Section 5.1

Integration: "An Overview of the Area Problem"

All graphics are attributed to:

- *Calculus,10/E* by Howard Anton, Irl Bivens, and Stephen Davis

Chapter Overview

- In this chapter we will begin with the idea of what "area" means and we will study two approaches to defining and calculating areas.
- After that, we will discuss the Fundamental Theorem of Calculus and how it relates tangent lines and areas.
- Later, we will study more rectilinear motion (position, velocity, acceleration along an s-axis) and integrals involving logarithms.

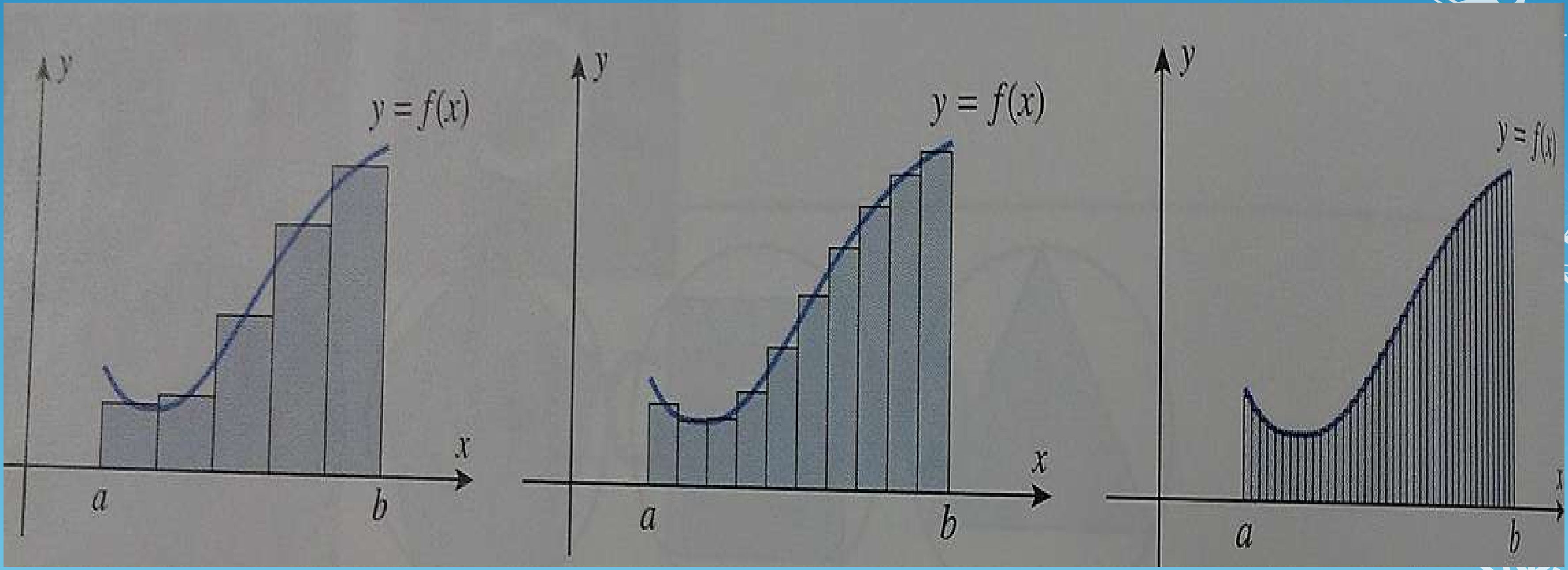
The Area Problem

- Formulas for the areas of polygons, such as squares, rectangles, triangles, and trapezoids, were well known in many early civilizations.
- However, the problem of finding formulas for regions with curved (curvilinear) boundaries caused difficulties for early mathematicians.
- The bottom of page 316 and the top of page 317 describes some work for the area formula of a circle involving different mathematicians and limits that you might find interesting, as well as a brief biography of Archimedes on page 318.

The Rectangle Method for Finding Area

- Divide the interval [a,b] into n equal subintervals and construct a rectangle on each subinterval from the x-axis to the curve f(x).
- Find the area of each rectangle.
- Find the sum of the areas of all of the rectangles in the interval [a,b].
- This total will be an approximation to the exact area under the curve over the interval [a,b].
- The more rectangles you divide the interval [a,b] into, the more accurate your approximation will be compared to the exact area.
- The limit as the number of rectangles, n, approaches infinity equals the exact area under the curve on [a,b].

Visual representation of how much closer the approximation gets as the number of rectangles in the interval increases.



The Antiderivative Method for Finding Area

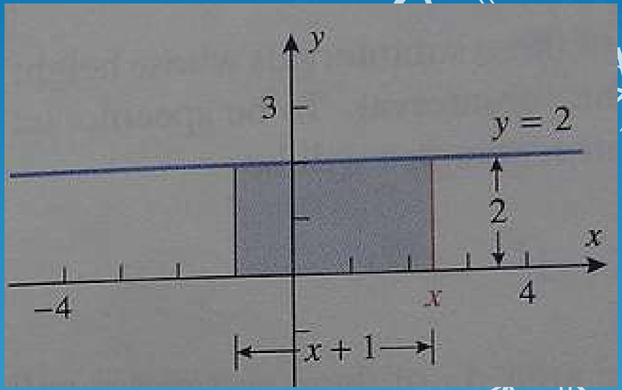
- When we use the rectangle method, we often end up with limits that we do not know how to compute or limits that are very long and difficult to compute.

- Therefore, the antiderivative method was discovered to make those problems possible.

- The derivative of the area equals the original function:

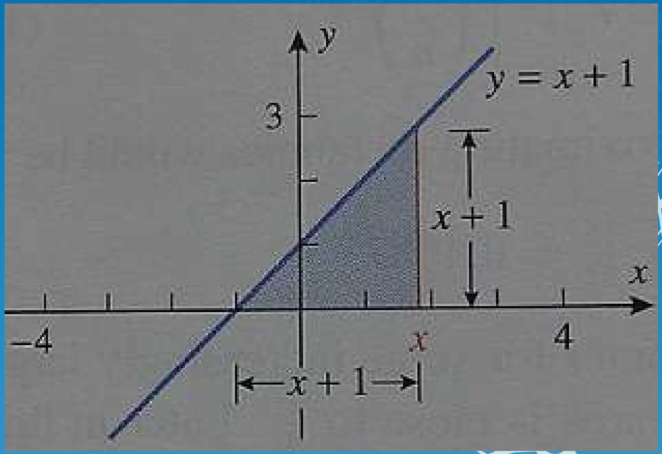
$$A' = f(x)$$

Confirming A′ = f(x) Using Geometry Examples



- Example 1: Find the area A(x) between the graph of f(x)=2 and the interval [-1,x] and find the derivative A′(x) of this area function.
- As you can see from looking at the graph, the area is a rectangle and we can find the area A(x) = base * height.
- A(x) = (x-(-1))(2) = (x+1)*2 = 2x + 2
- The derivative of the area function above A(x) = 2x + 2 is A′(x) = 2 which is equal to the original function.
- Therefore, A′(x) = f(x).

Confirming A' = f(x) Using Geometry Examples



- Example 1: Find the area A(x) between the graph of f(x)=x+1 and the interval [-1,x] and find the derivative A'(x) of this area function.
- As you can see from looking at the graph, the area is a triangle and area A(x)=(1/2)base*height.
- A(x)=(1/2)(x-(-1))(x+1)=(1/2)(x+1)(x+1)

  $=(1/2)\, x^2 + x + 1/2$
- The derivative of the area function above is

  A'(x) = x+1 which is equal to the original f(x).
- Therefore, A'(x) = f(x).

The Rectangle Method and the Antiderivative Method Compared

- NOTE: There is a third geometry example on page 320 with a different line, and the antiderivative method works for higher degree polynomial functions and more sophisticated functions.
- The rectangle method and the antiderivative method are very different approaches to solving the same problem of finding area.
- The antiderivative method is usually quicker and more efficient.
- The rectangle method makes more sense intuitively and we will use a similar method to find volume, length of a curve, surface area, etc.

Vacation!!