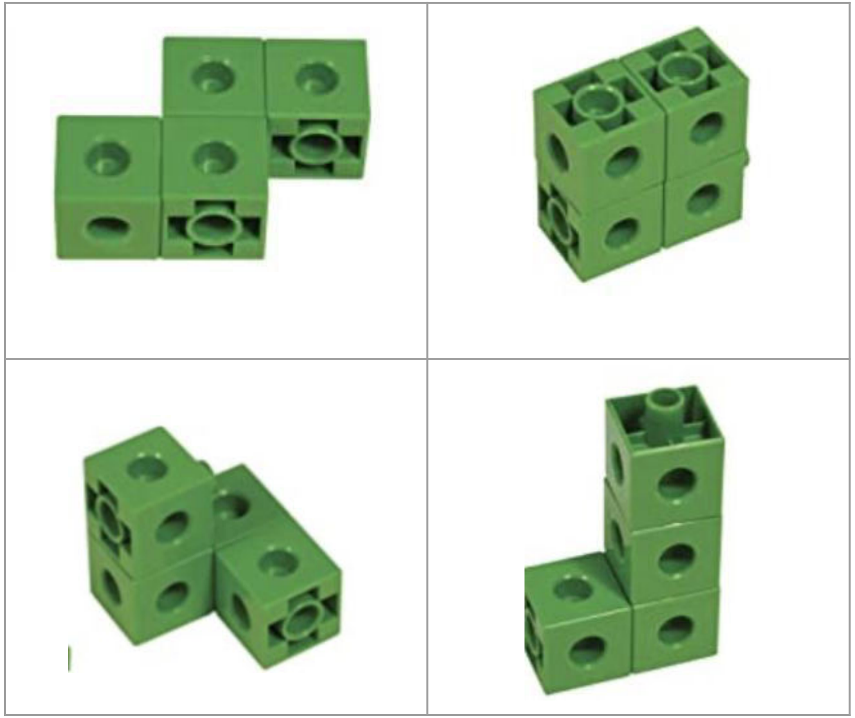
Week of Inspirational Math

Teaching Through Problems Worth Solving (Grade 8, Grade 2, and Grade 3)

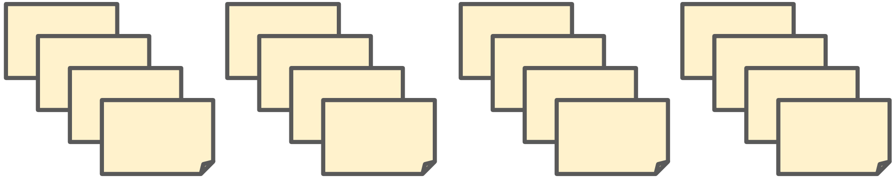
Nrich.maths.org

https://playwithyourmath.com/

Credit: Carrie Sutton pg. 52

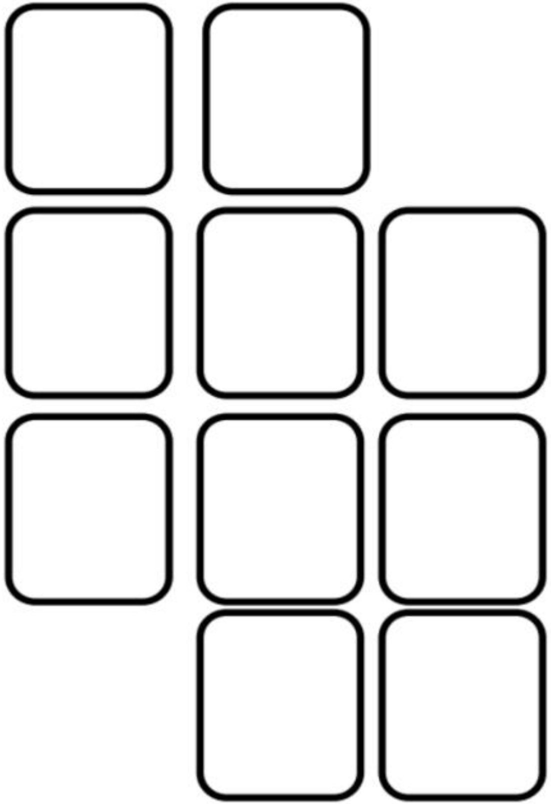


Credit: Peter Liljedahl *Building Thinking Classrooms*, pg. 143



You have 16 candies and 4 jars. Place the candies in the jars so that each jar has either 3 or 6 candies in them. Are there some things that are not possible?

Credit: Peter Liljedahl *Building Thinking Classrooms*, pg. 80



Credit: Adapted from Crown copyright, 2000, *Mathematical Problems for Able Pupils.pdf* retrieved from https://www.egfl.org.uk/sites/default/files/maths%20puzzles%20all.pdf

My name________________________________
I have ____ counters.
I have _____ (more than/less than/ the same as) my partner.
My partner has _____ counters.

My name________________________________
I have ____ counters.
I have _____ (more than/less than/ the same as) my partner.
My partner has _____ counters.

The Answers Are

1	2	3	4	5	6	7	8	9	10

+ + − −

Progression of Non-curricular Tasks (Kinder+)

- Counting Collections (Concrete)
- Count on Me (Concrete)
- Estimating Dots (Concrete/Visual)

Sources for these tasks and where to find more:

Week of Inspirational Math

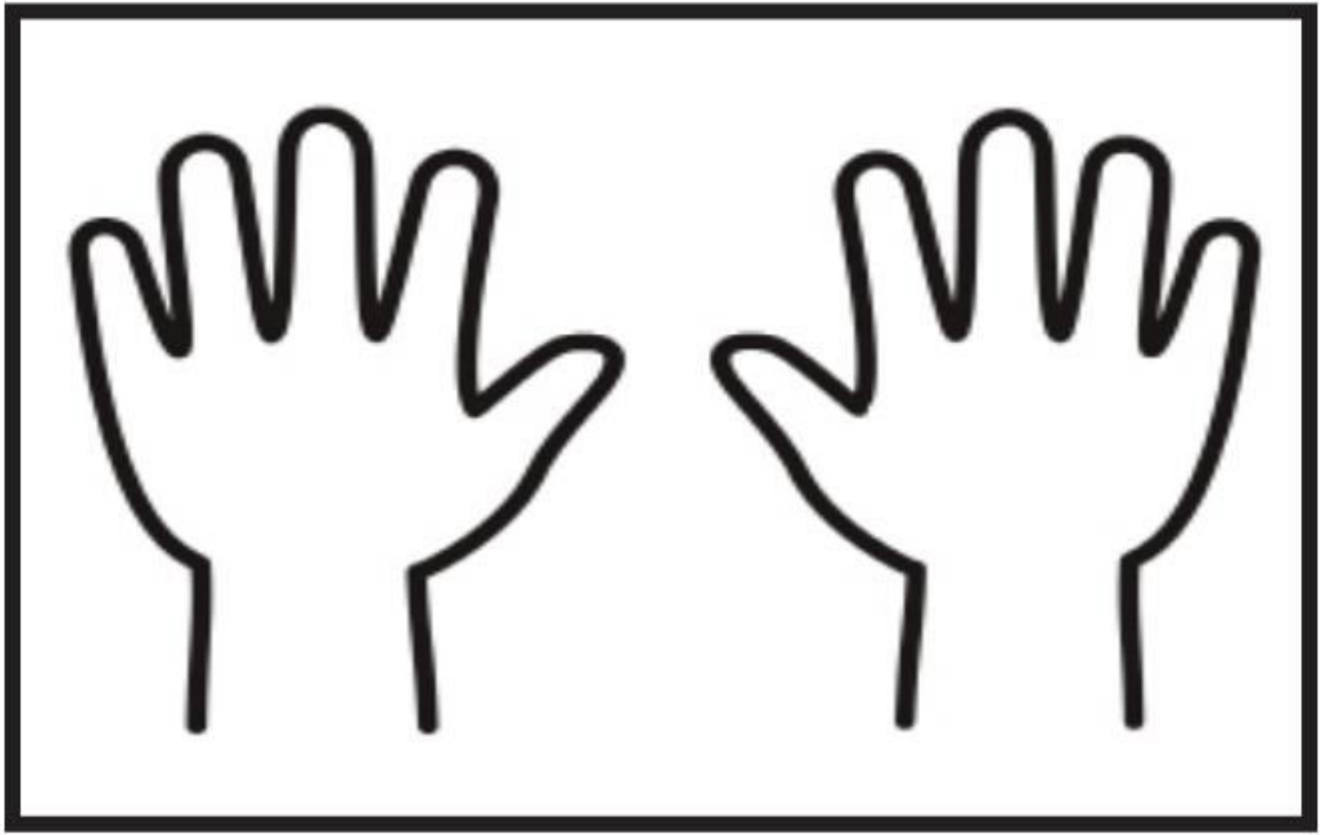
Teaching Through Problems Worth Solving (Grade 8, Grade 2, and Grade 3)

Nrich.maths.org

https://playwithyourmath.com/

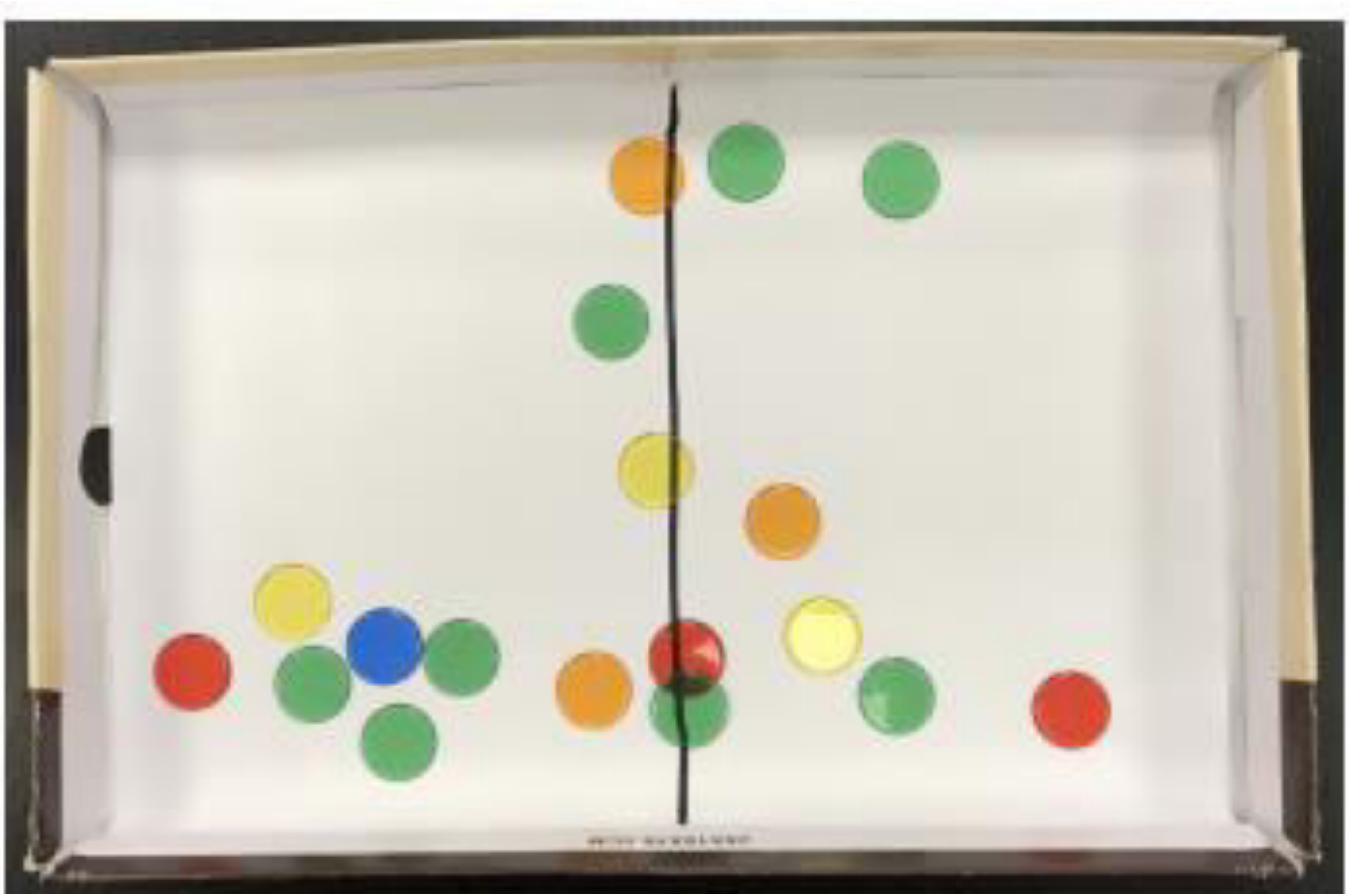
Link back to this page

Credit: nrichsmaths



Credit: youcubed.org

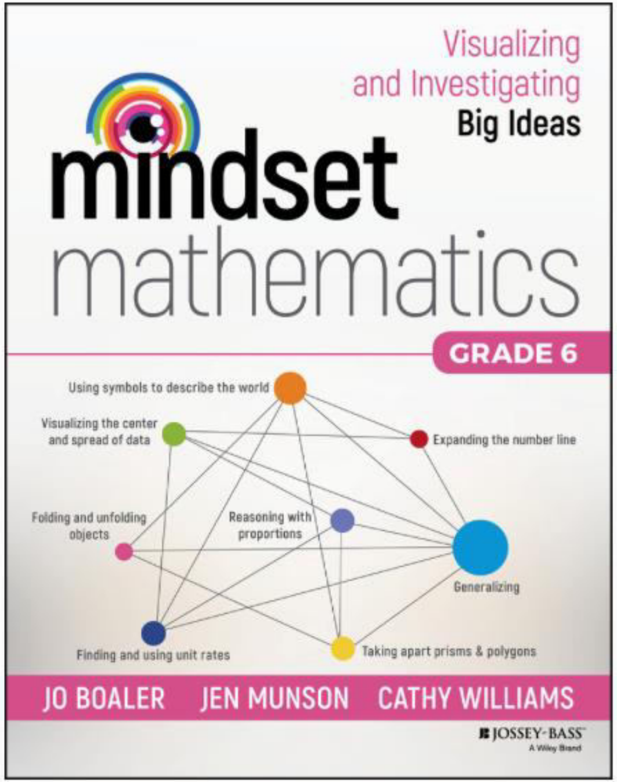
Credit: youcubed.org

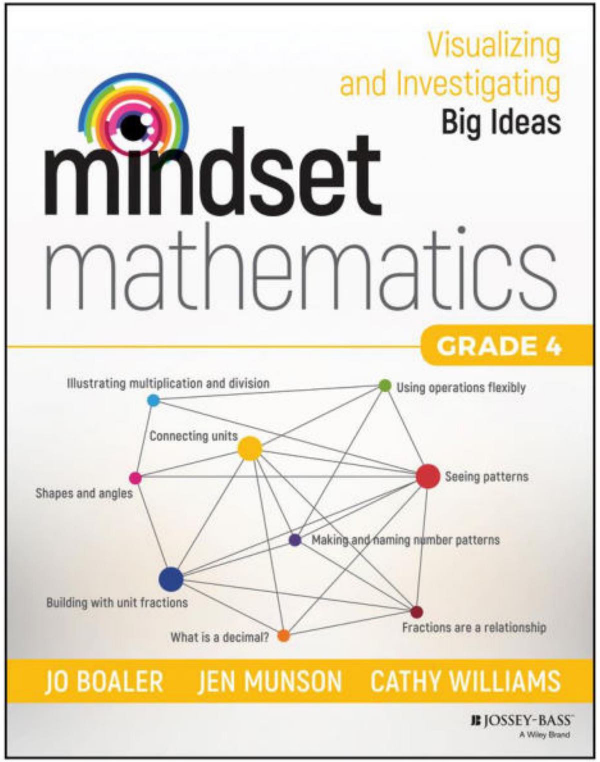


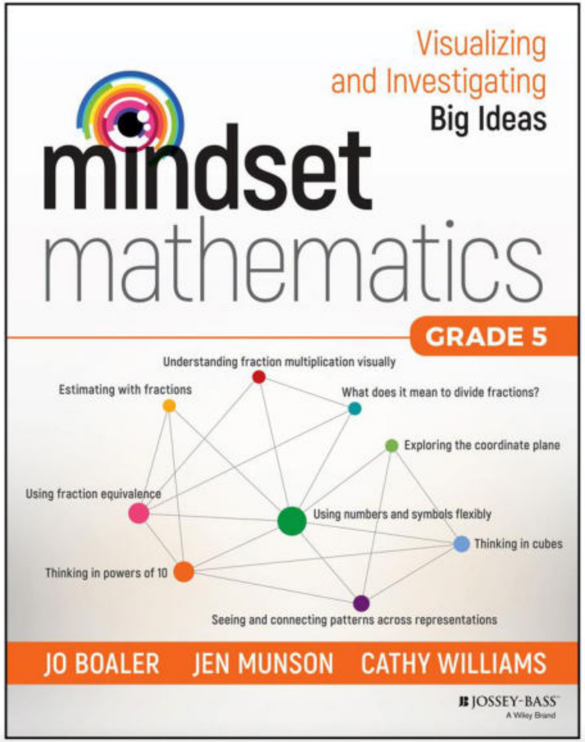
Curricular Tasks

PrBL: Emergent Math (by grade level)
Collection by grade level (Canada Standards)

Big Ideas Tasks (Connections across standards)







Open Middle: each underline represents a hyperlink

4th Grade	5th Grade	6th Grade	7th Grade	8th Grade
Interactive Decks	Interactive Decks	Interactive Decks	Interactive Decks Interactive Decks	Interactive Decks Interactive Decks
website w/solutions	website w/solutions	website w/solutions	website w/solutions	website w/solutions
Notice&Wonder	Notice&Wonder	Notice&Wonder	Notice&Wonder	Notice&Wonder

Open Middle: each underline represents a hyperlink

Kinder	1st Grade	2nd Grade	3rd Grade
Interactive Decks	Interactive Decks	Interactive Decks	Interactive Decks
website w/solutions	website w/solutions	website w/solutions	website w/solutions
Notice&Wonder	Notice&Wonder	Notice&Wonder	Notice&Wonder

Curricular Tasks (1st Grade)

$$\boxed{} = 6+3+2$$

Curricular Tasks (4th Grade)

2.49

3.52

2.23

4.31

95 cents

1.55

5.73

62 cents

2.07

2.15

99 cents

1.09

Curricular Tasks (5th Grade)

2.49

3.52

2.23

4.31

95 cents

1.55

5.73

62 cents

2.07

2.15

99 cents

1.09

3 Act Math

Robert Kaplinsky' (K-12)	Kendra Lomax (K-3)	Dan Meyer's (6-12)
Mike Wiernicki (K-8)	Kristen Acosta (K-5)	Andrew Stadel (6-12)
Kyle Pearce (3-12)	Catherine Castillo(K-5)	Dane Ehlert (6-12)
Geoff Krall (3-12)		Jon Orr (6-12)

Downloaded resources by educators in Brick, New Jersey Full slide decks for Grades K-8

Using Modes of Engagement to Manage Flow

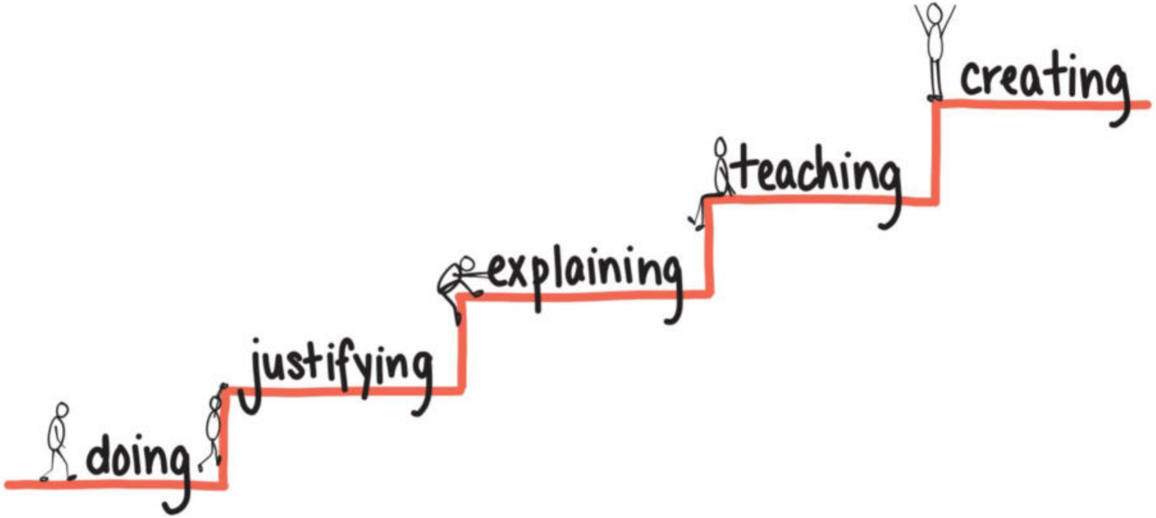


Figure 9.9 Modes of engagement that increase challenge.

Building Thinking Classrooms, **Peter Liljedahl p. 159**

3-Act Task (Kindergarten K.OA.1,2,3)

Humpty Dumpty

Credit to Graham
Fletcher, gfletchy.com

Too low

Too high

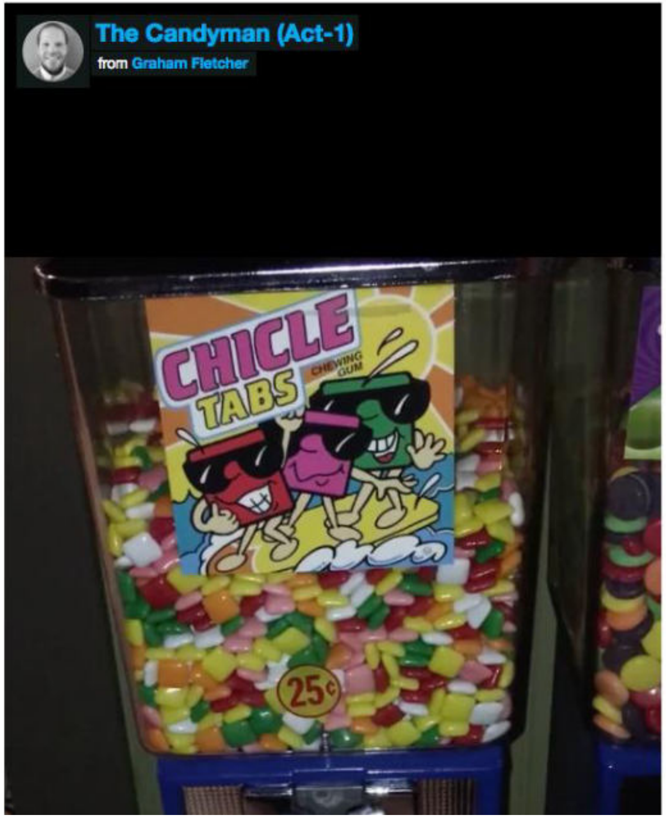
There were 9 eggs in the carton to begin.

3-Act Task (Kindergarten K.CC.A,B)

Candy Man

Desmos Version: Link

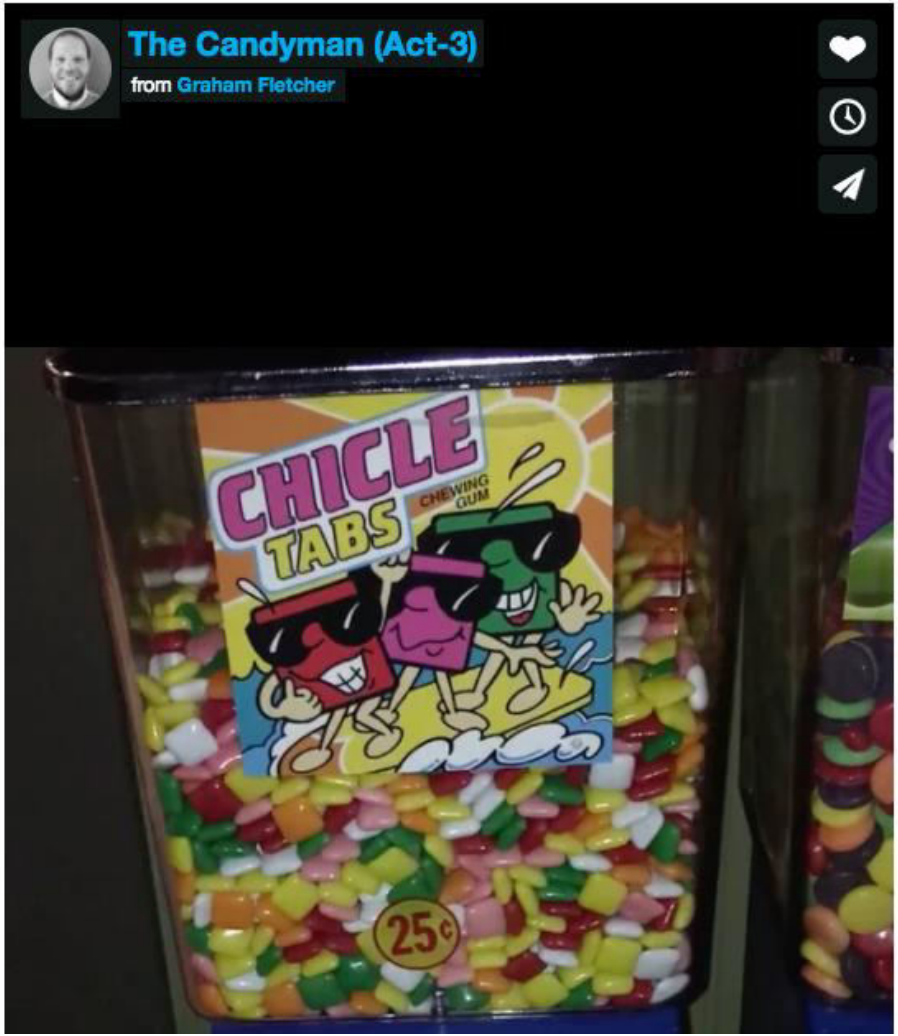
Credit to Graham Fletcher, gfletchy.com



Too low

Too high


4-orange
2-red
2-yellow
2-white
0-pink


The Candyman (Act-3)
from Graham Fletcher
CHICLE
TABS
CHEWING GUM
25¢

3-Act Task (Kindergarten K.MD.3, 1.OA.6)

M & M Spill

Credit to Kristen Acosta

Too low

Too high

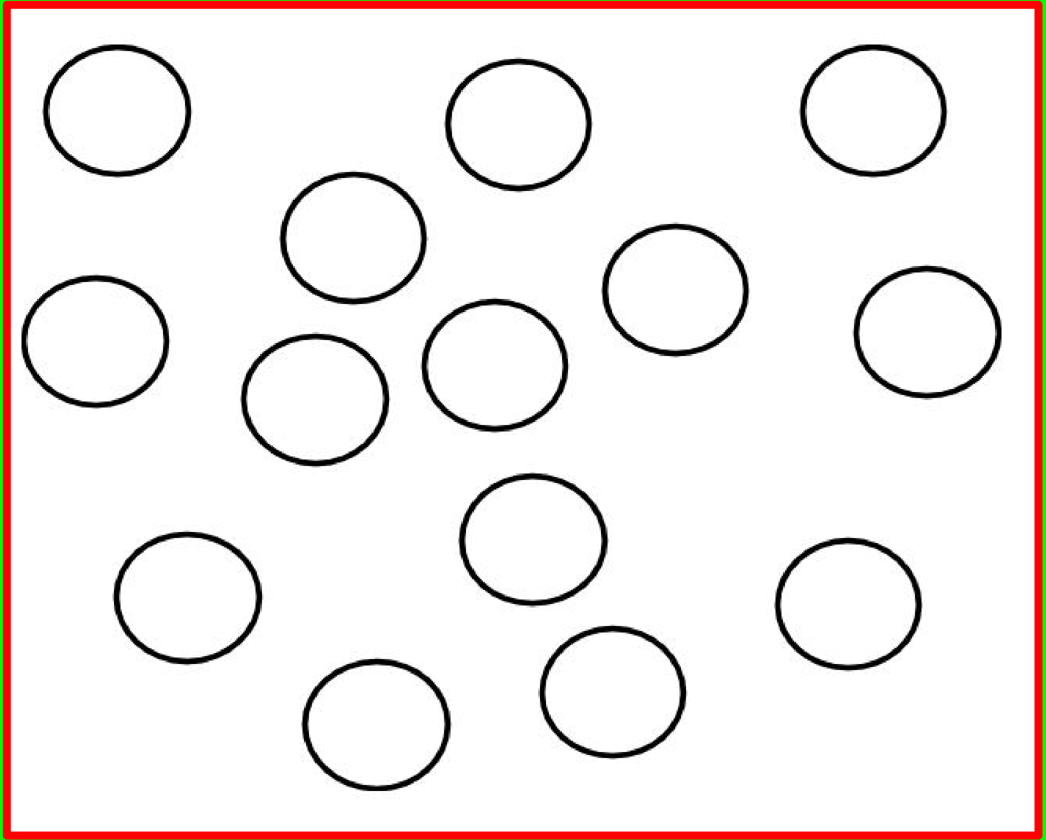
There was....

- 6 **orange** candies
- 4 **brown** candies
- 3 **yellow** candies
- 1 **blue** candy

Modified Activity

Color what we know

6 orange
3 yellow
4 brown
1 blue



14 = 6 orange + 4 brown + 3 yellow + 1 blue

Extension

Extension

Extension

3-Act Task (1st Grade 1.NBT.1)

Counting Squares

Credit to Graham Fletcher, gfletchy.com

Too low

Too high


00:00
vimeo


Counting Squares (Act 3)
from Graham Fletcher
01:08
vimeo

Extension Challenge:

3-Act Task (1st Grade 1.OA.5, 1.NBT.2)

Fun Size

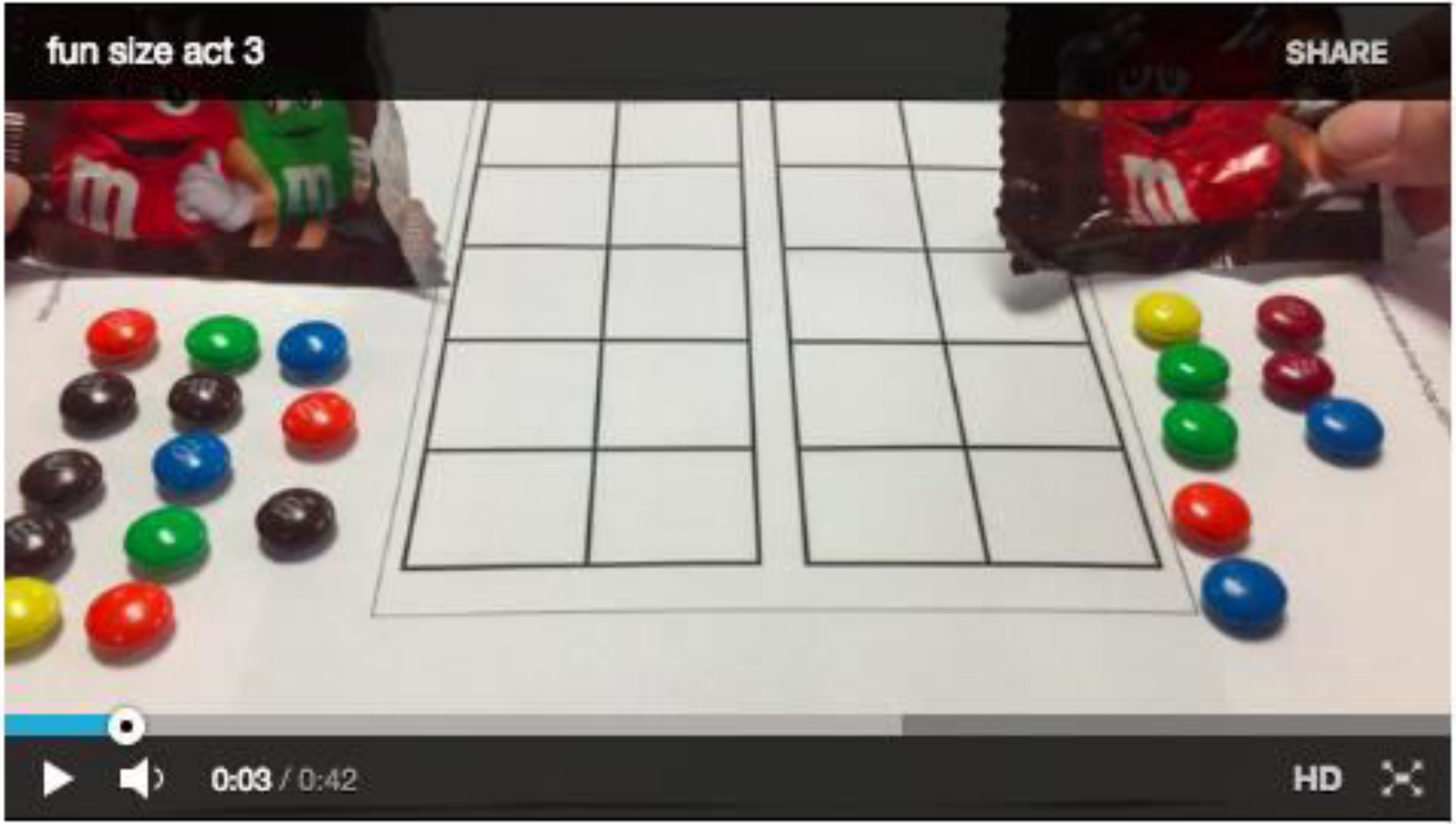
Credit to Kristen Acosta

Too low

Too high

1^{ST} BAG

2^{ND} BAG


fun size act 3
SHARE
0:03 / 0:42
HD

3-Act Task (1st Grade 1.G.3, 3.NF.1)

Sliced Up

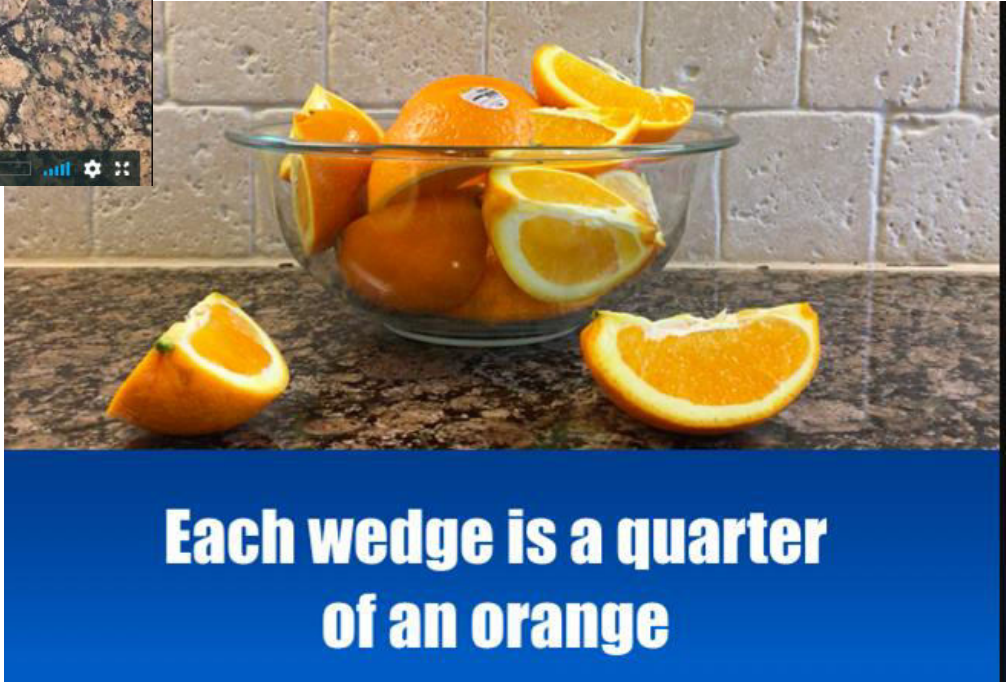
Desmos Version: Link

Credit to Graham Fletcher, gfletchy.com

Too low

Too high


00:14
00:05


Each wedge is a quarter
of an orange

Each wedge is a quarter of an orange


Sliced Up (Act-3)
from Graham Fletcher
00:23
vimeo

3-Act Task (1st Grade 1.NBT.1,4)

Pringle Ringle

Desmos Version: Link

Credit to Graham Fletcher, gfletchy.com

Too low

Too high

78 Pringles
came in the can
Pringles
Todays special
SALT & VINEGAR
Flavored
Crujientes de papa

14 Pringles left
in the can
Pringles
Todays special
SALT & VINEGAR
Flavored
Crujientes de papa

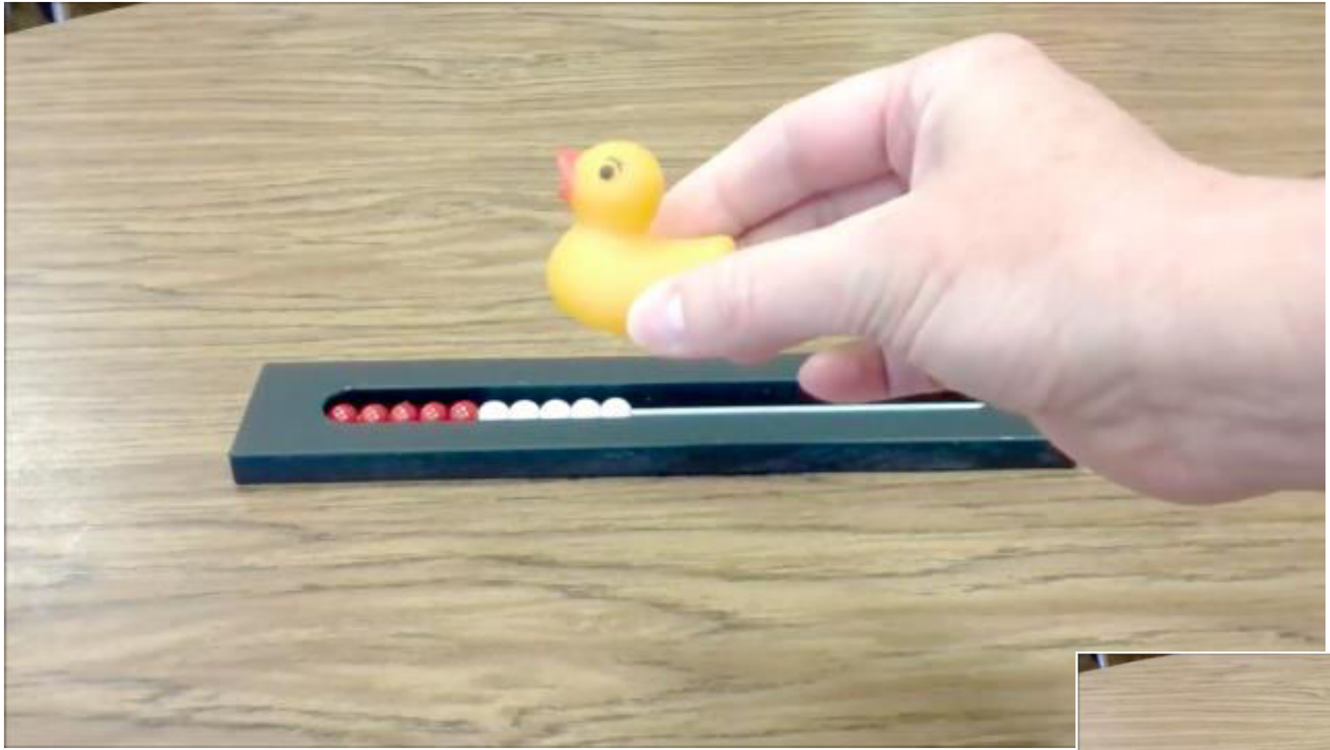

the Pringle Ringle (Act-3)
from Graham Fletcher
Pringles
SALT & VINEGAR
00:45
HD
vimeo

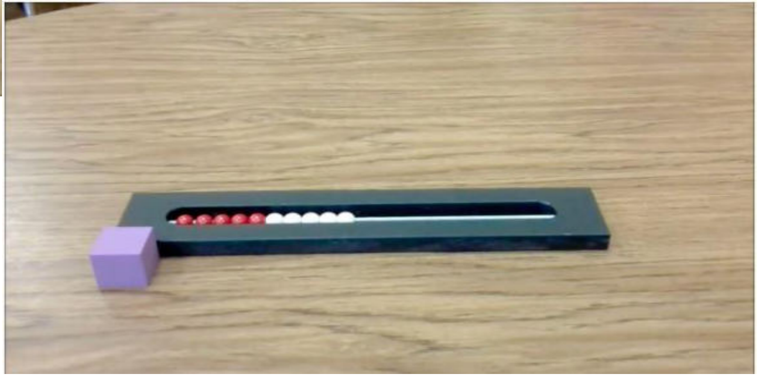
3-Act Task (2nd Grade 2.MD.2)

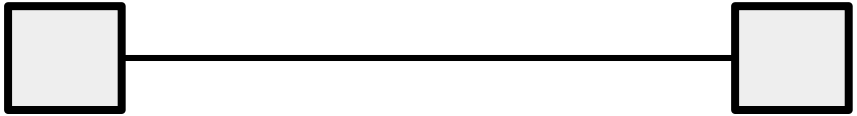
All the Little Duckies

Credit to Catherine Castillo

Desmos Version: Link



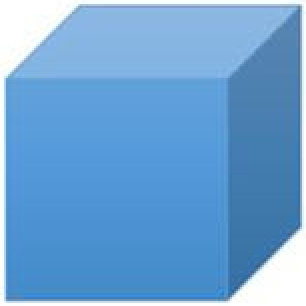


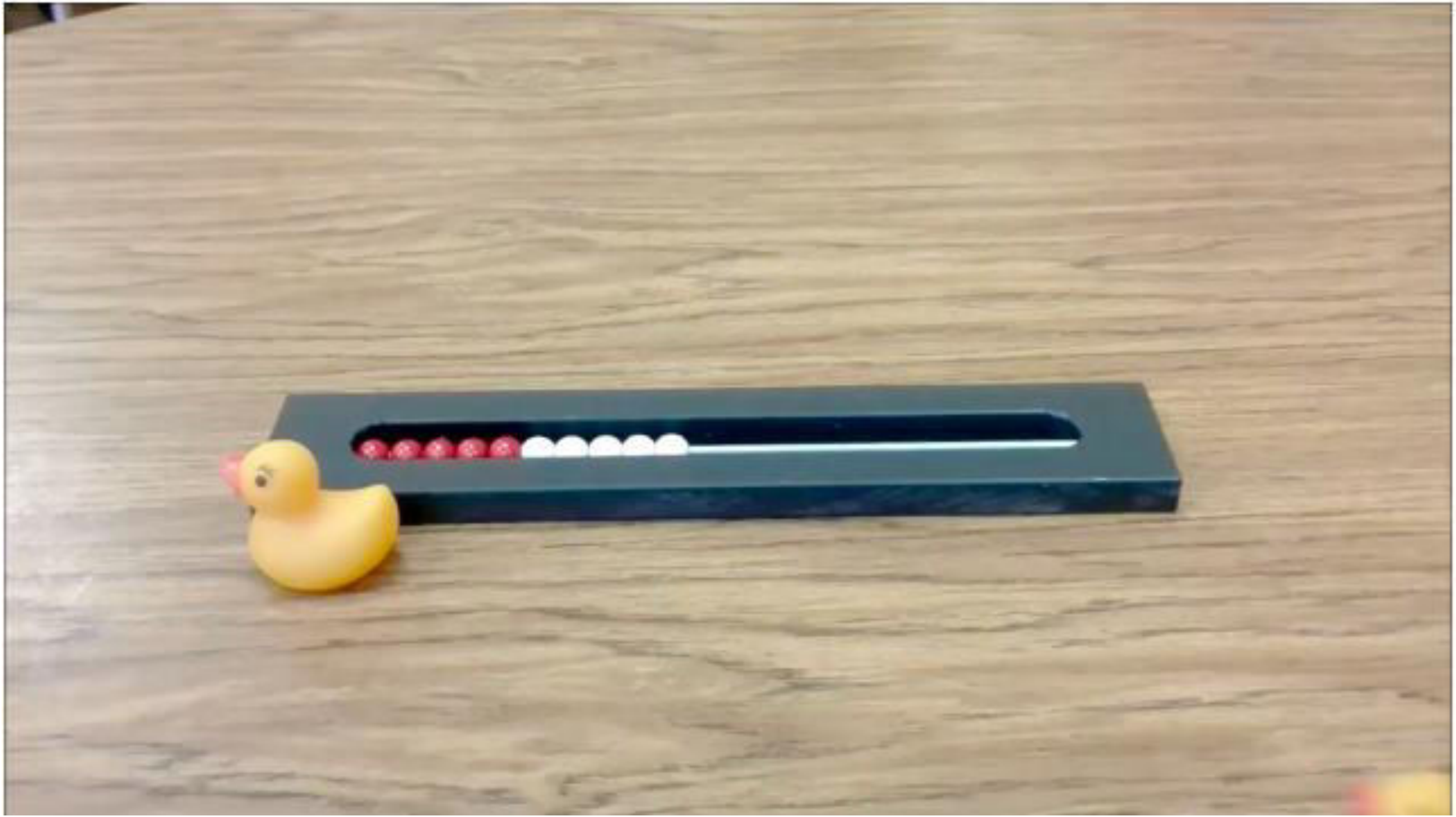


Too low

Too high

- If you measured 2 math racks, you would need 16 objects (ducks and cubes).
- You would use 4 more cubes than ducks.



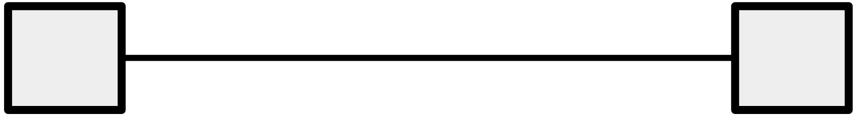


3-Act Task (2nd Grade 2.NBT.6)

Don't Spill the Beans

Credit to Catherine Castillo

Desmos Version: Link



Too low

Too high


20
Pink


20 + 20 + 20 + 20 + 20 = 100
20, 40, 60, 80, 100

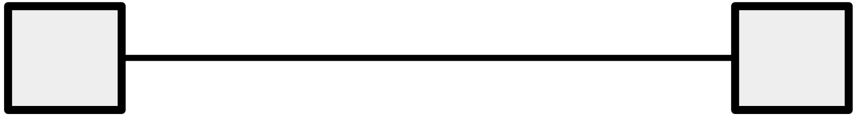
Extension:

3-Act Task (2nd Grade 2.NBT.6 & 2.MD.8)

Snack Machine

Desmos Version: Link

Credit to Graham Fletcher, gfletchy.com



Too low Too high


AMOUNT DEPOSITED
0.00
MAKE ANOTHER SELECTION
USE EXACT CHANGE
00:14


60 ¢

Extension:

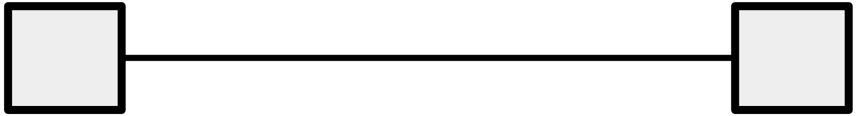
CASCADE
STYLE
JALAPEÑO
NY TWISTS
60¢
B3
60¢
B5
60¢
B7
60¢
B9
75¢
Grandma's
Grandma's
Grandma's
VANILLA
Famous Amos
Knott's
RASPBERRY
75¢
C3
75¢
C5
75¢
C9
75¢

3-Act Task (3rd Grade 3.OA.3)

Fruit and Nut

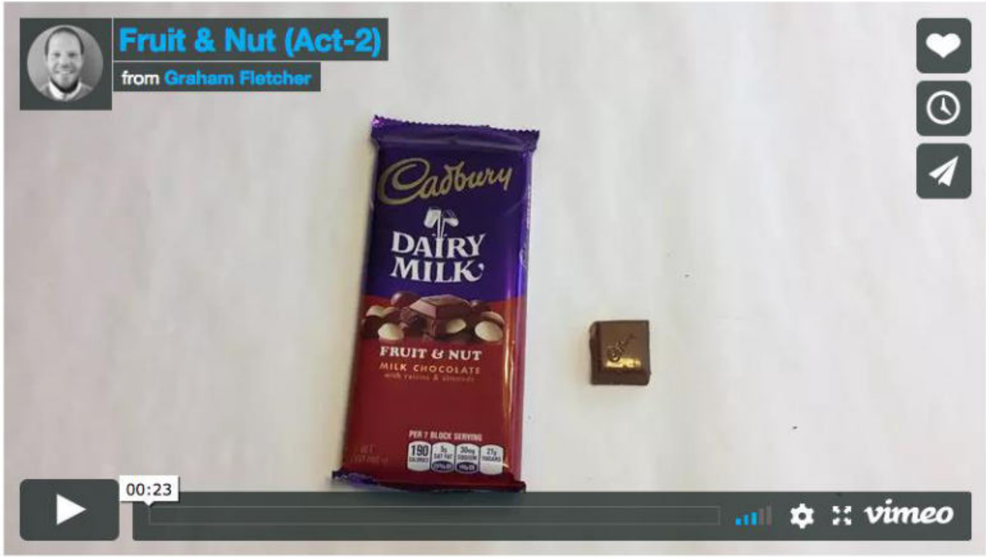
Desmos Version: Link

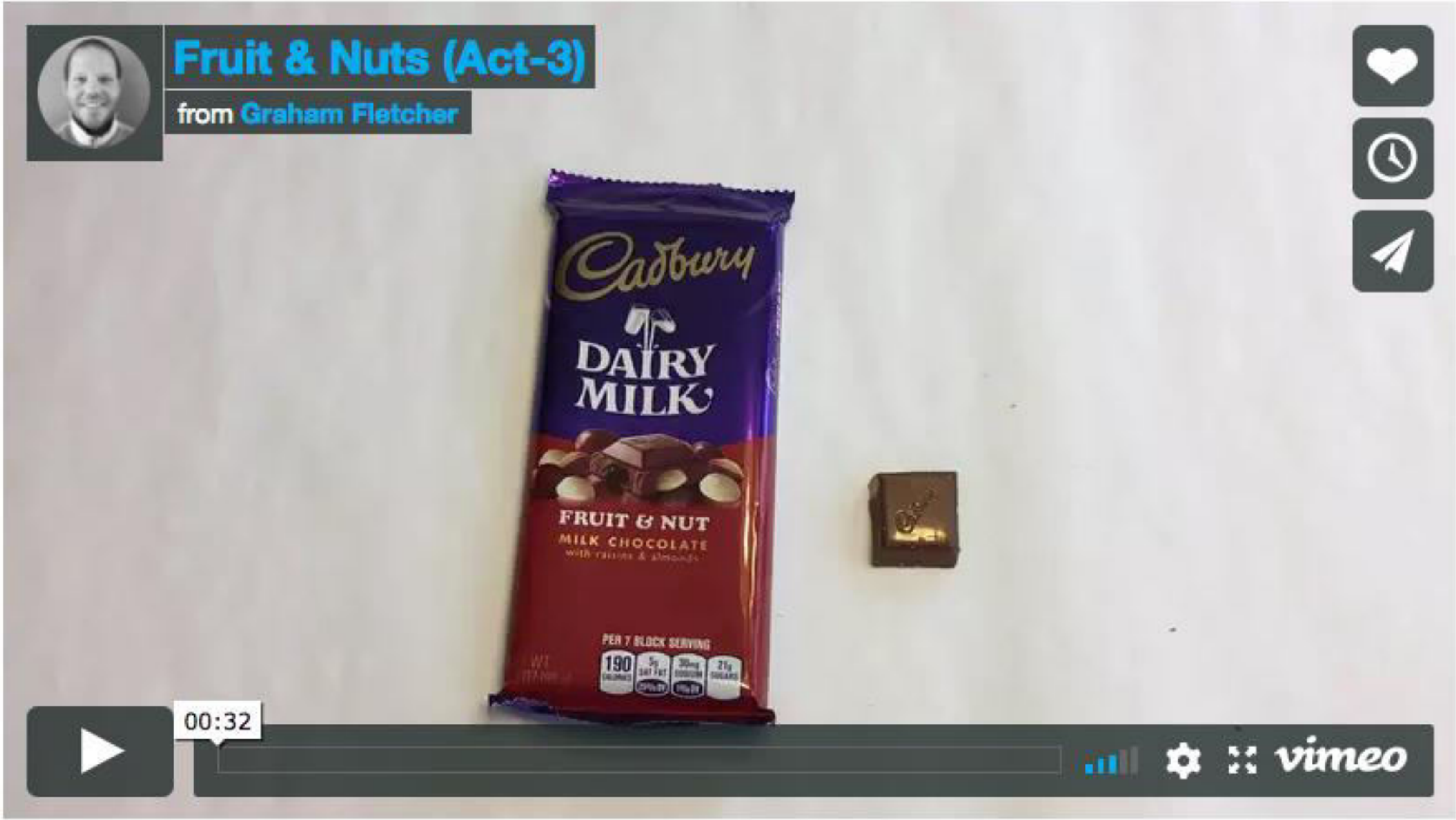
Credit to Graham Fletcher, gfletchy.com



Too low

Too high

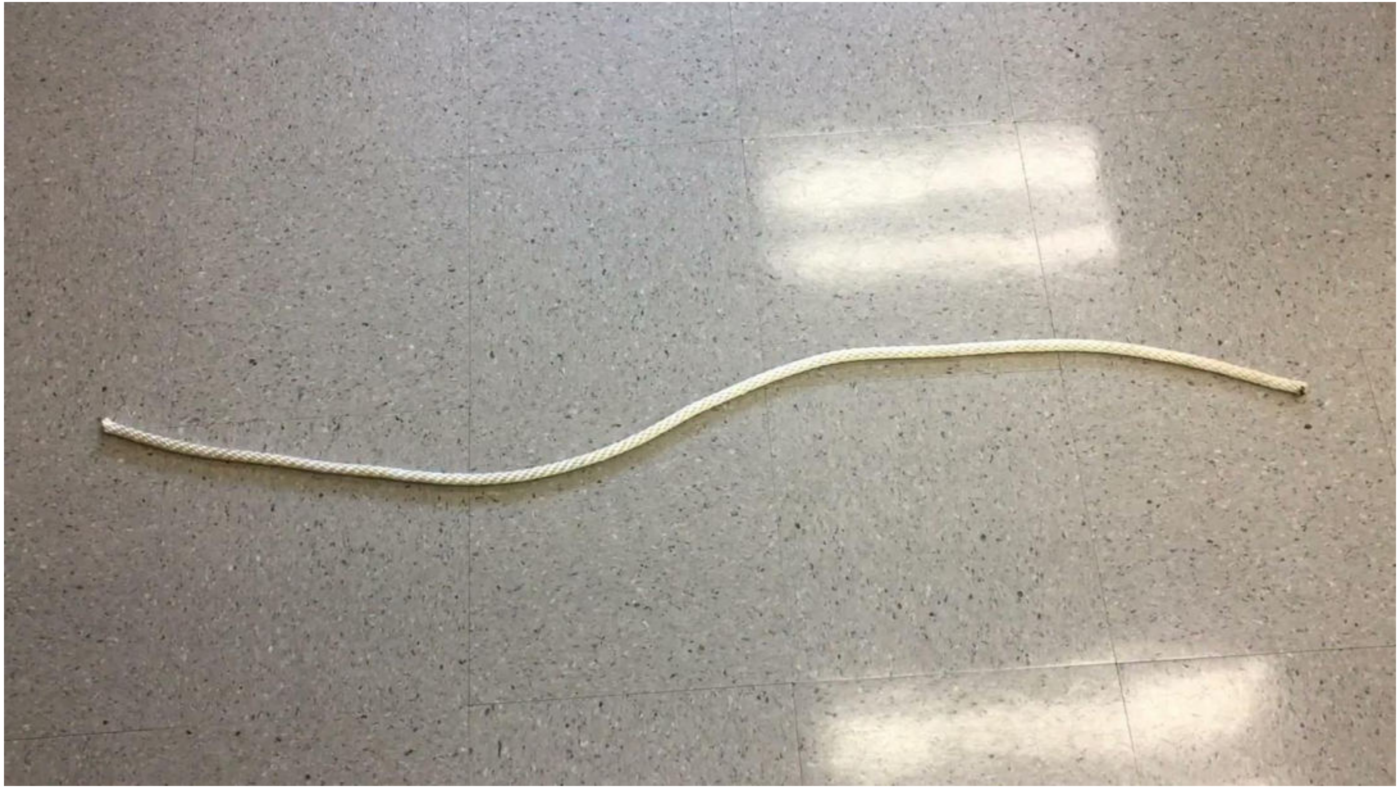

Fruit & Nut (Act-2)
from Graham Fletcher
Cadbury
DAIRY MILK
FRUIT & NUT
MILK CHOCOLATE
00:23
vimeo


Fruit & Nuts (Act-3)
from Graham Fletcher
Cadbury
DAIRY MILK
FRUIT & NUT
MILK CHOCOLATE
PER 7 BLOCK SERVING
190
00:32
vimeo

3-Act Task (3rd Grade 3.OA.7,8)

Knotty Rope

Credit to Graham Fletcher, gfletchy.com

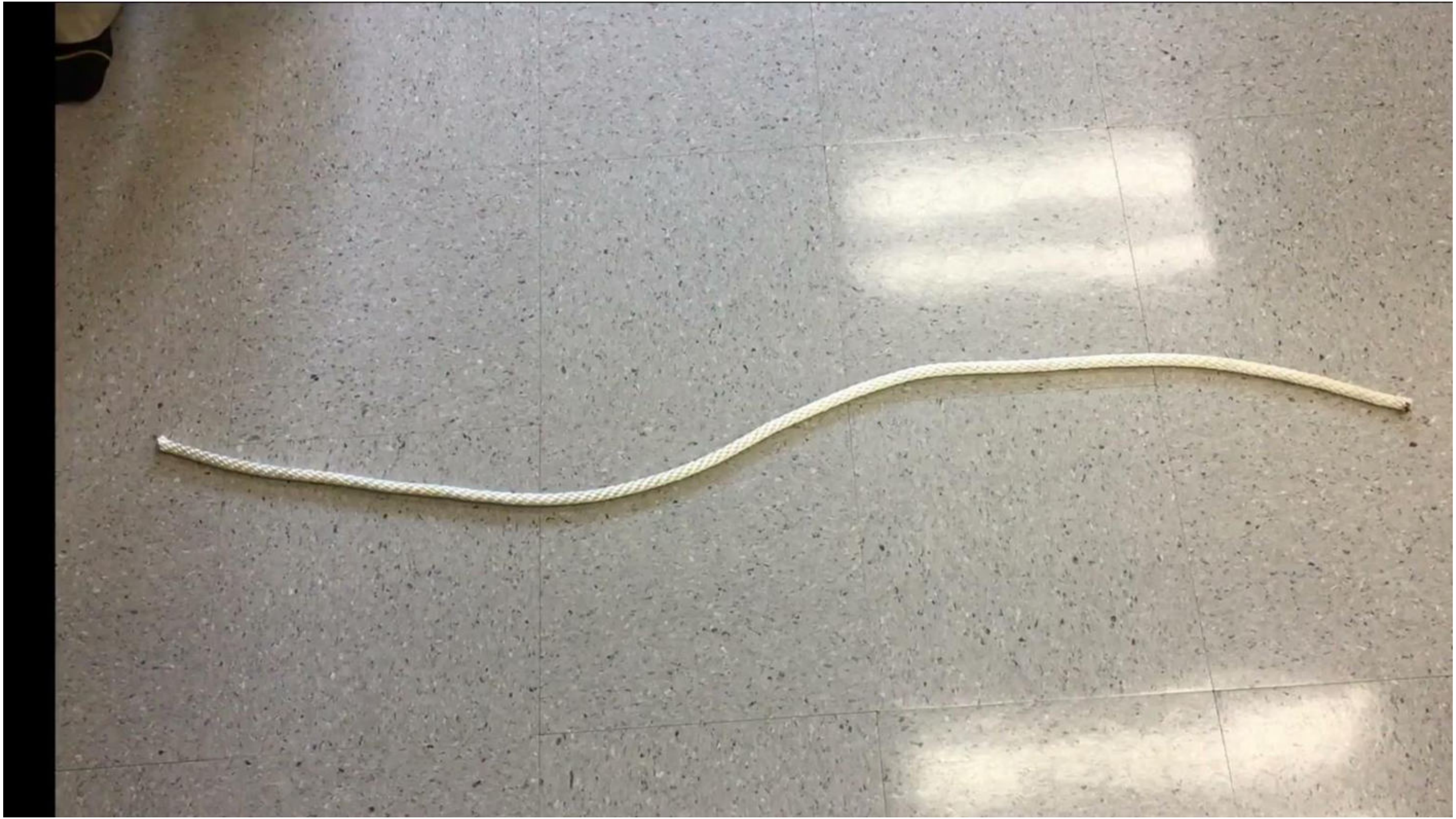


Lower limit

Upper limit

The length of the rope with no knots

The length of the rope with one knot



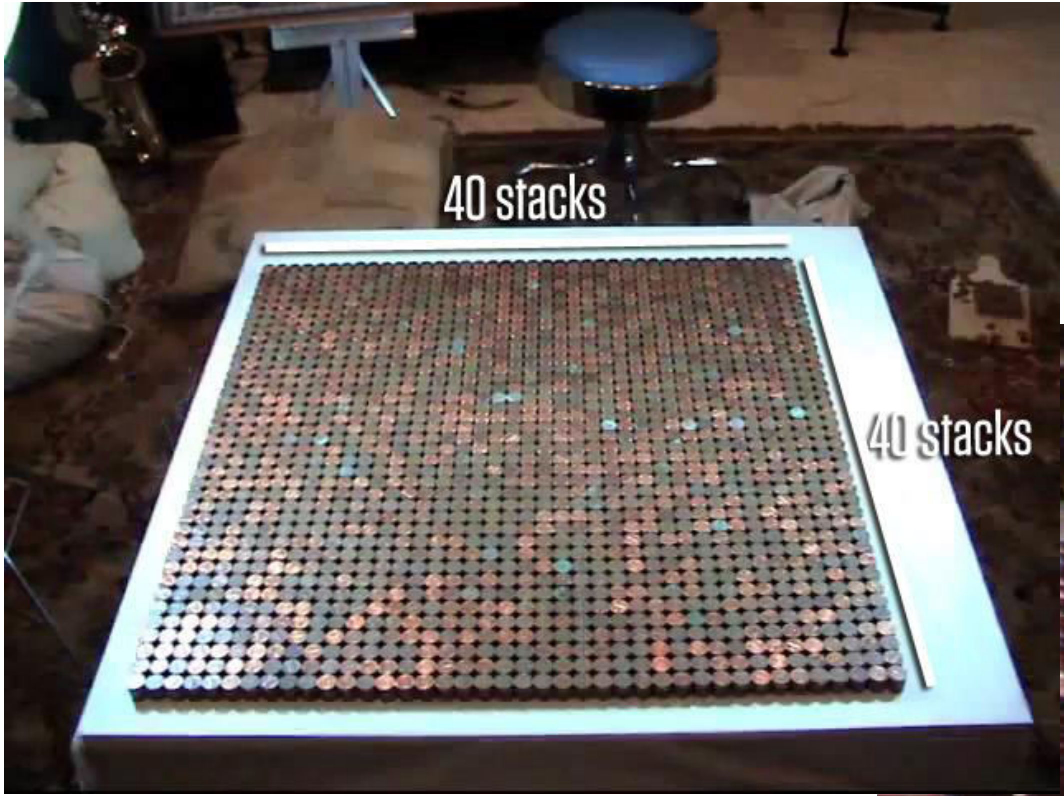
3-Act Task (3.MD.7)

Credit to Dan Meyer, http://threeacts.mrmeyer.com/pyramidofpennies/

Pyramid of Pennies

Lower limit

Upper limit


40 stacks
40 stacks

13 pennies

UNITED STATES MINT

Standard View

Specifications

Composition:	Copper-Plated Zinc: 2.5% Cu, Balance Zn
Weight:	2.500 g
Diameter:	0.750 in., 19.05 mm
Thickness:	1.55 mm
Edge:	Plain

2011 Lincoln One-Cent Obverse

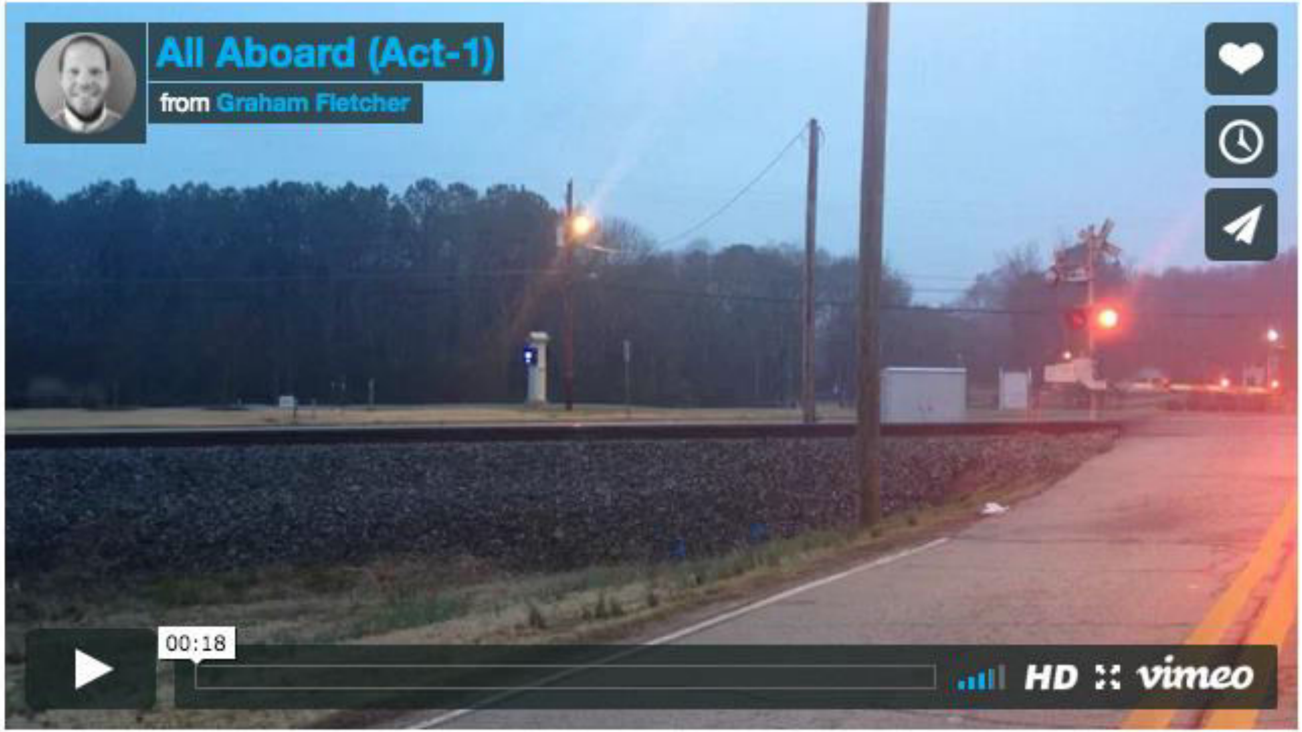
2011 Lincoln One-Cent Reverse

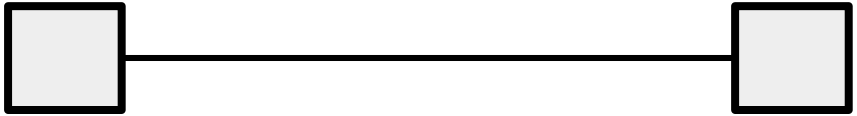
- Main Pyramid Completed; April 16th 2006
- 287,820 pennies,

3-Act Task (4th Grade 4.NBT.4)

All Aboard

Credit to Graham
Fletcher, gfletchy.com



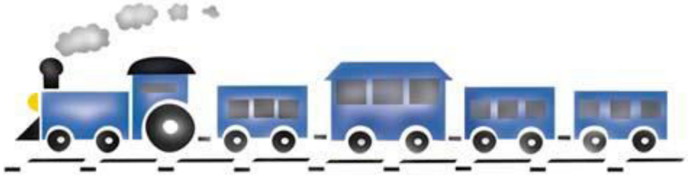


Too low

Too high

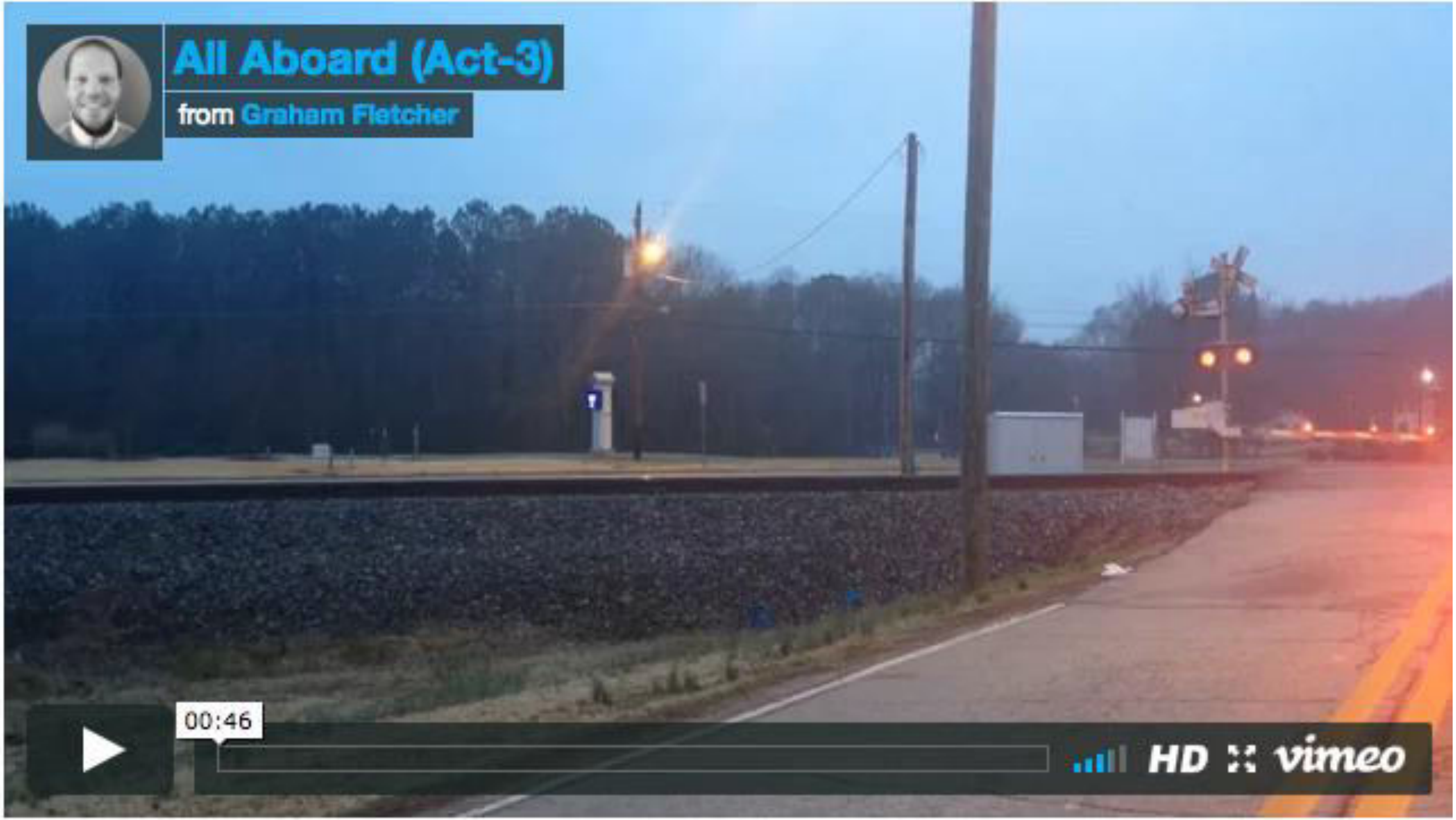
There's 70 train cars.

There's 2 locomotives.



It takes 10 seconds for 7 train cars to pass.

It takes 3 seconds for 1 locomotive to pass.

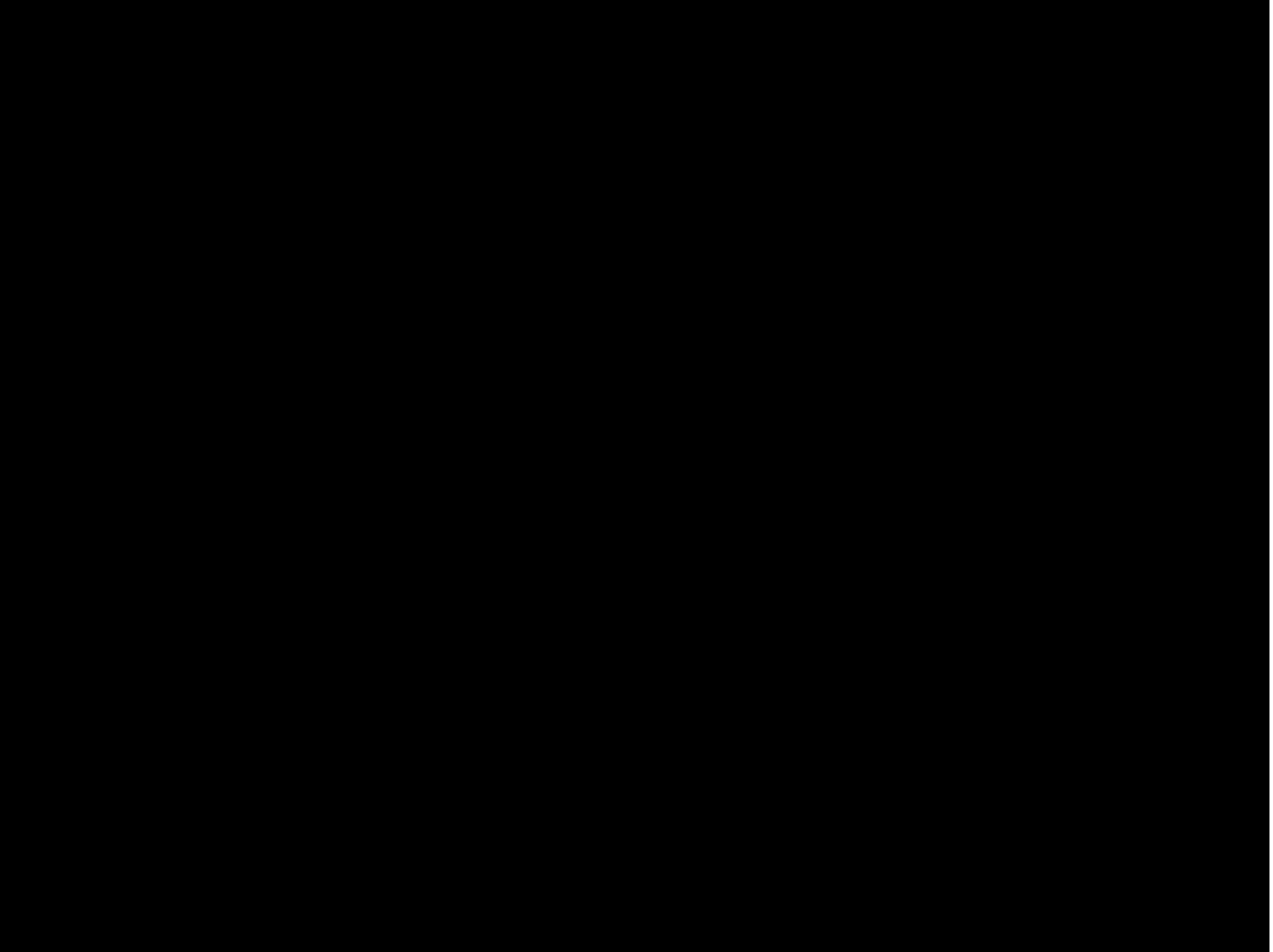

All Aboard (Act-3)
from Graham Fletcher
00:46
HD
vimeo

3-Act Task (4th Grade) NF.4

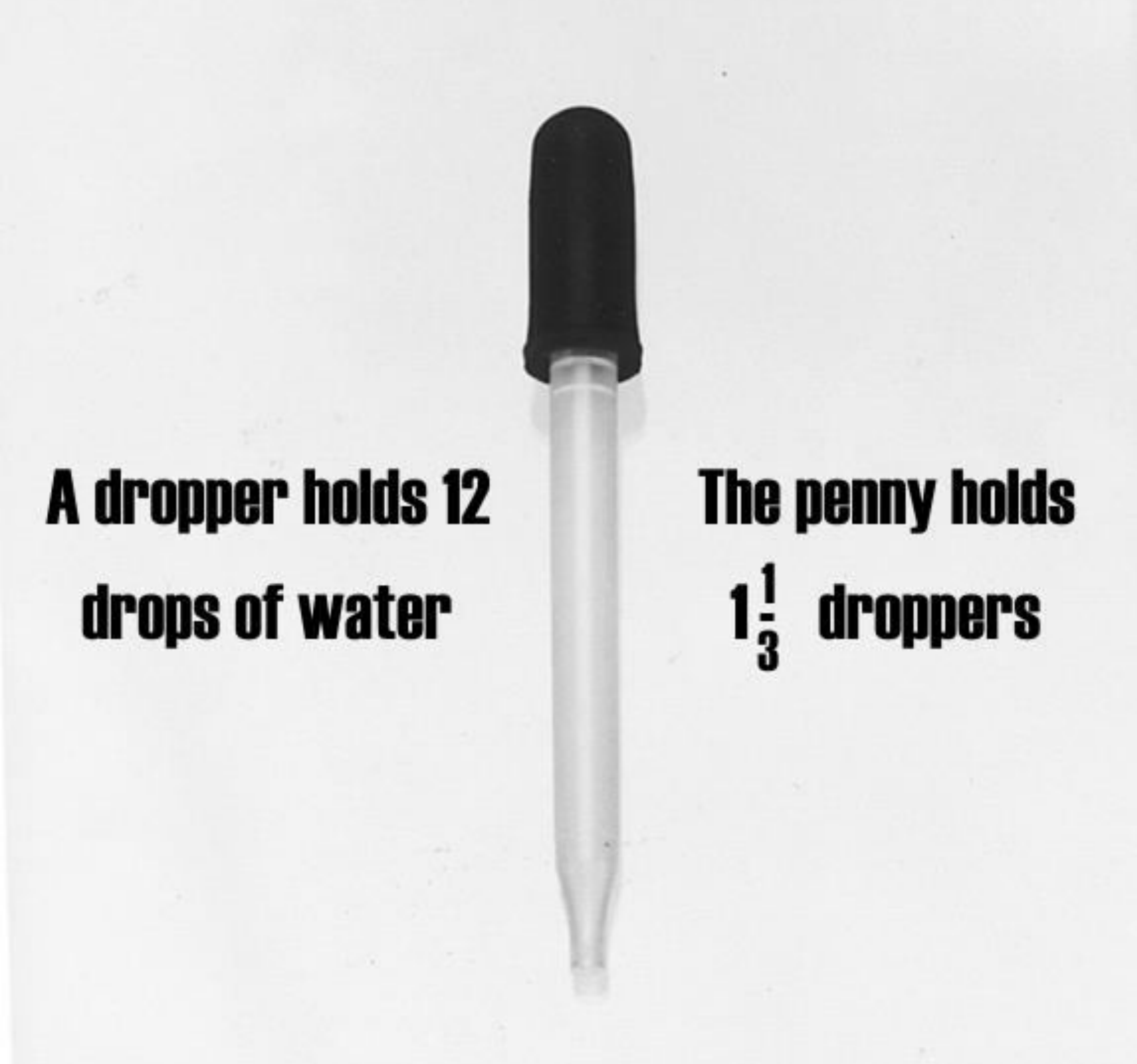
Drip Drop

Desmos Version: Link

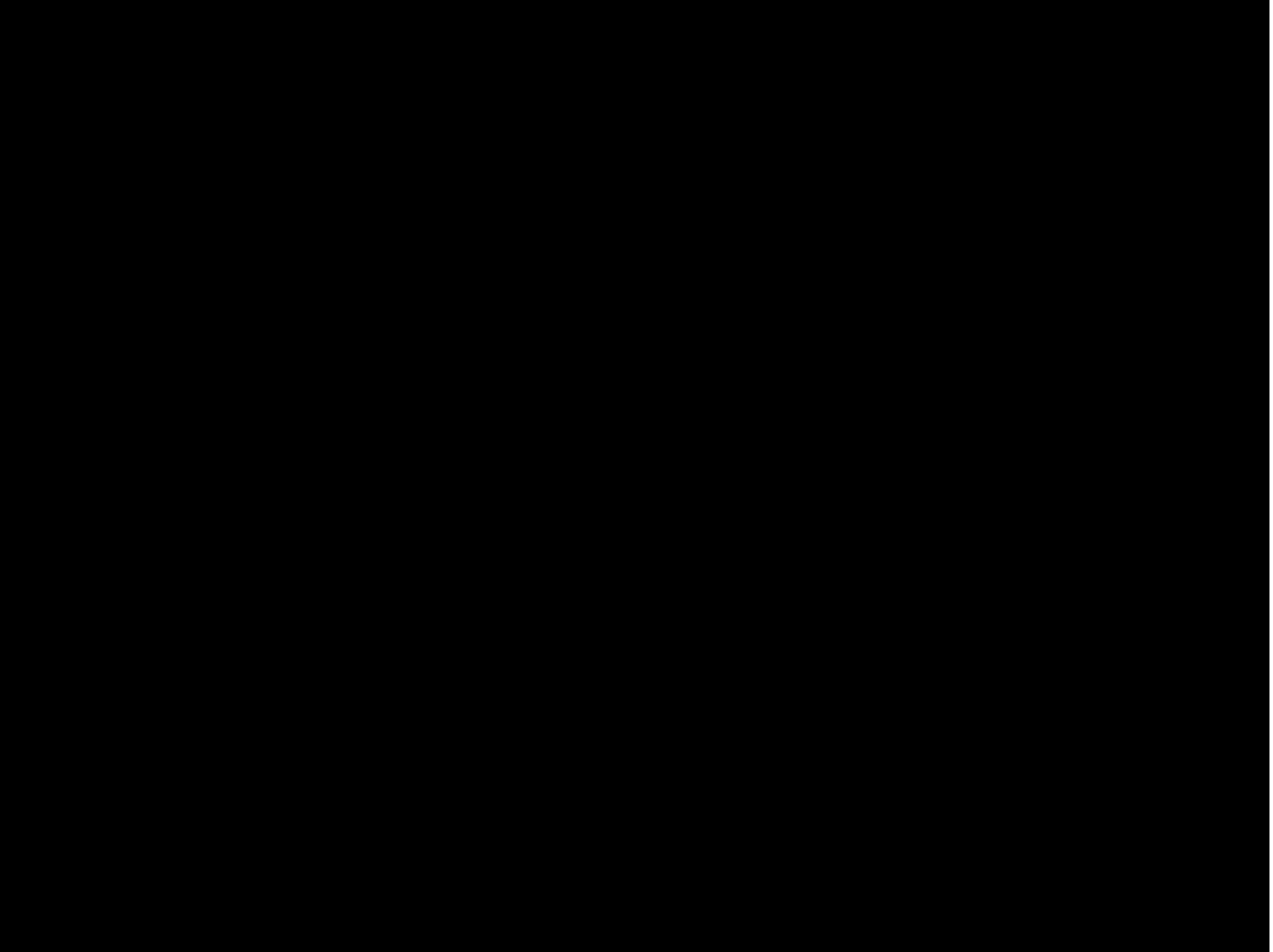
Credit to Graham Fletcher, gfletchy.com



Credit to Graham Fletcher, gfletchy.com



Credit to Graham Fletcher, gfletchy.com



Credit to Graham Fletcher, gfletchy.com

3-Act Task (4th Grade 4.NBT.5/OA.3)

Krispy Kreme Me

Desmos Version: Link

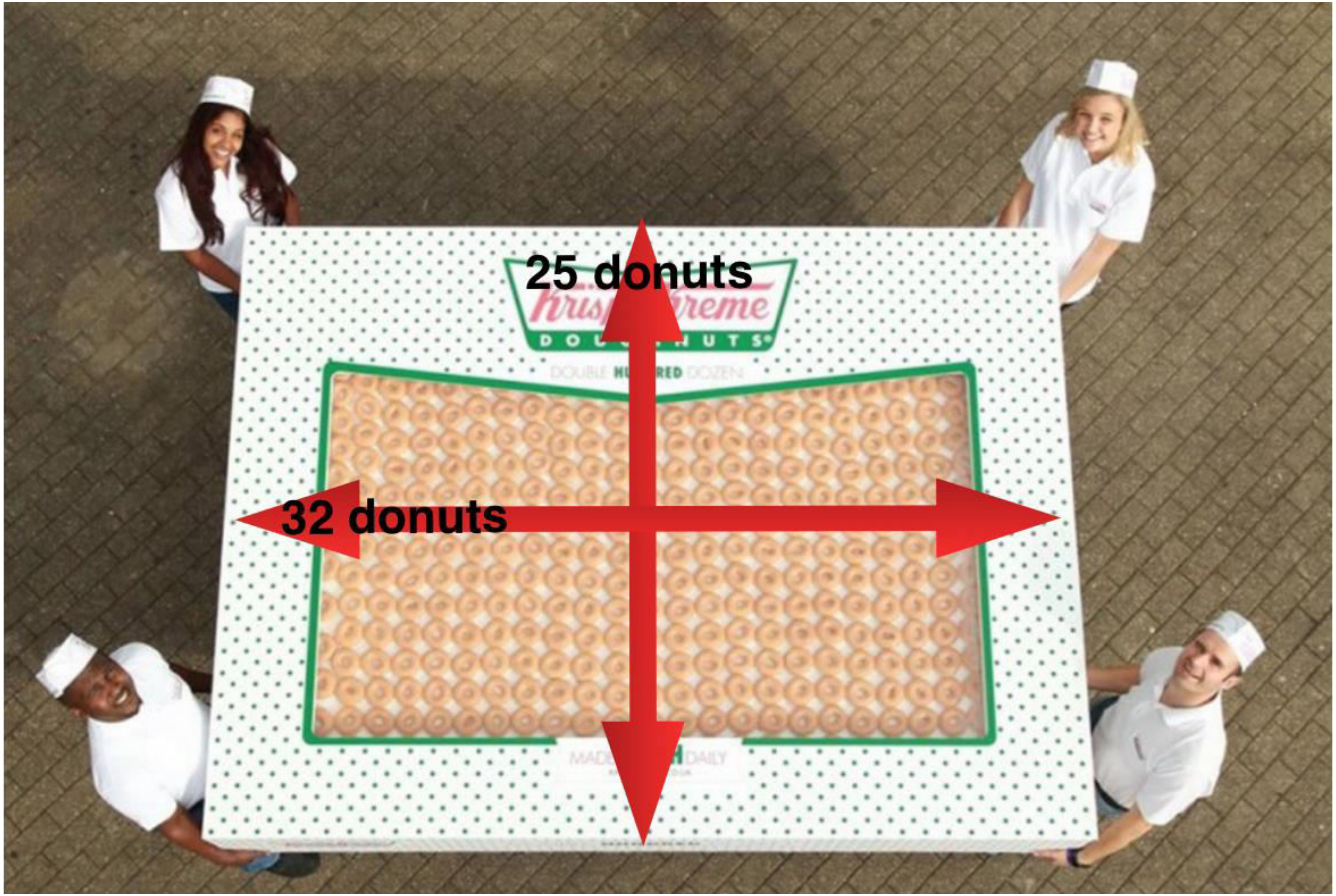
Credit to Graham Fletcher, gfletchy.com

Lower limit

Upper limit

From: **HayleyHutchison**
07/10/2014 13:21:36

Hi Graham,
We hope the information below helps:
The box was created to allow 3 x layers of [redacted] doughnuts ([redacted] x [redacted] doughnuts on each layer)
Each doughnut is approximately 89 millimetres in diameter.
The box is 3000 mm x 2300 mm which allows a gap between each doughnut to sit comfortably.
We do not have any schematics to share as they are for internal use only but we wish you all the best with your class.
Kind regards,
Krispy Kreme


25 donuts
32 donuts
DOUBLE HU RED DOZEN

If your school bought this box of doughnuts to split between 8 classes, how many would each class get?

$$\begin{array}{r} 2{,}400 \\ -\quad 80 \\ \hline 2{,}320 \\ -\quad 80 \\ \hline 2{,}240 \end{array}$$

The box was created to allow 3 x layers of 800 doughnuts (25 x 32 doughnuts on each layer)
Each doughnut is approximately 89 millimetres in diameter.
The box is 3000 mm x 2300 mm which allows a gap between each doughnut to sit comfortably.

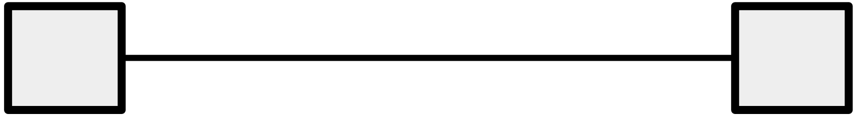
2400 donuts

3-Act Task (5th Grade 4.NF.4 ⇨ 5.NF.4)

Do the Dew

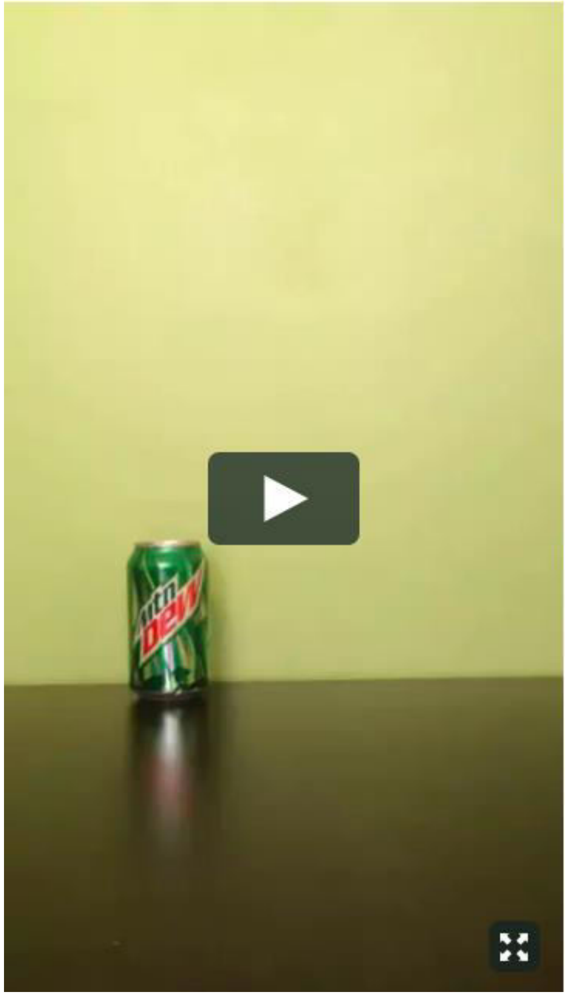
Desmos version: Link

Credit to Graham Fletcher, gfletchy.com



Too low

Too high



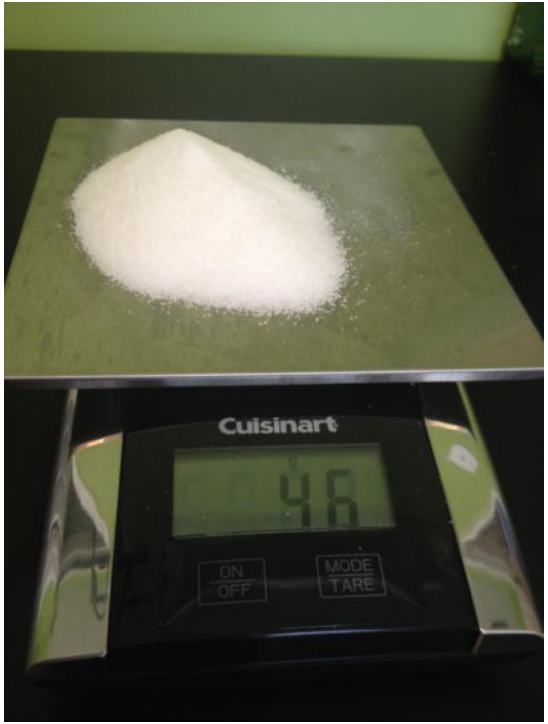
Nutrition Facts
Valeur nutritive
Per 355 mL / par 355 mL

Amount / Teneur	% Daily Value / % valeur quotidienne
Calories / Calories 170	
Fat / Lipides 0 g	0 %
Sodium / Sodium 45 mg	2 %
Carbohydrate / Glucides 46 g	15 %
Sugars / Sucres 46 g	
Protein / Protéines 0 g	
Vitamin C / Vitamine C	2 %

Not a significant source of saturated fat, trans fat, cholesterol, fibre, vitamin A, calcium or iron.

Source négligeable de lipides saturés, lipides trans, cholestérol, fibres, vitamine A, calcium et fer.

CAFFEINE CONTENT: 51 mg/355 mL
TENEUR EN CAFÉINE : 51 mg/355 mL



46 grams of sugar = ⅕ cup sugar

⅕ cup sugar = 1 can of Mt. Dew

$\frac{1}{5}$

$\frac{2}{5}$

$\frac{3}{5}$

$\frac{4}{5}$

1

$1\frac{1}{5}$

$1\frac{2}{5}$

$1\frac{3}{5}$

$1\frac{4}{5}$

2

$2\frac{1}{5}$

$2\frac{2}{5}$

Extension

Approx. ½ the sugar found in a can (weird).

2 Liters

3-Act Task (5th Grade 5.NBT.7)

BK Meal Deal

Credit to ….Mr. Kraft
adapted by Chelsea McClellan

Original Chicken Sandwich Burger King

$4.89

Our Original Chicken Sandwich Burger King menu prices is lightly breaded and topped with a simple combination of shredded lettuce and creamy mayonnaise on a sesame seed bun. Meal comes in medium and large sizes. Your choice of a side of piping hot, thick cut French Fries or golden Onion Rings and a fountain drink of your choice to make it a meal.

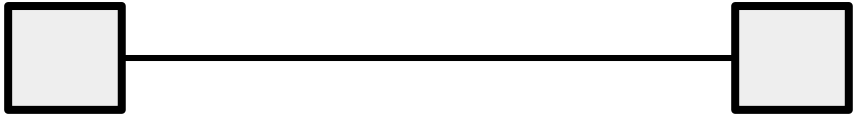
Original Chicken Sandwich Meal

$10.90

Our Original Chicken Sandwich is lightly breaded and topped with a simple combination of shredded lettuce and creamy mayonnaise on a sesame seed bun. Meal comes in medium and large sizes.

▼

Your choice of a side of piping hot, thick cut French Fries or golden Onion Rings and a fountain drink of your choice to make it a meal.



Lower limit Upper limit

Original Chicken Sandwich Meal

$10.90

Our Original Chicken Sandwich is lightly breaded and topped with a simple combination of shredded lettuce and creamy mayonnaise on a sesame seed bun. Meal comes in medium and large sizes.

▼

Your choice of a side of piping hot, thick cut French Fries or golden Onion Rings and a fountain drink of your choice to make it a meal.

Medium Sprite

$3.39

Enjoy an Ice-Cold, Crisp Sprite® Refreshment. Also Available In Zero Sugar. Shop Online! Classic, Cool, Crisp Lemon-Lime Taste That's Caffeine Free With 100% Natural Flavors.

Original Chicken Sandwich Burger King

$4.89

Our Original Chicken Sandwich Burger King menu prices is lightly breaded and topped with a simple combination of shredded lettuce and creamy mayonnaise on a sesame seed bun. Meal comes in medium and large sizes. Your choice of a side of piping hot, thick cut French Fries or golden Onion Rings and a fountain drink of your choice to make it a meal.

Fries at Burger King

$3.60

Our fries at Burger King menu prices signature piping hot, thick cut Salted French Fries are golden on the outside and fluffy on the inside.

Meal Deal: $10.90 Sandwich only: $4.89

Fries only: $3.60

Drink only: <u>$3.39</u>

$11.88

The Meal Deal saves us: $0.98

Classic Chicken Sandwich Combo

$9.29

A juicy, lightly breaded crispy chicken breast with crunchy lettuce, tomato, mayo, and the perfect pickles, all on a toasted bun. It’s a flawless blend of nostalgia and excitement—kinda like your all-time favorite song, only better ‘cause you can eat it.

Classic Chicken Sandwich

$6.29

A juicy, lightly breaded crispy chicken breast with crunchy lettuce, tomato, mayo, and the perfect pickles, all on a toasted bun. It’s a flawless blend of nostalgia and excitement—kinda like your all-time favorite song, only better ‘cause you can eat it.

Dr Pepper

$2.29

Grab a Dr. Pepper drink to go at a Wendy's near you that's a signature blend of 23 flavors making every sip truly unique and truly refreshing.

French Fries

$2.39

Natural-cut, skin-on, sea-salted fries served hot and crispy. The world loves them for a reason.

Sandwich Meal

$10.99

A boneless breast of chicken seasoned to perfection, freshly breaded, pressure cooked in 100% refined peanut oil.

Chicken Sandwich

$6.09

A boneless breast of chicken seasoned to perfection, freshly breaded, pressure cooked in 100% refined peanut oil and served on a toasted, buttered bun with dill pickle chips. Also available on a multigrain bun.

Dr Pepper

$2.79

Dr Pepper is a carbonated soft drink. It was created in the 1880s by pharmacist Charles Alderton in Waco.

Fries from Chick fil A

$3.05

Waffle-cut potatoes cooked in canola oil until crispy outside and tender inside. Sprinkled with Sea Salt.

Crispy Chicken Sandwich Meal

$6.29

Mcdonalds Menu Lunch Meal with Crispy, juicy and tender perfection. Southern style fried chicken on a toasted, buttered potato roll, topped with crinkle-cut pickles and served with our World Famous Fries® and your choice of an icy soft drink.

McChicken

$2.49

McDonalds menu item with juicy chicken patty, topped with shredded lettuce and just the right amount of creamy mayonnaise, all chicken sandwiches at Mcdonald's served on a perfectly toasted bun.

Sprite®

$1.59

Sprite® is a delicious lemon-lime fountain drink and is available in sizes extra small, small, medium, and large.

Large Fries

$1.00

Mcdonalds Menu Dollar offer valid thru 12/31/22 at participating McDonald's. Valid 1x/week. Refer to McD app for details. Mobile Order & Pay at Participating McDonald's. McD app download and registration required.

	Task Sequence	Visual Solution	Written Solution **Hint:** Suggest expanded form if Ss struggle with a strategy	Consolidation Sequence/ Notes & Hints - connect drawings to written form
1	Draw and label 4, 0.4, and 0.04	□□□□ ||||	4 0.4 0.04	Formative to check they understand changing the referent
2	0.4 + 0.5	|||| |||||	0.9	A. like parts added to like parts
3	4.4 + 0.5	□□□□|||| |||||	4 + 0.4 + 0.5 = 4+ 0.9 = 5.9 OR 4 + 0.9 from problem 2	B. highlight adding the 0.5 to the 4 in the tenths place;
4	4.4 + 3.6	□□□□|||| □□□||||||	4.4 = 4 + 0.4; 3.6 = 3 + 0.6; 8.0 = 7 + 1	
5	4.85 + 3.6	□□□□|||||||| ..... □□□||||||	4.85 = 4 + 0.8 + 0.05 +3.6 = 3 + 0.6 + 0.00 8.45 = 7 + 1.2 + 0.05	C. Regrouping when necessary like with whole numbers, each place can only fit a max of "9"
6	4.85 - 3.6	□□□□||||||||	4.85 = 4 + 0.8 + 0.05 -3.6 = -3 - 0.6 - 0.00 1.25 = 1 + 0.2 +0.05	
7	5.12 - 3.6	□□□□ |||| |||||| | ..	4 1.1 5.12 = ~~5~~ + ~~0.1~~ + 0.02 -3.6 = -3 - 0.6 - 0.00 1.52 = 1 + 0.5 + 0.02	D. Regrouping when necessary when subtracting; annotate regrouping
8	5.12 - 3.68	□□□□ |||| |||||| xxxxxxxx	4 1.~~1~~ 0.12 5.12 = ~~5~~ + 0.1 +~~0.02~~ -3.68 = -3 - 0.6 -0.08 1.44 = 1 + 0.4 +0.04	
9	4.05 - 3.68	□□□ ||| |||||| xxxxxxxx..	0.9 3 ~~1.0~~ 0.15 4.05 = ~~4~~ +~~0.0~~ + 0.05 -3.68 = -3 - 0.6 - 0.08 0.37 = 0 + 0.3 + 0.07	E. Regrouping when necessary when subtracting; annotate regrouping
10	4.05 - 1.263	❐❐❐ □□□□□ □□□□|||| ||||| |||| xxx	0.9 0.14 3 ~~1.0~~ ~~0.15~~ 0.010 4.05 = ~~4~~ + ~~0.0~~ +~~0.05~~ + ~~0.000~~ -1.263 = -1 - 0.2 - 0.06 - 0.003 2.787 = 2+ 0.7 + 0.08 + 0.007	Extension: Change the referent when you need a model for thousandths . = 0.001; | = 0.01; □ = 0.1; ❐ = 1

Thin Sliced Problems to Follow Up…Link here

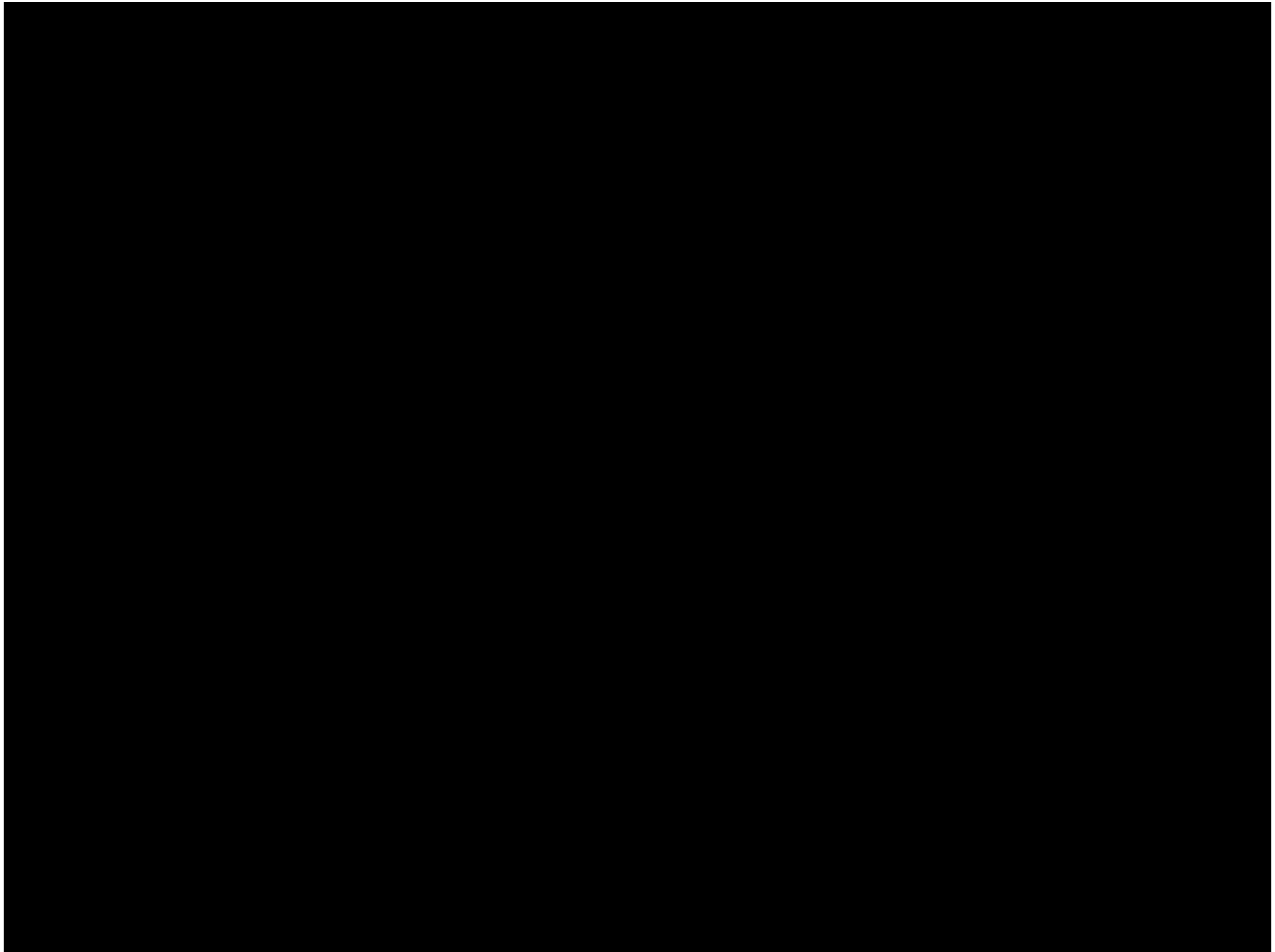
Word Problem for 3 Reads Strategy (Try it Verbally)

Jill bought items costing \$3.45, \$1.99, \$6.59, and \$12.98. She used a coupon worth half-off 2 items. If Jill had \$50.00 when she went into the store, how much did she have when she left?

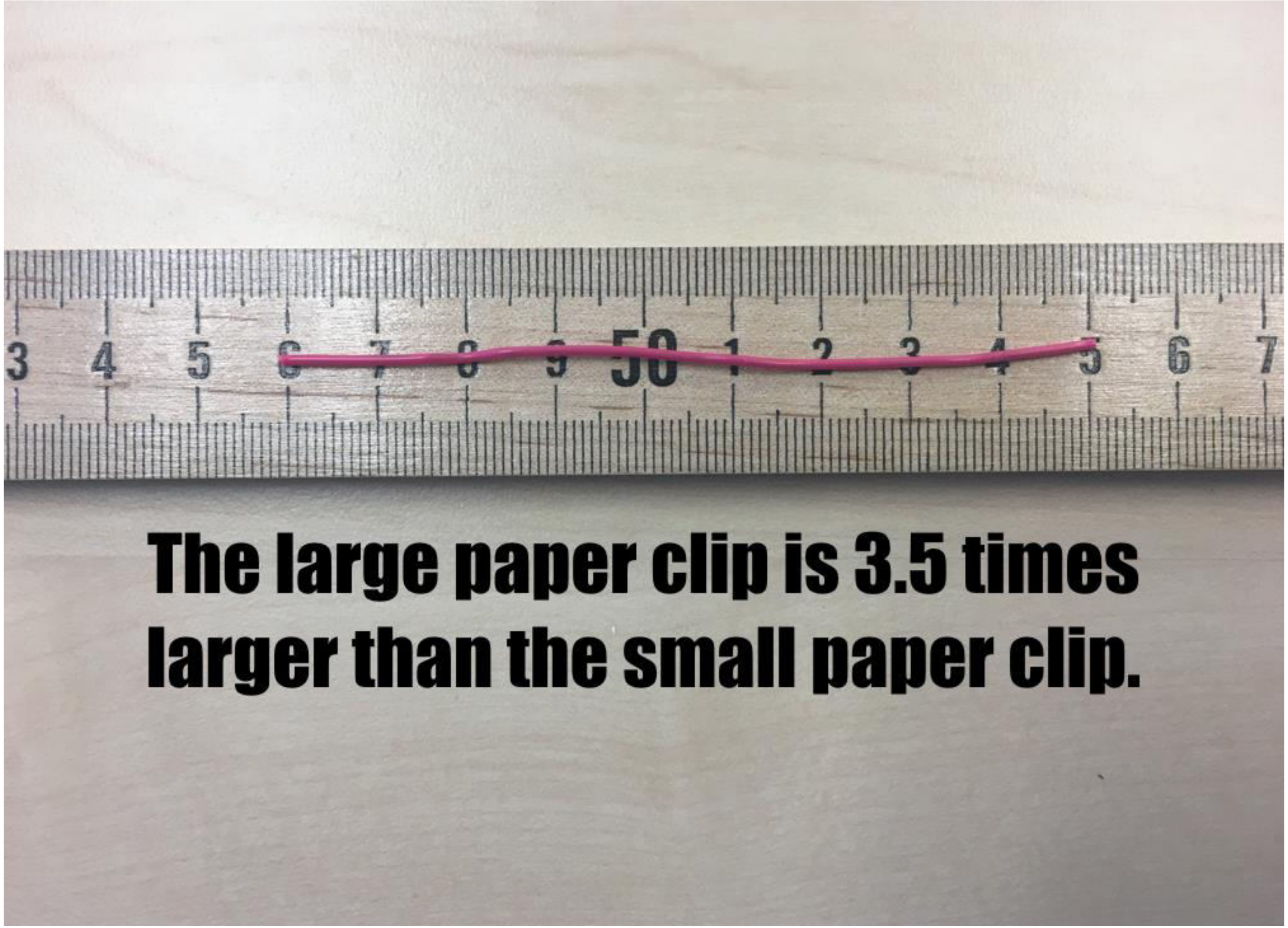
5.NBT.7: 3-Act Task (5th Grade)
Multiplying decimals (scaling)

Straighten Up

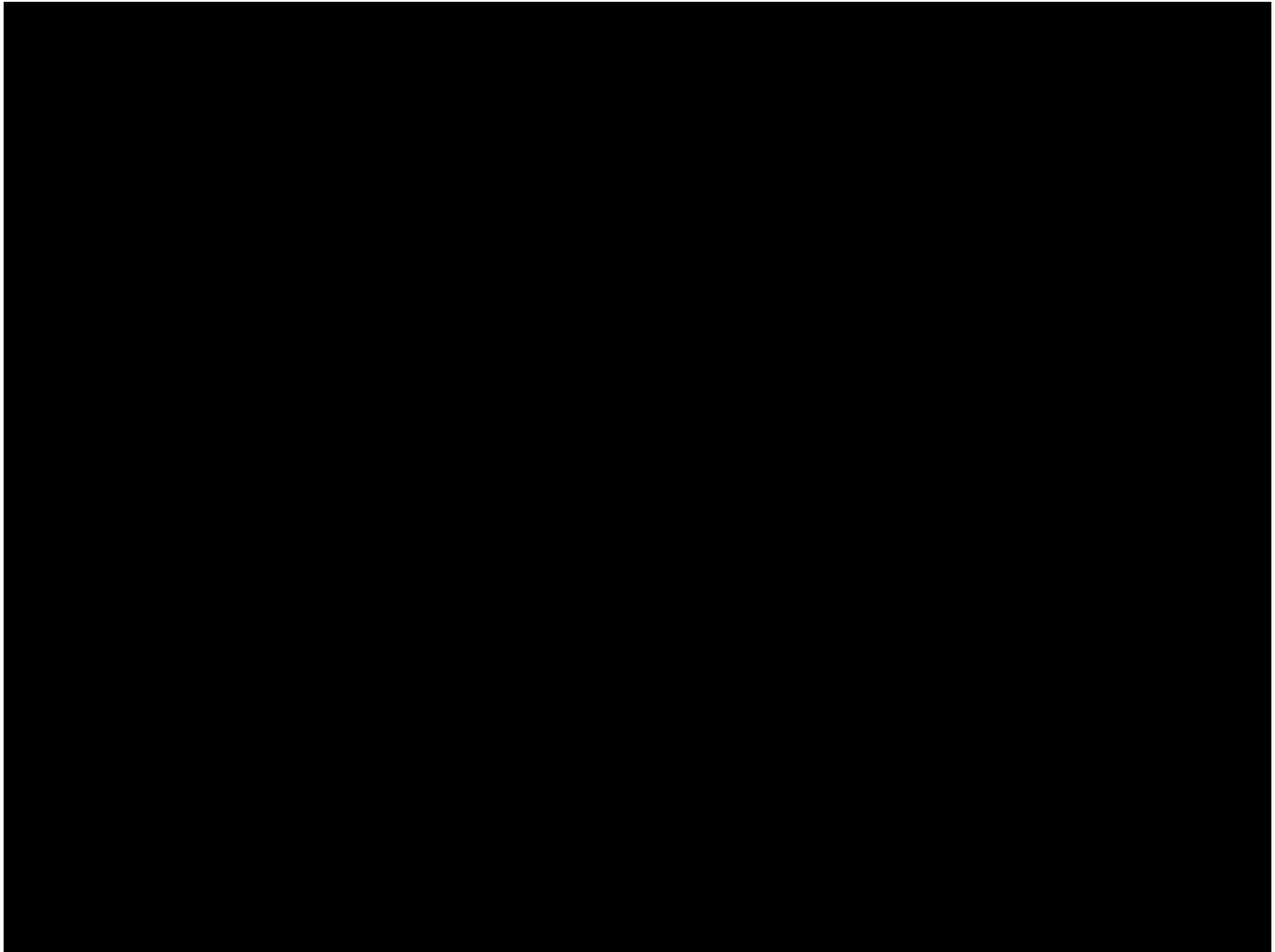
Credit to Graham Fletcher, gfletchy.com



Credit to Graham Fletcher, gfletchy.com



Credit to Graham Fletcher, gfletchy.com



Credit to Graham Fletcher, gfletchy.com

4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

   a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.*

   b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)*

9. Grade-four expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

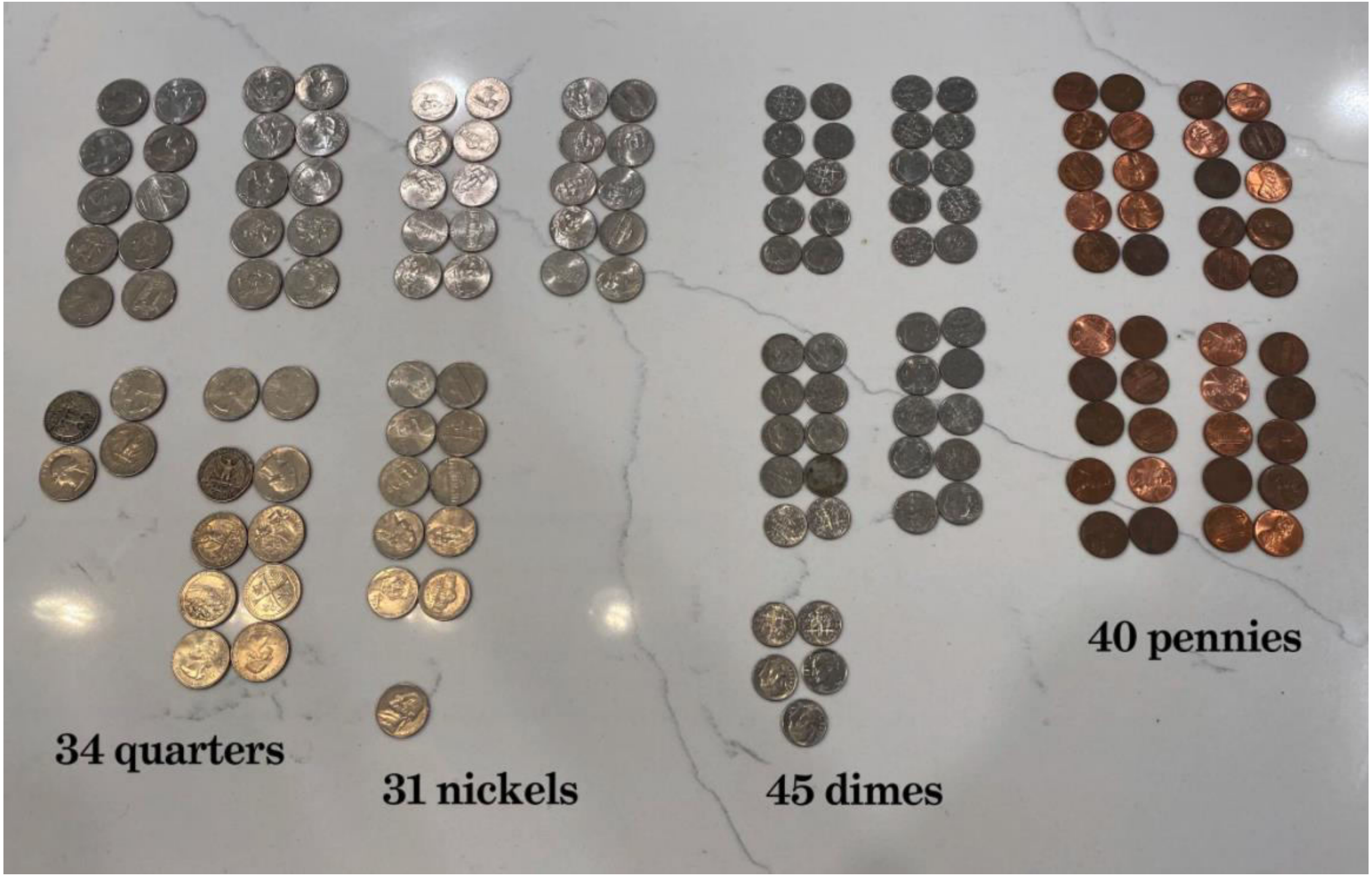
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

3-Act Task: 5NBT.7 Dividing Decimals

Towers of Coins

Credit to Kristen Acosta


$8.50
$1.55
$4.50
$0.40


34 quarters
31 nickels
45 dimes
40 pennies

3-Act Task (4-6 Grade 6.RP.3, 4th/5th Division)

Rope Jumper

Desmos Version: Link

Credit to Graham Fletcher, gfletchy.com

Lower limit

Upper limit

ギネス世界記録公認
奇跡の瞬間100連発!
高速!なわとび女子高生
回数
0
TIME
0:00

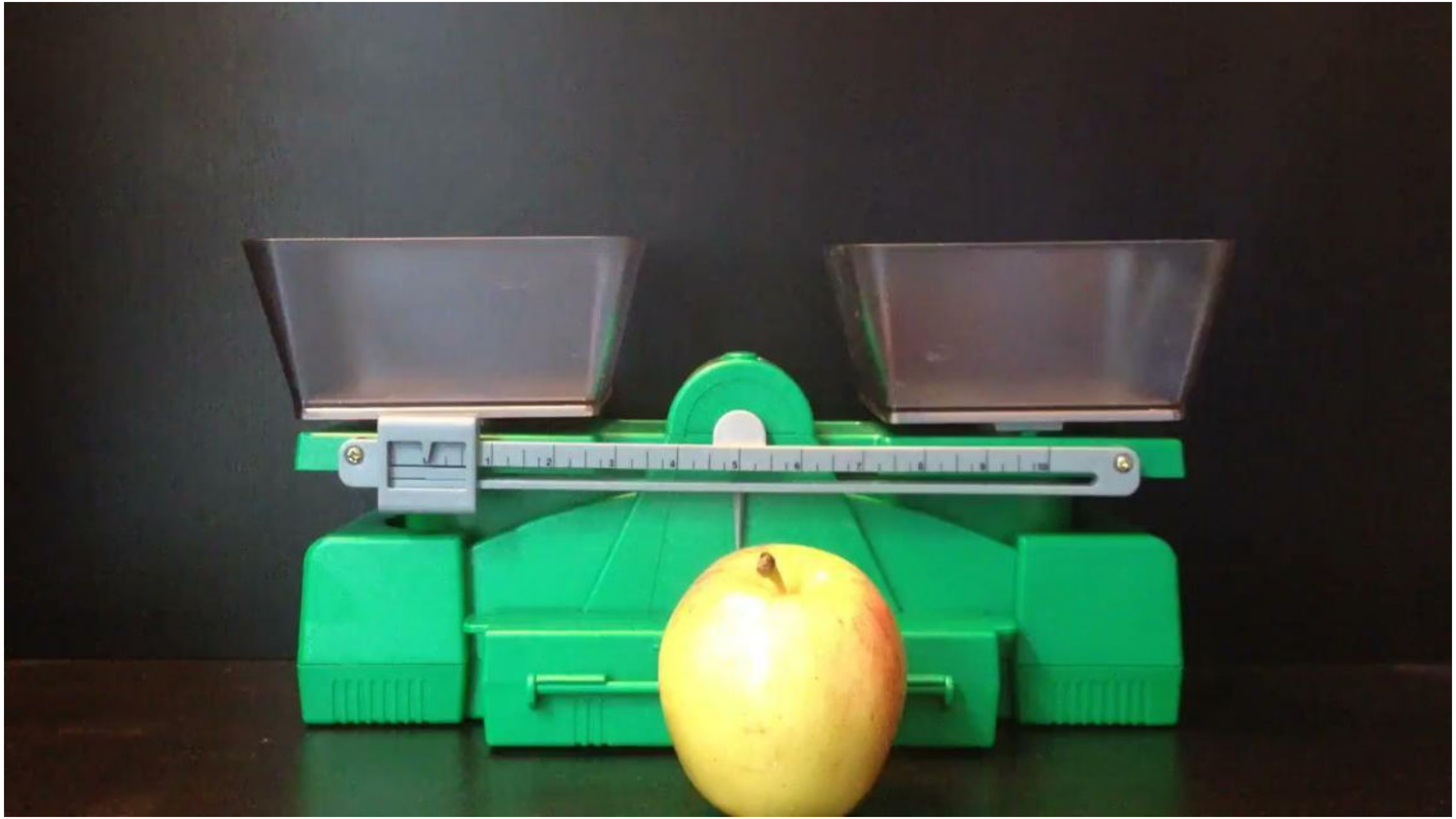

ギネス世界記録に挑戦!
高速!なわとび女子高生
回数
14
TIME
0:03


ギネス世界記録公認
奇跡の瞬間100連発!
世界一小さな女子高生の家
高速!なわとび女子高生
回数
0
TIME
0:00

3-Act Task (6th Grade 6.NS.1)

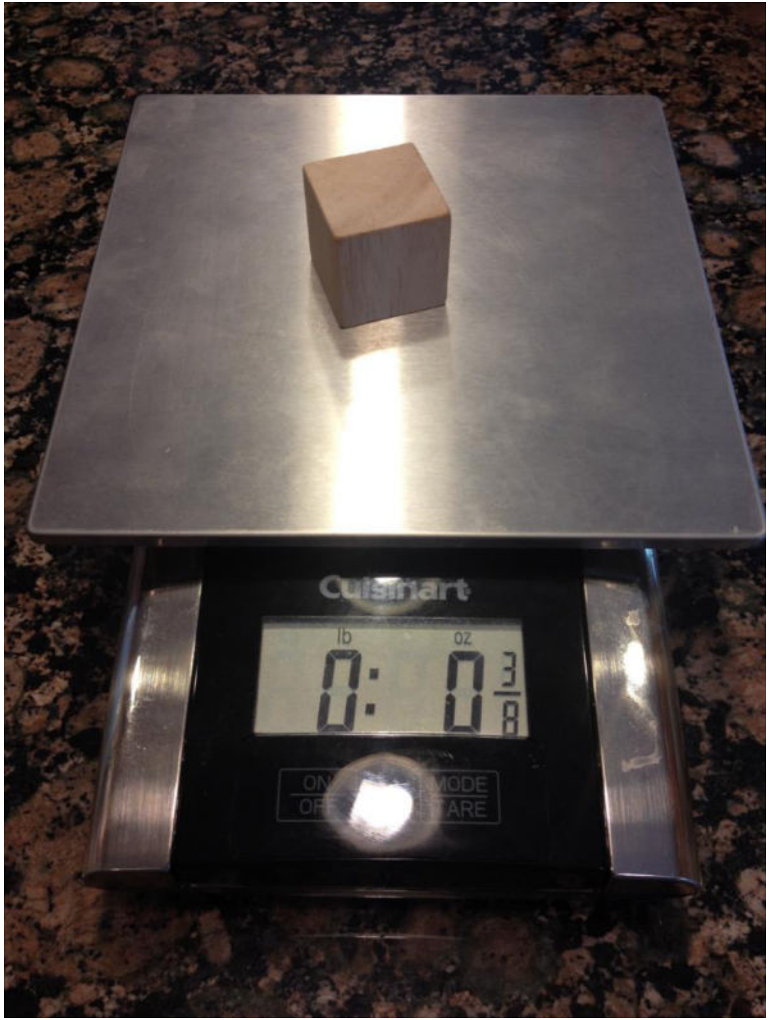
Apple

Credit to Graham Fletcher, gfletchy.com



Lower limit   Upper limit


Cuisinart
lb
oz
0: 5 1/4
MODE

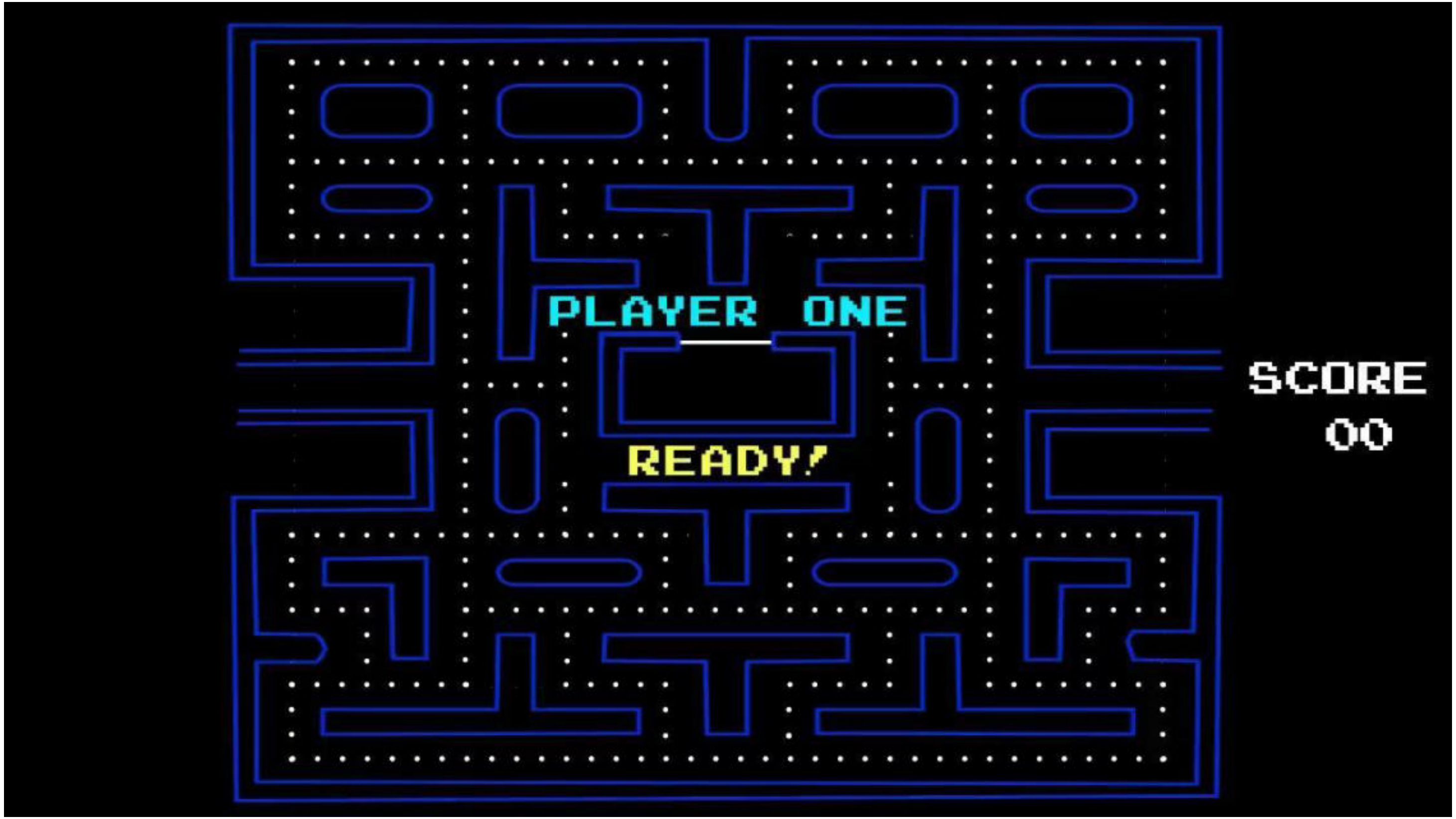

Cuisinart
lb
oz
0: 0 3/8
MODE
TARE

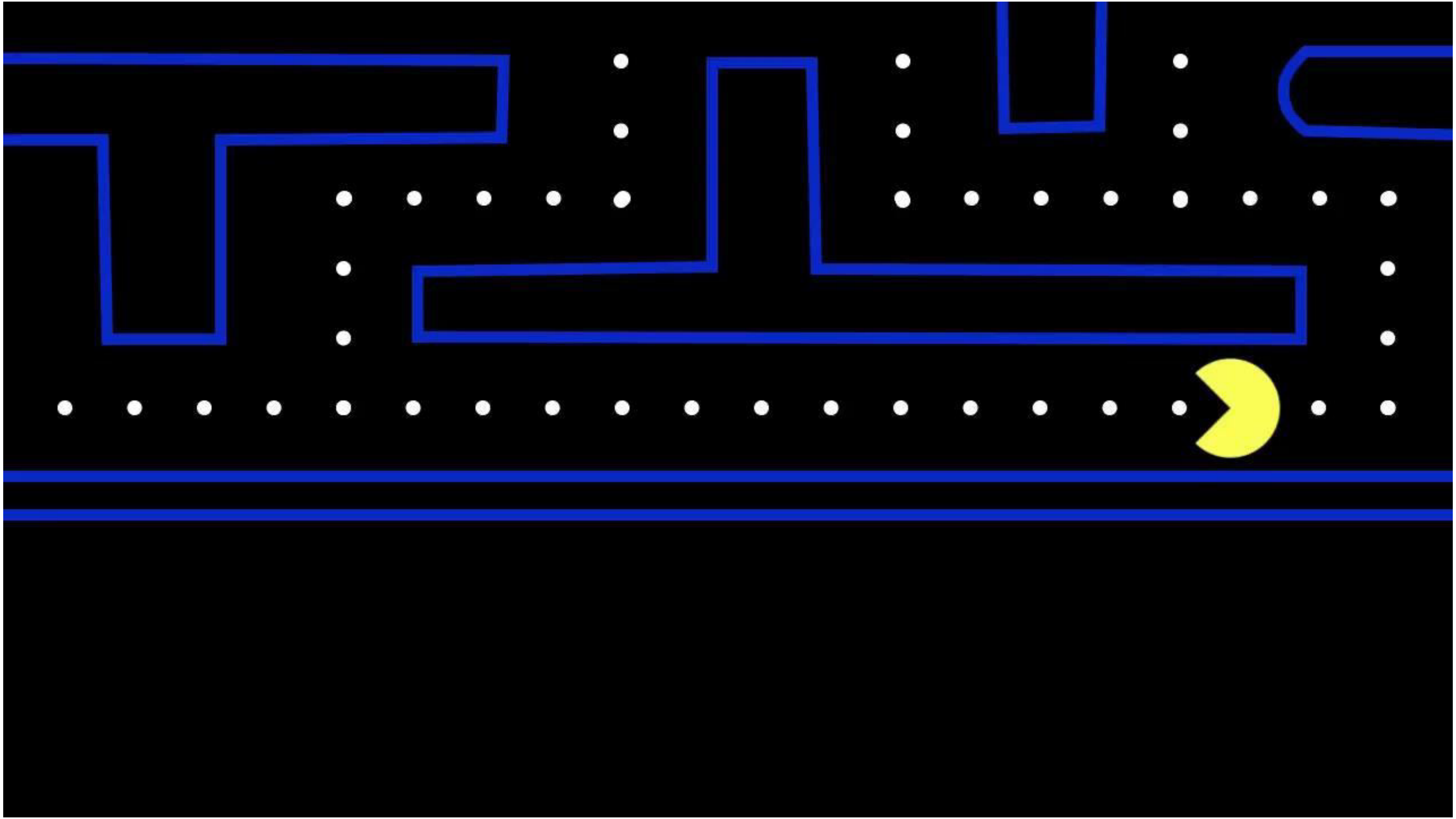

168.00
ON/OFF
TARE
UNIT

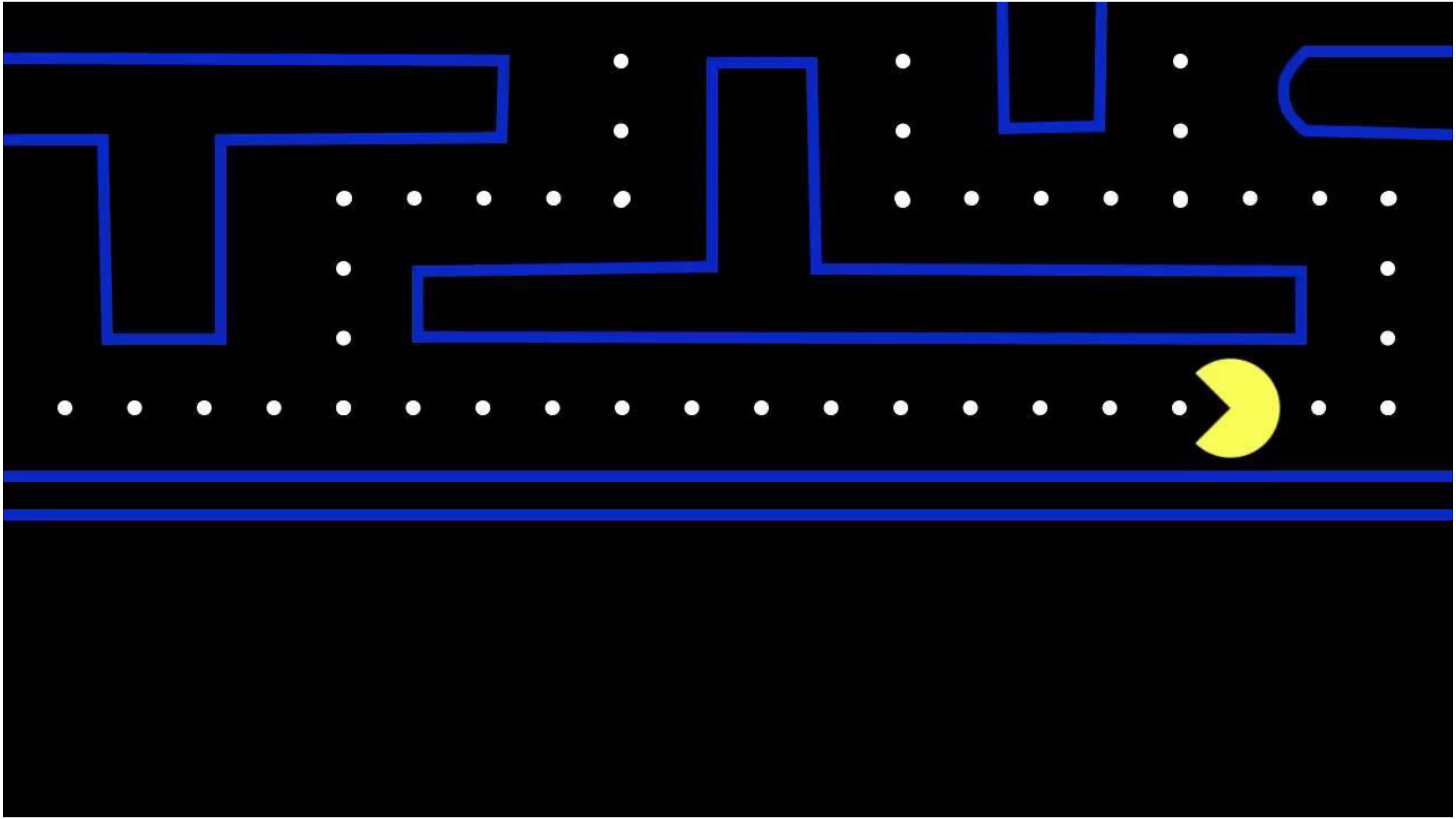
3-Act Task (6th Grade 6.RP)

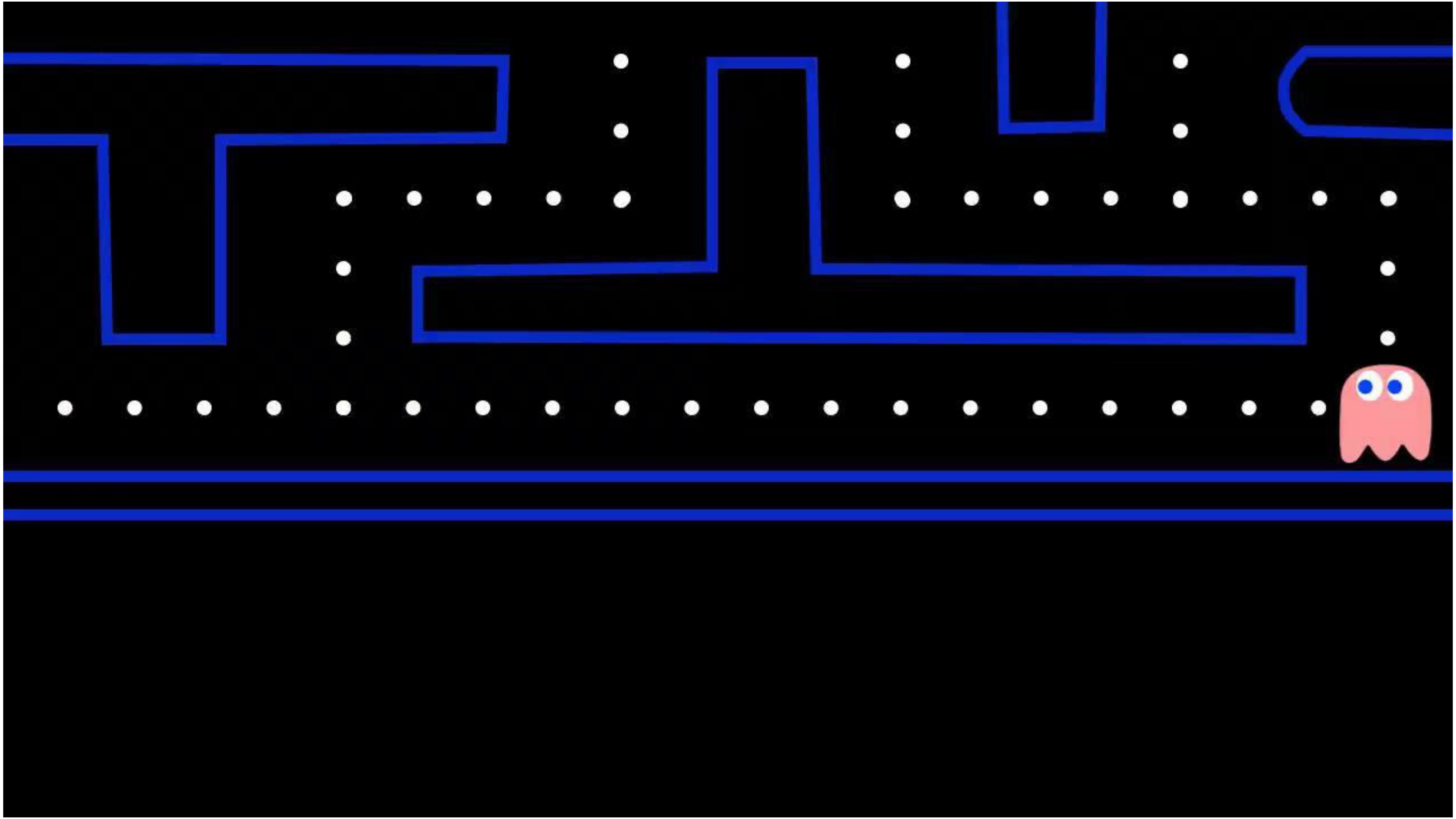
Pac Man

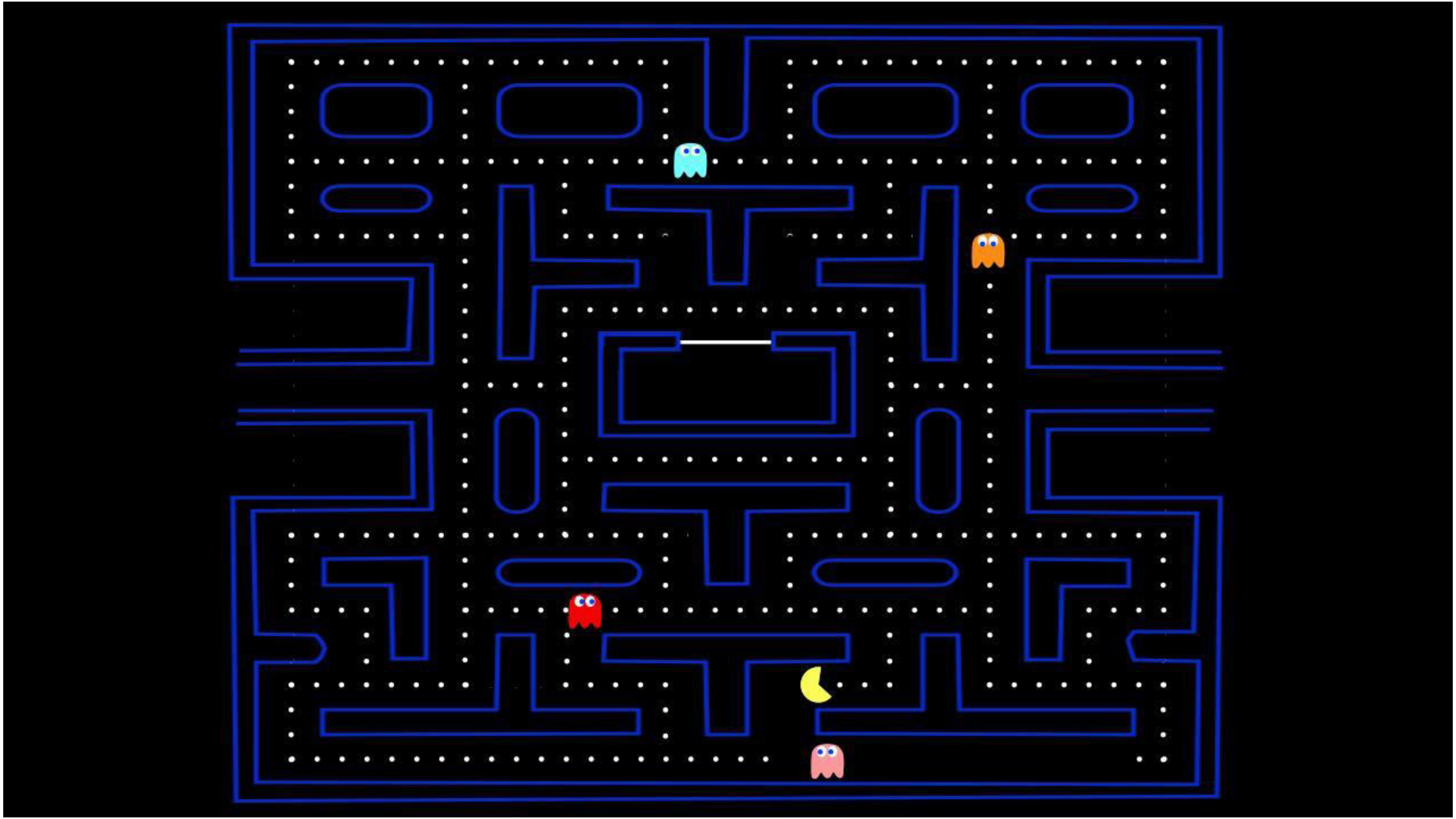
Credit to Dane Ehlert:
https://whenmathhappens.com/2017/01/04/pac-man/

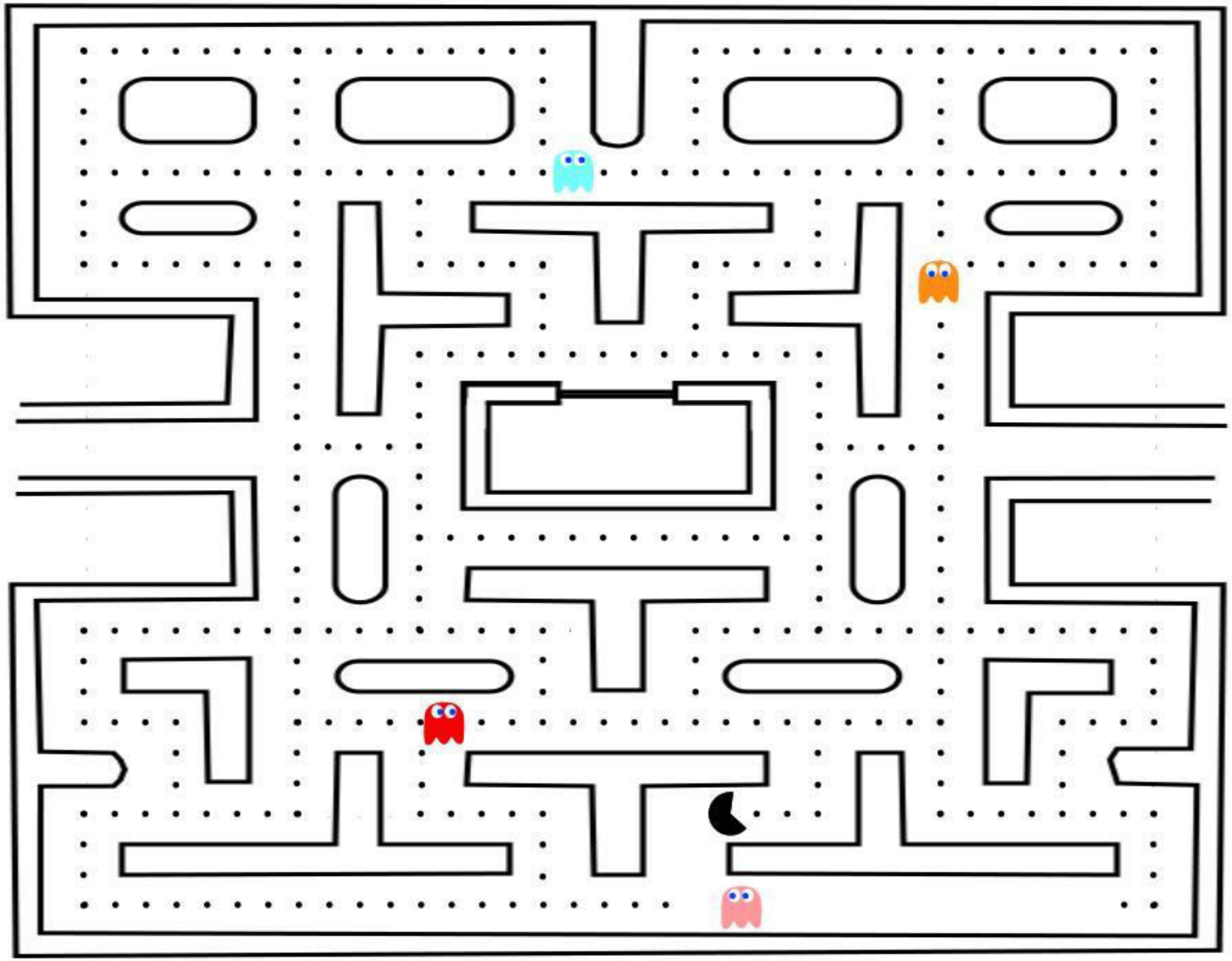

PLAYER ONE
READY!
SCORE
00

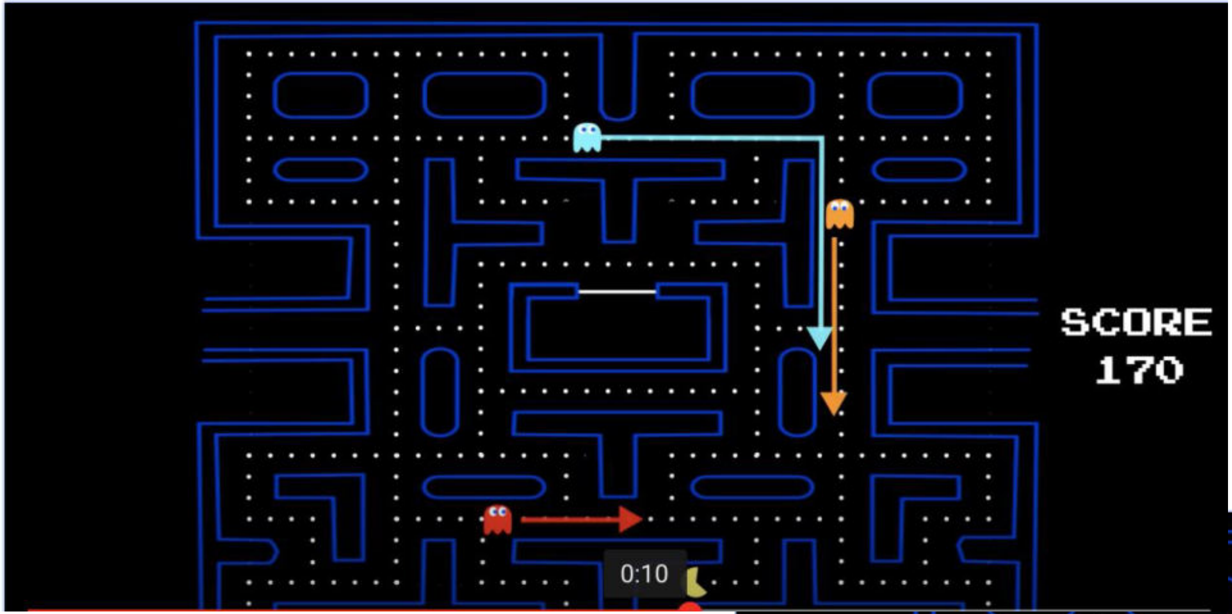


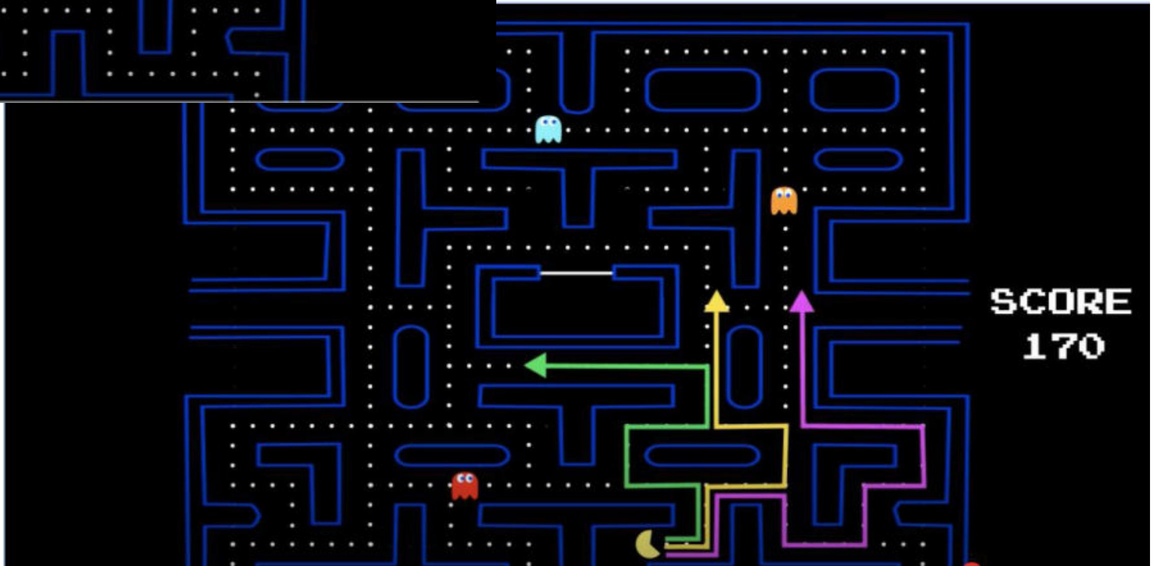


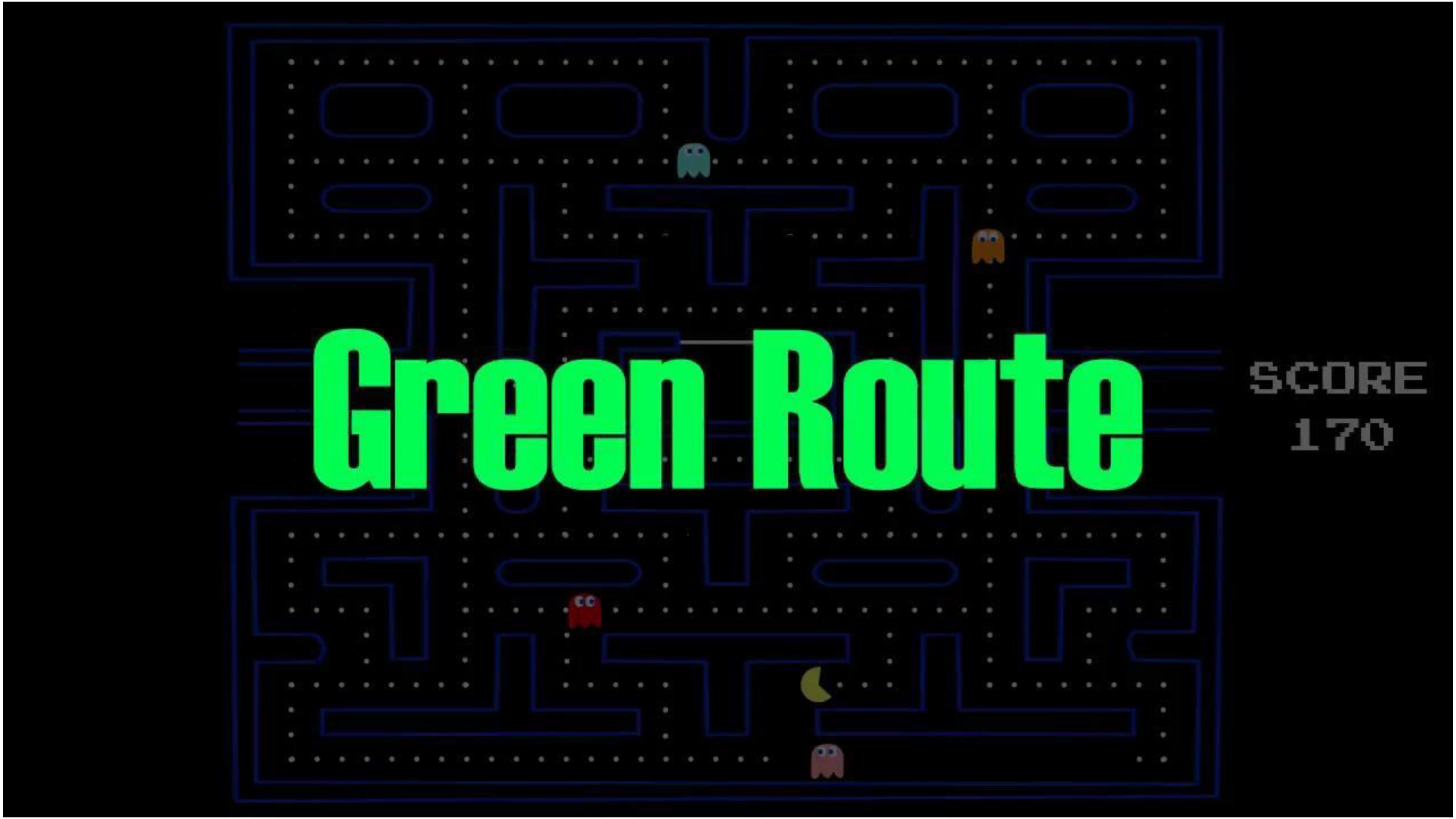







SCORE
170
0:10


SCORE
170

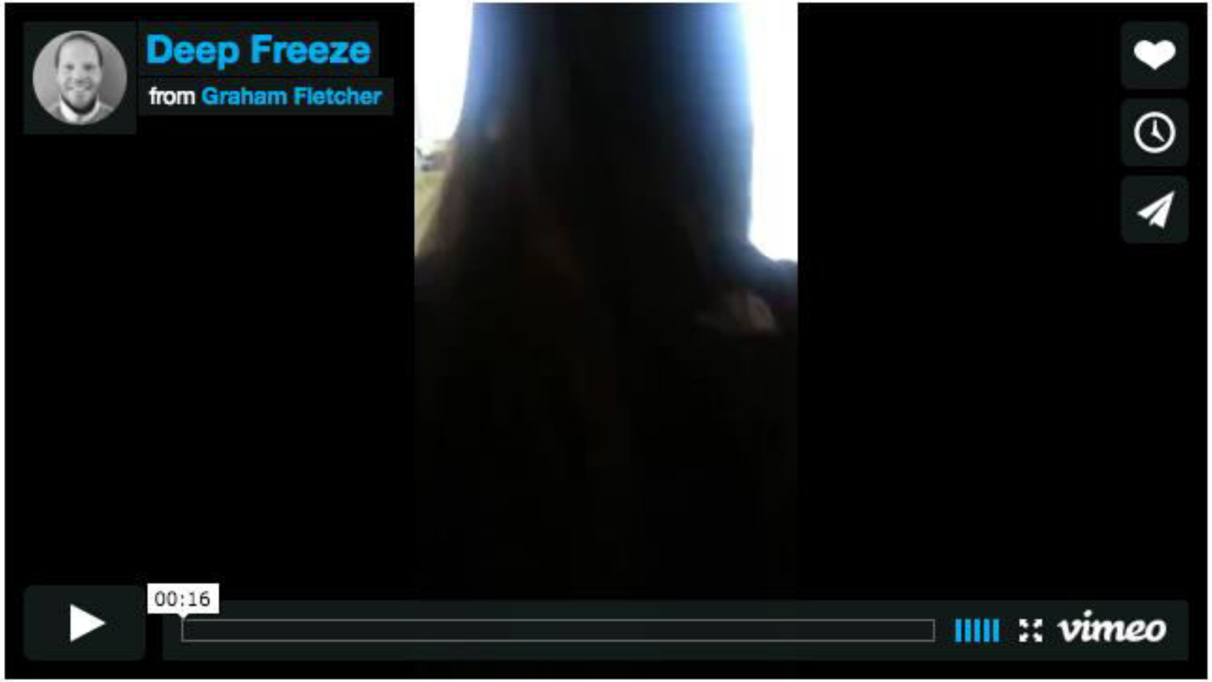

Green Route
SCORE
170

3-Act Task (7th Grade 7.NS.3)

Deep Freeze

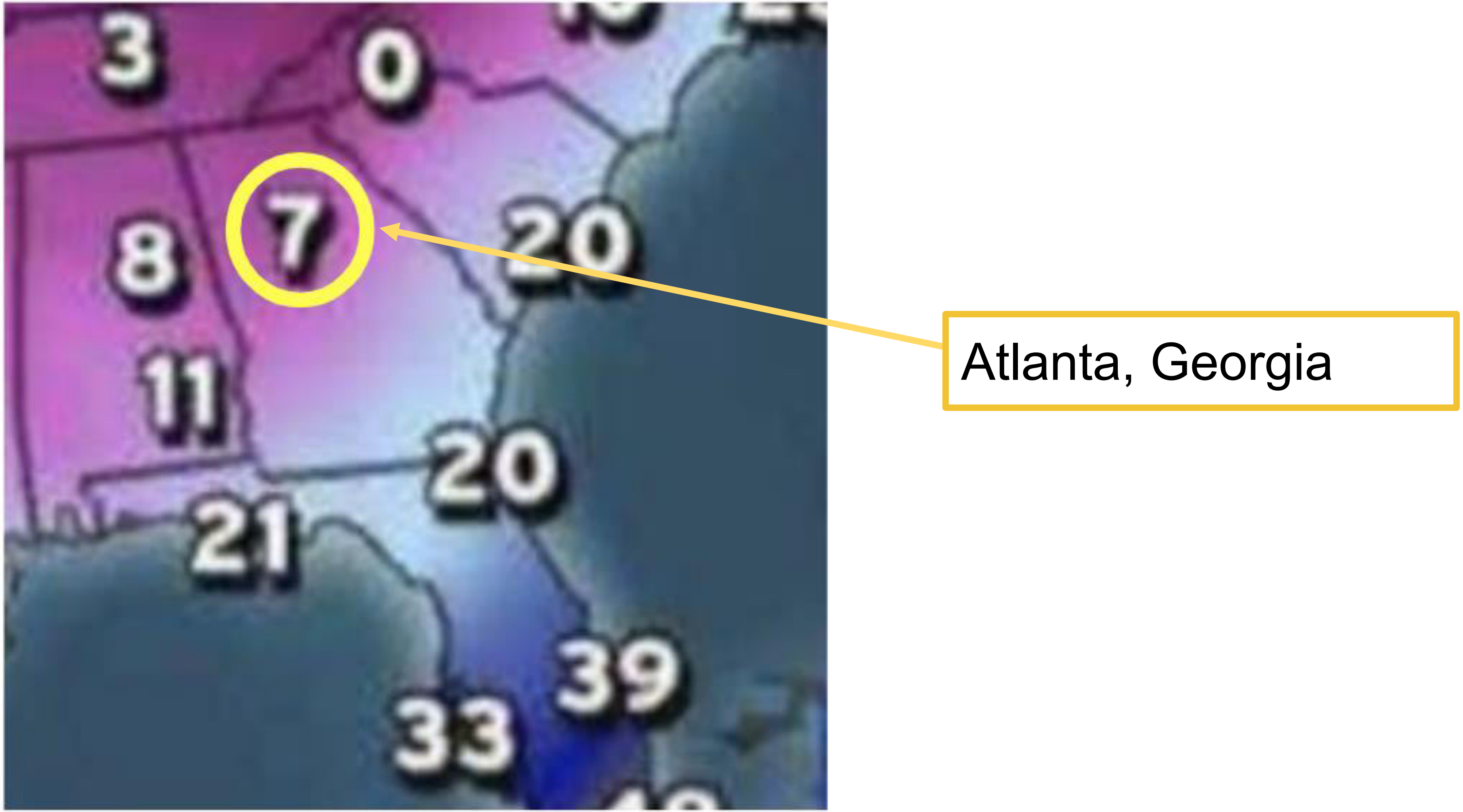
Credit to Graham Fletcher, gfletchy.com

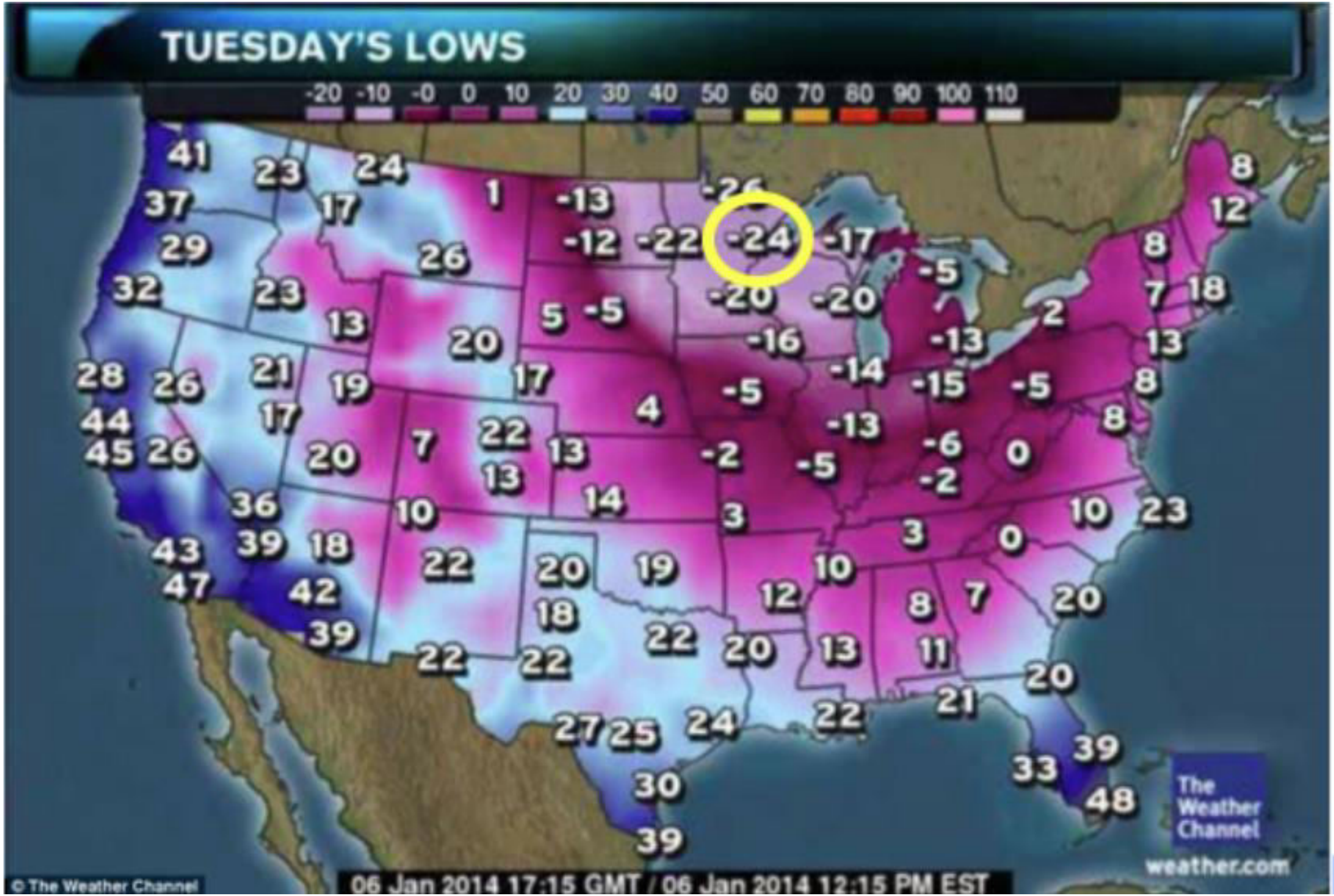
Watch the video



Lower limit

Upper limit


3
0
8
7
20
11
20
21
39
33
Atlanta, Georgia


TUESDAY'S LOWS
-20 -10 -0 0 10 20 30 40 50 60 70 80 90 100 110
41
23
24
37
17
1
-13
29
-12
-22
-24
-17
8
12
26
8
32
23
13
5
-5
-20
-20
-5
7
18
20
-16
-13
2
13
28
26
21
19
17
-5
-14
-15
-5
8
44
17
4
-13
8
45
26
20
7
22
13
-2
-5
-6
0
13
14
3
-2
36
10
10
23
43
39
18
22
20
19
10
3
0
47
42
18
12
8
7
20
39
22
22
22
20
13
11
20
21
27
25
24
22
39
33
48
30
39
The Weather Channel
weather.com
© The Weather Channel
06 Jan 2014 17:15 GMT / 06 Jan 2014 12:15 PM EST

Boston sees a 63-degree swing in two days

By Matt Rocheleau | GLOBE STAFF FEBRUARY 17, 2016

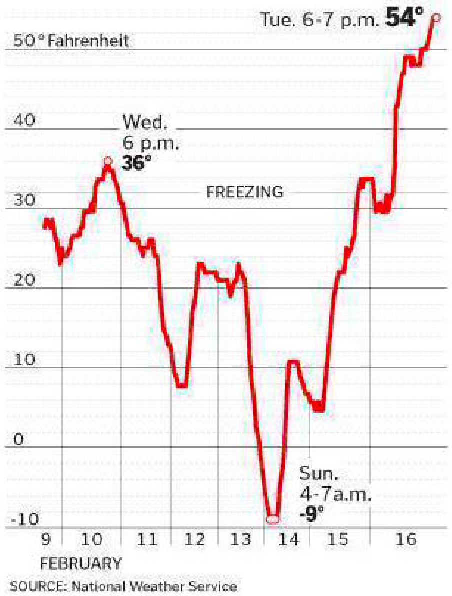
Talk about your fickle weather.

Late Saturday morning at Logan Airport, the high temperature reached the low 20s, according to data from the National Weather Service.

About 17 hours later, just before 4 a.m. Sunday, the mercury bottomed out at minus 9 degrees.

Temperatures warmed up slightly Sunday, and even more Monday. And by 6 p.m. Tuesday, it was a balmy 54 degrees.

That was a 63-degree swing from Sunday morning.



3-Act Task (8th Grade 8.G.9, 6.RP)

Commercial Break

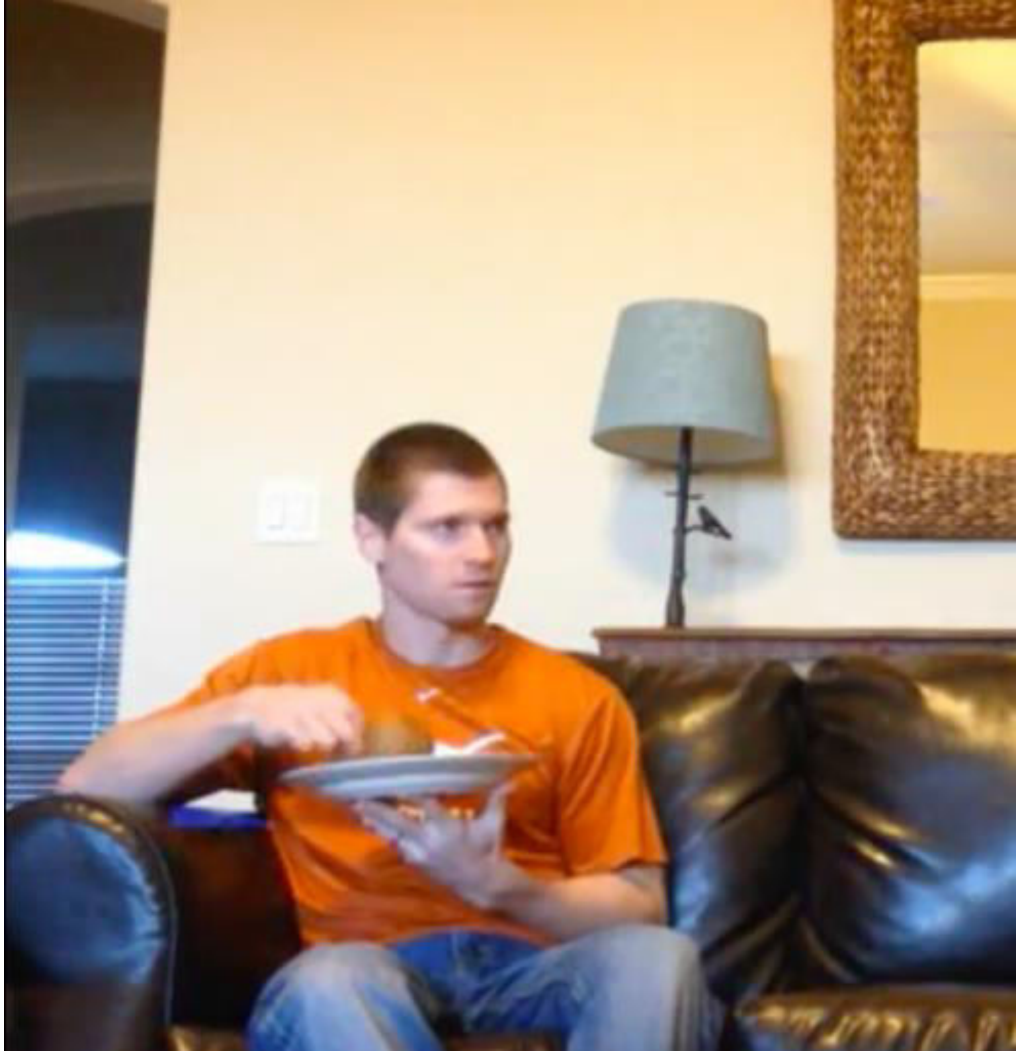
Credit to Dane Ehlert, gfletchy.com

Lower limit

Upper limit

Opening Bread

Sandwich

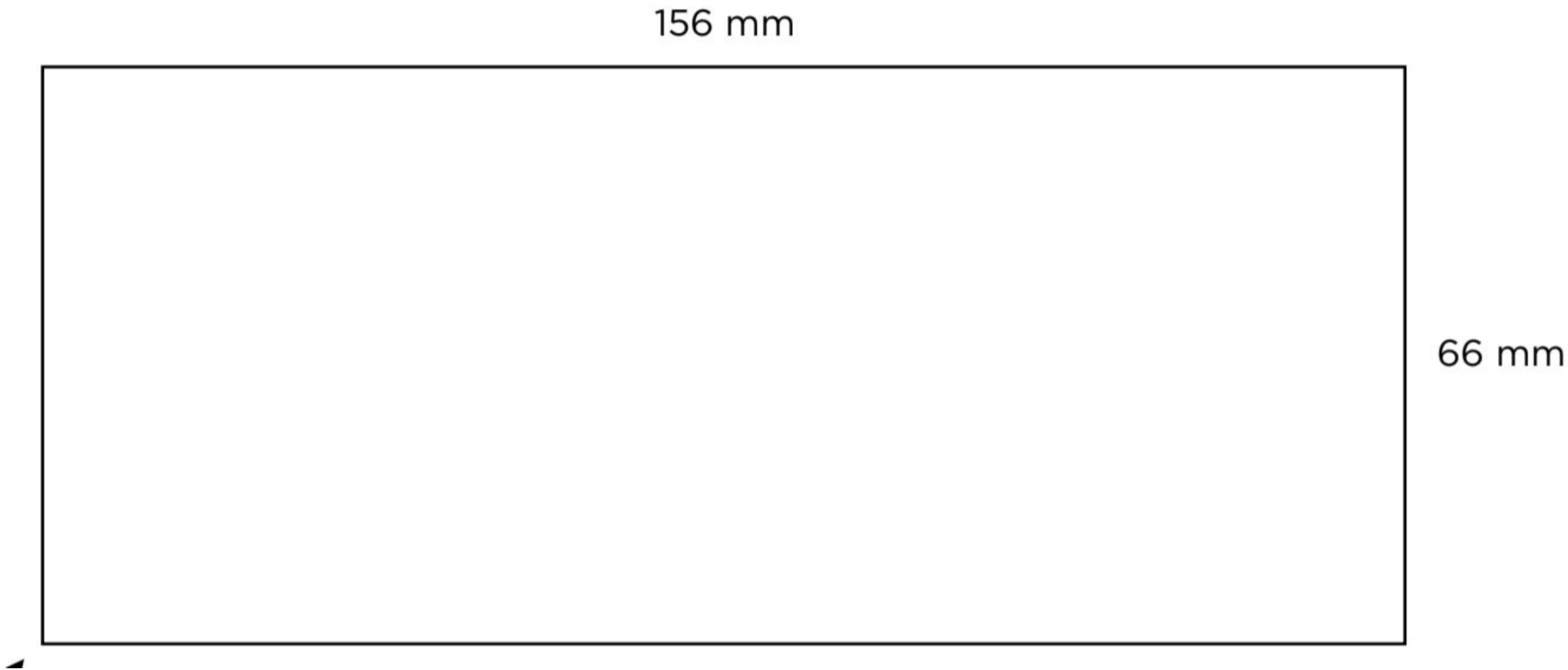


Credit to Dan Meyer, https://mrmeyer.com/threeacts/shrinkingdollar/

3-Act Task

Incredible Shrinking Dollar

Credit to Dan Meyer, https://mrmeyer.com/threeacts/shrinkingdollar/


156 mm
66 mm

Framework Connections for 3-Act Task: Shrinking Dollar

5th: In preparation for grade-six work with ratios and proportional reasoning, fifth-grade students interpret multiplication as scaling (resizing) **[5.NF.5▲]** by examining how numbers change as the numbers are multiplied by fractions. Students should have ample opportunities to examine the following cases: (a) that when multiplying a number greater than 1 by a fraction greater than 1, the number increases; and (b) that when multiplying a number greater than 1 by a fraction less than one, the number decreases. This is a new interpretation of multiplication that needs extensive exploration, discussion, and explanation by students.

6th: A critical area of instruction in grade six is to connect ratio, rate, and percentage to whole-number multiplication and division and use concepts of ratio and rate to solve problems. Students' prior understanding of and skill with multiplication, division, and fractions contribute to their study of ratios, proportional relationships, unit rates, and percentage in grade six. In grade seven, these concepts will extend to include scale drawings, slope, and real-world percent problems. (**6.RP▲**)

Thin Slicing Using Connecting Representations

A Routine easily adapted from Card Sorts and Textbooks

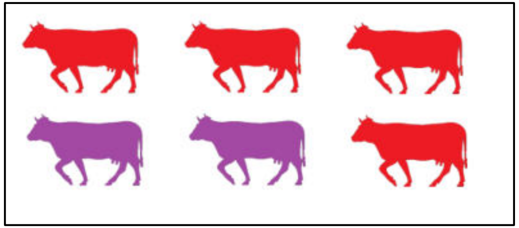
- angry
- loving
- sad

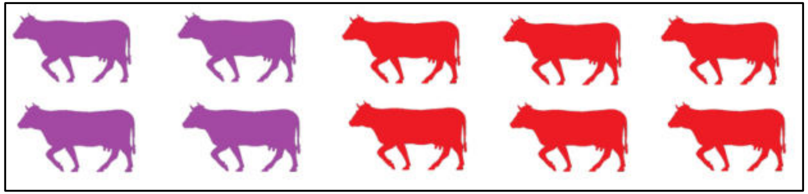
I noticed……………..so I…………

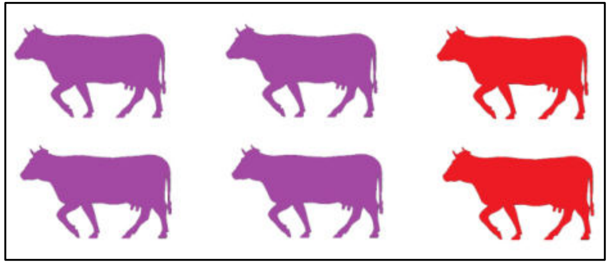
6th Focus

2 out of 3 of my cows are purple.

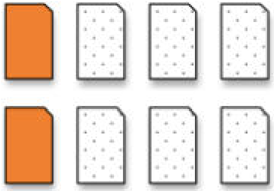
The purple to red ratio for my cows is 2 to 3.



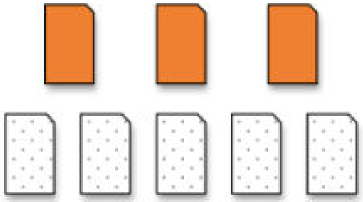




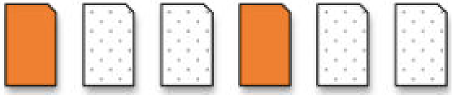
O Shade in:



B



A



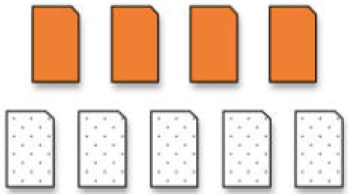
G

For every orange there are 2 sodas

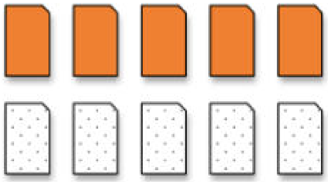
I

One fourth of the mixture is orange

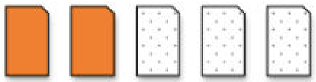
C



M Shade in:



D



H

Orange : Soda
= 4 : 5

F

Half of the mixture is orange

L

For every soda there is $\frac{2}{3}$ orange

K

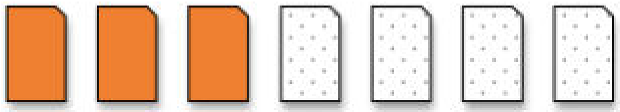
For every orange there is $1\frac{1}{3}$ soda

J

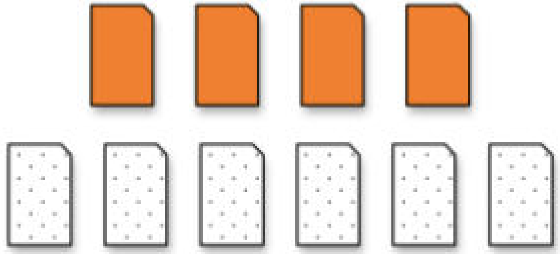
$\frac{2}{3}$ of the mixture is soda

P

Shade in:



E



7th Focus

$d = 6t$

The nacho sales made \$32.50 in 5 hours.

Rachel walked 45 feet in 9 seconds.

$d = 5t$

Sally walked 42 feet in 7 seconds.

t	P
0	0
2	12.50
4	25
6	37.50

t	d
0	0
2	10
4	20
6	30

t	P
0	0
2	13
4	26
6	39

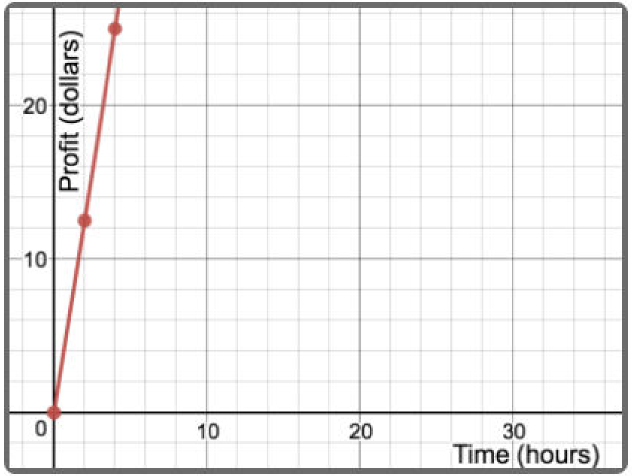
The nacho sales made \$32.50 in 5 hours.

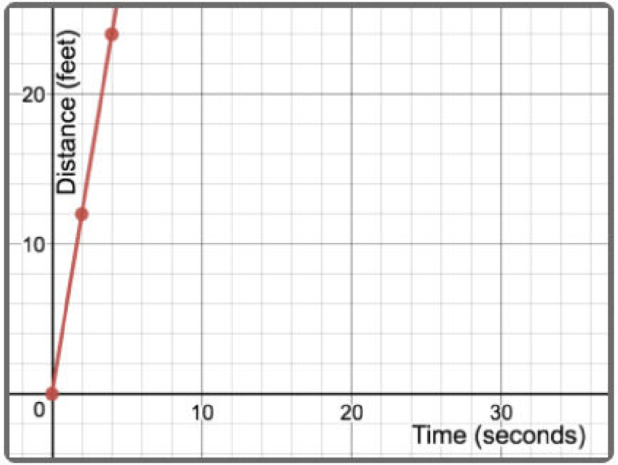
The snow cone sales made \$50 in 8 hours.

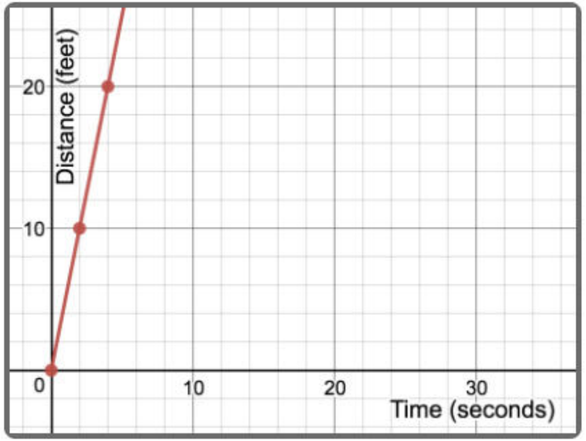
t	d
0	0
2	12
4	24
6	36

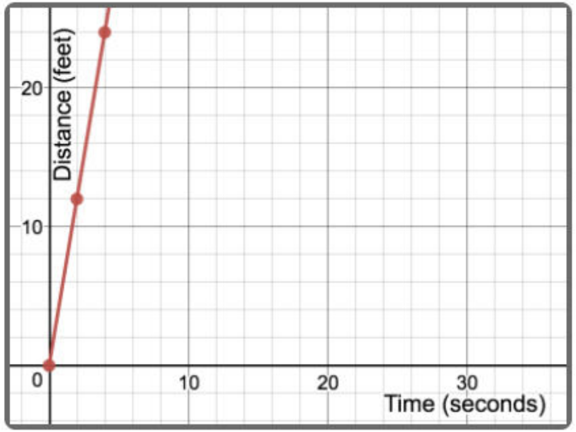
t	P
0	0
2	12.50
4	25
6	37.50

t	d
0	0
2	10
4	20
6	30









The snow cone sales made $50 in 8 hours.

Rachel walked 45 feet in 9 seconds.

Sally walked 42 feet in 7 seconds.

8th Focus

Demyla went to the store and bought notebooks and pens. Notebooks were 7 a package and pens were 2. She bought 3 things and spent $11.

Omar scored 2 pointers and 3 pointers in the basketball game. He made 7 baskets.

Larry went to the store and bought apples and oranges. Apples were 2 a pound and Oranges were 3 a pound. He bought 7 pounds of fruit and spent $19.

$$2x + 3y = 19$$

$$x + y = 7$$

$$2x + 3y = 17$$

$$x + y = 7$$

) $y = x + 2$

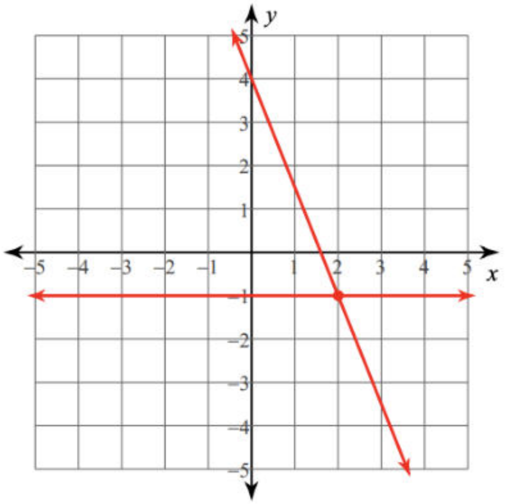
$x = -3$

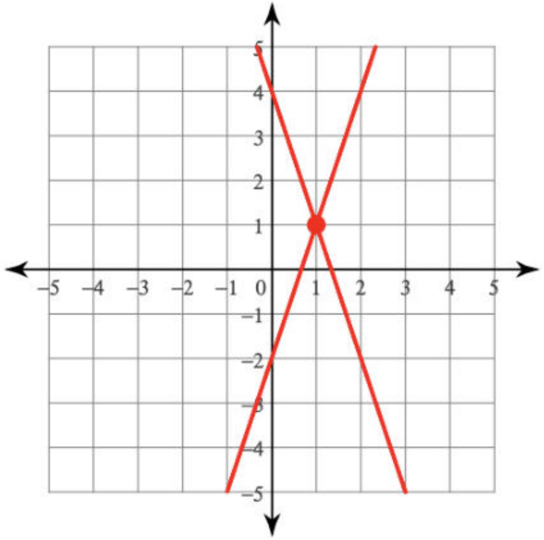
) $y = -3x + 4$

$y = 3x - 2$

$y = -1$

$y = -\frac{5}{2}x + 4$





) $4x + y = 1$

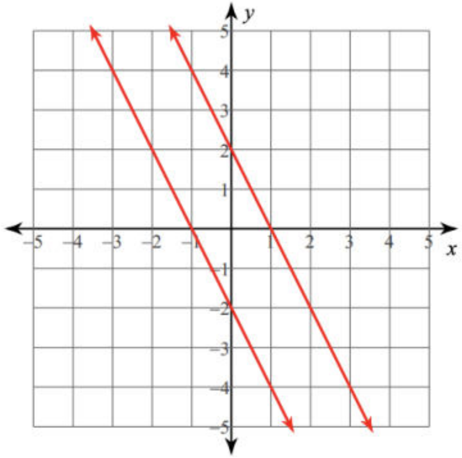
$x + y = -2$

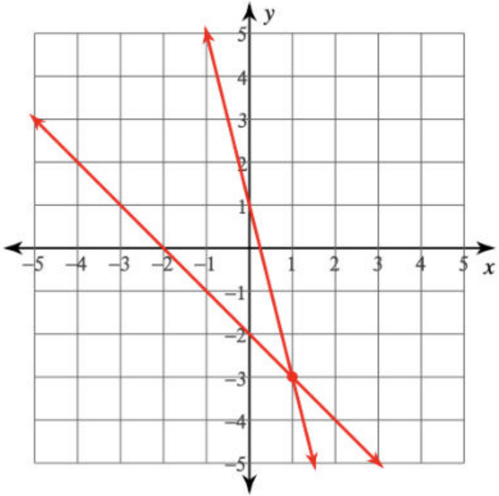
$y = -3x + 4$

$y = 3x - 2$

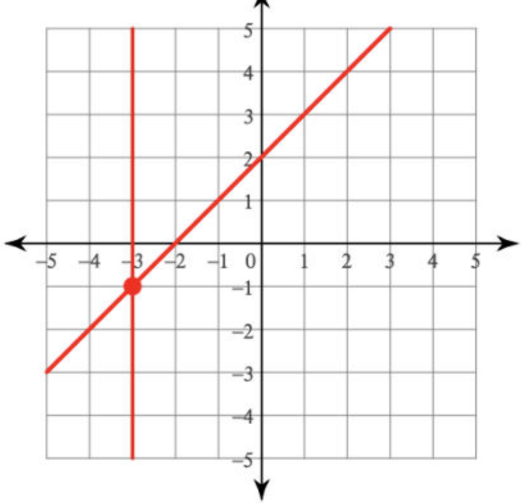
$y = -2x + 2$

$y = -2x - 2$

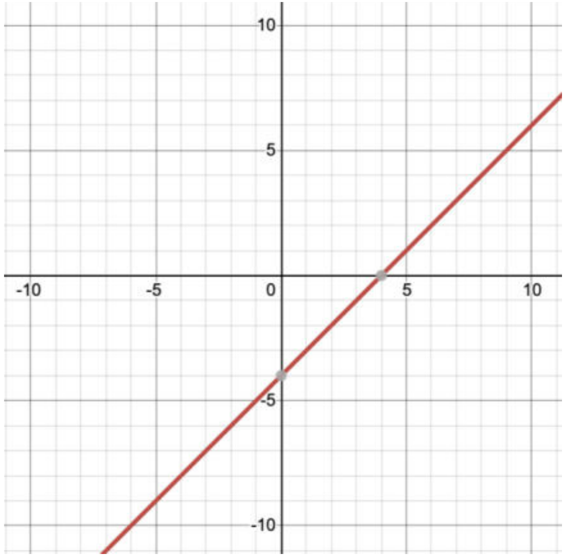




$$-2x = -8 - 2y$$
$$-2y - 8 = -2x$$



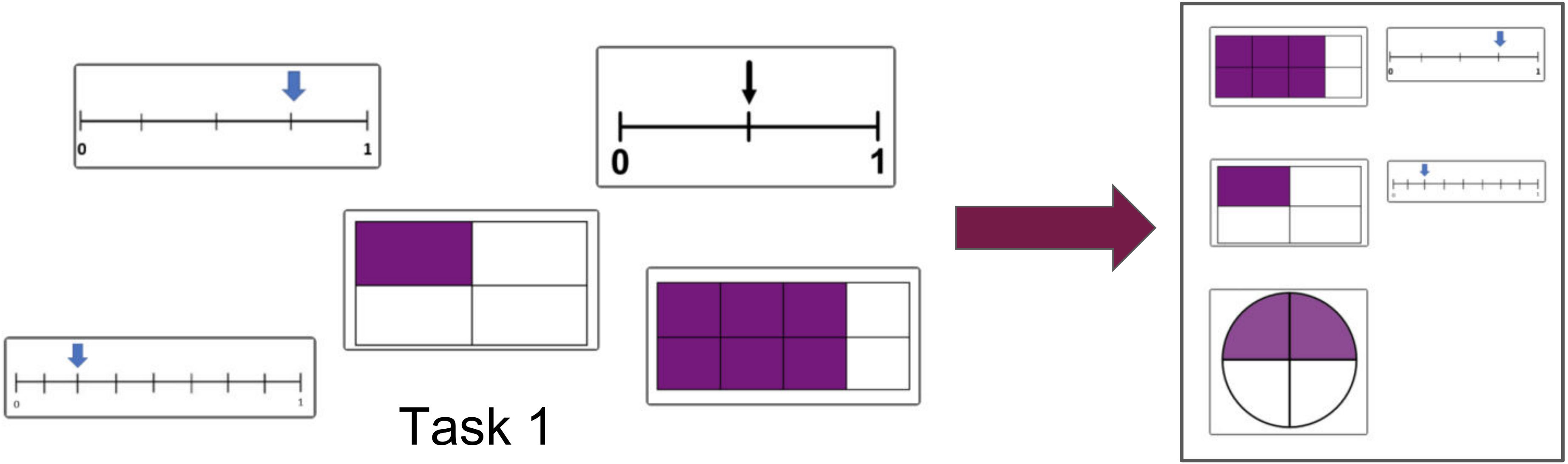
$$y = x + 2$$
$$x = -3$$



$$y = -2x + 2$$
$$y = -2x - 2$$

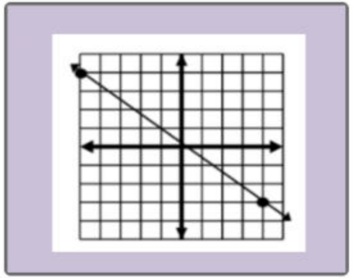
Resources for Card Sorts/Connecting Representations

- Card Sorts (Any sets where you can keep 2 matching sets and one representation, equation, word problem/statement, table, or graph left w/out its match.)
- Example for Fractions/Decimals ([Desmos]) To find Card sorts to use as thin slicing by connecting representations, open a tab and type desmos activity+**topic** card sort (for example: desmos activity+fraction card sort)

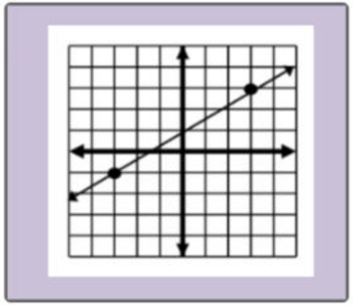


Task 1

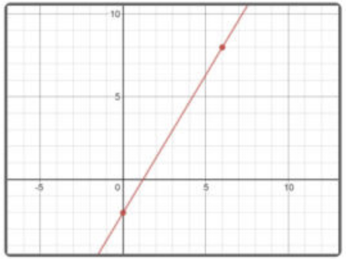
Negative



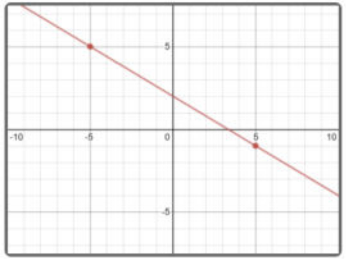
Positive



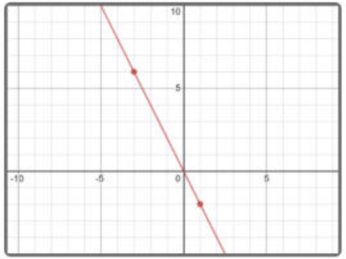
Zero



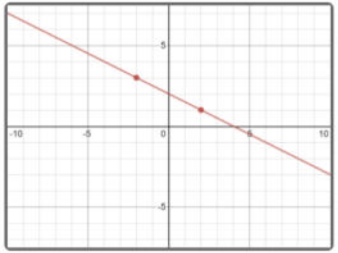
$m = \frac{5}{3}$



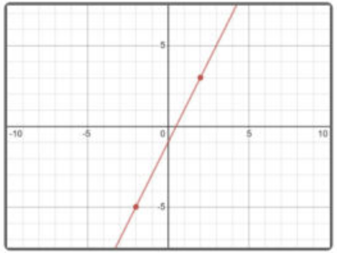
$m = -\frac{3}{5}$

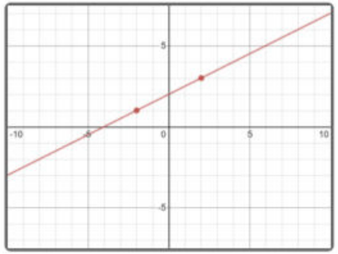


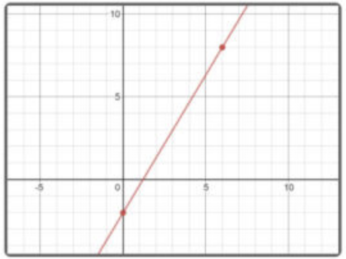
$m = -\frac{1}{2}$



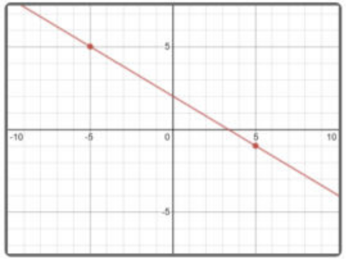
$m = 2$



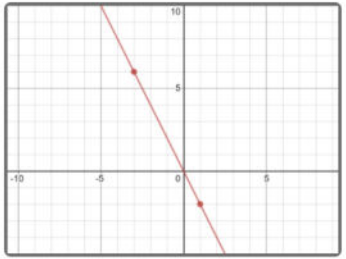




$m = \frac{5}{3}$



$m = -\frac{3}{5}$



8.EE.B.5-6

Matching Equations and Lines Using Connecting Representations

A Routine for Thin Slicing…

Connecting Representations

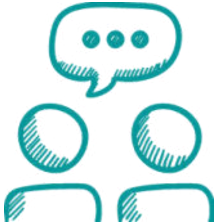
WHAT: Match visuals to expressions by **chunking** and **connecting** to math you know

WHY: To "think like mathematicians", to use mathematical *structure* to match two different representations.

Credit to: http://www.fosteringmathpractices.com/connecting-representations/

Connecting Representations

Think

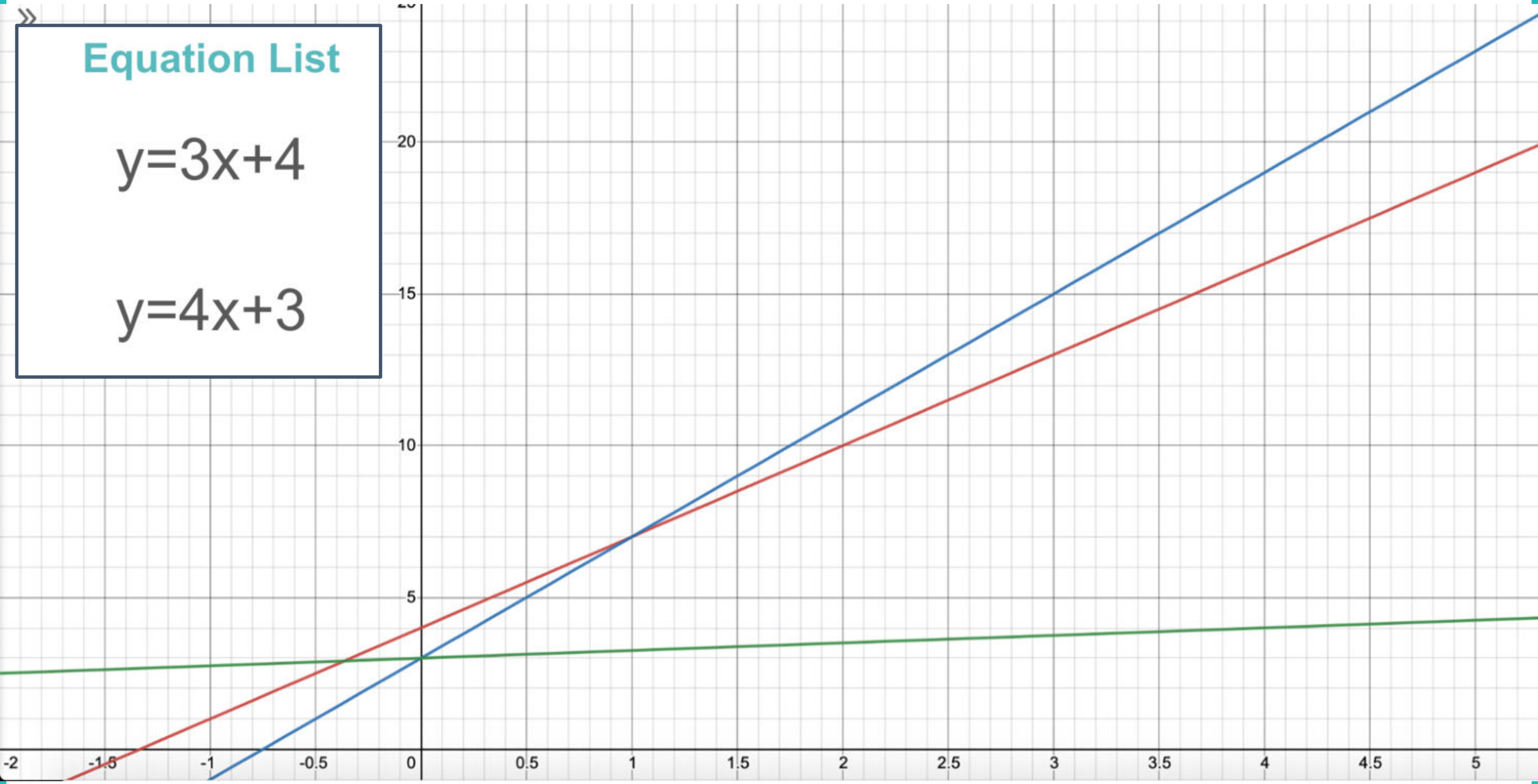


Make connections

Share & study connections

Create representation

Reflect on learning


Equation List
y=3x+4
y=4x+3
20
15
10
5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Think

Ask yourself…

What part of the *visual* will help me connect to a chunk of the *equation*?

What about the *equation* will help me connect to the *visual*?

Make Connections

"I noticed… so I looked for..."

"... connects to ... because..."

Share and Study Connections

We noticed… so we ...
We knew... so we...

They noticed… so they ...
They knew... so they...

Create a Representation

THINK

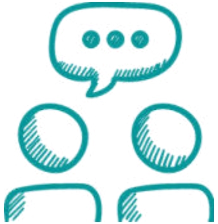
Ask yourself…

What do I notice about this equation?

How can I chunk this equation

into pieces I can represent with a visual?

Create a Representation



PAIR

Share your interpretations of the equation.
Together, create a matching visual

Create a Representation

SHARE

They noticed…..so they

When they saw….it made them think of….so they…

Meta-Reflection

A. When interpreting a *visual / expression*, I learned to pay attention to…

A. When connecting representations, I learned to ask myself…

A. A new mathematical connection I made was…

Possible Extensions from here….

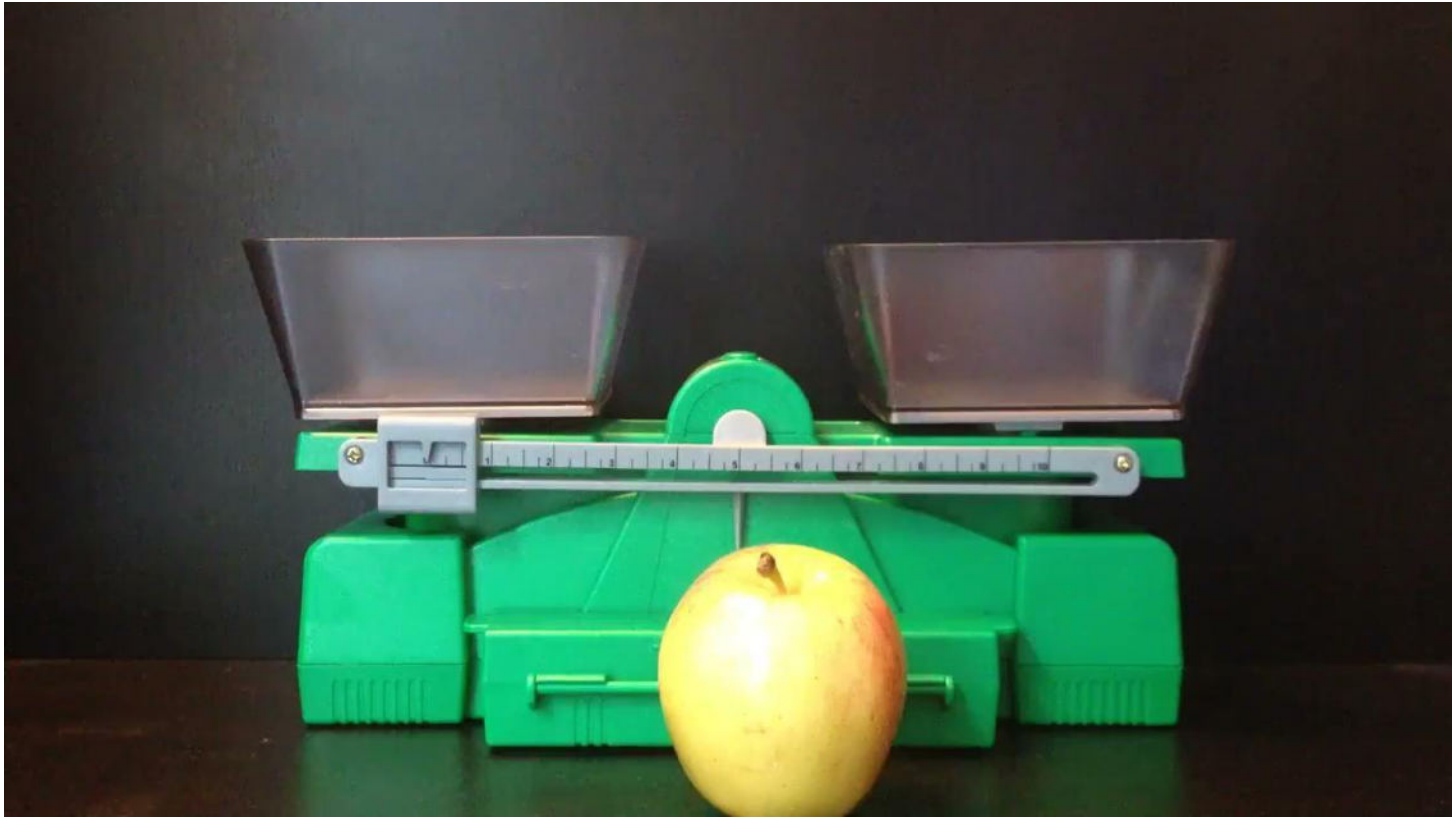
- Give them a line to graph (I was trying to keep it on this graph, but I guess it doesn't have to)
  - y=2x+1
  - y=(3/2)x+2
  - y=-2x+4
  - y=(-1/3x)+10
  - A line that has the same slope as the blue line and a y-intercept of 8
  - A line that has the same y-intercept as the red line and a slope of 4/5

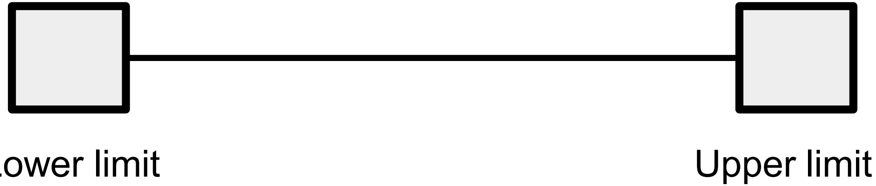
Thick Sliced Curriculum Task: (Use for conceptual understanding/big ideas)

Apple

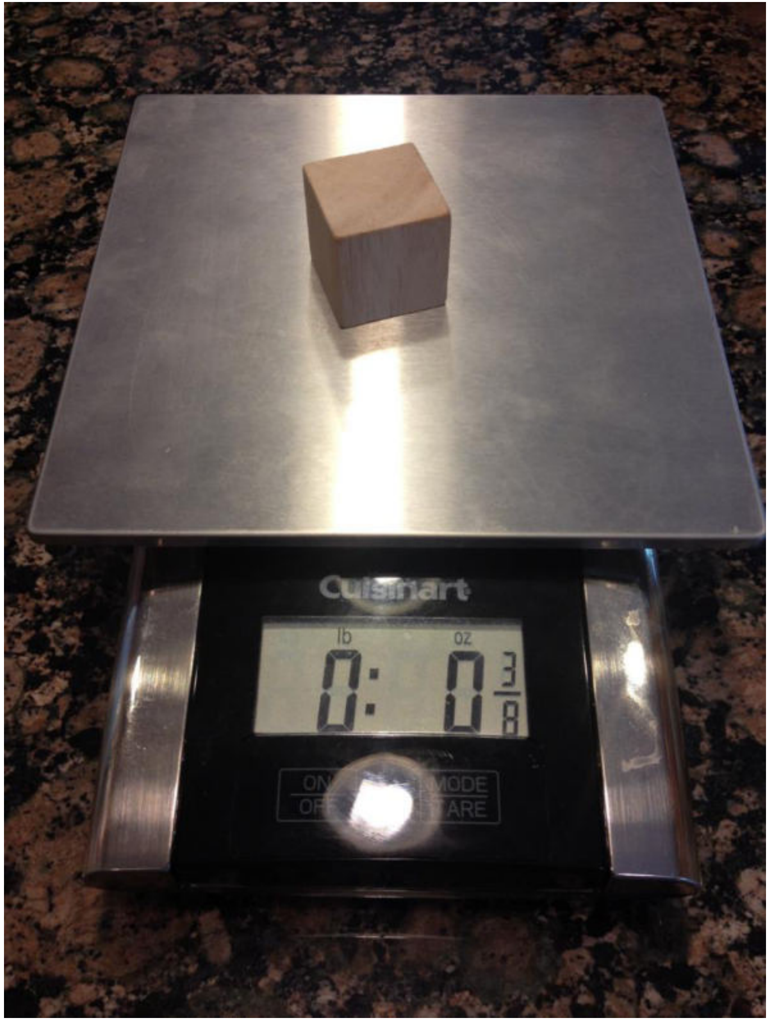
3-Act Task (6th Grade 6.NS.1)

Credit to Graham Fletcher, gfletchy.com




Lower limit
Upper limit


Cuisinart
lb
oz
0: 5 1/4
MODE
TARE

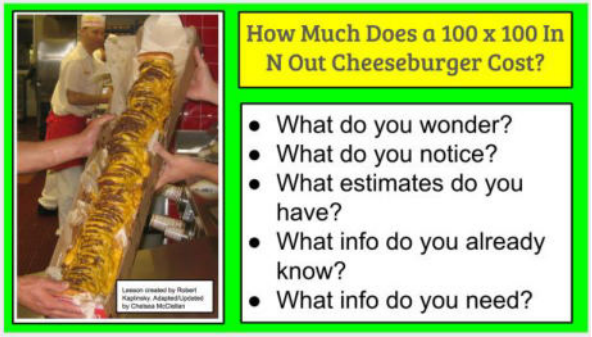

Cuisinart
lb
oz
0: 0 3/8
MODE
TARE


168.00
ON/OFF
TARE
UNIT

Thick Sliced: Problem-based Lessons

http://bit.ly/BARBIEzipline

http://bit.ly/100x100InNout



http://bit.ly/Mullet-y

http://bit.ly/MotorcycleTicket

http://bit.ly/DoritoRoulette

http://bit.ly/Real-LifeJenga

Thin Slicing Examples

Thin Slicing Example: The Unusual Bakers Task

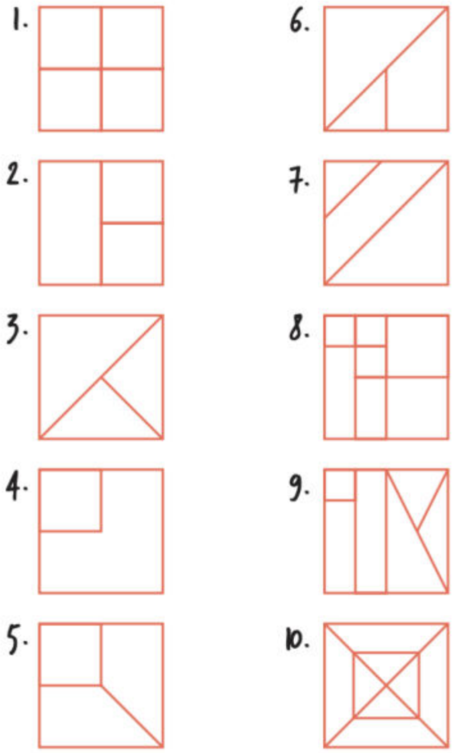
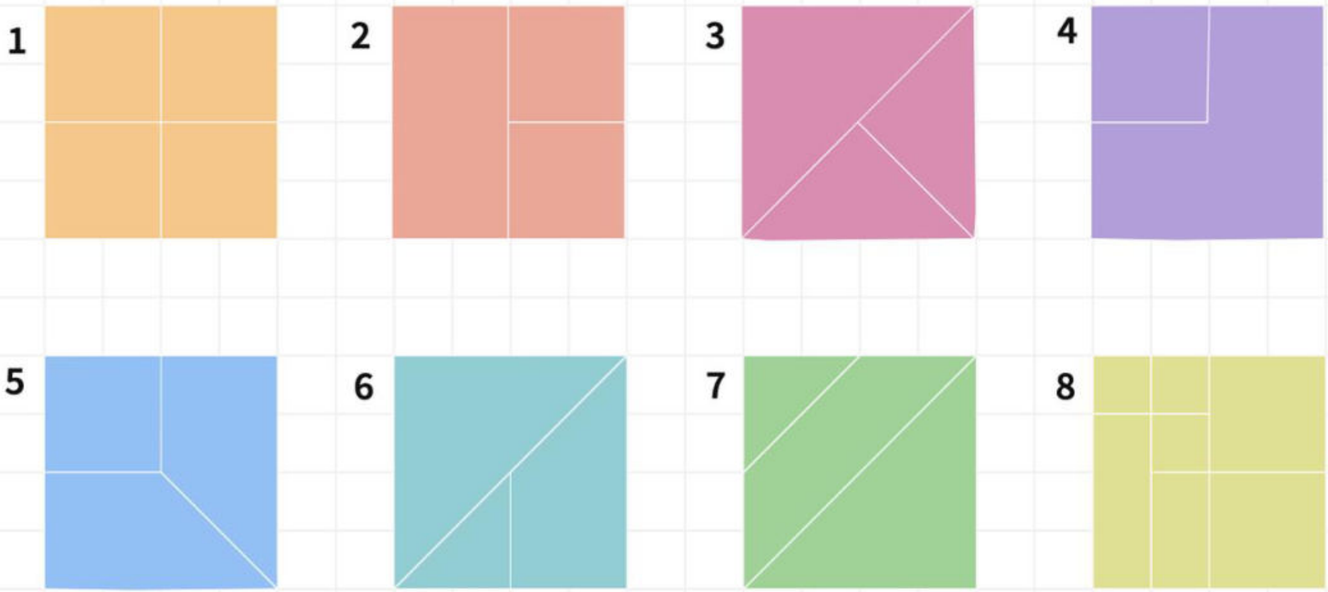
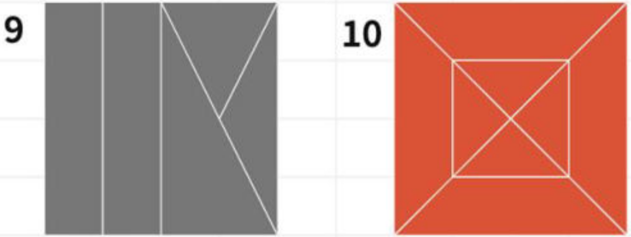


Figure 9.5 The unusual baker's cakes.

- What is the minimum amount of information you can give during the launch without teaching them something?
- Start with a bit of review
- Extend the thinking
- How will you keep them in flow?





Mrs. G's geometry class is having a bake sale.
A whole cake is $10. What is the **price of each piece**?

adapted from The Unusual Baker Task NCTM 2012 as reference in Building Thinking Classroom by @normabgordon

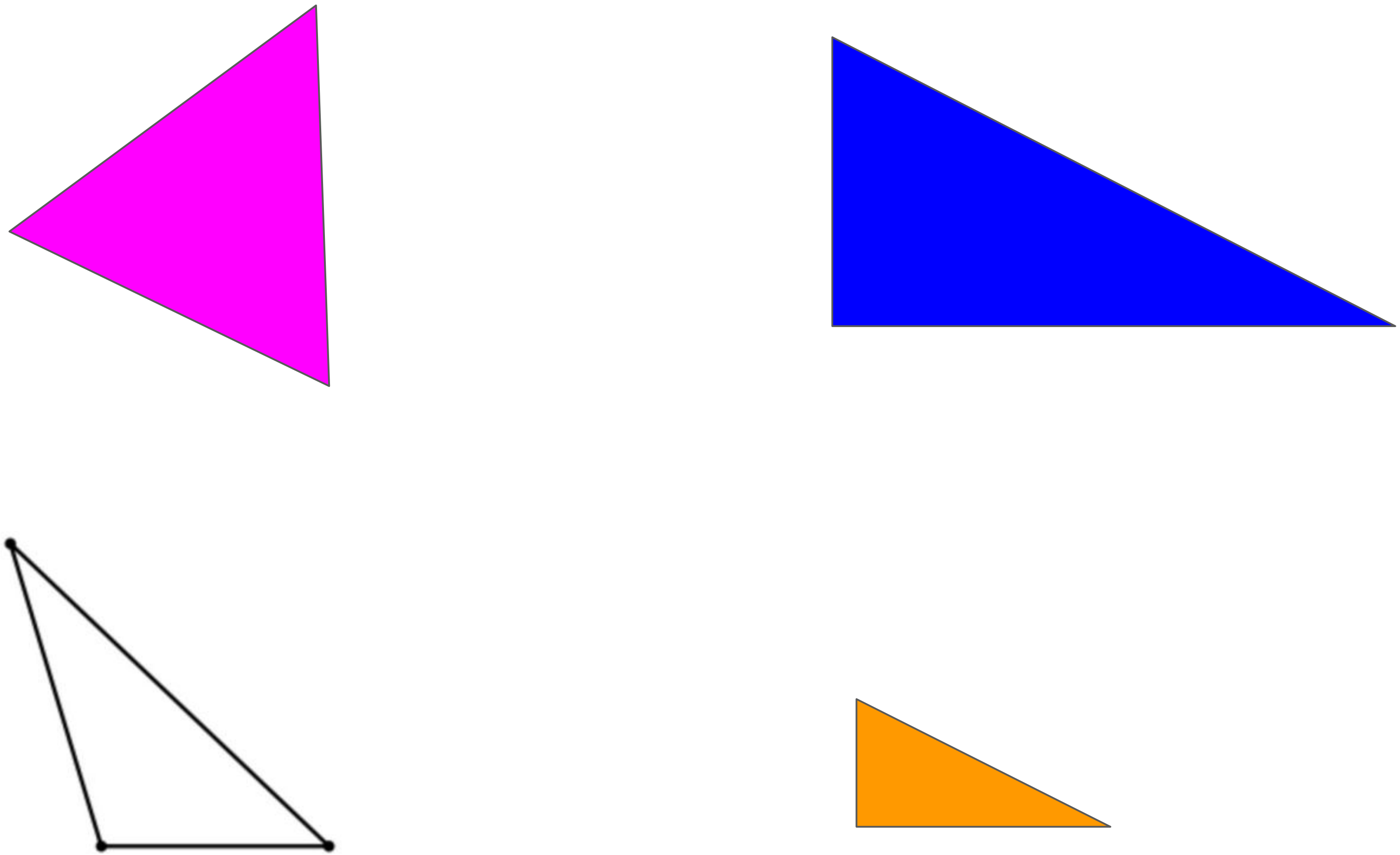
Stretching Trees (%s)

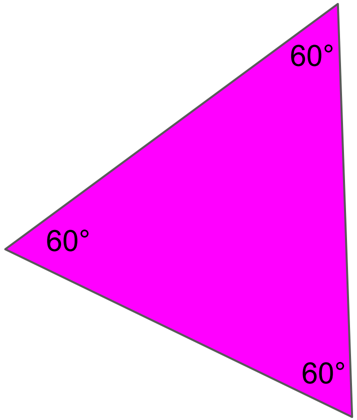
Stretching Trees Task

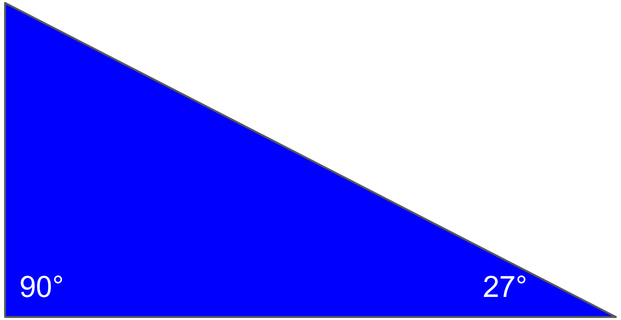
Stretching Trees: Alternative Launch

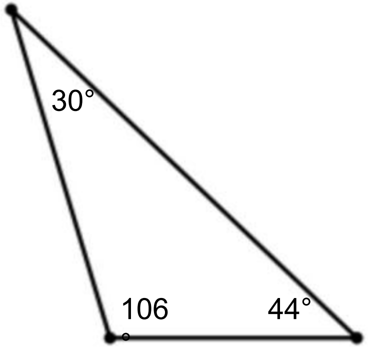
⅕ of 100 units

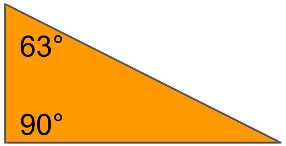
Triangle Sums




60°
60°
60°


90°
27°


30°
106°
44°


63°
90°

$\square\square + \square\square + \square\square$

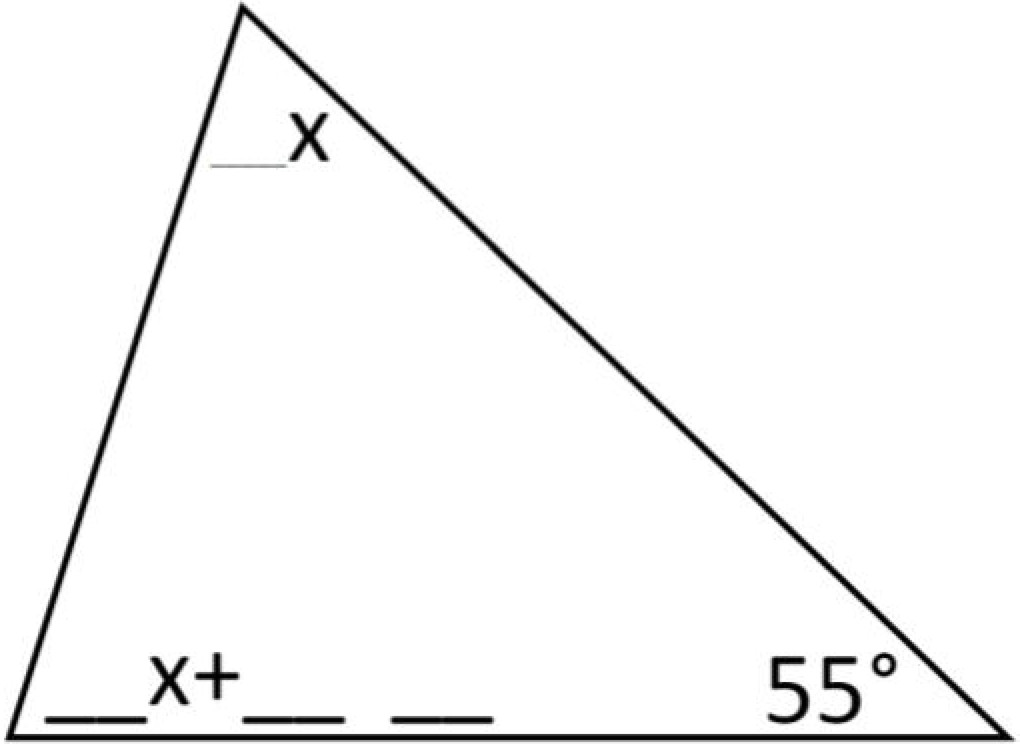
0	1	2	3	4	5	6	7	8	9

OPEN MIDDLE - Triangle Sum Theorem

Directions: Using the digits 1-9 at most one time each, fill in the blanks so that when you solve for x, it is a whole number.

Tags 8.G.5 DOK 3: STRATEGIC THINKING FRANCO D. ADKINS

1	2	3	4	5
6	7	8	9	



Value of *x* (type in below)

Equations w/Variables on Both Sides (one, no, and infinite solutions)

[Slide Deck for complete lesson]

Whole Numbers Place Value
Open Middle Thinking Classroom

1a) Using only the digits 1-9 one time each, what is the **largest** 3 digit number you can make?

1b) Using only the digits 1-9 one time each, what is the **smallest** 3 digit number you can make?

1c) How can you prove it?

2a) Using only odd numbers one time each,
what is the **largest** 3 digit number you can make?

2b) How can you prove it?

3a) Using only even numbers up to two times each,
what is the **smallest** 3 digit number you can make?

3b) How can you prove it?

4a) Using only the digits 1-9 one time each, what is the **closest 3 digit number to** 400 you can make?

4b) How can you prove it?

5a) Using only the digits 1-9 one time each,
what is the **largest** 4 digit number you can make?

5b) Using only the digits 1-9 one time each,
what is the **smallest** 4 digit number you can make?

5c) How can you prove it?

6a) Using only the digits 1-9 one time each, what is the **closest 4 digit number to 8000** you can make?

6b) How can you prove it?

7a) Using only the digits 1-9 one time each, what is the **closest 4 digit number to 1927** you can make?

7b) How can you prove it?

<u>Differentiation</u>
-if struggling: give a hint then once solved give a similar question
-if advanced past these: choose different target numbers or numbers to ten thousands, hundred thousands, or millions

Decimal Place Value Open Middle Thinking Classroom

$ $\square . \square \square$

1a) Using only the digits 1-9 one time each, what is the **highest** price you can make?

1b) Using only the digits 1-9 one time each, what is the **lowest** price you can make?

1c) How could you prove it?

$ □ . □ □

2a) Using only even numbers one time each, what is the **highest** price you can make?

2b) How can you prove it?

$ □ . □ □

3a) Using only the digits 1-9 one time each, what is the **closest price to $3.00** you can make?

3b) How could you prove it?

$ $\square$. $\square$ $\square$

4a) Using only the digits 1-9 one time each, what is the closest price to **$8.88** you can make?

4b) How could you prove it?

$ $\square\square.\square\square$

4a) Using only the digits 1-9 one time each, what is the **highest** price you can make?

4b) Using only the digits 1-9 one time each, what is the **lowest** price you can make?

4c) How could you prove it?

$ □□.□□

5a) Using only odd numbers one time each, what is the **lowest** price you can make?

5b) How can you prove it?

$ □□.□□

6a) Using only the digits 1-9 one time each, what is the **closest price to $60.00** you can make?

6b) How could you prove it?

$ $\square\square.\square\square$

7a) Using only the digits 1-9 one time each, what is the closest price to **$22.22** you can make?

7b) How could you prove it?

<u>Differentiation</u>

-if struggling: give a hint then once solved give a similar question

-if advanced past these: choose different target numbers or use numbers to more challenging place values

Thin Slicing Examples in progress:

5th Grade Team: Powers of Ten Dividing Decimals

6th Grade: Order of Operations - Gemini Puzzles

Multiple Grades: Increasingly Difficult Questions

3 Reads Launch

Orange Problem

A grocer was asked how many oranges he had sold that day. He replied:

> "My first customer said I'll buy half your oranges and half an orange more."
>
> He then said, "My second customer said the same thing… I'll buy half your oranges and half an orange more."
>
> Then he stated, "My third customers said the same thing... I'll buy half your oranges and half an orange more."
>
> Finally, he stated, "When I had filled all three orders I was sold out and I did not have to cut a single orange all day."

How many oranges had the grocer sold in all?

What if there were four customers? Five customers? Ten customers? Any number of customers?

A grocer was asked how many oranges he had sold that day. He replied:

"My first customer said I'll buy <u>part of</u> your oranges and <u>part of</u> an orange more."

He then said, "My second customer said the same thing…I'll buy <u>part of</u> your oranges and <u>part of</u> an orange more."

Then he stated, "My third customers said the same thing…I'll buy <u>part of</u> your oranges and <u>part of</u> an orange more."

Finally he stated, "When I had filled all three orders I was sold out and did not have to cut a single orange all day."

A grocer was asked how many oranges he had sold that day. He replied:

"My first customer said I'll buy half your oranges and half an orange more."

He then said, "My second customer said the same thing…I'll buy half your oranges and half an orange more."

Then he stated, "My third customers said the same thing…I'll buy half your oranges and half an orange more."

Finally he stated, "When I had filled all three orders I was sold out and did not have to cut a single orange all day."

How many oranges had the grocer sold in all?

_5_ children get onto a bus. In each hand, they have _5_ bags. In each bag there are _5_ cats. Each cat has _5_ kittens.

How many legs are on the bus?

Together, Evan, Katie, and McKenna had $\$$ 865 when they left to go shopping. Evan spent 2/5 of his money. Katie spent $\$$ 40 . McKenna spent twice as much as Evan. They each have the same amount of money left.

How much money did each take shopping with them?

Kaden is on a geocache hunt. His GPS tells him that he is __40__ meters away from the treasure. He walks __24__ meters due west. The GPS compass now tells him that the treasure is due south from where he is standing.

How far south does he need to go to find it?

A fruit salad consists of blueberries, raspberries, grapes, and cherries. The fruit salad has a total of 280 pieces of fruit. There are two times as many raspberries as blueberries, three times as many grapes as cherries, and four times as many cherries as raspberries.

How many cherries are there in the fruit salad?

Three-tenths of the wooden tiles were painted blue and One-fourth of them were painted green. Half of the remaining tiles were painted red and half were painted yellow.

If 300 tiles are blue, how many are there of each of the other colors?

Tom planted vegetables in a rectangular garden that was 25 feet long and 15 feet wide. He used 1/3 of the area for corn and 1/5 of it for peas.

How many square feet are left for other vegetables?

Tom planted vegetables in a rectangular garden that was 2.5 feet long and 15 feet wide. He used 1/3 of the area for corn and .2 of it for peas.

How many square feet are left for other vegetables?

A slab of soap on one pan of a scale balances $\frac{3}{4}$ of a slab of soap and a $\frac{3}{4}$ pound weight on the other pan.

How much does the full slab of soap weigh?

A baker used 12 cups of batter to make muffins. It took 2/3 cup of batter to make 1 muffin.

How many muffins did the baker make?

A bag of flour and 3 equal-sized bags of sugar have a total weight of 26 pounds. The bag of flour weighs 8 pounds.

What is the weight of each bag of sugar?

A Reflection...

On a scale of 1-5: 1 → 5

Very *Unlikely* … Very Likely

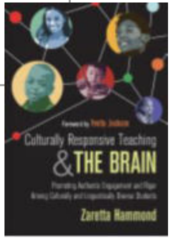
1) How **likely** is it that you will *offer an idea* to the group while thinking about our task(s) today?

1) How **likely** is it that the group will *decide to use one of your ideas* in a solution path for today's task(s)?

Where is the Equity?

The Dependent Learner	The Independent Learner
• Is dependent on the teacher to carry most of the cognitive load of a task always • Is unsure of how to tackle a new task • Cannot complete a task without scaffolds • Will sit passively and wait if stuck until teacher intervenes • Doesn’t retain information well or “doesn’t get it”	• Relies on the teacher to carry some of the cognitive load temporarily • Utilizes strategies and process for tackling a new task • Regularly attempts new tasks without scaffolds • Has cognitive strategies for getting unstuck • Has learned how to retrieve information from long-term memory

From Zaretta Hammond’s Culturally Responsive Teaching and the Brain: Promoting Authentic Engagement and Rigor among Culturally and Linguistically Diverse Students



Behind the Curtain

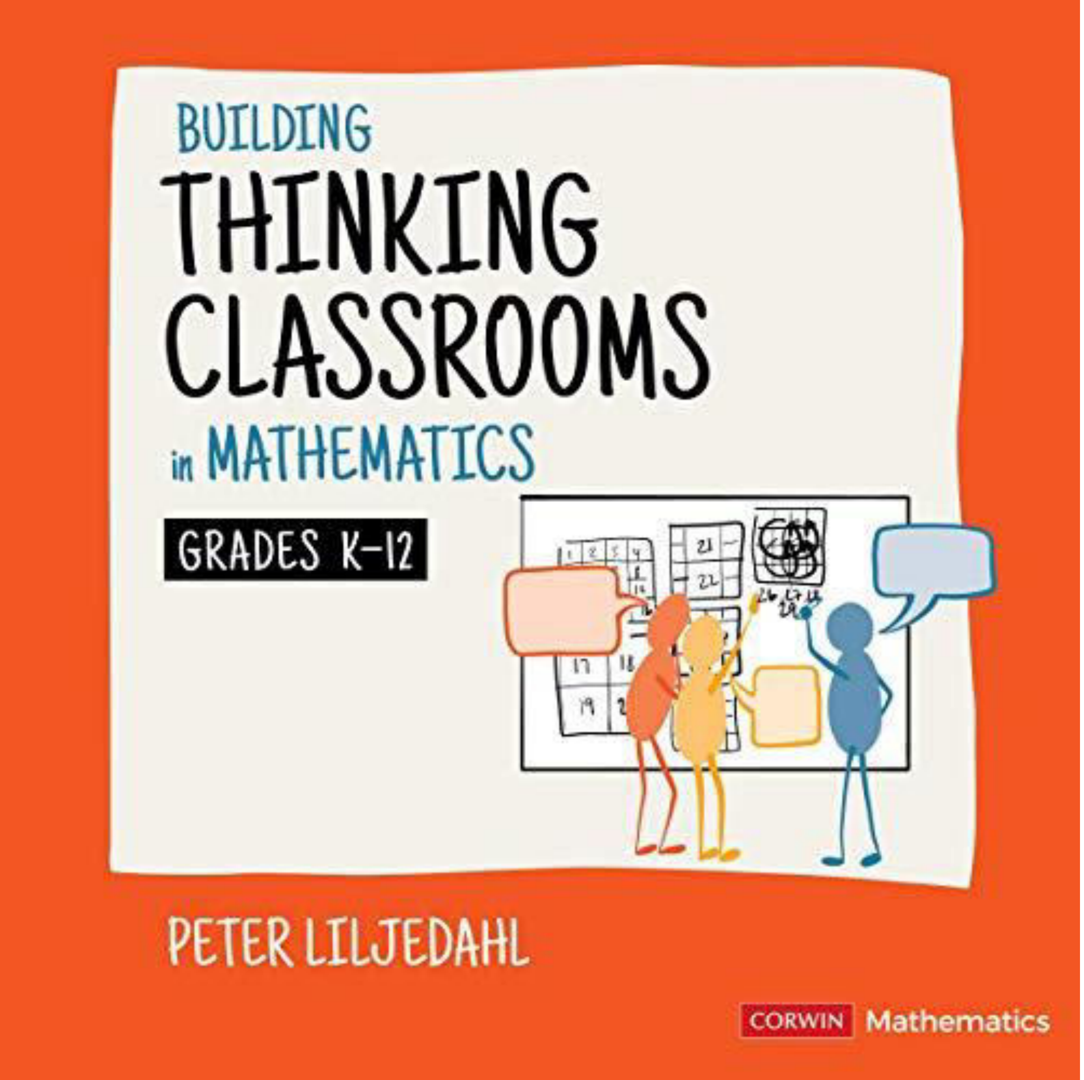
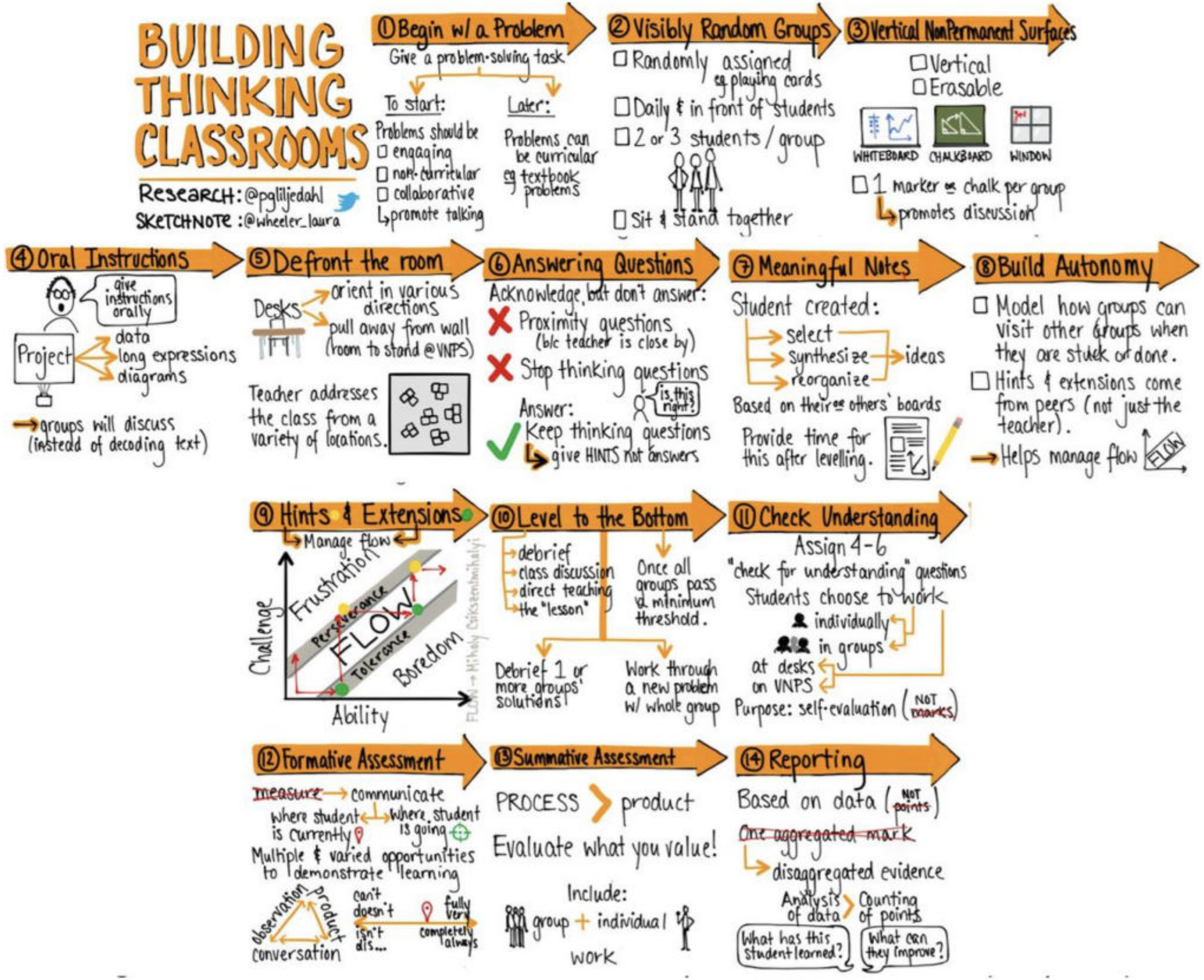
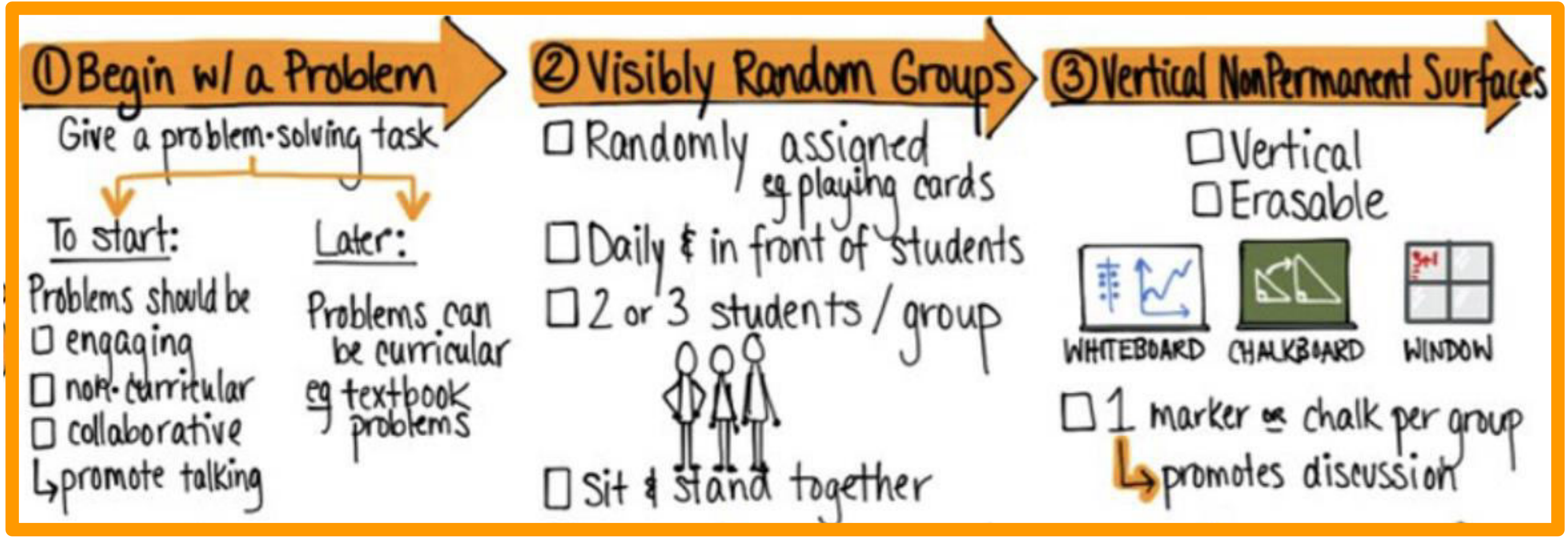


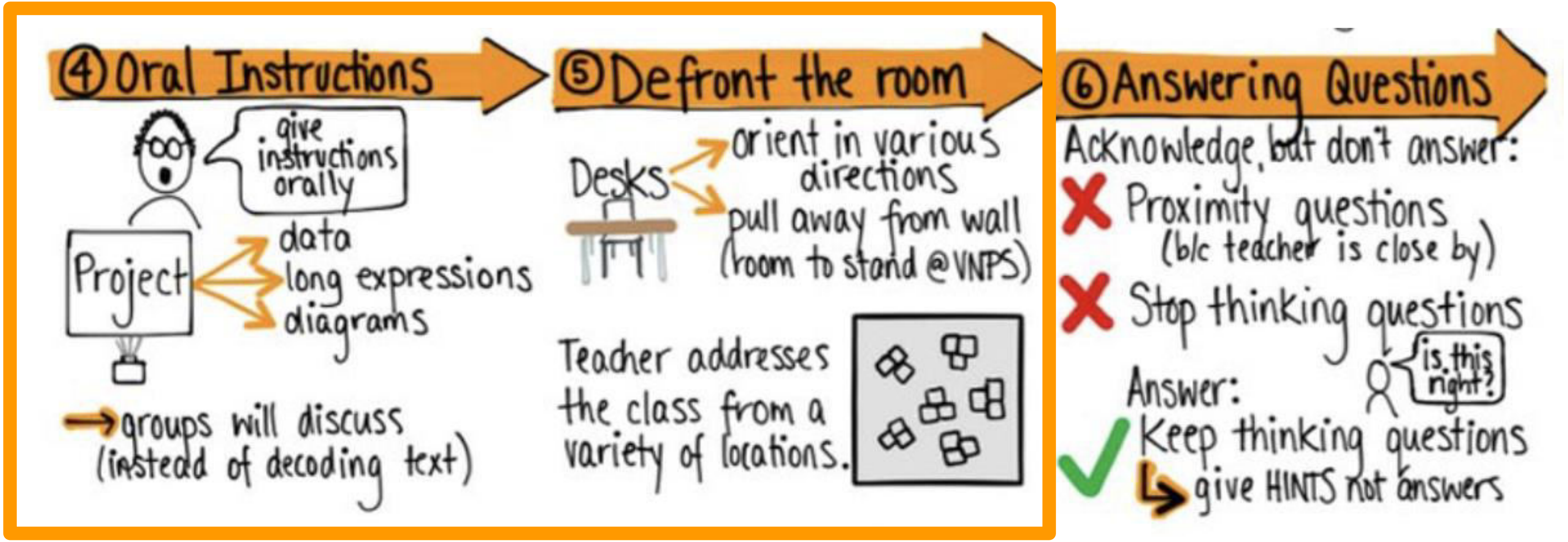
Image Source: amazon.com


BUILDING THINKING CLASSROOMS
RESEARCH: @pgliljedahl
SKETCHNOTE: @wheeler_laura
① Begin w/ a Problem
Give a problem-solving task
To start:
Problems should be
engaging
non-curricular
collaborative
promote talking
Later:
Problems can be curricular
eg textbook problems
② Visibly Random Groups
Randomly assigned
eg playing cards
Daily & in front of students
2 or 3 students / group
Sit & stand together
③ Vertical Non-Permanent Surfaces
Vertical
Erasable
WHITEBOARD
CHALKBOARD
WINDOW
1 marker or chalk per group
promotes discussion
④ Oral Instructions
give instructions orally
Project
data
long expressions
diagrams
groups will discuss (instead of decoding text)
⑤ Defront the room
Desks
orient in various directions
pull away from wall (room to stand @ VNPS)
Teacher addresses the class from a variety of locations.
⑥ Answering Questions
Acknowledge, but don't answer:
Proximity questions (b/c teacher is close by)
Stop thinking questions
is this right?
Answer:
Keep thinking questions
give HINTS not answers
⑦ Meaningful Notes
Student created:
select
synthesize
reorganize
ideas
Based on their or others' boards
Provide time for this after levelling.
⑧ Build Autonomy
Model how groups can visit other groups when they are stuck or done.
Hints & extensions come from peers (not just the teacher).
Helps manage flow
FLOW
⑨ Hints & Extensions
Manage flow
Challenge
Ability
Frustration
Perseverance
FLOW
Tolerance
Boredom
FLOW → Mihaly Csikszentmihalyi
⑩ Level to the Bottom
debrief
class discussion
direct teaching
the "lesson"
Once all groups pass a minimum threshold.
Debrief 1 or more groups' solutions
Work through a new problem w/ whole group
⑪ Check Understanding
Assign 4-6 "check for understanding" questions
Students choose to work
individually
in groups
at desks
on VNPS
Purpose: self-evaluation (NOT marks)
⑫ Formative Assessment
measure → communicate
where student is currently
where student is going
Multiple & varied opportunities to demonstrate learning
observation
product
conversation
can't
doesn't
isn't
dis...
fully
very
completely
always
⑬ Summative Assessment
PROCESS > product
Evaluate what you value!
Include:
group + individual
work
⑭ Reporting
Based on data (NOT points)
One aggregated mark
disaggregated evidence
Analysis of data
Counting of points
What has this student learned?
What can they improve?

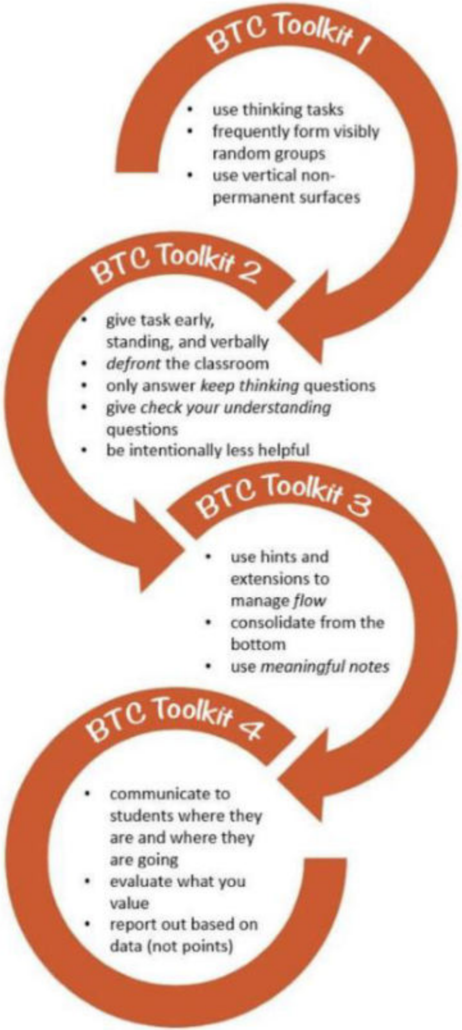
Toolkit #1 Implement Together



Toolkit #2 Order of Implementation not important

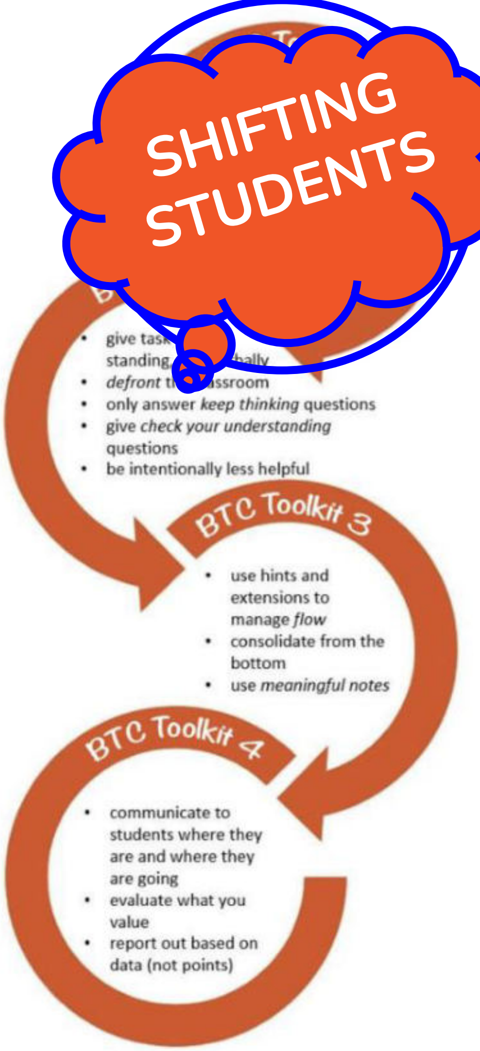


How do we Build a Thinking Classroom?

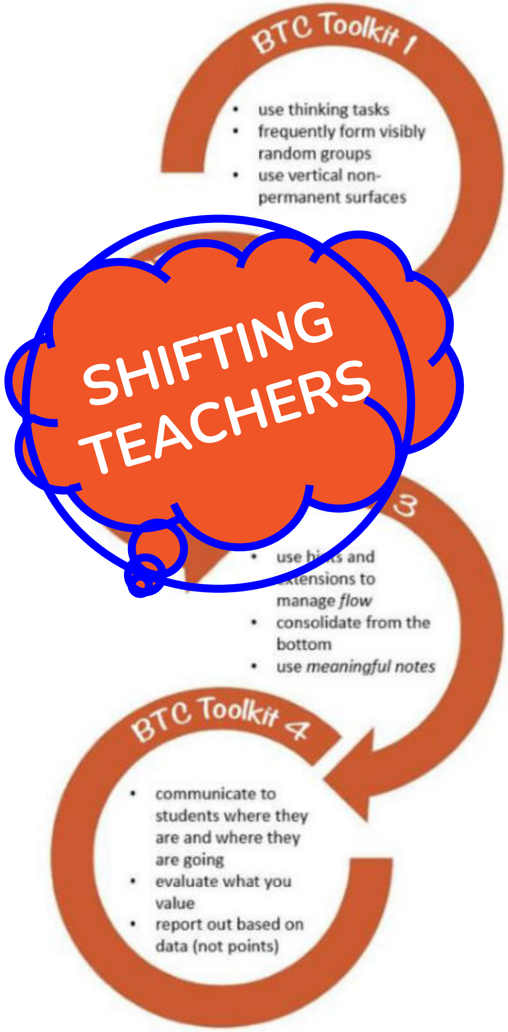


- use thinking tasks
- frequently form visibly random groups
- use vertical non-permanent surfaces

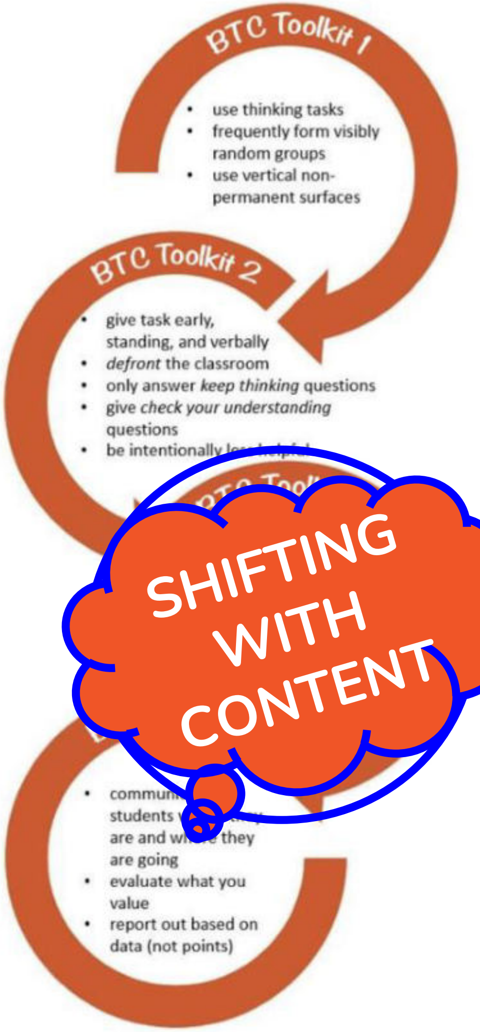
Evaluate what you value
(use co-constructed rubrics)



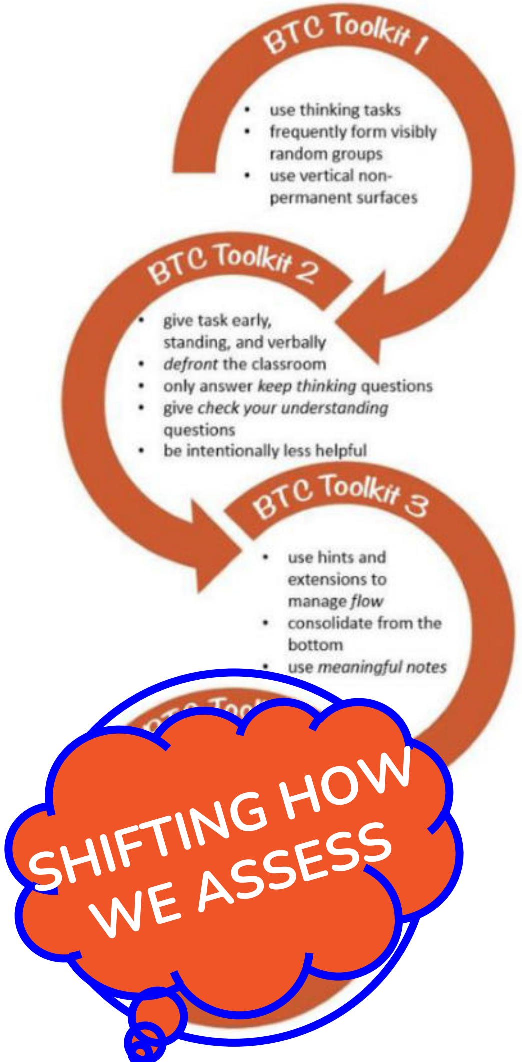
- give task early, standing, and verbally
- *defront* the classroom
- answer only *keep thinking* questions
- give *check your understanding* questions
- be intentionally less helpful



- use hints and extensions to manage *flow*
- consolidate from the bottom
- use *meaningful* notes



- communicate to students where they are and where they are going
- evaluate what you value
- report out based on data (not points)



BTC: Navigation Tool for Implementation (Self-Reflection)

Building Thinking Classrooms: Navigation Tool   Teacher: Grade: Date:

	Initial Implementation	Partial Implementation	Full Implementation
Toolkit #1 Practices: Setting the Stage for Thinking	☐ Non-curricular (NC) tasks used to introduce BTC re-norming (Ch.1) ☐ Visibly Random Grouping (some students try to go to another group) (Ch. 2) ☐ Students (Ss) working in groups at VNPS (Ss need help finding group, wait for prompt to get started)	☐ At least 3-5 NC tasks used. Waited until students seemed ready to shift to curricular tasks ☐ VISIBLY Random Grouping (no students switching groups) ☐ Ss working in groups of 2-3 at VNPS (every S knows where to go and most get started on the task w/out prompting) (Ch. 3)	☐ 4-6+ NC tasks used to create culture. NC task used to introduce each new practice. ☐ VISIBLY Random Grouping (all Ss willing to work with any group) ☐ Ss working in groups of 2-3 at VNPS (Students get started on the task together or checking in with another group if stuck.)
Toolkit #2 Practices: Fine-tuning and Building Capacity for Student Autonomy/Agency	☐ Desks are defronted in groups (Ch. 4) ☐ Ss hands are up and they ask the teacher questions rather than others in their group or other groups. T answers stop thinking/proximity questions. (Ch. 5) ☐ *Verbal launch is less than 10 minutes (Ch. 6)* ☐ *Students are standing for the launch (Ch. 6)* ☐ *A set of math problems are given as independent work (Ch.7)* ☐ Ss are turn taking as they solve the tasks rather than collaborating. ☐ Students wait for the teacher to tell them if they are correct and get the next task from the teacher.	☐ The only "front" of the room is a smartboard. ☐ Few S hands up. T directs Ss to check in with another group before answering questions. ☐ T answers some proximity questions. ☐ *Verbal launch is less than 7 minutes.* ☐ *Students stand and turn and talk during launch* ☐ *A set of mild, med, spicy math problems are given to Check Your Understanding (not collected or graded)* ☐ The student w/the pen is the scribe and records what is discussed collaboratively. ☐ Ss are beginning to get next tasks from other groups rather than the teacher. ☐ Ss seek input from T & others.	☐ T addresses the whole class from different places in the room. ☐ No hands up. Students check in w/ each other and then other groups w/out being directed by a Teacher ☐ T only answers keep thinking questions. ☐ *Verbal launch is less than 7 minutes.* ☐ *Students stand and turn and talk during launch* ☐ *Ss are invited to decide where to start on a set of mild, med, spicy math problems that are responsive to that lesson's stopping point (CYU) *Ss have access to the answers* ☐ Groups make sure that everyone in the group understands before moving on to the next task. ☐ Students get the next math task from other groups. ☐ Students discuss and seek input from others rather than the T.

Toolkit #3 Practices: Making Sense of Curricular Content Goals/Standards	☐ Launch includes direct instruction on how to do the first task before giving the first task at the VNPSs ☐ Tasks given are a set of problems exactly like the problem modeled in the launch (Ch. 9) ☐ Teachers provide direct instruction when students get stuck (Ch. 9) ☐ There is a brief discussion telling students about the objective of the lesson ☐ Students may or may not be standing. ☐ *Students are given time to make notes on what is important to them after the consolidation (Ch. 11)*	☐ Limited direct instruction on a specific strategy or process to use during the set of tasks is given (Ch. 9) ☐ Tasks given show some progression of complexity but may not be closely connected ☐ Students are given hints that increase challenge OR increase ability through questioning when groups get stuck ☐ Teacher leads a discussion using specific examples or ideas that consolidate the lesson intentions (Ch. 10) ☐ Students are standing next to a Turn & Talk partner and given opportunities to discuss ideas as the T facilitates the discussion. ☐ *Students are given time to make notes on what is important to them and calibrate with other students to add or remove content*	☐ Launch includes a review of prior knowledge that will build capacity to enter the first task WITHOUT limiting students to a specific strategy or showing them how to solve the first task ☐ Tasks given move students through progressively more complex concepts by TYPE that are closely connected and vary by only one thing ☐ Students are directed to other groups to discuss ideas BEFORE any hints are given. T is able to keep Ss in FLOW ☐ T is able to facilitate a consolidation that matches the types of tasks given (Divergent or Convergent) in order to close the lesson. ☐ The consolidation provides Turn & Talk opportunities to notice and name strategies and sequences so that structure, formalization, and order emerges for the lesson outcome. ☐ *Students are given time to make meaningful group notes from a 2-part to 4-part graphic organizer that includes a scaffolded example, another example, a choice, and things to remember.* ☐ *Ss make their own notes version after the group notes.*

Key to Chapters for support on these practices: ***Please note italics for colors that are similar.***

Chapter 1
Chapter 2
Chapter 3
Chapter 4
Chapter 5
Chapter 6
Chapter 7
Chapter 8
Chapter 9
Chapter 10
Chapter 11

Building Thinking Classrooms

Toolkit 1

Chapter Overview

- The Issue
- The Problem
- Toward a Thinking Classroom
- FAQ
- Macro- and Micro- Moves
- Questions to think about

Toolkit 1 - Practices 1 through 3

1. What Types of Tasks We Use
2. How We Form Collaborative Groups
3. Where Students Work

Practice 1

What types of tasks we use

Once a word problem is decoded, the mathematics is often trivial, procedural, and analogous to the mathematics that was taught that day.

pp. 25

If we want our students to think, we need to give them something to think about.

Problems (tasks which students do not already have the tools to solve) *precede* teaching of the focal mathematics which are necessitated by the problem. That is, the major point of a problem is to raise questions that can be answered, and promote students using their intuition, before learning new mathematical ideas (Deslauriers, McCarty, Miller, Callaghan, & Kestin, 2019).

Towards a Thinking Classroom: 3 Types of Tasks

- Non-curricular
  - Low- floor, High-ceiling, Open middle
- Scripted curricular task
- As-is Curricular

Macro Moves

Begin the lesson (first 5 minutes) with a thinking task.

Micro Moves

- The first three to five thinking tasks you use should be non-curricular, highly engaging thinking tasks.
- Shift to curriculum thinking tasks.
  - Begin by asking a question about prior knowledge.
  - Ask a question that is an extension of that prior knowledge.
  - Ask students to do something without telling them how.

Practice 2

How we form collaborative groups

No matter how strategic a teacher is in their groupings, when there is a mismatch between their goals and students' individual goals, it means some students will be unhappy and will disengage.

pp. 41

The Issue: Reasons Teachers Group Students

Educational Goals

- Pedagogy
- Productivity
- Peacefulness

Social Goals

- Diversity
- Integration
- Socialization

95%

of students either grouped themselves or attempted to group themselves, in order to **socialize**.

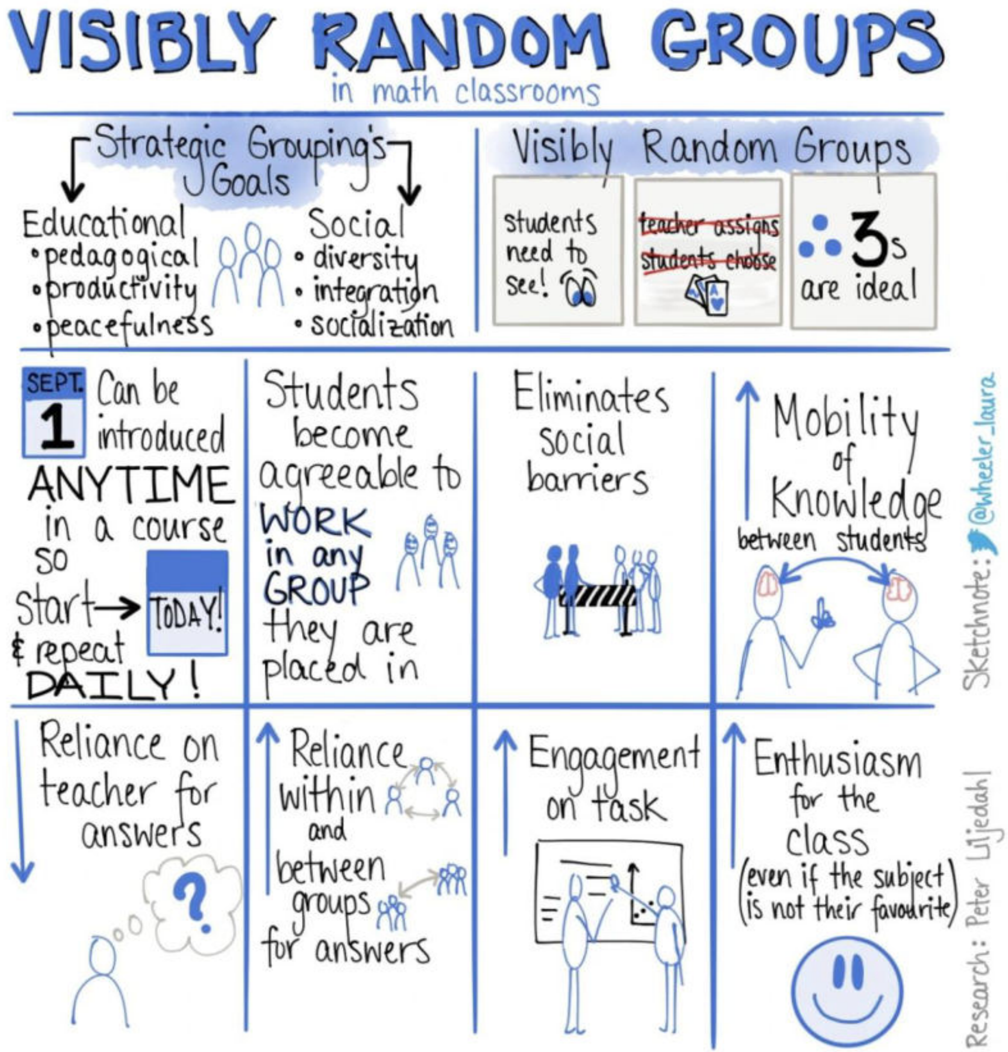
THE ISSUE: REASON STUDENTS GROUP

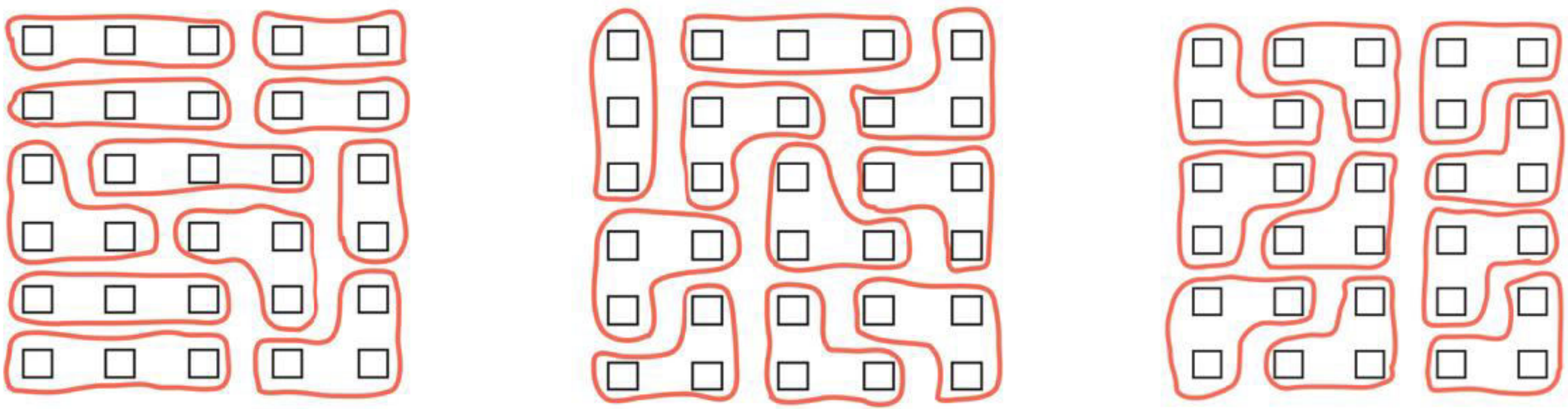
Towards a Thinking Classroom: Visibly Random Grouping

- Visibly random
  - Necessary for students to perceive and believe in randomness.
- Frequent
  - About every hour
- Grade 3 and up, optimal group size = 3
  - Enough redundancy and diversity (language, interest, experience, knowledge
- K-2, optimal group size = 2

Results of Frequently Visible Random Grouping

- Willingness to collaborate.
    - Resistance typically gone by week 3.
- Elimination of social barriers.
- Increased knowledge mobility.
- Increased enthusiasm for mathematics learning.
- Reduced social stress.


VISIBLY RANDOM GROUPS
in math classrooms
Strategic Grouping's Goals
Educational
• pedagogical
• productivity
• peacefulness
Social
• diversity
• integration
• socialization
Visibly Random Groups
students need to see!
~~teacher assigns~~
~~students choose~~
3s are ideal
SEPT. 1
Can be introduced ANYTIME in a course so start → TODAY! & repeat DAILY!
Students become agreeable to WORK in any GROUP they are placed in
Eliminates social barriers
Mobility of Knowledge between students
Reliance on teacher for answers
?
Reliance within and between groups for answers
Engagement on task
Enthusiasm for the class (even if the subject is not their favourite)
Sketchnote: @wheeler_laura
Research: Peter Liljedahl



Creating VRG with Fixed Seating

Small Class Size due to Absences? (Below 12 students)

- **Consider Random Groups of 2 that are “near” each other**
  - Having pairs in close proximity will increase the likelihood that ideas will spread from one group to another.
  - If they form a group of 4 by choice, it works much better than if you put them in a group of 4 to begin with.
- **Put up VNPSs around the room that are for the Teacher to use**
  - This space is for displaying ideas that are not coming up in the less diverse thinking of having less groups.
  - These should not be completed strategies, or solutions. Instead they display ways of thinking, representing, and notating such as: graphs, visuals, backward thinking.

Macro Moves

Frequently form visibly random groups.

Micro Moves

- K-2 form groups of 2, Grades 3-12 form groups of 3.
- Set method of randomization such that it tells students where to go.
- Randomize such that the students know that you know what group they are in.

Practice 3

Where students work

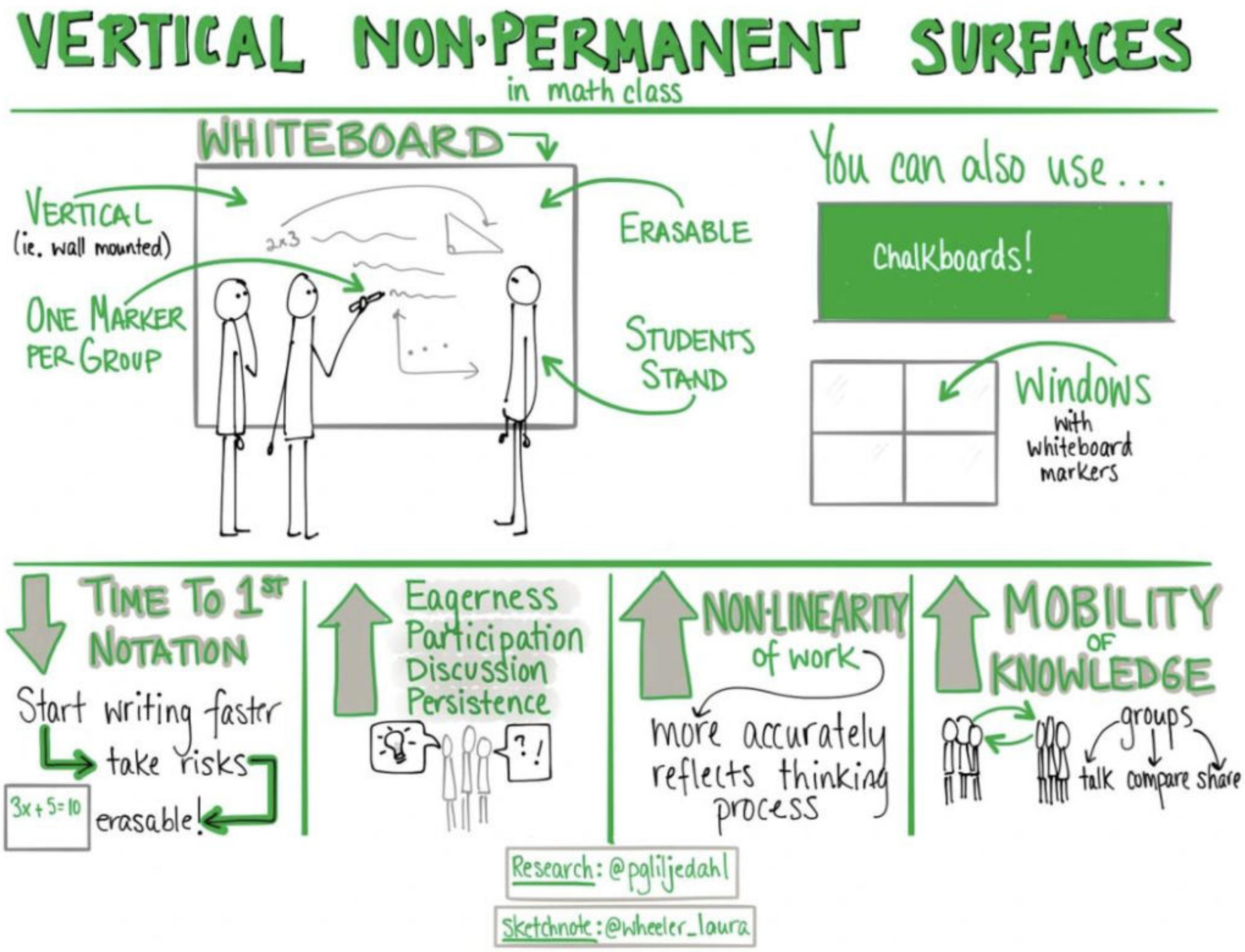
When students are sitting, they feel anonymous. And when students feel anonymous, they are more likely to disengage.

The Issue

- Students sit to:
    - Take notes
    - Do now-you-try-one tasks
    - Do homework
- Students bring the same behavior, level of energy, engagement and attention to these activities.

A Thinking Classroom: Vertical Non-Permanent Surfaces

- De-emphasize the correct answer
  - Ability to erase
- Increased knowledge mobility
  - Public thinking = cross-pollination of ideas
- Eliminates anonymity
  - The more things between the teacher and student, the more anonymity
- Teacher can see everything


VERTICAL NON-PERMANENT SURFACES
in math class
WHITEBOARD
VERTICAL
(ie. wall mounted)
ONE MARKER PER GROUP
ERASABLE
STUDENTS STAND
2x3
You can also use...
Chalkboards!
Windows with whiteboard markers
TIME TO 1ST NOTATION
Start writing faster
take risks
erasable!
3x + 5 = 10
Eagerness
Participation
Discussion
Persistence
?!
NON-LINEARITY of work
more accurately reflects thinking process
MOBILITY OF KNOWLEDGE
groups
talk
compare
share
Research: @pgliljedahl
Sketchnote: @wheeler_laura

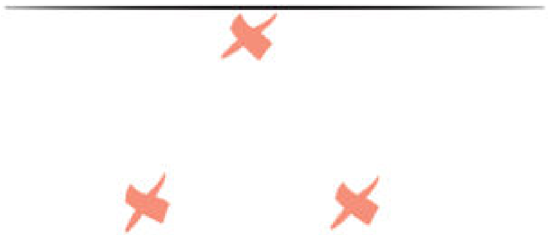
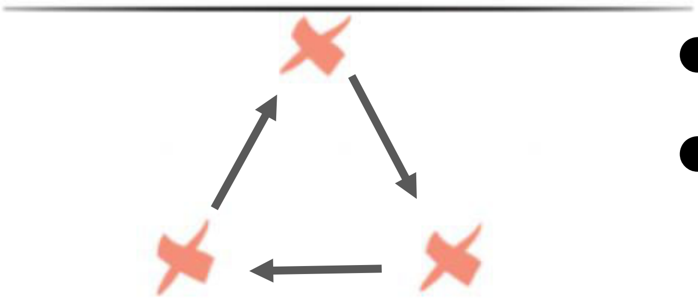


Figure 1.1 Masking Tape Triangle at VNPS



- **Frequently rotate on cue.**
- **Student at VNPS records group thinking but does NOT record their own.**

Modifications:

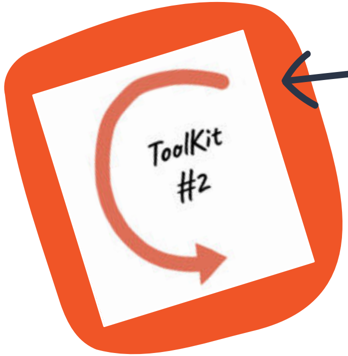
- Fixed Seating: Use a Knowledgefeed Document (Google Doc)
  - Students, Groups, and Teacher can add thoughts, images, screenshots of work, etc. (Mobilizing Knowledge)
  - This document should be non-thematic and non-chronological.
  - Let it be messy and full of errors...rough draft thinking encourages mistakes and risk-taking.
  - Let it look and feel chaotic. (Defronting)
  - Use the knowledgefeed to be less helpful. Point students to places within the document and ask them to make sense of it and discuss within the group. (Hints and Extensions/Mobilizing Knowledge)
  - Use the knowledgefeed to give the next task. Point student groups to this in the Knowledgefeed, and be less helpful. (Hints and Extensions)
  - Use the knowledgefeed during *consolidation from the bottom*.

Macro Moves

Use vertical non-permanent surfaces (VNPSs)

Micro Moves

- Have only one marker per group.
- Move the marker around within the group.
- Sometimes have the rule that the person writing cannot write any of their ideas.
- Hold groups responsible for the learning of every member of the group.
- Have groups in close proximity to each other.
- Talk to the students about valuing wrong ideas & not erasing others' work.

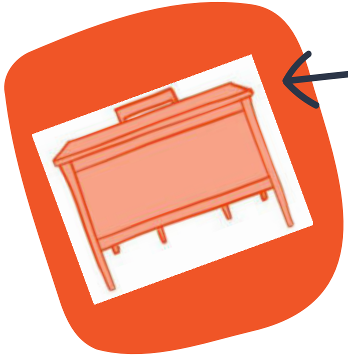


Building Thinking Classrooms

Toolkit 2

Toolkit 2 - Practices 4 through 8

4. How we arrange the furniture
5. How we answer questions
6. When, where, and how tasks are given
7. What homework looks like
8. How we foster student autonomy



Practice 4

How we arrange furniture

Thinking is messy. It requires a significant amount of risk taking, trial and error, and non-linear thinking. It turns out that in super organized classrooms, students don't feel safe to get messy in these ways.

The Issue: Room set up - sets expectations

- Different classroom setups allow for different types of learning.
- If the classroom is super organized, generating thinking is difficult.
- Students don't feel safe taking risks and sharing messy thinking in overly organized classrooms.
- Overly chaotic classrooms become a distraction as well.

What experience would you expect from each room?

Towards a Thinking Classroom: Defront the Classroom

- Ensure that every chair in the room is facing a different compass direction.
- Organize (not too chaotic, not straight or symmetrical) in a way that says thinking, collaboration, and risk-taking are expected.
- Move your desk away from the traditional “front.”

Macro Moves

Defront the classroom.

Micro Moves

- Cluster desks and tables away from the vertical surfaces.
- Position desks and tables so that chairs point in all different directions.
- Try not to stand at what used to be the front.
- Move around the room when you are talking to the students.

Practice 5

How we answer questions

A typical teacher will answer between 200 and 400 questions a day.

The Issue: We are much too helpful

- Answering every question does not promote student thinking.
- Students ask 3 types of questions.
  - Proximity (when you are close by - confirms role as student)
  - Stop Thinking (asked to get you to do the thinking for them)
  - Keep Thinking (asked so they can continue to engage)
- Answering proximity or stop thinking questions is antithetical to building a thinking classroom.

Towards a Thinking Classroom: Be less helpful!!!

- Answer only “keep thinking” questions.
- Avoid answering any questions in the first 5 minutes after launching a task.
- Respond to questions with a question.
- Smile and walk away.

10 Back pocket Responses to Proximity and Stop Thinking Questions:

1. Isn't that interesting?
2. Can you find something else?
3. Can you show me how you did that?
4. Is that always true?
5. Why do you think that is?
6. Are you sure?
7. Does that make sense?
8. What is something else you could try?
9. Why don't you try another one?
10. Are you asking me or telling me?

Building Thinking Classrooms, **Peter Liljedahl pgs. 89-90**

Macro Moves

Answer only keep- thinking questions.

Micro Moves

- When asked a proximity or stop-thinking question that you do not want to answer, answer with a question:
  - Isn't that interesting? Can you find something else? Show me how you did that? Is that always true? Why do you think that is? Are you sure? Why don't you try something else? Why don't you try another one? Does that make sense? Are you asking me or telling me?
- When asked a proximity or stop-thinking question, smile and walk away.
- Talk to students about the three types of questions they ask and the types of questions you will answer AFTER you have already implemented the practice

Practice 6

When, where, and how tasks are given

Students have been socialized to believe that questions are assigned from the textbook after they have first been shown how to do them.

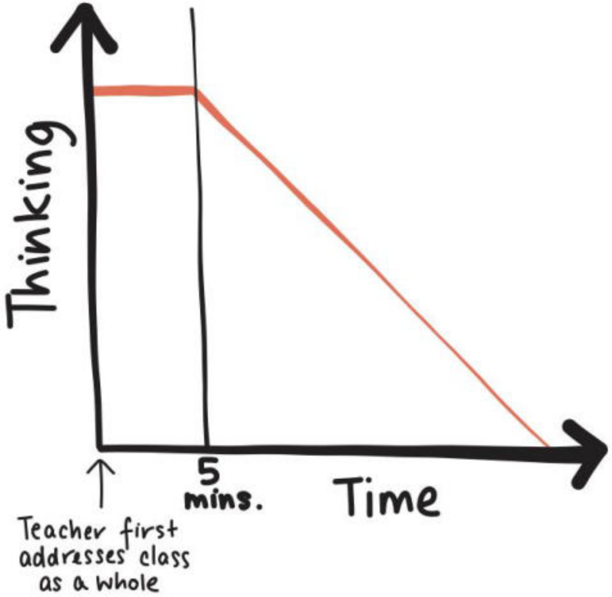
The Issue

- The internet is full of resources and rich tasks.
- “Where can I get good tasks” is a common question we ask; however, we are really asking how to teach our curriculum AND engage our students.
- Engaging our students lies not in the task, but what we do with it.
- Teachers tend to give tasks in one of 3 ways:
  - Project it or write it on a vertical surface
  - Give it as a handout
  - Assign it from a textbook or workbook.

Towards A Thinking Classroom: Verbal, Standing, 3-5 Min

- Having students stand, loosely clustered around the teacher, creates a higher-energy and active environment for the students.
- Giving tasks verbally produced more thinking- sooner and deeper- and generated fewer questions at every grade level.
- Know that the longer you talk, and the longer they listen, the less likely you are going to be able to get them to think.

Toward a Thinking Classroom- Some Visuals



Macro Moves

- Give the first thinking task in the first 3-5 minutes after you begin the lesson.
- Give the thinking task with students standing loosely clustered around you.
- Give the instructions and thinking task verbally.

Micro Moves

- Identify and create locations in the room where there is enough space for all the students to stand.
- Try to use different locations around the room for presenting tasks.
- If new knowledge is needed to do the first task, think about what the minimum new knowledge needed is, as well as the minimum things that need to be said and written to pass on that knowledge.
- When giving a task, write on the board only the details that the students would otherwise need to remember - quantities, measurements, geometric shapes, data, etc.
- When the students have started working, ask yourself if what is written on the board would make sense to a student who comes in late.

Practice 7

What homework looks like

When students who got help from a tutor or parent were asked how they would do if a pop quiz based on the homework were given, 90% of the students said they would fail.

The Issue- A huge disconnect in objectives

- The students who need to to their homework don't, and the ones who do their homework are the ones who don't really need to do it.
- There is a huge disconnect between what teachers and students see as the objectives of homework.
  - Teachers say it's a chance for students to test their understanding, learn from their mistakes, and to find what they need more help with.
  - Students say its for practice, points and the teacher.

STUDENTING BEHAVIORS FOR HOMEWORK

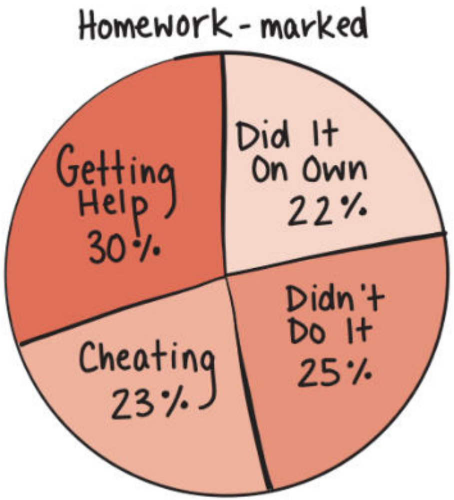


Figure 7.1 Studenting behaviors when homework is marked.

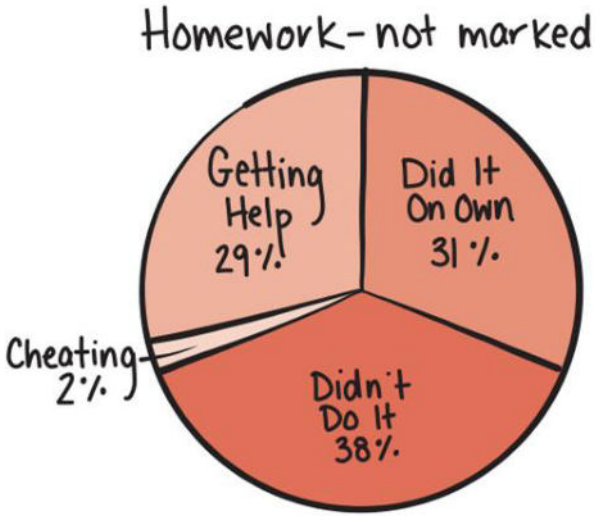


Figure 7.2 Studenting behaviors when homework is not marked.

Toward a Thinking Classroom: Check Your Understanding Questions

- Stop calling it homework and start calling it “check your understanding questions”
- In order for them to be a safe space for students, they can’t be checked or marked.
- Answers need to be provided at the same time as the questions were given.
- These are an opportunity for students to check their understanding.
  - Avoid using words like “practice” or “assignment” to describe them to students.

Implementation Steps

1. Prepare the questions.

- Identify a set of problems that will give students a chance to demonstrate whether they understand the concepts of a lesson or unit. These may be the same problems that you would traditionally have assigned as "homework."
- Provide answers along with the questions.
  - Access to the answers will help students gauge their own levels of understanding.
  - Do not provide worked solutions initially - that can get in the way of students doing their own thinking - but make them available a day or two after the questions are shared. If possible, share the worked solutions on a class website or somewhere that students can access them independently.
- Provide guidance about where to find more information in case students want support in making sense of any of the questions.
  - This could include material from the current or previous lessons/units that the class has studied, or supplemental materials like Khan Academy videos or Desmos activities.
  - If possible, make yourself available for individual or small group support as well, either during or outside of class.

Credit to Better Lesson: Link

2. Present the questions to students.

- Frame the questions to students as an opportunity to check their own understanding.
  - Call the problem set "Check Your Understanding Questions."
  - Use the phrase "this is your opportunity" to message that the work is for the students and their learning, not for the teacher or anyone else.
- Invite students to talk to each other about which of the questions they think are most important or useful to try.
  - This helps focus student attention on the questions without creating pressure from the teacher.

3. Invite students to work on the questions of their choice.

- If possible, make time for students to work on the Check Your Understanding Questions during class time.
  - Give students the autonomy to decide whether they want to work on the questions independently, or with others.
  - Make it clear to students that they can always continue to work on the questions outside of class.

Taken directly from Better Lesson: Link

- Do not collect the work, or check the questions for completion, or grade them for accuracy.
    - Do not refer to the Check Your Understanding Questions as practice, or as an assignment. Focus on the word "opportunity."

4. Make this "Check Your Understanding Questions" approach part of your daily or weekly routine.

Taken directly from Better Lesson: Link

Social Distancing: Launching a Thinking Task

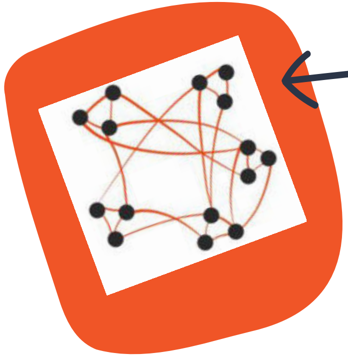
- Send students to their “x” at VNPS before you launch
- Try to stay in the middle of the room as much as possible for proximity to the student groups.
- Use a rotating whiteboard or project what you would record on a VNPS as you launch from your hand-held device.
- Project (if using a smart/digital white board) only enough to get the point across without giving too many written directions.
- OR...Use additional VNPS on each wall for recording the images, equations, etc...that accompany what you say verbally. Move around the room and record parts on different spaces.

Macro Moves

Give students an opportunity to do check-your-understanding questions.

Micro Moves

- Give students an opportunity to do check-your-understanding questions.
- Do not mark it.
- Do not check it.
- Do not ask about it.
- Don't use words like PRACTICE OR ASSIGNMENT.
- Use phrases like THIS IS YOUR OPPORTUNITY.
- Provide answers at the same time when you give check-your-understanding questions.
- Provide worked solutions a day or so after giving check-your-understanding questions.
- Give students a chance to discuss which questions they think are important for everyone to do.



Practice 8

How we foster student autonomy

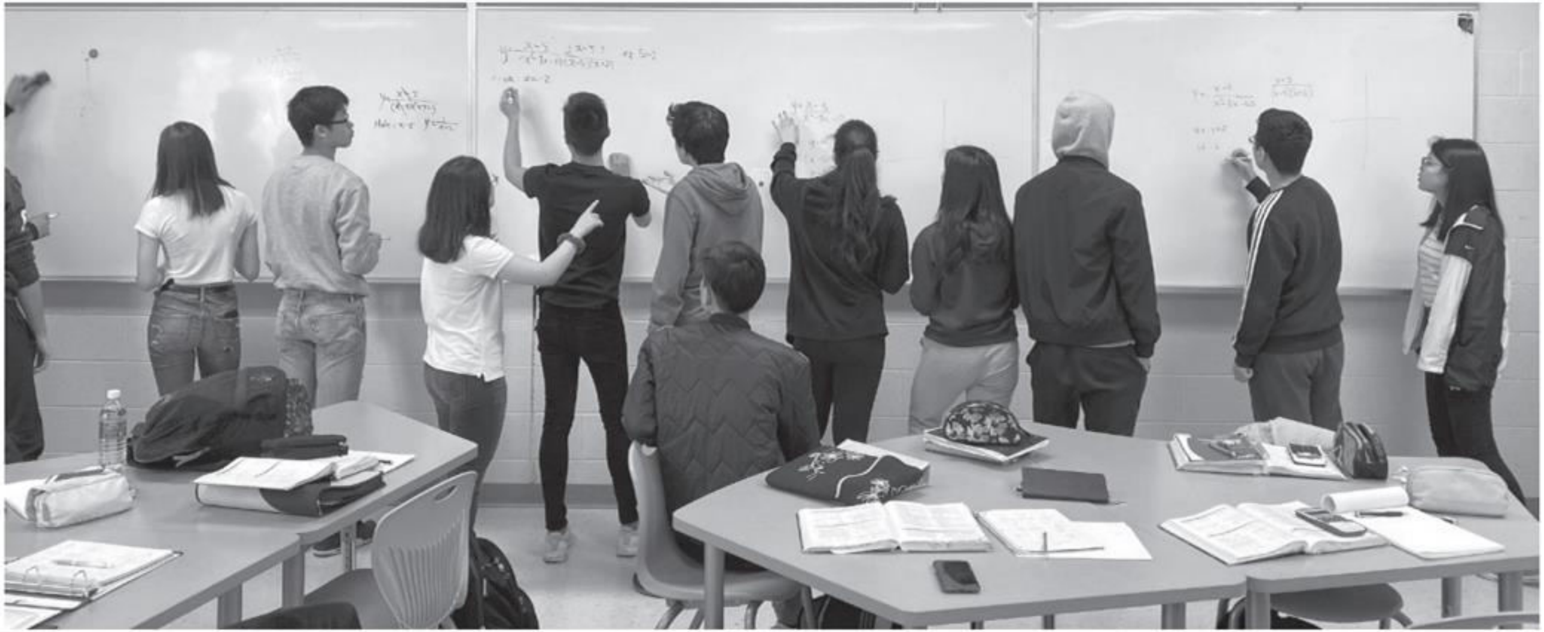
The amount of thinking students were required to do, and did, was sharply reduced in situations where their actions were managed - even micromanaged.

The Issue

- In a traditional classroom the teacher has a lot of control around what is happening for all students in each moment.
- The stronger the structure of the class, the lesser the need for the students to be independent - and the lesser the need for students to have autonomy.
- Lack of autonomy is synonymous with lack of choice. And lack of choice reduces the need for students to think.

Towards A Thinking Classroom: Knowledge Mobility

- In order for these practices to work well, students need to take on much more responsibility for their learning
- Passive interaction- looking at other groups' work
- Active interaction- talking to other groups
- Rather than being the source of knowledge in the room, work to mobilize the knowledge already in the room
- If you need help, get it. If you need another question to work on, find it.



Where do you see passive interaction? How about active?

Macro Moves

Mobilize knowledge

Micro Moves

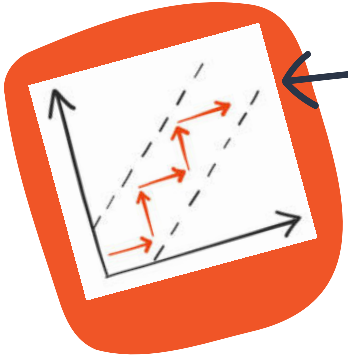
- Model passive interactions by helping groups to see what others are doing.
- Model active interactions by suggesting groups talk to each other.
- Be deliberately less helpful.
- Don’t say or show anything another group could.

Building Thinking Classrooms

Toolkit 3

Toolkit 3 - 9 through 11

9. How we use hints and extensions
10. How we consolidate a lesson
11. How students take notes



Practice 9

How we use hints and extensions

Mathematics teaching, since the inception of public education, has largely been built on the idea of synchronous activity.

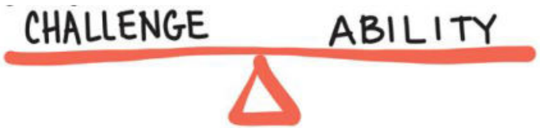
Decades of work on differentiation is build on the realization that students need teaching built on the idea of asynchronous learning.

The Issue: Synchronous vs. Asynchronous

- Students need teaching built on the idea of asynchronous activity - activities that meet the learner where they are and are customized for their particular pace of learning.
- Education today has been built on the idea of synchronous activity allowing teachers to transmit large amounts of content to groups of 20-30+ students at the same time.
- Differentiating learning opportunities for every student every day is overwhelming for teachers.

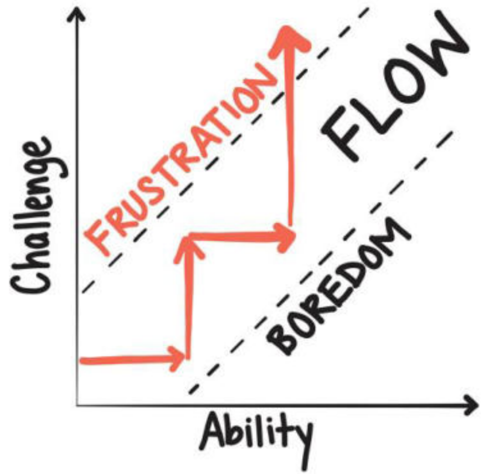
Towards a Thinking Classroom: Create & Manage flow

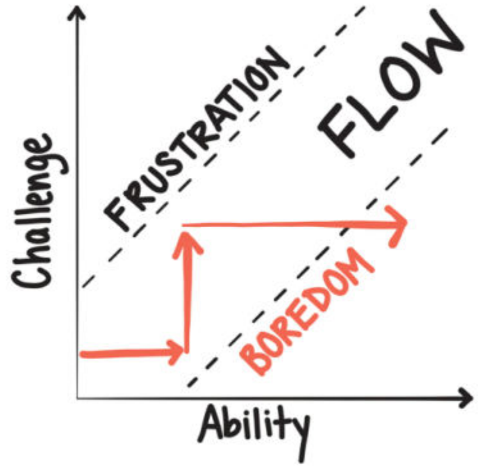
- If students are thinking, they will be engaged. If they are engaged, they will be thinking.
- The optimal experience is achieved when:
  - there are clear goals every step of the way.
  - there is immediate feedback on one's actions.
  - there is a balance between the ability of the doer and the challenge of the task.



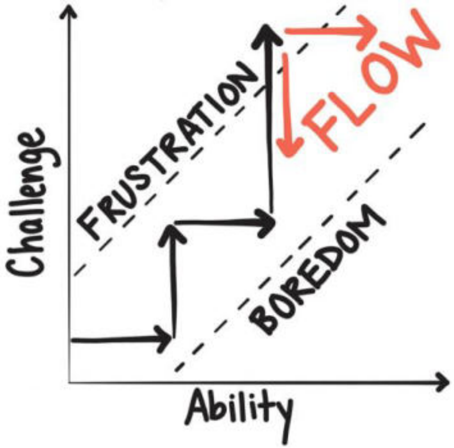
Flow Basics: Timing Matters

- Increasing the challenge of a task before students fully grow ability = frustration
- Waiting too long to increase challenge = boredom

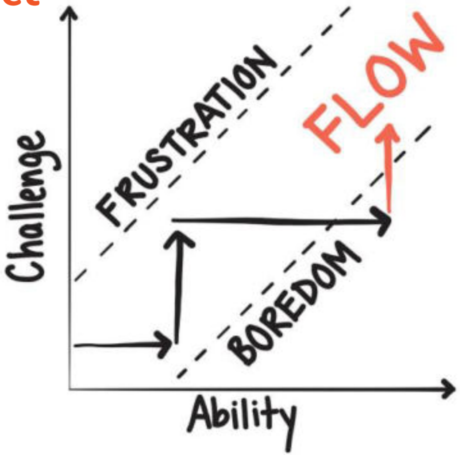




Hint to Decrease Challenge: Short-term Effect



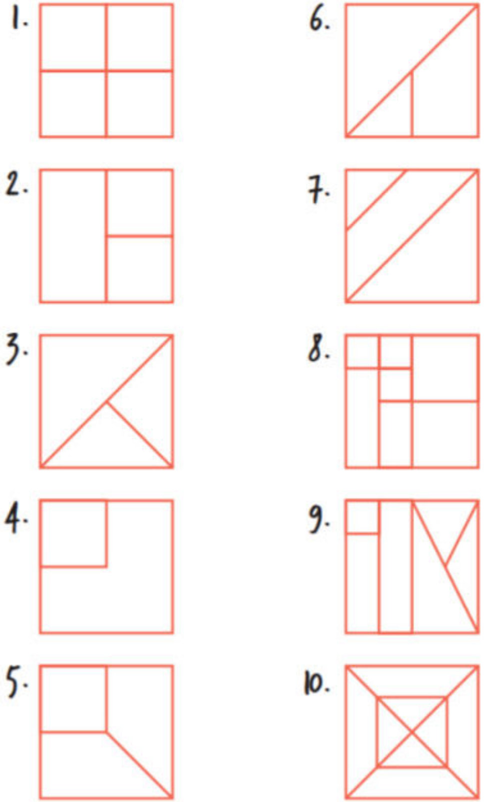
Hint to Increase Ability: Long-term Effect



SWEET SPOT:

Use **Hints** and Extensions to maintain the balance between challenge and ability.

Using Extensions to Maintain Flow

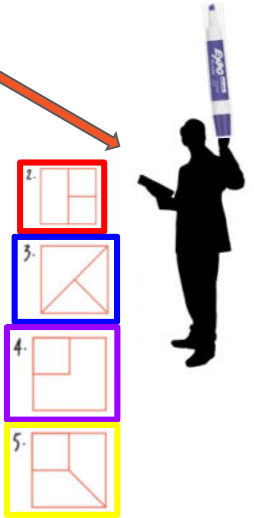


1. $(x+2)(x+3) = x^2 + 5x + 6$
2. $(\quad)(\quad) = x^2 + 7x + 6$
3. $(\quad)(\quad) = x^2 + 7x + 12$
4. $(\quad)(\quad) = x^2 + 14x + 24$
5. $(\quad)(\quad) = x^2 + 10x - 24$
6. $(\quad)(\quad) = x^2 + 4x - 12$
7. $(\quad)(\quad) = x^2 - x - 12$
8. $(\quad)(\quad) = x^2 - 2x - 24$
9. $(\quad)(\quad) = x^2 - 6x - 16$
10. $(\quad)(\quad) = x^2 - 0x - 16$
11. $(\quad)(\quad) = x^2 - 25$
12. $(\quad)(\quad) = x^2 - 49$
13. $(\quad)(\quad) = x^2 - 10x + 24$
14. $(\quad)(\quad) = x^2 - 13x + 12$

From Building Thinking Classrooms

Beginning of Lesson…just after the LAUNCH.

List of thin sliced problems to write on boards (and then groups "borrow" as needed, or printed single problems to post on boards as groups finish the previous problem.



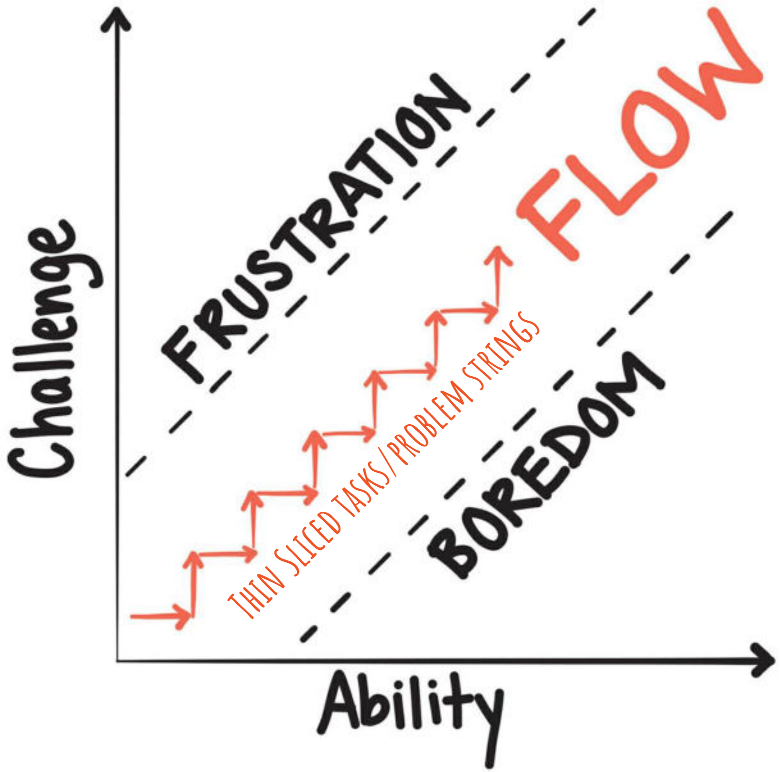
Purple expo marker (or whatever color) that ONLY the teacher uses to write the next problem on a group's board and draw boxes around work NOT to be erased.

Notice that everyone has problem #1 on their boards as this was the first challenge given after the 5 minute "launch."

During the lesson…

Notice that groups are working on different problems, but may still have previous problems on their boards to refer to. ***To the left, we see a student going to another group to "get" the information for Task #4 to bring back to their group.***

The other problems have been posted or written at the TOP of a group's board AS A GROUP FINISHES the previous task. Every other group must go on a ***scavenger hunt*** to find the next problem and bring the image, equation, etc back to their group to work through together.



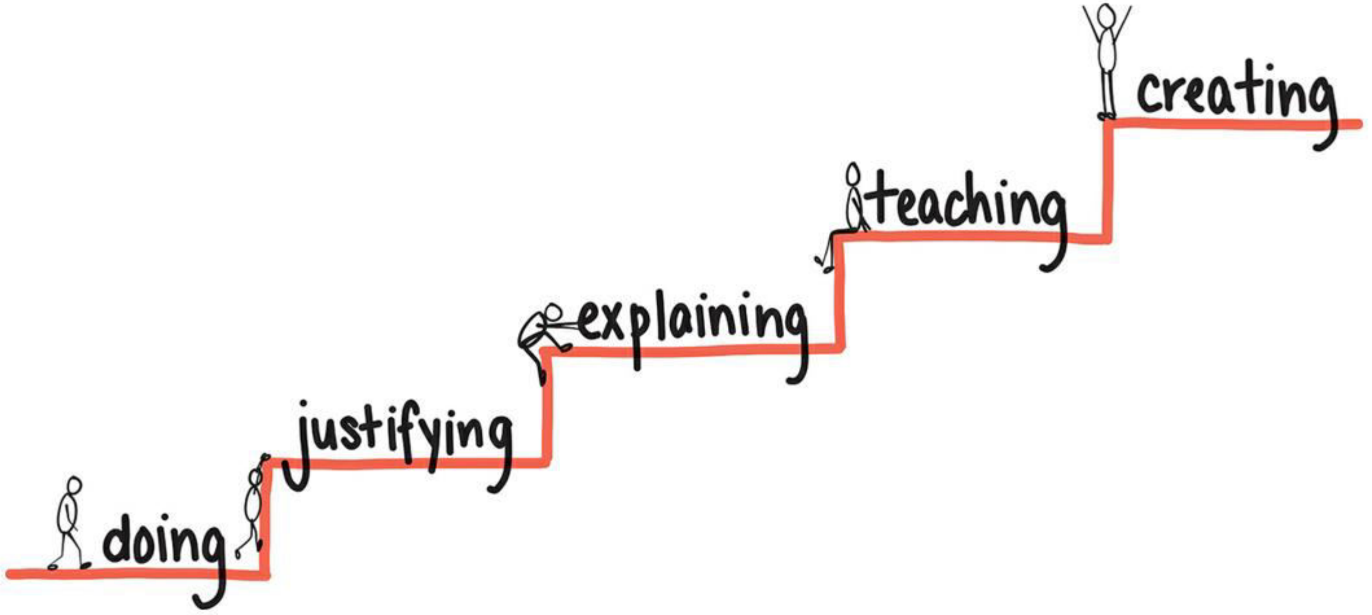
Sweet Spot:

Use Hints and **Extensions** to maintain the balance between challenge and ability.

Towards a Thinking Classroom: Create Thin Sliced Sequences

Benefit: Avoid Frustration and Boredom

Use shifts in the mode of engagement to maintain Flow



MACRO MOVES

Build and maintain flow through the asynchronous use of hints and extensions.

MICRO MOVES

- Make groups responsible for learning of every member of the group.
- Have groups write the task they are working on at the top of their vertical surface.
- Start with low-challenge tasks to ensure the groups start in flow.
- Create sequences of tasks that get incrementally more challenging by varying one thing at a time.
- Create a parallel sequence of tasks.

Practice 10

How we consolidate a lesson

Students began to mistake being shown how to do it for learning, and they mistook having it in their notes for knowledge.

If all students could learn by having us just tell them how to do it, we would not have any of the problems that we have in mathematics education today.

The Issue: Telling and showing does not work

- Waiting 4 minutes and 22 seconds after giving a problem and then going over it step-by-step causes students to *fake* and *stall*, rather than think.
- Spending time going over the most advanced problems in a sequence with students that have not yet solved them (leveling to the top is too big a cognitive jump and results in less engagement.
- Having students present to the class, unless there is a punitive structure in place, causes others to zone out.

Towards a Thinking Classroom: Consolidate from the Bottom

- Consolidation follows the same path as the extensions of increasingly challenging tasks in your sequence.
- Start with the solutions that all students got to.

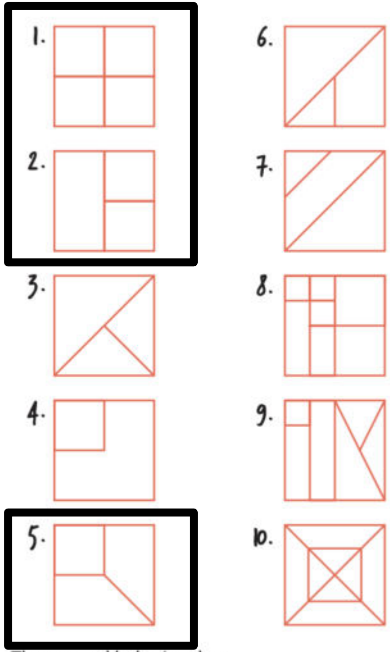


Figure 9.5 The unusual baker's cakes.

50% of the students looked at their cell phones for 50% of the time while sitting.

Keep students standing: 3 Methods of consolidation

STANDING!!

Teacher leads a general discussion but writes nothing down.

Teacher leads a detailed discussion while recording on the board.

Teacher leads a detailed discussion using student work on the VNPSs.

Tips for Keeping Kids Engaged:

- Keep it short (Start with 5 minutes and build up to 10)
    - Only consolidate curricular tasks
- Involve other students when discussing a groups' work
    - Give students quiet think time to make sense of the work
    - Invite them to discuss a specific part of the work (Why did they do……here?)
- Give students something to do during/after consolidation
    - Let students add to their Meaningful Notes
    - Have them discuss an idea with someone NOT near them to increase movement
    - Let them respond or build on an idea with a personal mini-board
- Focus on connections and similarities/differences when looking at multiple pieces of student work
    - Look in depth at the first board, then look for connections with the others lifted
- Set Norms for a class discussion

Social Distancing: Consolidating from the Bottom

- Sever the attachment between student group and work shown on the VNPS by having student groups rotate 2 stations clockwise OR counterclockwise before consolidation.
- Before consolidation from the bottom, have the new student group try to interpret what the student group was thinking as shown on the board.
- Do NOT take pictures of the work and have students sit down before you begin the consolidation from the bottom.

Small Class Size due to Absences? (Below 12 students)

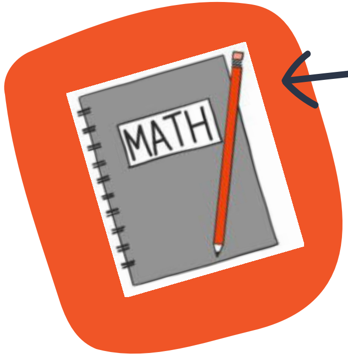
- **Consolidate from the Bottom**
  - Consider including displayed ways of thinking from your VNPS by connecting it to the thinking of student groups as you select and sequence during the discussion.
  - If there is an idea that is absent from the collective thinking of your groups that you would like to focus on as part of the consolidation, you may switch to leading a discussion at a VNPS where you are bringing that idea to the forefront.

Macro Moves

Consolidate from the bottom

Micro Moves

- Lock in student thinking by drawing a box around it with your red marker.
- Use hints to get missing ideas up on the vertical surfaces.
- Select and sequence students' work for guided gallery WALK.
- Keep the students standing.
- Keek the students walking.
- Spend more time on the foundation ideas at the beginning of the consolidation.
- Do not let students present their own work.



Practice 11

How students take notes

The continual effort to track, write, and keep up with the teachers' thinking requires a huge amount of cognitive effort, which causes students to fall further behind - to the point where they just stop listening and trying to make sense of what they are writing.

The Issue: Students cannot keep up with you *and* think

- 14% of students interviewed did not take notes at all because it was difficult to take notes and listen at the same time.
- More than 50% of the students taking notes were NOT keeping up...they were taking **dead notes**, tuning out and just copying with no particular structure.

Copying dead notes is a mindless activity.

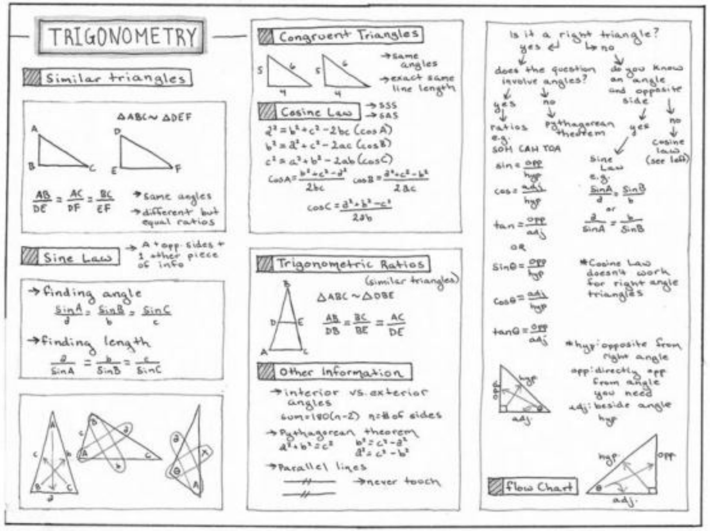
The Issue: Why students don't take notes if required

- They will not write notes on something that they do not find interesting or important.
- They will not write notes on something they know they can find elsewhere–like a PowerPoint presentation, an article, or the textbook.
- They will not write notes on things they think they will remember.

Towards A Thinking Classroom: Notes By Them - For Them

- Pose note-taking as an opportunity to write to their *Future Forgetful Selves*.
- Encourage students to begin as a group to decide what are the most important strategies and/or worked solutions to record in their notes to help them remember these ideas.
- Give them graphic organizers.
- Don’t check them, collect them, or grade them.

Examples of Meaningful Notes Templates



Graphic Organizer with cells as demarcations.

LR2 Characteristics of Linear Relations — Linear Relations

Linear	Non-linear	First differences
Line/Curve of best fit	Rate of change	Initial Value
Direct variation	Partial variation	Create tables of values, graphs, & equations

Graphic Organizer with prelabeled cells to demarcate different subtopics

Task(s) that caused us the most frustration:

Why? What did you think about that helped?

Strategies that helped us find new fractions of the cake:

Strategies other groups used that I want to remember:

Common equivalent fractions, decimals, percents:

Notes to Future Forgetful Self:

Worked Solution(s):	What I tried that didn't work: (...didn't work because...)
Assumptions I made: (We noticed...so we...)	What helped me get "unstuck?" (We got stuck on...so we...)

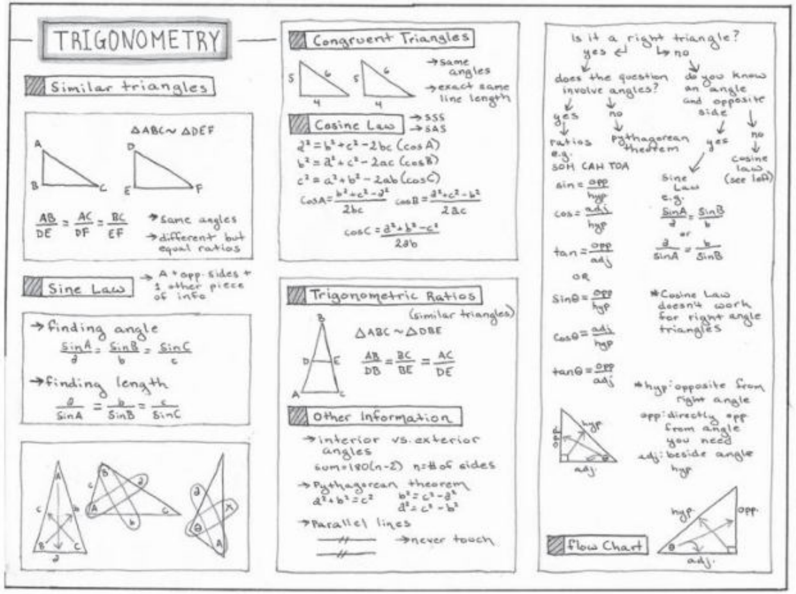


Figure 11.3 Type II: Graphic organizer with cells as demarcations.

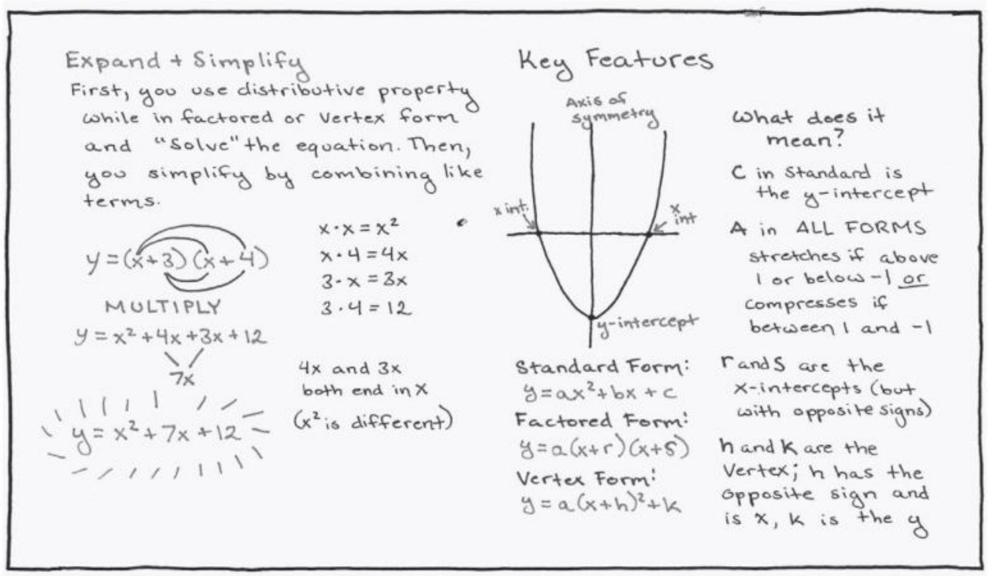


Figure 11.7 Example student notes.

Credit: *Building Thinking Classrooms*, pgs. 196 and 201

Name ____________________ # __________ Date _______________

Meaningful Notes

What do you need to record today for your future forgetful self?

Today's Topic:

VOCABULARY

???

EXAMPLES TO REMEMBER

Anything else? Formulas? Special steps?

Name ____________________ # __________ Date _______________

Meaningful Notes

EXAMPLES TO REMEMBER

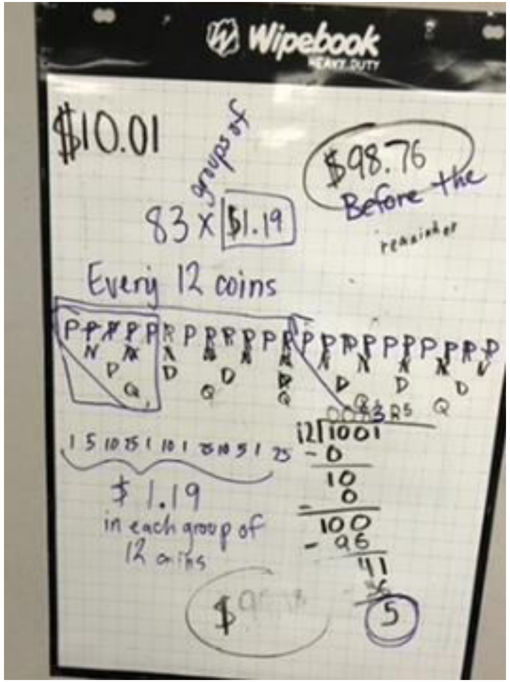
Anything else? Formulas? Special steps?

Vocabulary?

Today's Topic

3 COMPETENCIES FOR STUDENTS TO PRODUCE MEANINGFUL NOTES

- **Creation**: producing a worked example.
- **Annotation**: using small phrases and side examples as signposts on the journey through a worked example.
- **Selection**: choosing an appropriate question to form the basis of a worked example.



Meaningful notes: tips for scaffolding

Creation:
Give a specific question, topic, or subtopic for students to use as a guide to construct a worked example.

Annotation:
Begin by having them annotate complete, but incorrect, worked examples.

Selection:
Give a list of questions from which to choose one or two to develop.

Remind them that there are several on the VNPSs to choose 1 or 2 from.

Macro Moves

Have students write meaningful notes.

Micro Moves

- Emphasize that meaningful notes are BY THEM - FOR THEM.
- Prompt students to write NOTES TO THEIR FUTURE FORGETFUL SELVES.
- Use graphic organizers.
- Have students collaboratively write NOTES TO THEIR FUTURE FORGETFUL SELVES.
- Emphasize the importance of worked examples:
  - Give students choices of worked examples.
  - Have them correct incorrect worked examples.
  - Make sure to annotate.
- Give tasks 3 weeks later that require students to use their meaningful notes.

Building Thinking Classrooms

Toolkit 4

Toolkit 3 - Practices 12 through 14

12. What we choose to evaluate
13. How we use formative assessment
14. How we grade

Practice 12

What we choose to evaluate

We need to put our evaluation where our mouth is. We need to start evaluating what we value.

The Issue: Students do not come "ready to be thinkers"

- Competencies necessary for students to be successful in a thinking classroom:
  - Perseverance
  - Willingness to take risks
  - Ability to collaborate
- Students know what we value by what we evaluate.

Towards a Thinking Classroom: Evaluate what you value

- Co-construct a T-chart of productive and unproductive descriptors on 1 competency of focus
- Create a 3-column rubric based on the descriptors for the competency of focus.
  - Use the student language as much as possible.
  - Use an arrow as a continuum at the top of the rubric
  - No more than 5 descriptors and only ONE competency
- Use the rubric with students the next day.

CO-CONSTRUCTED T-CHART

- Discuss a noticing about a competency you want to develop.
- Ask for what productive descriptors look like.
- Ask for what unproductive descriptors look like.

Unproductive	Productive
• Group members hogging the marker • Excluding others from the group discussions • Being disrespectful to others who are sharing ideas • Using put downs to discourage other kids • Not open to hearing others' ideas	• Encouraging other kids in the group to share ideas • Showing the student respect when they talk • Sharing the marker fairly • Including everyone in the group as we solve each problem

Collaboration Rubric

• closed to others' ideas • disrespectful of others • actively excluding • hogging the marker • discouraging		• open to others' ideas • respectful of others • actively inclusive • sharing the marker • encouraging

BAD	GOOD
• Giving up when we get stuck	• Not giving up when it gets tough • Looking around for a hint • Asking the teacher for help

Figure 12.6 Coconstructed T-chart for developing perseverance rubric.

Figure 12.3: K–1 collaboration rubric.

- working alone


- not trying and/or giving up


- goofing off


- working together

- not giving up when it gets tough

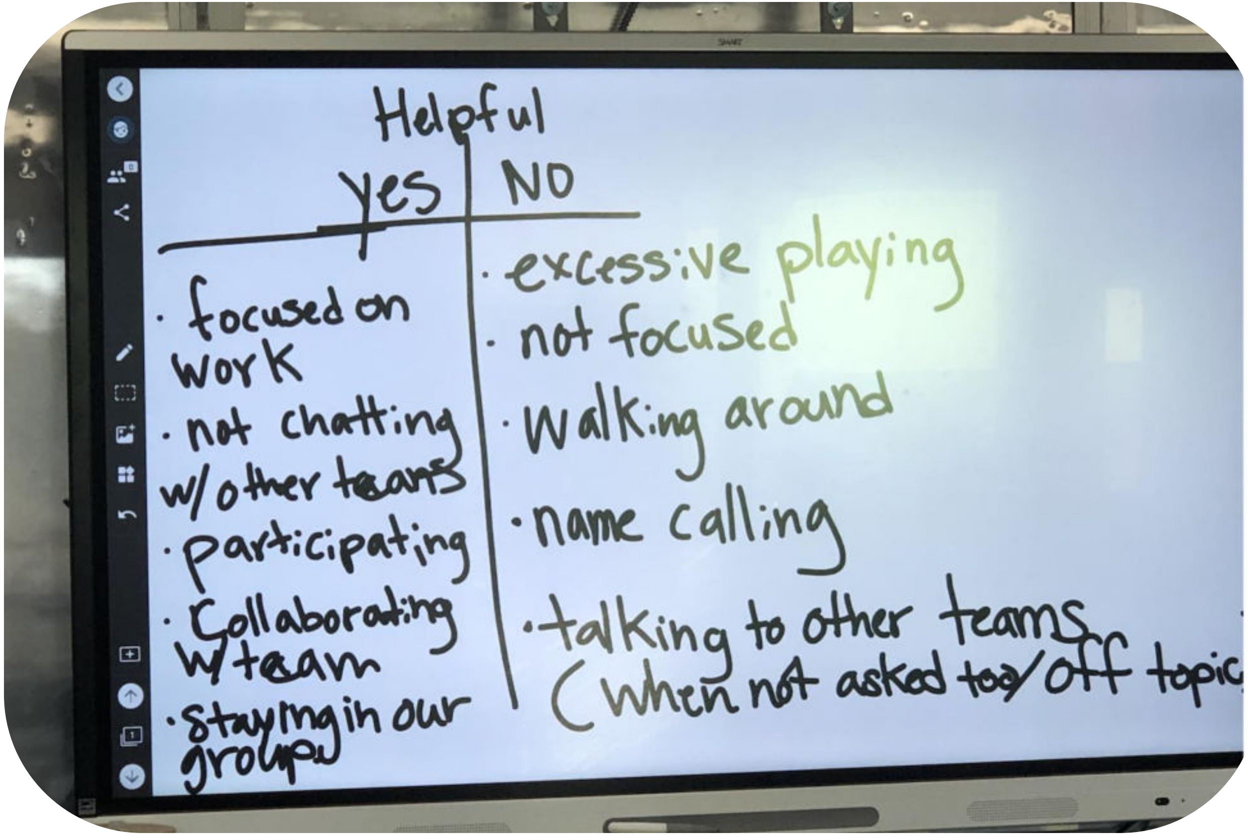
- focused and on track with the task


Self-Assessment
Date:

Perseverance	When a task is hard, I find a way to avoid working on it		I stick with a challenging task
	If I don’t know how to do something, I sit and wait for someone to tell me what to do		If I don’t know how to do something, I try something or look for a way to start
	I only do some of my classwork		I do all my classwork even when it is challenging

Collaboration	I hog the marker or I do only a bit of the task		I do my fair share of the work
	I work by myself		I lean in to my group and we work together
	I do all the talking or I do none of the talking		I help make sure everyone in the group has a voice and shares ideas, myself included
	I don’t think about the others in my group when I speak and act		I am respectful and encouraging to my group members
	I let the group move on even if I don’t understand that work, I don’t check the understanding of others		I ask questions and clarify to make sure all members of the group understand, including myself

Willingness to Take Risks	I wait until I know everything to do before I try something on a math task		I “give it a go” and try to find a way to start a task
	I stay quiet if I’m not sure what to do		I speak up in my group, even if I’m not sure my thinking is correct
	I hide my group’s work		I share my group’s thinking with other groups, even if it might not be fully correct yet
	I only do what I have to in class		I look for ways to challenge myself or try new methods


Helpful
yes
NO
· focused on work
· not chatting w/other teams
· participating
· Collaborating w/team
· staying in our group
· excessive playing
· not focused
· walking around
· name calling
· talking to other teams (when not asked too/off topic

Once students see what behaviors are expected, and that these behaviors are valued, the students begin to see them as valuable as well.

Macro Moves

Evaluate what you value

Micro Moves

- Construct t-charts with your students.
- Turn t-chars into 2 or 3 column rubrics.
    - Assess one competency at a time.
    - Use an arrow instead of labels.
    - Keep language to a minimum.
    - Preserve student voices.
    - Have no more that 5 indicators.
- Use exemplars when co-constructing rubrics for PRODUCIBLES.

Practice 13

How we use formative assessment

Although the information that is flowing to the student is the same as that flowing to the teacher, the recipients of the information and what they can make of it vary greatly.

The Issue: Students do not see the subtopics

>90 %

Students who scored 90% or better on the unit test could name and delineate the subtopics of the unit.

75-90%

Students who scored between 75 and 90% knew there were subtopics but could not name and delineate them.

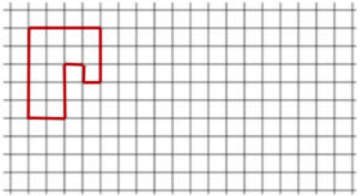
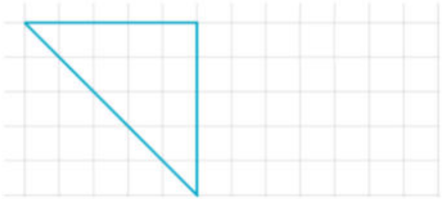
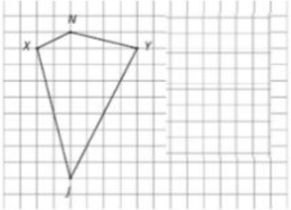
<70%

Students who scored below 70% said that the unit was one big topic.

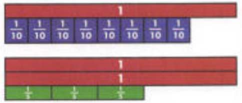
In order for assessment to equally inform teaching *and* learning, we need to find ways to help students see mathematical topics as collections of subtopics, sections, and/or special cases the way teachers do.

Towards a Thinking Classroom: Tools for Self-Evaluation

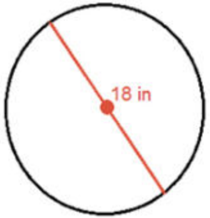
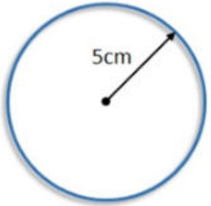
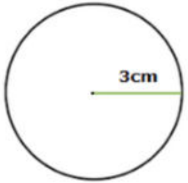
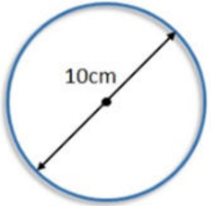
- Must communicate where they are AND where they are going.
- Be explicit about the list of outcomes that constitute a unit of study to help students see that it is comprised of a collection of topics.
- Break each (subtopic)outcome down by conceptual complexity: *Basic*, *Intermediate*, *Advanced*.

<table>
<tr><th>Stretched Trees</th><th>Introductory</th><th>Intermediate</th><th>Complex</th></tr>
<tr><td>Drawing Scaled Copies</td><td>Draw a scaled copy of the polygon using a scale factor of 200%
</td><td>Draw a scaled copy of the triangle using a scale factor of 50%
</td><td>Draw a scaled copy of the polygon using a scale factor of 1/3

</td></tr>
<tr><td>Constant of Proportionality (table)</td><td>Bob rides his bike at a constant rate. What is the constant of proportionality? Complete the table.
<table><tr><th>Hours Biked</th><th>Miles Traveled</th></tr><tr><td>1</td><td>12</td></tr><tr><td>2</td><td></td></tr><tr><td>3</td><td></td></tr><tr><td></td><td>48</td></tr></table></td><td>There is a proportional relationship between the number of dozen eggs and the cost. What is the constant of proportionality? Complete the table.
<table><tr><th colspan="2">Cost of Dozens of Eggs</th></tr><tr><th>Dozen</th><th>Cost ($)</th></tr><tr><td>6</td><td></td></tr><tr><td></td><td>15</td></tr><tr><td>14</td><td>21</td></tr><tr><td>16</td><td></td></tr></table></td><td>In a drink recipe there is a proportional relationship between the amount of grape juice and the amount of peach juice. What is the constant of proportionality? Complete the table.
<table><tr><th>grape juice (cups)</th><th>peach juice (cups)</th></tr><tr><td>5</td><td>2</td></tr><tr><td>10</td><td></td></tr><tr><td></td><td>12</td></tr><tr><td>2.5</td><td></td></tr></table></td></tr>
<tr><td>Percent Increase and Decrease</td><td>There were 950 students that attended Day last year. This year there are 4% less students. How many students are at Day this year?</td><td></td><td>When The Jonas Brothers played their very first concert only 15 fans attending. At their second concert 18 fans attended. What was the percent increase in fans?</td></tr>
</table>

Equivalent Fractions	Draw two fractions that are equivalent to ¼.	Using number sentences, show two ways to get equivalent fractions for ⅔.	Explain how you know that 3/12 is equivalent to 1/4.
Estimate + Add Fractions	$\frac{5}{6} + \frac{2}{6} =$	$\frac{5}{9} + \frac{5}{6} =$	$\frac{?}{?} + \frac{?}{?} = ?$ Using the digits 1-9 at most once, make this equation true so that the answer is a whole number.
Estimate + Add Mixed Numbers	$4\frac{1}{4} + 3\frac{3}{4} =$	$3\frac{7}{8} + 2\frac{5}{6} =$	$?\frac{?}{6} + ?\frac{?}{3} = ?\frac{?}{18}$ Using the digits 0-9 at most once, make this equation true.

Estimate + Subtract Fractions	$\frac{7}{12} - \frac{5}{12} =$	$\frac{3}{4} - \frac{3}{5} =$	$\frac{?}{?} - \frac{?}{?} = \frac{1}{6}$ Determine two fractions you could subtract to get ⅙. BUT the denominators must be two different numbers to begin.
Estimate + Subtract Mixed Numbers	$4\frac{1}{5} - 2\frac{4}{5} =$	$6\frac{1}{4} - 2\frac{3}{10} =$	$?\frac{?}{?} - ?\frac{?}{?} = ?$ Using the numbers 1-9 at most once, make this equation true where the answer is a whole number.
Problem Solve	What is the sum of the model below: 	Subtract the sum of 4 ¾ and 5 ⅔ from 12 ½.	See problem below...

Credit to Tracey Bean

Circles	Introductory	Intermediate	Complex
Circumference of a circle	Find the approximate circumference of the circle. Use 3.14 for pi. 	Find the exact circumference of the circle. 	A go-kart has a wheel with a diameter of 10 inches. In one trip around the go-kart track the wheel rotates 6715 times. Approximately how long is the go-kart track in inches?
Area of a circle	Find the approximate area of the circle. Use 3.14 for pi. 	Find the exact area of the circle. 	Below you will see an example of the target used in the Olympics for archery. The target measures 122cm in diameter. What is the exact area of each color? 
Circumference to Area and Area to Circumference		The approximate circumference of this lime is 9.42in. What is the approximate area of the top of the lime? 	A farmer bought a round field for his horses. He knows the area of the field is 625pi square yards. He needs to fence in the field. Exactly how many yards of fencing will he need to purchase.

Credit to @andburnett123

Towards a Thinking Classroom: Students Self-Reflect

Legend

✓ solved **Correctly**

S mostly correct, **Silly** mistake

X solved **Incorrectly**

N **Not** attempted, blank

H solved correctly with **Help**

G questions solved in **Group**

FRACTIONS	BASIC	INTERMEDIATE	ADVANCED
Definitions	1abc		
Add and subtract proper fractions	7a	7c	7e
Add and subtract mixed fractions	7b	7d	7f
Multiply and divide proper fractions	2a	2b	3b
Multiply and divide mixed fractions	4a	4b	3c
Solve order of operation tasks with proper and mixed fractions	8	10	11, 13
Solve contextual problems involving fractions		9, 11	12
Estimate solutions for problems involving fractions	5abc	6a	6bc

Figure 13.3 Instrument for navigating where you are and where you are going for fractions.

Building Thinking Classrooms, **Peter Liljedahl p. 236**

FRACTIONS	BASIC	INTERMEDIATE	ADVANCED
Definitions	1abc ✓✓✗		
Add and subtract proper fractions	7a ✓	7c ✓	7e ✗
Add and subtract mixed fractions	7b ✓	7d ✓	7f N
Multiply and divide proper fractions	2a ✓	2b ✗	3b N
Multiply and divide mixed fractions	4a ✓	4b H	3c H
Solve order of operation tasks with proper and mixed fractions	8 ✓	10 ✗	11, 13 ✓ ✗
Solve contextual problems involving fractions		9, 11 ✓ ✓	12 N
Estimate solutions for problems involving fractions	5abc ✓✓✓	6a ✓	6bc H H

Figure 13.6 Student's record of how they did on the fractions practice test.

✓	Questions that are attempted and answered correctly
S	Questions attempted and mostly answered correctly, but have a silly mistake
H	Questions attempted and answered correctly with help from the teacher or a peer
G	Questions that are answered correctly within a collaborative group
X	Questions that are attempted and answered incorrectly
N	Questions not attempted

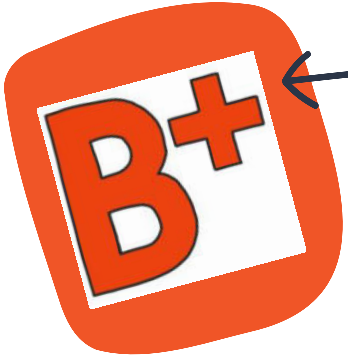
Building Thinking Classrooms, **Peter Liljedahl p. 240**

Macro Moves

Help students to see where they are and where they are going.

Micro Moves

- Construct a navigation instrument like the one in figure 13.1.
- Use headings that delineate question complexity levels (not student abilities).
- Use navigation instrument to help students record achievement on quizzes and review tests.
- Use the navigation instrument to help students record continuous progress on check-your-understanding questions.



Practice 14

How we grade

The fact that she didn't know how to do something in the beginning is expected - she is learn*ing*, not learn*ed*, and she shouldn't be punished for her early-not-knowing.

The Issue: Event-Based Grading is not Objective or Accurate

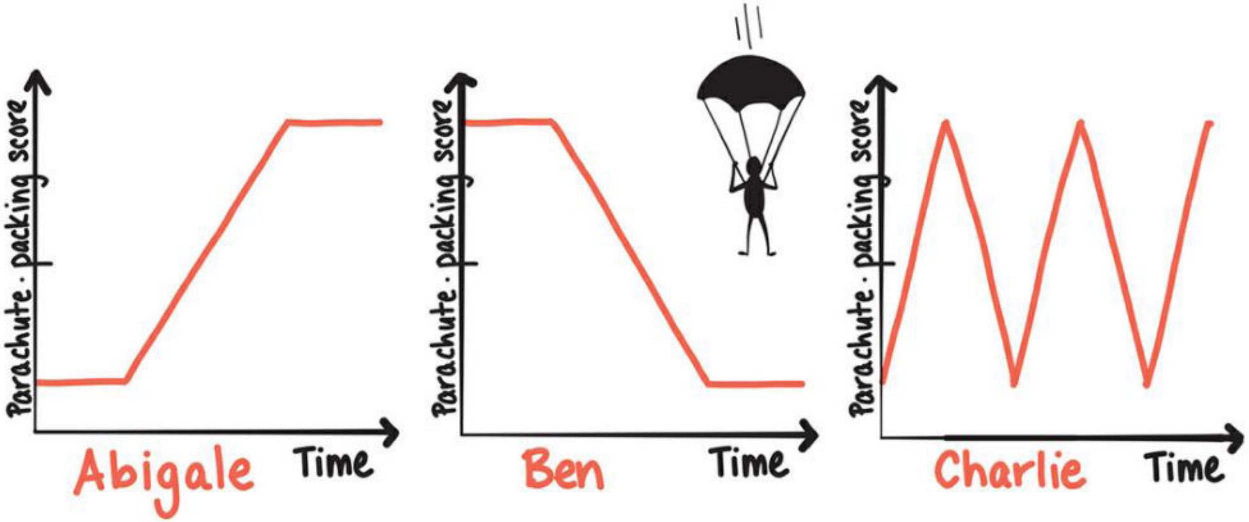


Figure 14.1 Parachute packers.

The fact that she didn't know how to do something in the beginning is expected - she is learn*ing*, not learn*ed*, and she shouldn't be punished for her early-not-knowing.

Towards a thinking classroom

BENJAMIN

REPEATING PATTERNS	BASIC	INTERMEDIATE	ADVANCED
Fill in the blanks in a repeating pattern	✓c ✓o ✓	X G ✓o ✓	✓
Create a repeating pattern	✓✓	✓✓✓✓✓	

Report out based on Data (not points)

BENJAMIN

REPEATING PATTERNS	BASIC	INTERMEDIATE	ADVANCED
Identify the core of a repeating pattern	✓ ✓ S ✓	X N H G_o	
Transfer a repeating pattern	X $\checkmark_o$ ✓		
Extend a repeating pattern	X $\checkmark_o$ ✓ ✓	N N H H	
Fill in the blanks in a repeating pattern	$\checkmark_c$ $\checkmark_o$ ✓	X G $\checkmark_o$ ✓	✓
Create a repeating pattern	✓ ✓	✓ ✓ ✓ ✓ ✓	

Figure 14.4 Benjamin's performance on the repeating patterns unit.

✓	Questions that are attempted and answered correctly
S	Questions attempted and mostly answered correctly, but have a silly mistake
H	Questions attempted and answered correctly with help from the teacher or a peer
G	Questions that are answered correctly within a collaborative group
X	Questions that are attempted and answered incorrectly
N	Questions not attempted
C	Data collected by **Conversation**
O	Data collected by **Observation**

BENJAMIN

REPEATING PATTERNS	BASIC	INTERMEDIATE	ADVANCED
Fill in the blanks in a repeating pattern	✓c ✓o ✓	X G ✓o ✓	✓
Create a repeating pattern	✓✓	✓✓✓✓✓	

"It turned out that tipping point is two consecutive demonstrations of attainment. That is, two positive data points were sufficient to match with teachers' subjective assessments of a student provided that the two positive data points were consecutive. So, whereas ✓ ✓ was enough to show attainment, ✓ X ✓ was not—more data may be needed."

Liljedahl, Peter. Building Thinking Classrooms in Mathematics, Grades K-12 (Corwin Mathematics Series) (p. 263). SAGE Publications. Kindle Edition.

ALICIA

FRACTIONS	BASIC	INTERMEDIATE	ADVANCED	OUT OF	MARK
Definitions	✓✓			2	2
Add and subtract proper fractions	✓✓	✓✓	✓✓	4	4
Add and subtract mixed fractions	✓X✓	✓SX✓✓	S✓✓	4	4
Multiply and divide proper fractions	XX✓✓	NNX✓X	✓✓✓	4	4
Multiply and divide mixed fractions	XX✓✓	XS	XXH✓✓	4	4
Solve order of operation tasks with proper and mixed fractions	XS	NNX	✓✓	4	4
Solve contextual problems involving fractions		N✓✓	✓XSX	4	3
Estimate solutions for problems involving fractions	XXN✓	XN✓S	✓✓✓	4	4
	2	3	4	30	29

In approximately 80% of the cases, the teacher awarded a grade that was 10%–15% higher than they had originally awarded through their event-based gradebook. The reorganization of points into data allowed the teachers to let go of outliers and early-not-knowing.

Liljedahl, Peter. Building Thinking Classrooms in Mathematics, Grades K-12 (Corwin Mathematics Series) (p. 265). SAGE Publications. Kindle Edition.

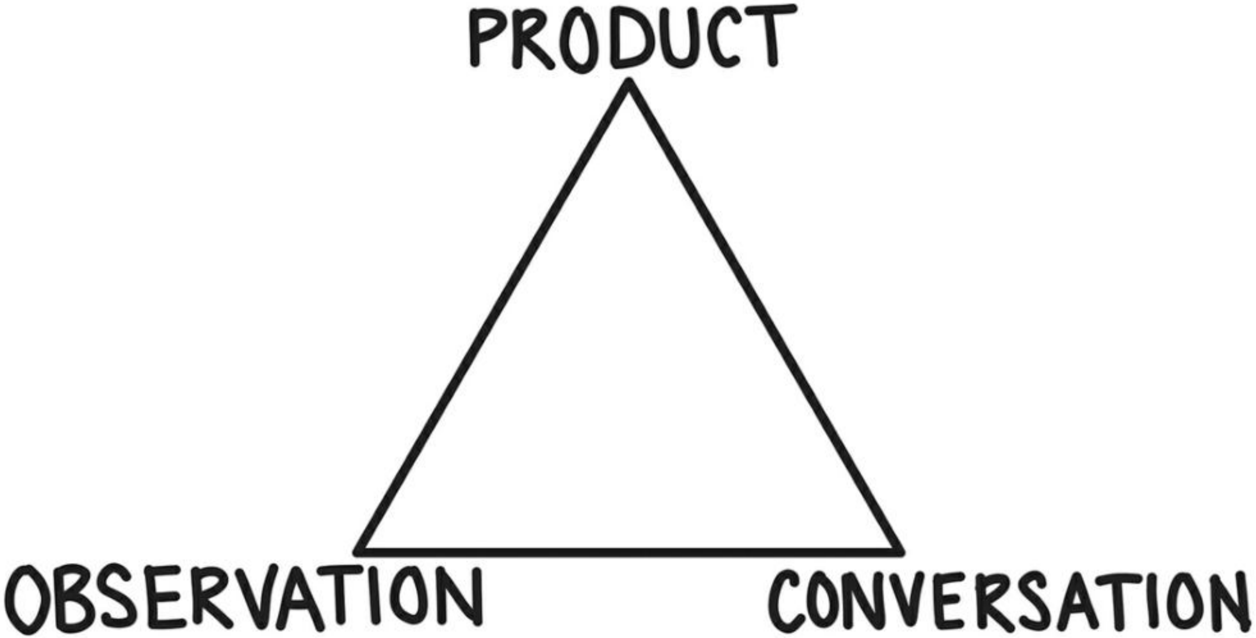


Figure 14.9 The COP framework as triangulation of data.

Liljedahl, Peter. Building Thinking Classrooms in Mathematics, Grades K-12 (Corwin Mathematics Series) (p. 272). SAGE Publications. Kindle Edition.

We accepted the idea of *differentiated instruction* a long time ago because we recognized that all students are different. If this is true, then we must also accept the idea of *differentiated assessment*.

Macro Moves

Grade based on DATA (not points).

Micro Moves

- Create instruments that delineate outcomes and complexity levels.
- Weight your outcomes.
- Gather observational and conversational data.
- Grade based on what the data are telling you.
- Be willing to ignore outliers and early not-knowing.

More Micro Moves

- Organize your tests so that all basic questions are on the first page, etc.
- Let students pick which page of a test they need to do.
- Introduce some from of collaborative testing.
- Set up portfolios as a way for students to evidence their learning.
- Allow some students to not have to take tests.

Pulling the 14 Practices together to Build a Thinking Classroom

Chapter 15

Once you are familiar with the 14 practices, then the question becomes, where to start? You cannot start with all of them at the same time. Where you start and what you do next turns out to matter.

Year 1 Order of Implementation

TK 1: Simultaneously

TK 2: After TK 1. No order. One at a time or concurrently.

TK 3: In order. One at a time.

TK 4: Ch. 15 must come after Ch. 14. Ch. 13 anytime.

Toolkit #1

Toolkit #2

Toolkit #3

Toolkit #4

Implement simultaneously

Toolkit #1

- Give thinking tasks
- Frequently form visibly random groups
- Use vertical non-permanent surfaces

“The three practices in the first toolkit, when implemented together, shock the system, shocks the students, and necessitate a different behavior.”

After Toolkit 1 is established. No order. Concurrently or one at a time.

Toolkit #2

- Defront the classroom
- Answer only keep thinking questions
- Give thinking task early, standing, and verbally
- Give check your understanding questions
- Mobilize knowledge

“Where the first toolkit is all about student behavior, the second toolkit is all about teaching practice...fine tuning the thinking classroom and laying the foundation for the third toolkit.”

Implement in order, one at a time.

Toolkit #3

- Asynchronously use hints and extensions to maintain flow
- Consolidate from the bottom
- Have students write meaningful notes

"Once the practices in the second toolkit have been implemented, you are ready to start creating flow in your classroom, and here is where you'll begin to reap the benefits."

Grade based on data after helping students see where they are and going. Evaluate what you value anytime.

Toolkit #4

- Evaluate what you value
- Help students see where they are and where they are going
- Grade based on data (not points)

"The reason these practices all ended up in the fourth toolkit is that assessment should be a reflection of a teacher's practice, and until this point, your teaching practice has been in flux."

Toolkit #2

- Defront the classroom
- Answer only keep thinking questions
- Give thinking task early, standing, and verbally
- Give check your understanding questions
- Mobilize knowledge

Toolkit #3

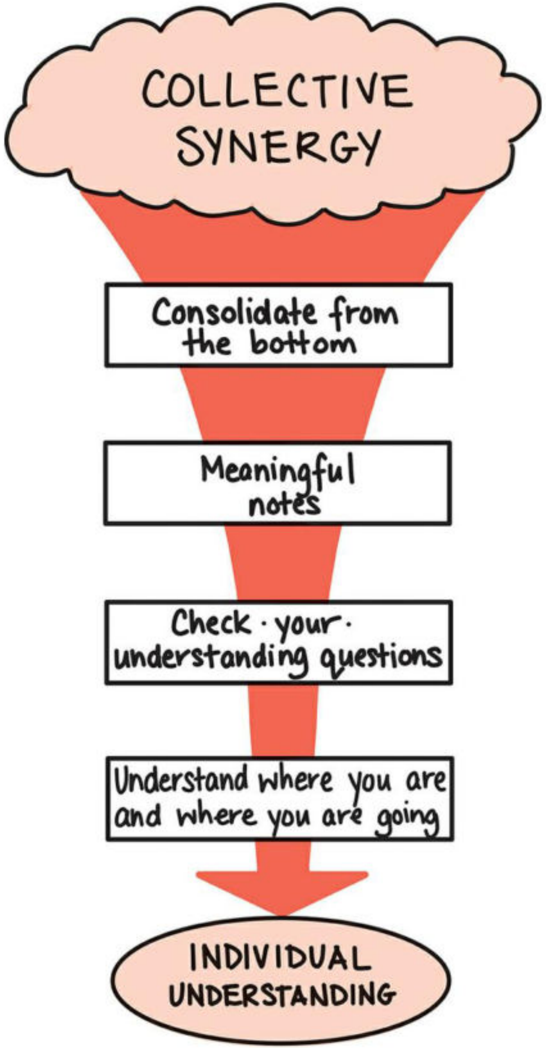
- Asynchronously use hints and extensions to maintain flow
- Consolidate from the bottom
- Have students write meaningful notes

Toolkit #4

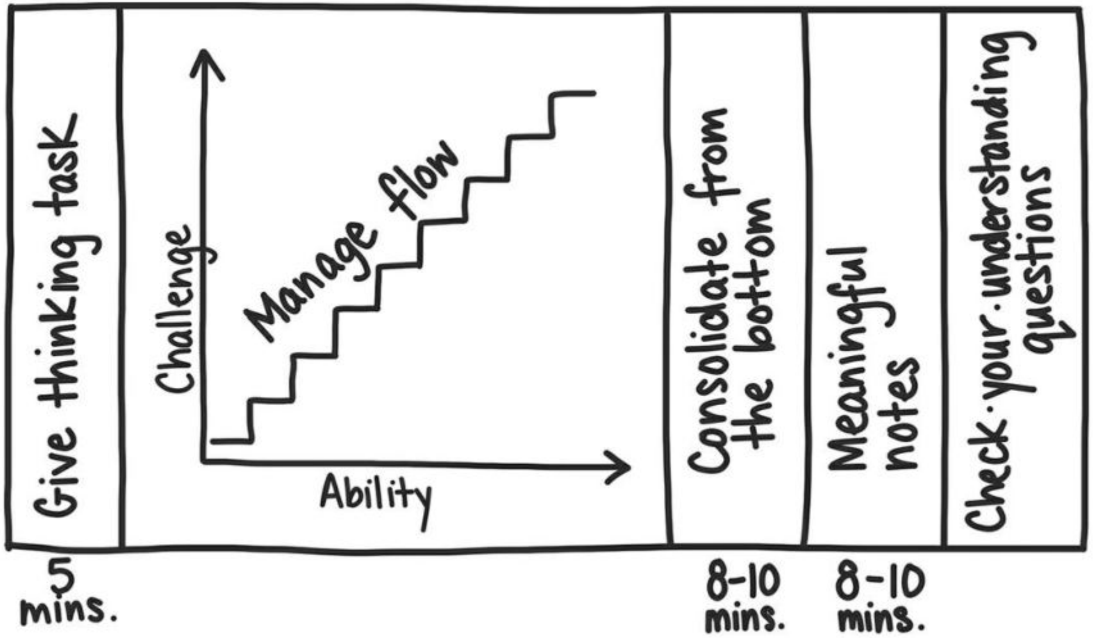
- Evaluate what you value
- Help students see where they are and where they are going
- Grade based on data (not points)

"For the transfer to individual understanding to occur, students need to take on more and more responsibility for their own learning."

"Although each of these practices helps, in some way, to move collective learning toward individual learning, they are most effective when all four work together."

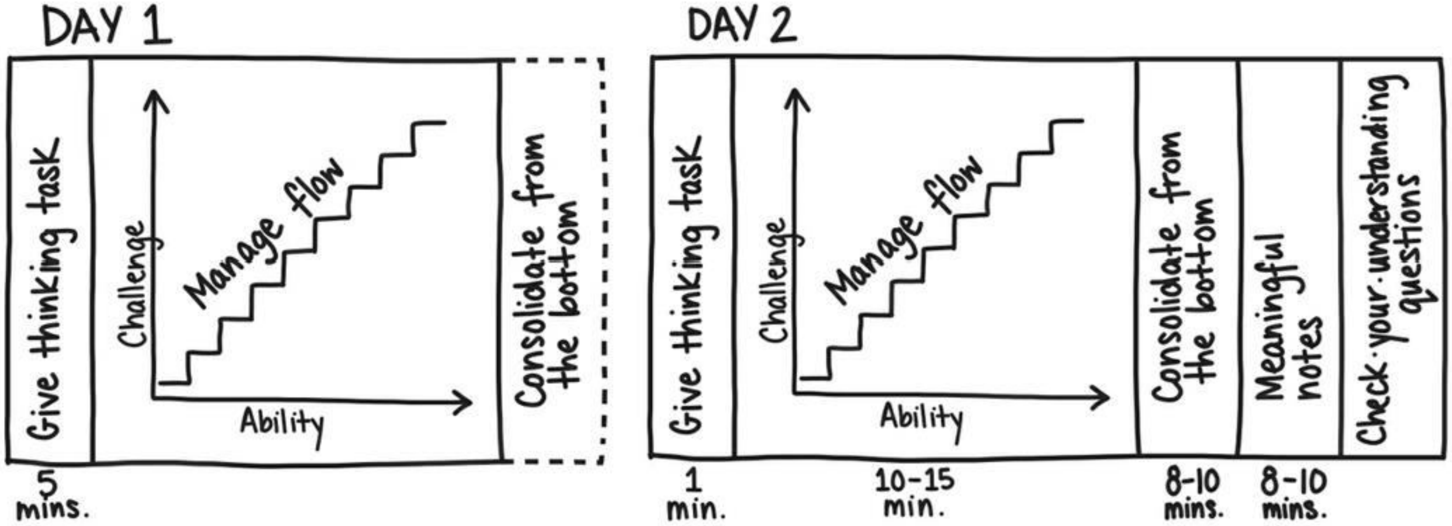


Typical Lesson sequence in a 65 minute Period:



- You begin the lesson by giving a task verbally with the students standing around you somewhere in the room, randomly grouping the students, and sending them off to their VNPSs.
- You then manage the flow in the room by using hints and extensions, while at the same time planning for consolidation.
- You keep the students in flow until the energy wains, at which point you consolidate from the bottom.
- This is followed by meaningful notes.
- Finally, you provide the opportunity to do check-your-understanding questions.

Typical Lesson sequence spread over 2 days:



Year 2 Order of Implementation

This is about rebuilding a thinking classroom and allows the teacher to implement most of the first 3 toolkits together. It is only dependent on the acclimatization rate of students.

This is the only practice from the first 3 Toolkits to wait on until the other practices are established.

- Give thinking tasks
- Frequently form visibly random groups
- Use vertical non-permanent surfaces
- Defront the classroom
- Answer only keep thinking questions
- Give thinking task early, standing, and verbally
- Mobilize knowledge
- Asynchronously use hints and extensions to maintain flow
- Consolidate from the bottom

- Give check-your-understanding questions
- Have students write meaningful notes
- Evaluate what you value
- Help students see where they are and where they are going
- Grade based on data (not points)

Try to avoid...

*Starting more gradually than recommended by the toolkit implementation.

Why?

If the change is too subtle, then the students' behaviors don't change and your effort has little impact.

*Implementing thinking classroom practices once or twice a week.

Why?

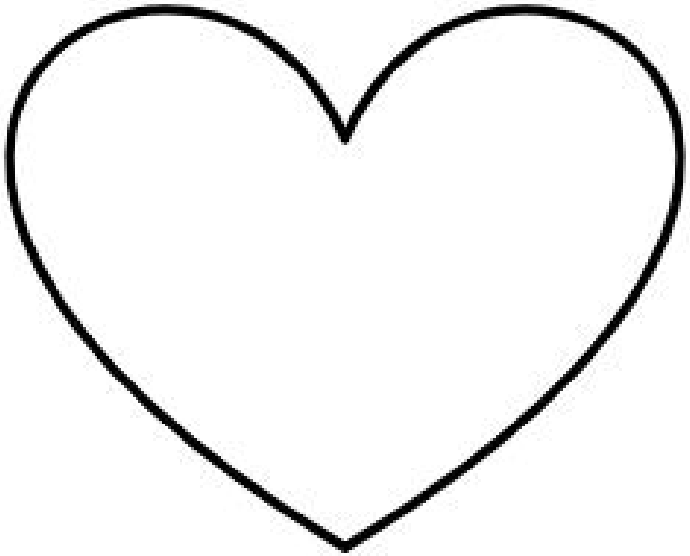
This will result in students seeing math class an thinking classrooms as two distinct events.

Adapting a Worksheet

Reflections with Shapes

Practice adapting a worksheet → slide deck → verbal launch

What does the word “symmetry” or “symmetrical” make you think of?

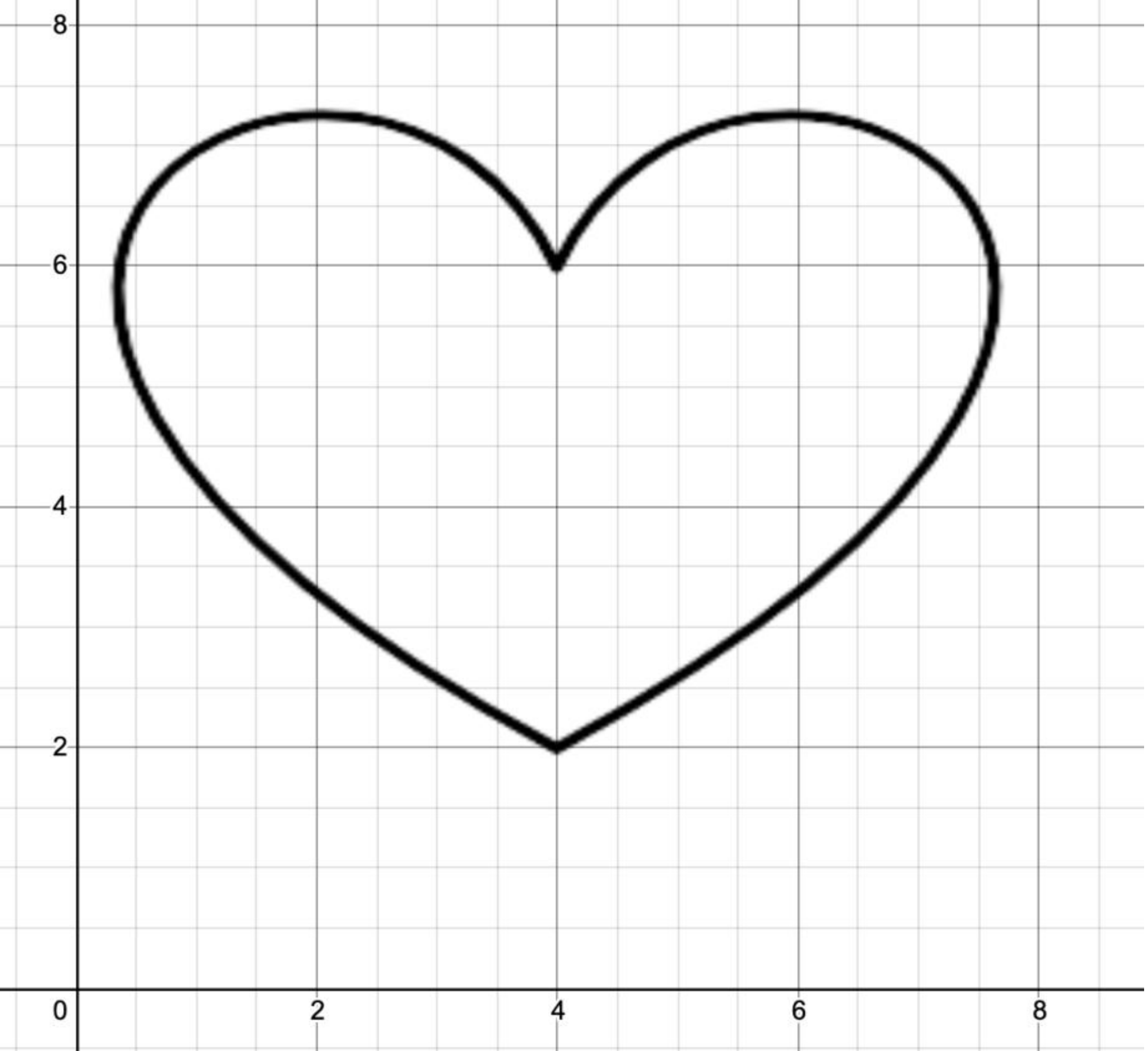


Does this shape have any lines of symmetry?

If so, how many? Draw them.

If not, how do you know it doesn't?

What quadrant will the image be in?

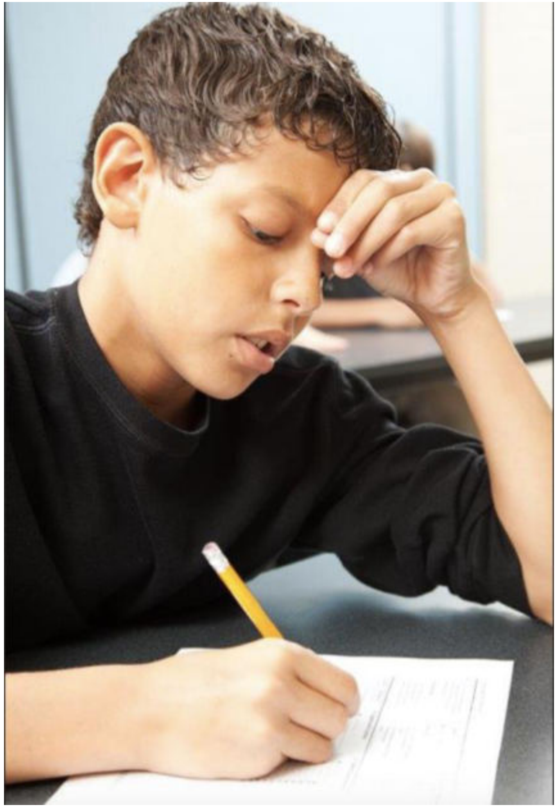


Please copy this heart on the coordinate plane on your white board.

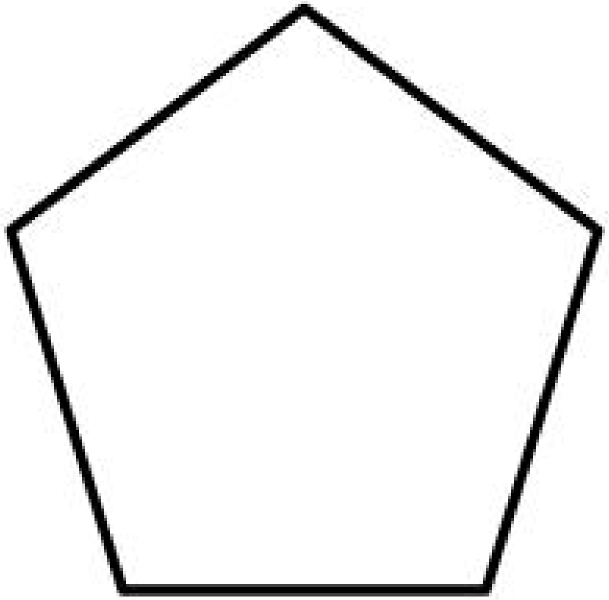
Then, reflect it over the x-axis using a different color.

How do you know you reflected it correctly over the x-axis?

Notes to your future forgetful self



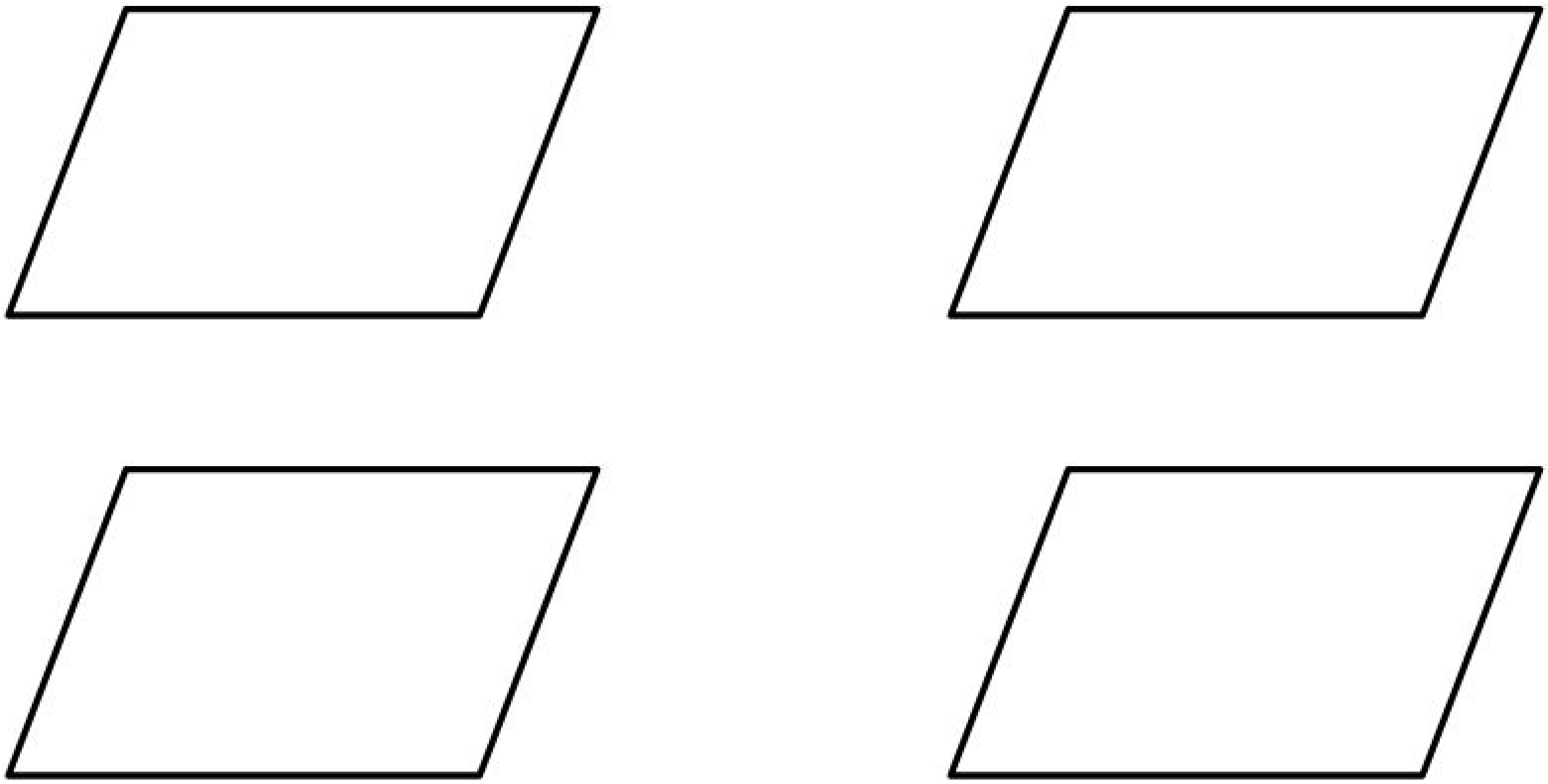
What did you learn in class today that you want to remember in the future?



Does this shape have any lines of symmetry?

If so, how many? Draw them.

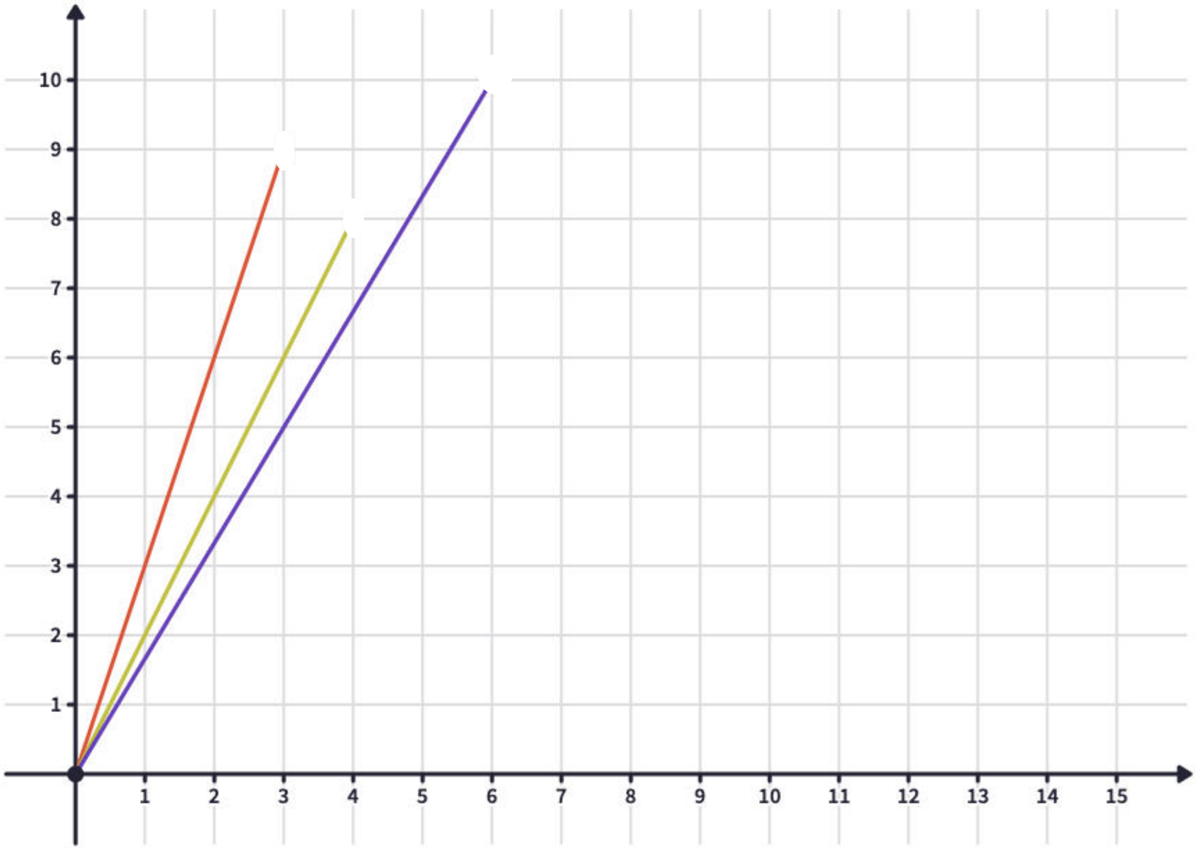
If not, how do you know it doesn't?



8.EE.B.5-6

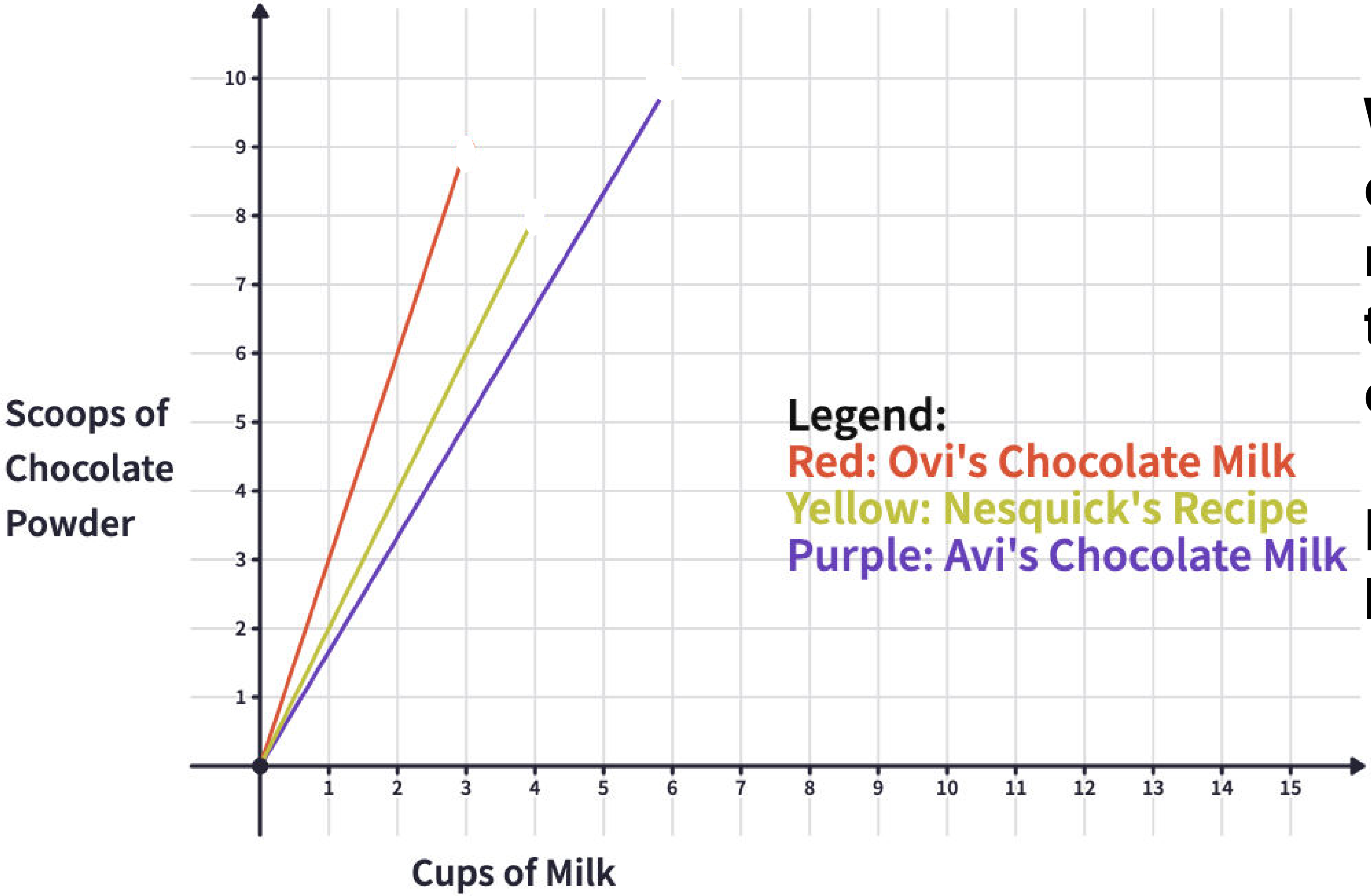
Thinking about Chocolate Milk

Thinking about Chocolate Milk…



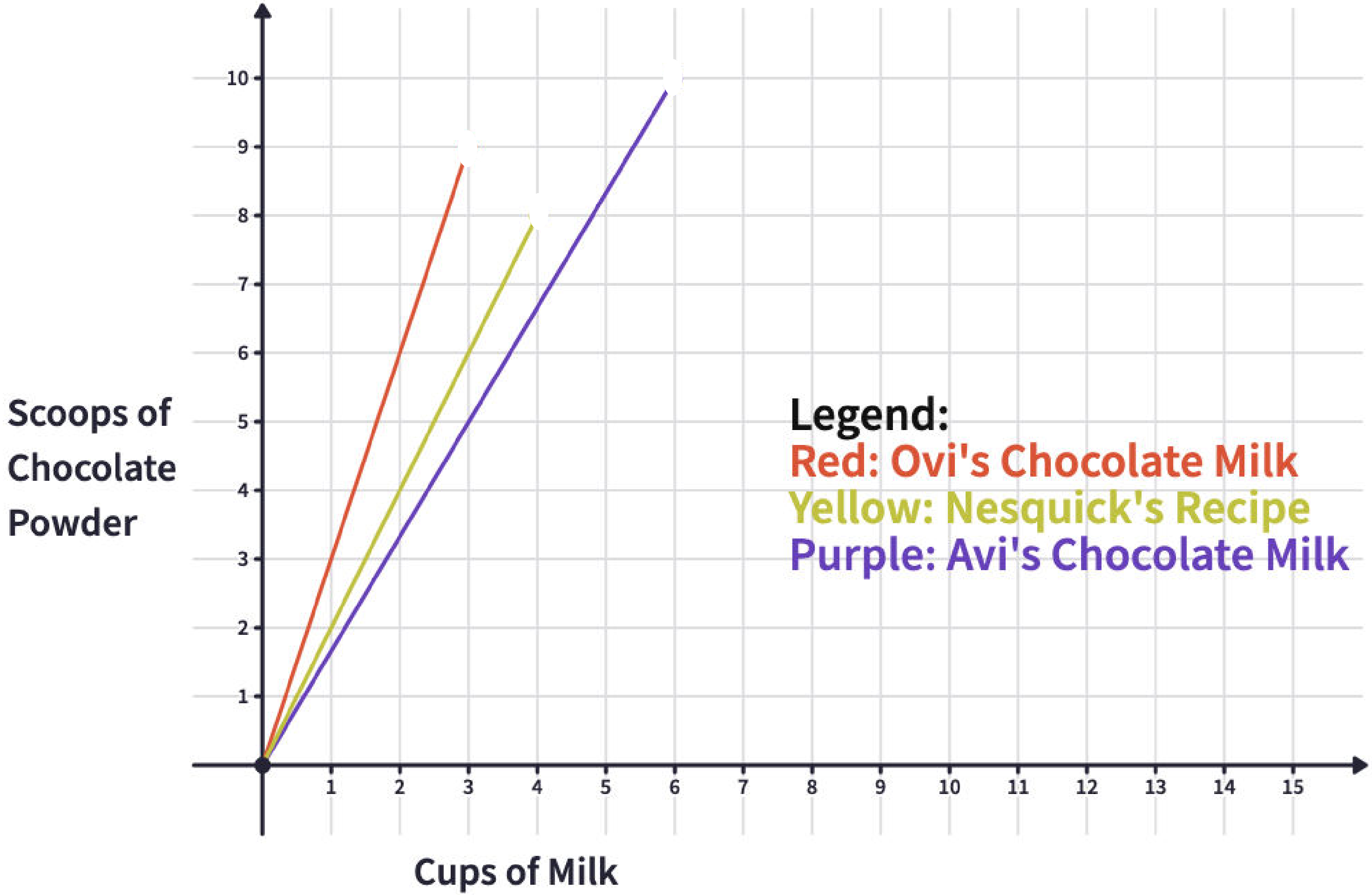
What do you notice?

What do you wonder?



Who's chocolate milk tastes the most chocolatey?

How do you know?



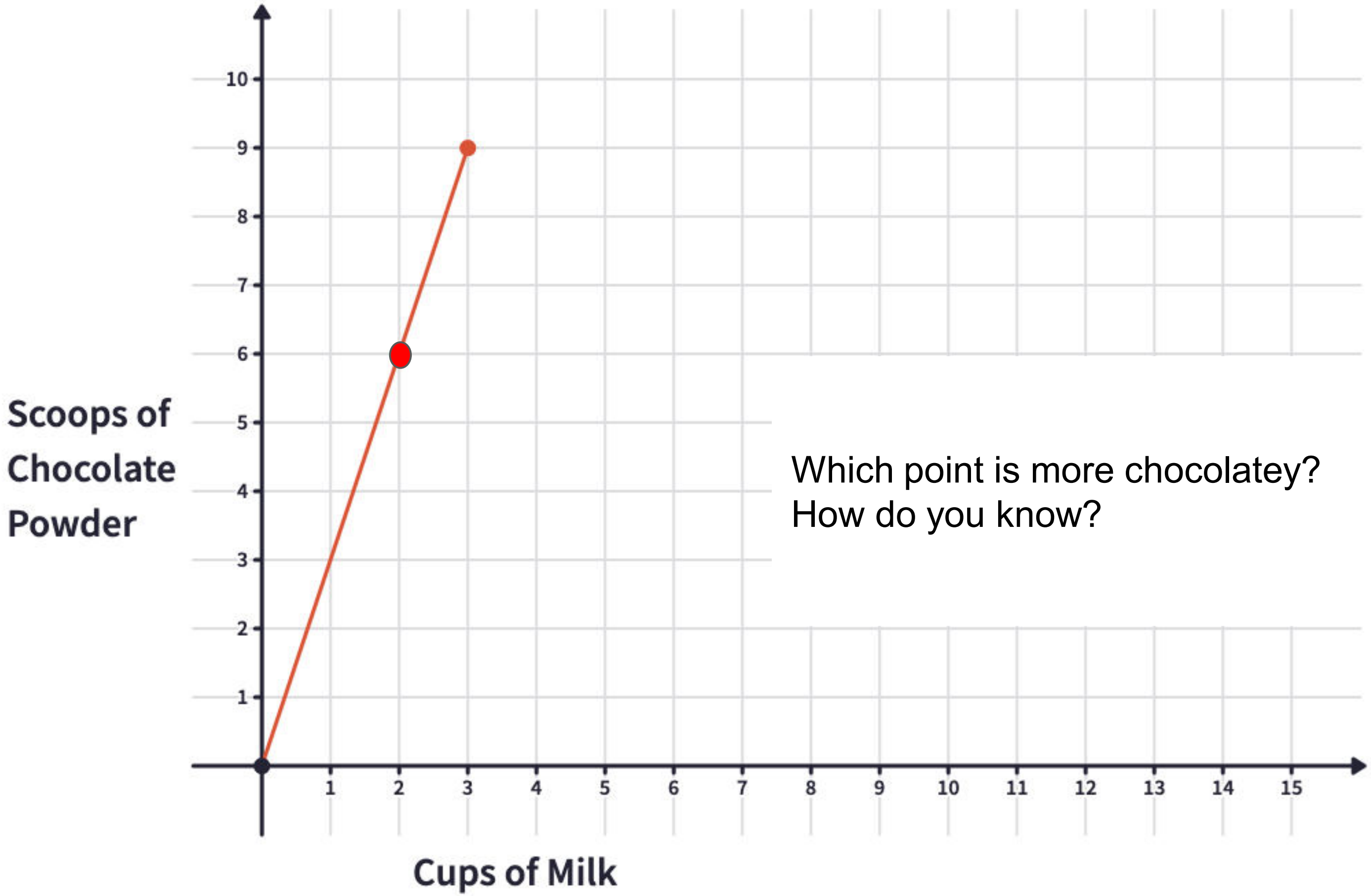
What is each recipe?

How did your group figure it out?

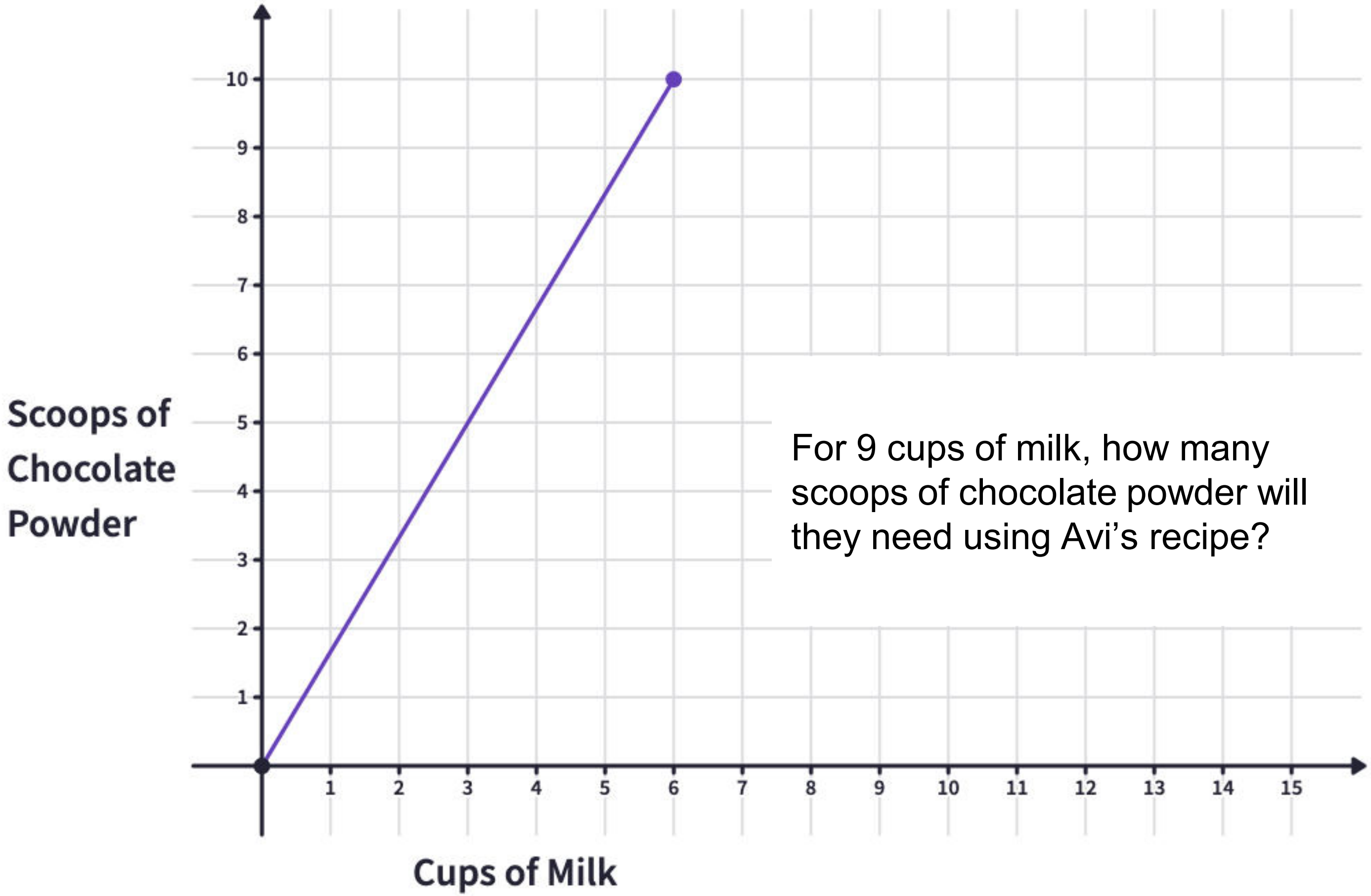
Exit Ticket

How would you describe slope to someone that wasn’t here today?

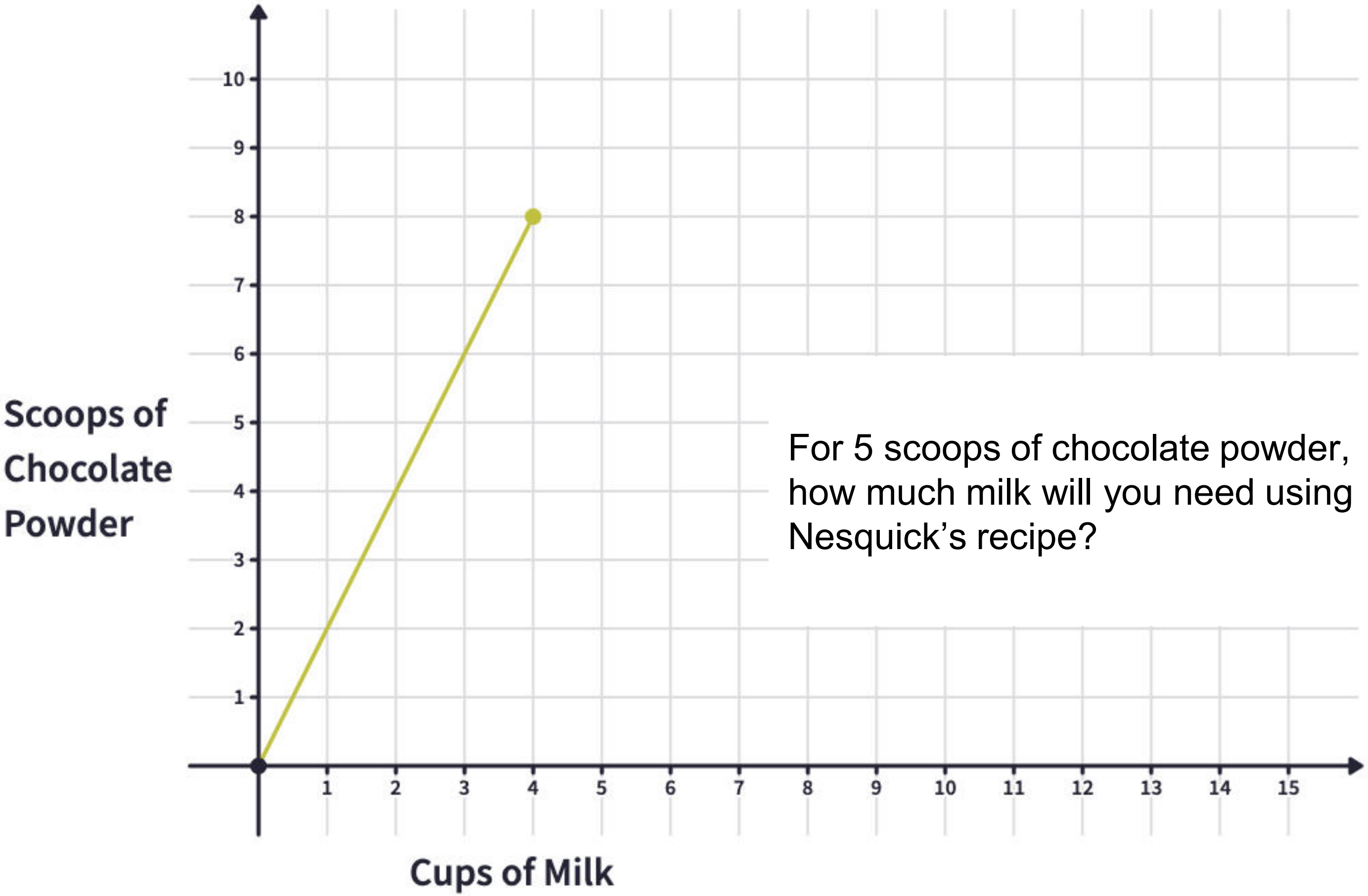
Possible Extensions



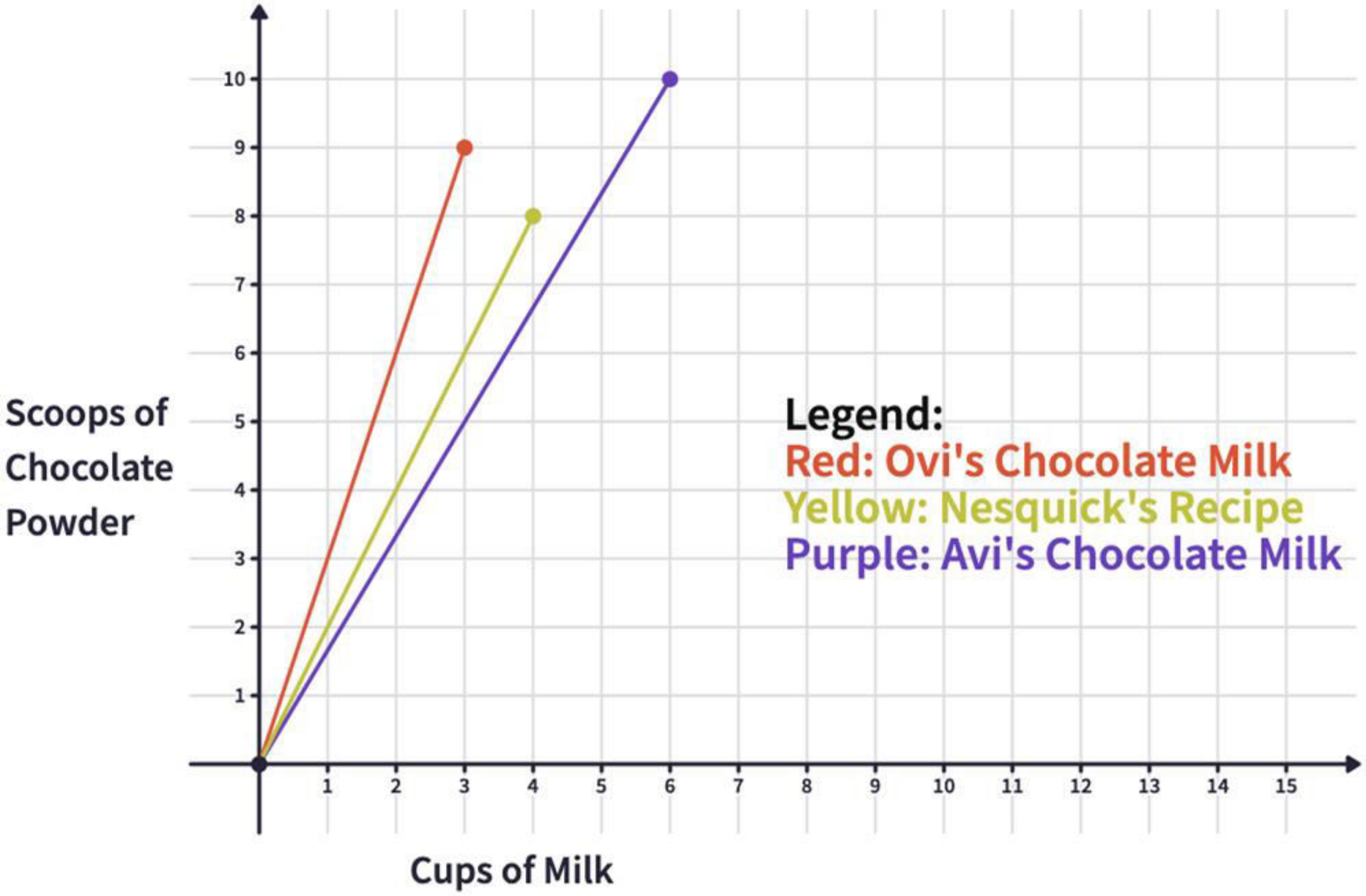
Which point is more chocolatey?
How do you know?



For 9 cups of milk, how many scoops of chocolate powder will they need using Avi’s recipe?



For 5 scoops of chocolate powder, how much milk will you need using Nesquick's recipe?



Your friend is making chocolate milk using 5 cups of milk and 10 scoops of chocolate powder. Did they create their own recipe or are they using someone else's recipe? How do you know?