

# Rotation, Gravity, Oscillation



Monday, November 23, 2009



Torque



# Torque and the see-saw

A see-saw is an example of a device that twists.

A force that causes a twisting motion, multiplied by its distance from the point of rotation, is called a torque.

***Torque*** is what makes a see saw fun.



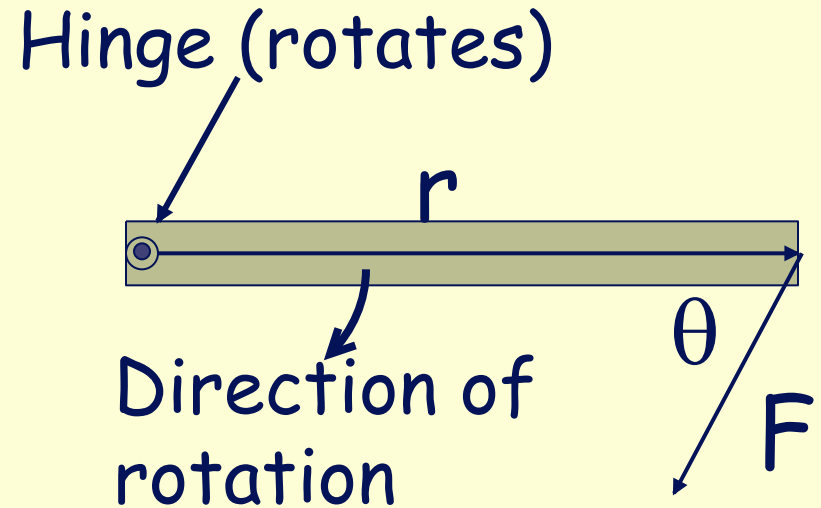
# Torque

If we know the angle  $\theta$  between  $F$  and  $r$ , we can calculate torque!

$$\tau = r F \sin \theta$$

- $\tau$  is torque
- $r$  is "moment arm"
- $F$  is force
- $\theta$  is angle between  $F$  and  $r$

The SI unit of torque is the Nm. You cannot substitute Joule for Nm in the case of torque.





# Sample Problem

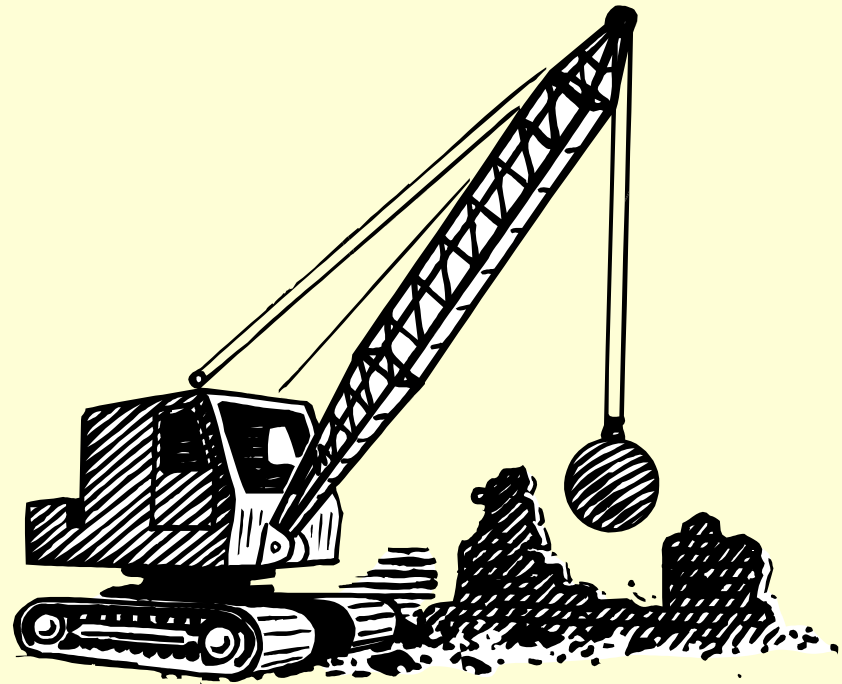
Consider the door to the classroom. We use torque to open it.

Identify the following:

- A. The point of rotation.
  - B. The point of application of force.
  - C. The moment arm ( $r$ ).
  - D. The angle between  $r$  and  $F$  (best guess).
- 

# Sample Problem

A crane lifts a load. If the mass of the load is 500 kg, and the crane's 22-m long arm is at a  $75^\circ$  angle relative to the horizontal, calculate the torque exerted about the point of rotation at the base of the crane arm.



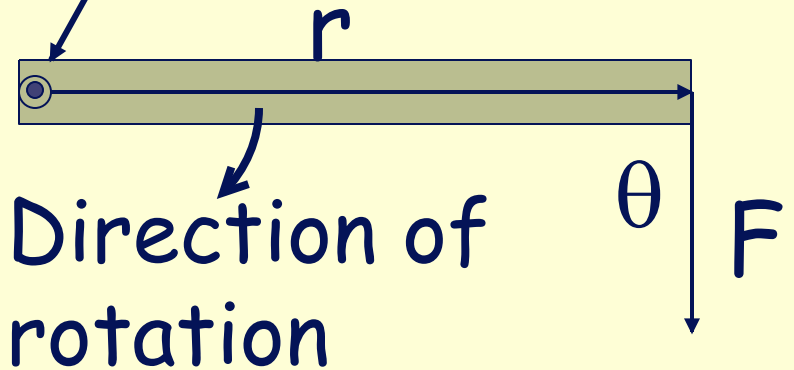
# Torque simplified

Usually,  $\theta$  will be  $90^\circ$ , and

$$\tau = r F$$

- $\tau$  is torque
- $r$  is "moment arm"
- $F$  is force

Hinge: rotates





# Problem

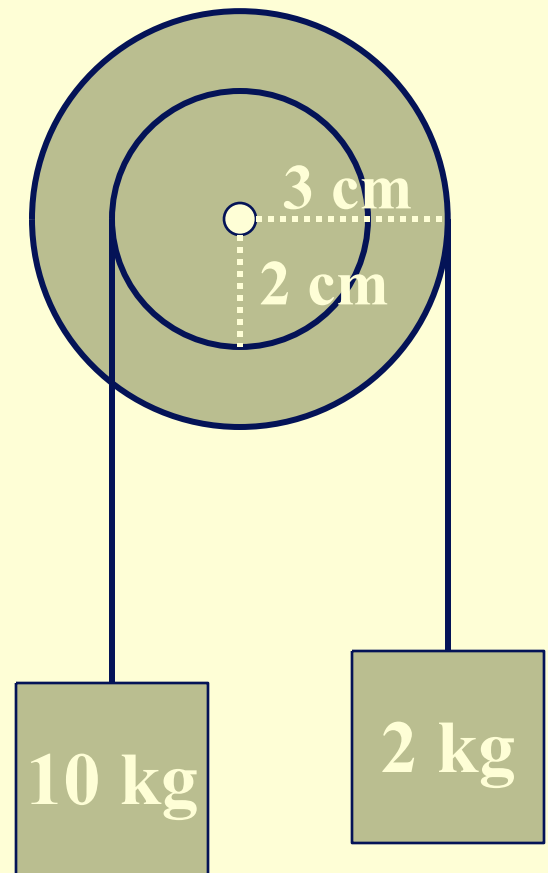
A standard door is 36 inches wide, with the doorknob located at 32 inches from the hinge. Calculate the torque a person applies when he pushes on the doorknob at right angles to the door with a force of 110 N. (Use 1 inch = 2.54 cm to calculate the torque in SI units).



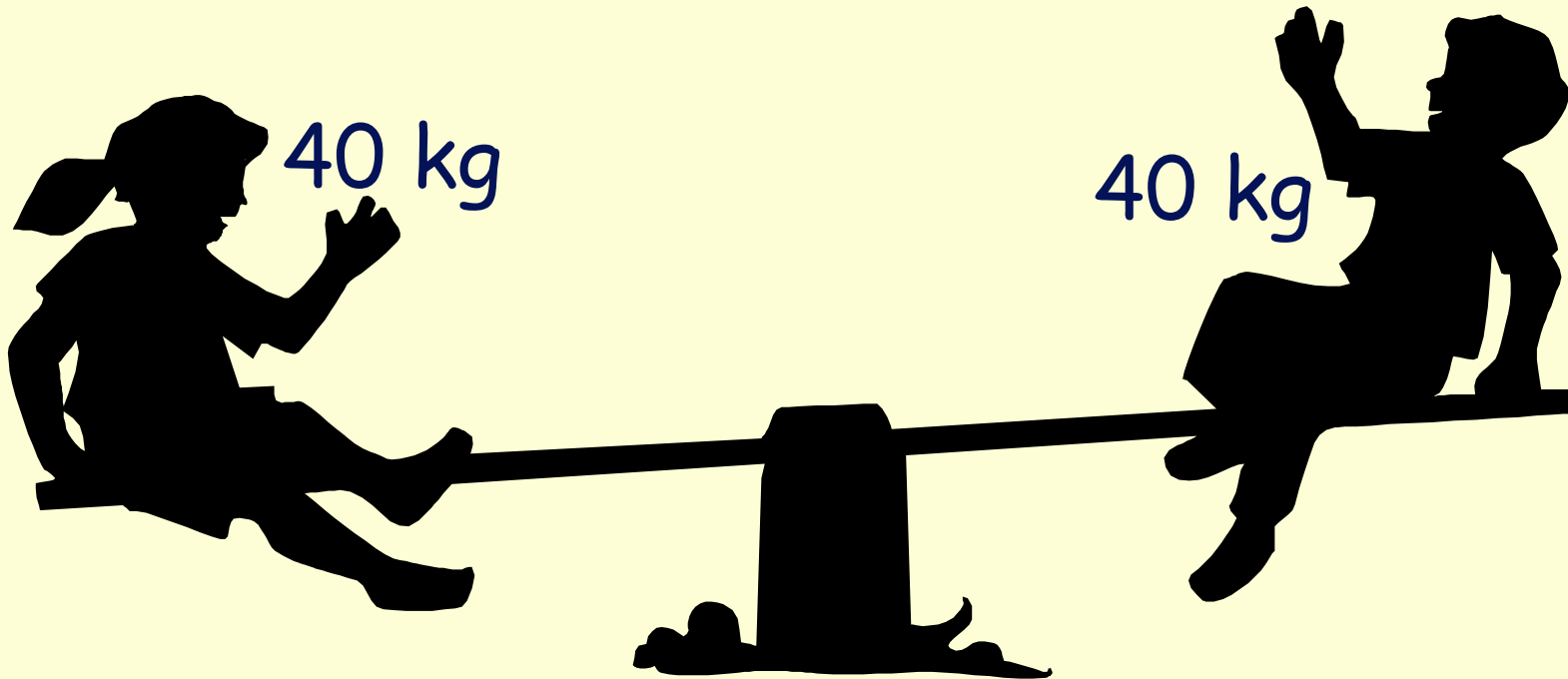
# Problem

A double pulley has two weights hanging from it as shown.

- A) What is the net torque?
- B) In what direction will the pulley rotate?



Now consider a balanced situation



$$\tau_{ccw} = \tau_{cw}$$

This is called *rotational equilibrium!*



# Sample Problem

A 5.0-meter long see saw is balanced on a fulcrum at the middle. A 45-kg child sits all the way on one end. Where must a 60-kg child sit if the see-saw is to be balanced?





# Sample Problem

A 5.0-meter long see saw is balanced on a fulcrum at the middle. A 45-kg child sits all the way on one end. And a 60-kg child sits all the way on the other end. If the see saw has a mass of 100 kg, where must the fulcrum be placed to attain a balanced situation?

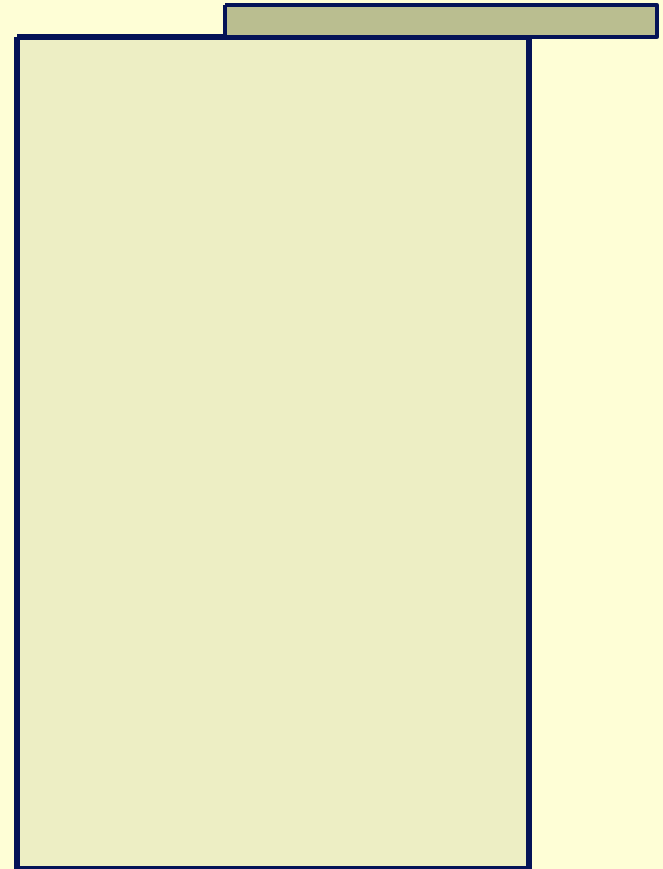
Check against notes; mass is different





# Sample Problem

A 10-meter long wooden plank of mass 209 kg rests on a flat roof with 2.5 meters extended out beyond the roof's edge. How far out on the plank can an 80-kg man walk before he is in danger of falling?



Tuesday, November 24,  
2009

Torque Lab



# Torque Lab - data collection

Create a "torque balance" with the meter stick, two "known masses" and one "unknown mass".

## Rules

- All masses, known and unknown, must be attached to clips.
- The meter stick cannot be balanced at the 50 cm point

## Data collected

- Positions on meter stick of all hanging masses, and position of fulcrum.
  - Masses of all "known" components. DO NOT MASS THE UNKNOWN!
  - DRAW A DIAGRAM THAT IS CLEARLY LABELED!
- 

- 
- *Calculate your unknown.*

We will complete the lab on the computers.





# Torque Lab II

Use Excel to determine if your unknown calculation was OK.

Turn in:

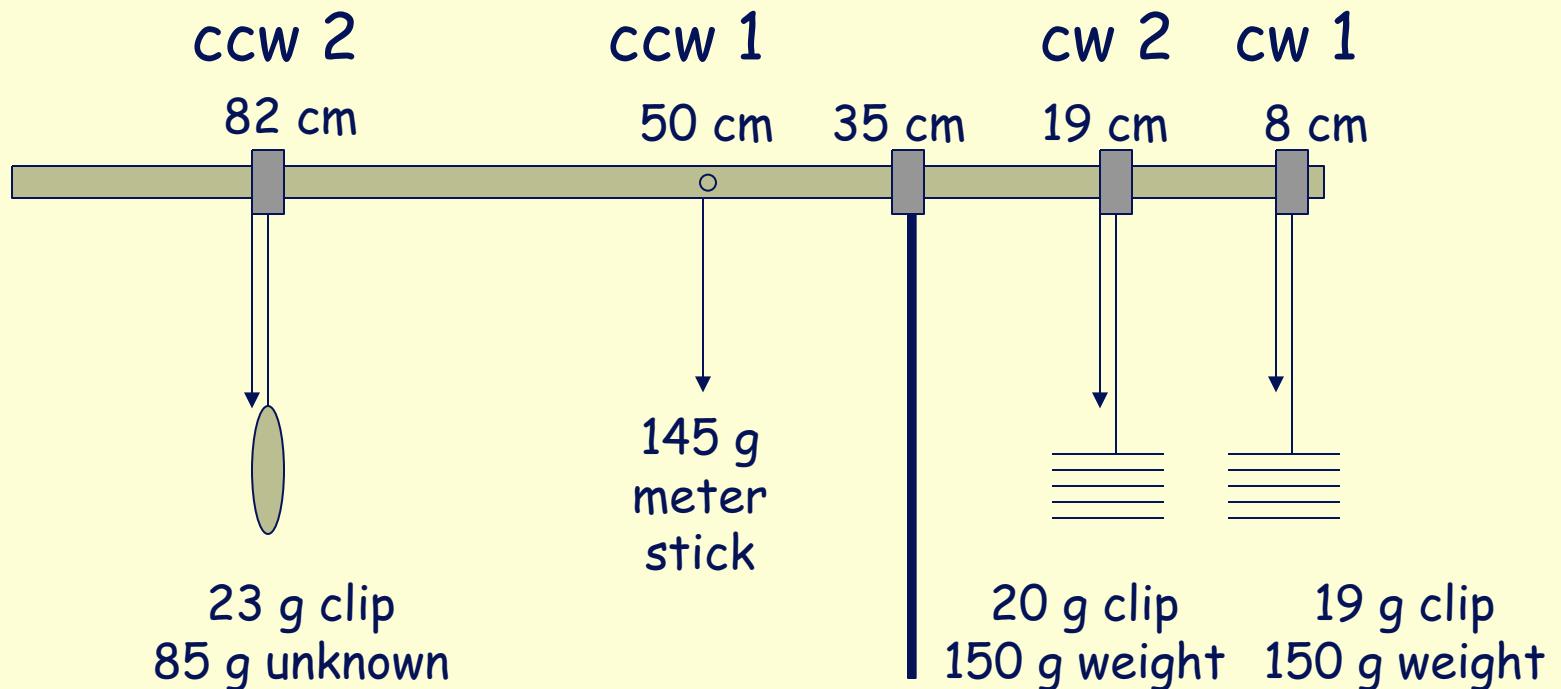
- Hand calculation of torque lab. This will include your diagram, your data, and your calculation of your unknown mass.
  - Torque lab spreadsheet. Submit in one of the following ways:
    - Drag and drop into the 'Prinkey' class temp folder.
    - Print and submit to me a hard-copy.
    - Email the spreadsheet to me.
- 

Tuesday, December 1, 2009

Universal Law of Gravity

# Torque lab tables

Let's take a minute to review the torque lab, and entry of the data into a spreadsheet and calculation with Excel.





# The Universal Law of Gravity

Newton's famous apple fell on Newton's famous head, and lead to this law.

It tells us that the force of gravity objects exert on each other depends on their masses and the distance they are separated from each other.



# The Force of Gravity

Remember  $F_g = mg$ ?

We've use this to approximate the force of gravity on an object near the earth's surface.

This formula won't work for planets and space travel.

It won't work for objects that are far from the earth.

For space travel, we need a better formula.





# The Force of Gravity

$$F_g = -Gm_1m_2/r^2$$

- $F_g$ : Force due to gravity (N)
- $G$ : Universal gravitational constant
  - $6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$
- $m_1$  and  $m_2$ : the two masses (kg)
- $r$ : the distance between the centers of the masses (m)

The Universal Law of Gravity **ALWAYS** works, whereas  $F = mg$  only works sometimes.





# Sample Problem

- A. How much force does the earth exert on the moon?
- B. How much force does the moon exert on the earth?





# Sample Problem

What would be your weight if you were orbiting the earth in a satellite at an altitude of 3,000,000 m above the earth's surface? (Note that even though you are apparently weightless, gravity is still exerting a force on your body, and this is your actual weight.)





# Sample Problem

Sally, an astrology buff, claims that the position of the planet Jupiter influences events in her life. She surmises this is due to its gravitational pull. Joe scoffs at Sally and says "your Labrador Retriever exerts more gravitational pull on your body than the planet Jupiter does". Is Joe correct? (Assume a 100-lb Lab 1.0 meter away, and Jupiter at its farthest distance from Earth).



Wednesday, December 2,  
2009

Gravitational Acceleration and  
Orbit



# Acceleration due to gravity

Remember  $g = 9.8 \text{ m/s}^2$ ?

This works fine when we are near the surface of the earth. For space travel, we need a better formula! What would that formula be?





# Acceleration due to gravity

$$g = GM/r^2$$

This formula lets you calculate  $g$  anywhere if you know the distance a body is from the center of a planet.

We can calculate the acceleration due to gravity anywhere!



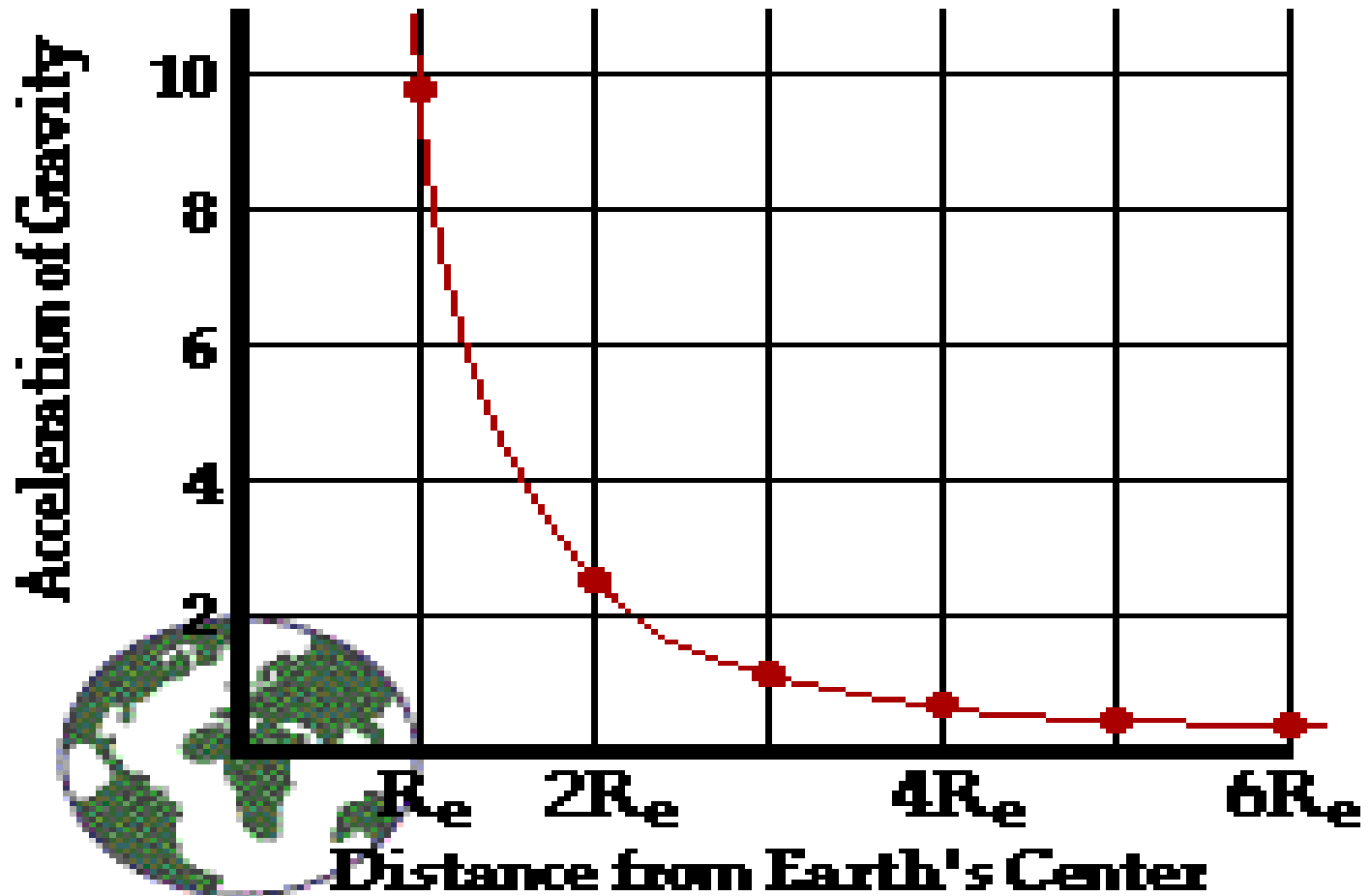


# Sample Problem

What is the acceleration due to gravity at an altitude equal to the earth's radius? What about an altitude equal to twice the earth's radius?



# Acceleration and distance





Surface gravitational acceleration  
depends on mass and radius.

| Planet  | Radius(m            | Mass (kg)              | g (m/s <sup>2</sup> ) |
|---------|---------------------|------------------------|-----------------------|
| Mercury | $2.43 \times 10^6$  | $3.2 \times 10^{23}$   | 3.61                  |
| Venus   | $6.073 \times 10^6$ | $4.88 \times 10^{24}$  | 8.83                  |
| Mars    | $3.38 \times 10^6$  | $6.42 \times 10^{23}$  | 3.75                  |
| Jupiter | $6.98 \times 10^6$  | $1.901 \times 10^{27}$ | 26.0                  |
| Saturn  | $5.82 \times 10^7$  | $5.68 \times 10^{26}$  | 11.2                  |
| Uranus  | $2.35 \times 10^7$  | $8.68 \times 10^{25}$  | 10.5                  |
| Neptune | $2.27 \times 10^7$  | $1.03 \times 10^{26}$  | 13.3                  |
| Pluto   | $1.15 \times 10^6$  | $1.2 \times 10^{22}$   | 0.61                  |





# Sample Problem

What is the acceleration due to gravity at the surface of the moon?





# Johannes Kepler

## (1571-1630)

Kepler developed some extremely important laws about planetary motion.

Kepler based his laws on massive amounts of data collected by Tycho Brahe.

Kepler's laws were used by Newton in the development of his own laws.



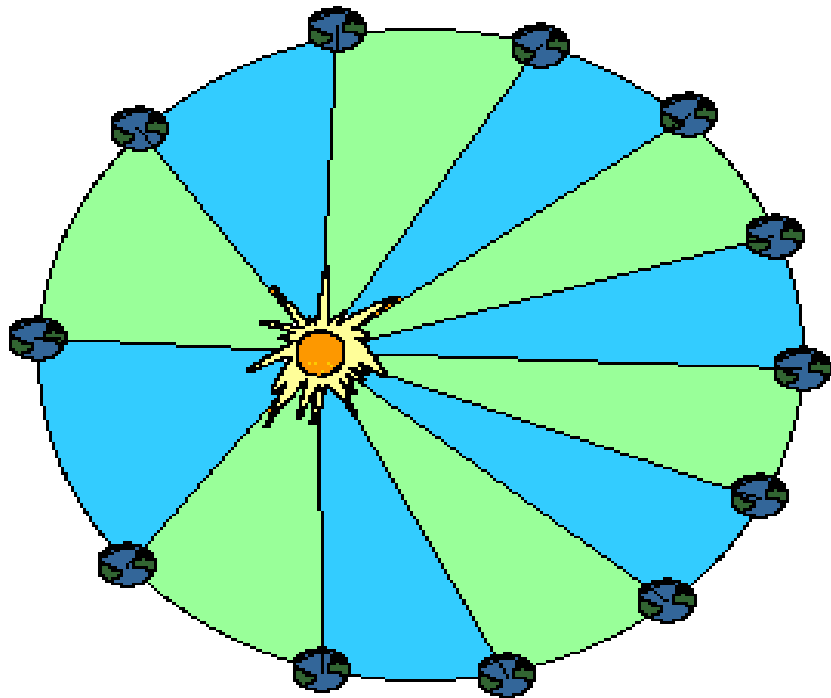


# Kepler's Laws

1. Planets orbit the sun in elliptical orbits, with the sun at a focus.
  2. Planets orbiting the sun carve out equal area triangles in equal times.
  3. The planet's year is related to its distance from the sun in a predictable way.
- 

# Kepler's Laws

**The Law of Equal Areas**



**An imaginary line drawn from the sun to any planet sweeps out equal areas in equal amounts of time.**

Let's look at a simulation of planetary motion at <http://surendranath.tripod.com/Applets.html>

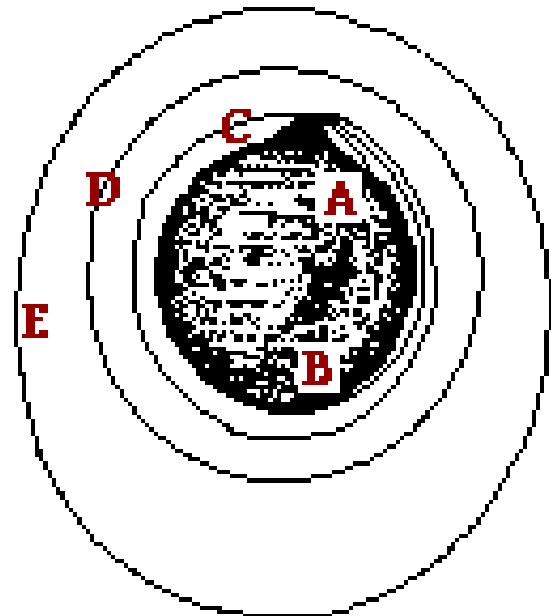


## Sample Problem (not in packet)

Using Newton's Law of Universal Gravitation, derive a formula to show how the period of a planet's orbit varies with the radius of that orbit. Assume a nearly circular orbit.



# Satellites

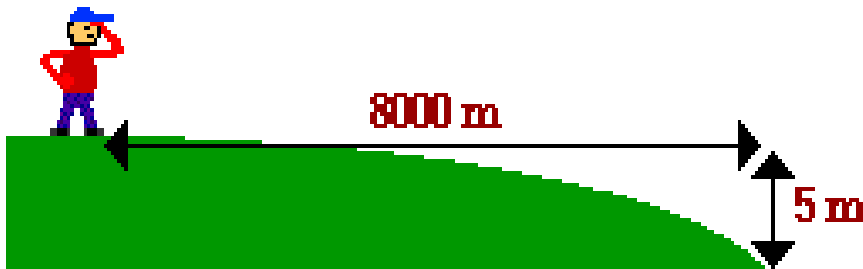


**A satellite is a projectile  
which falls around the  
earth instead of into it.**

# Orbital speed

At any given altitude, there is only one speed for a stable circular orbit.

From geometry, we can calculate what this orbital speed must be.



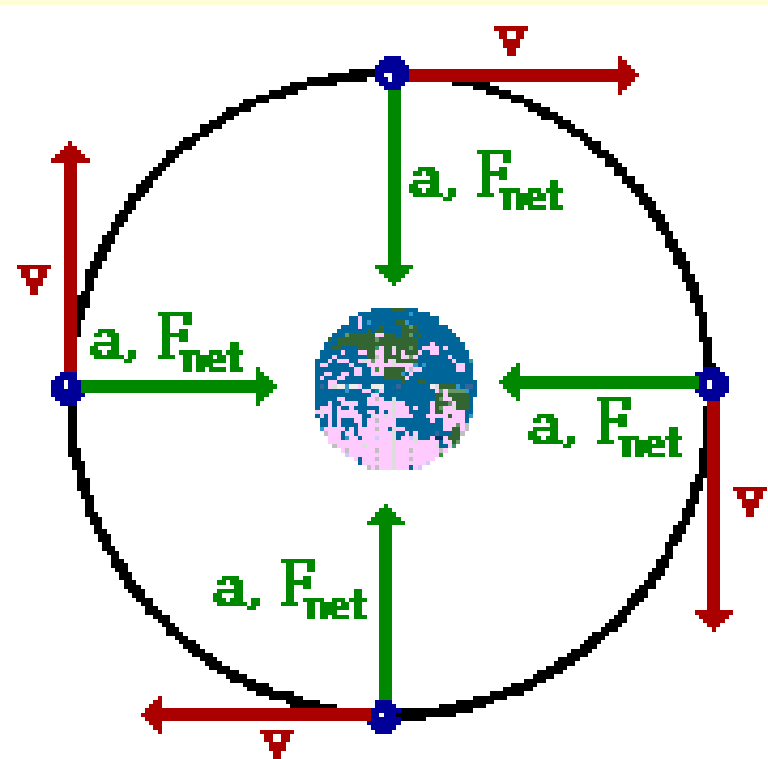
**For every 8000 meters along the horizon, the earth curves downward by 5 meters.**

At the earth's surface, if an object moves 8000 meters horizontally, the surface of the earth will drop by 5 meters vertically.

That is how far the object will fall vertically in one second (use the 1<sup>st</sup> kinematic equation to show this).

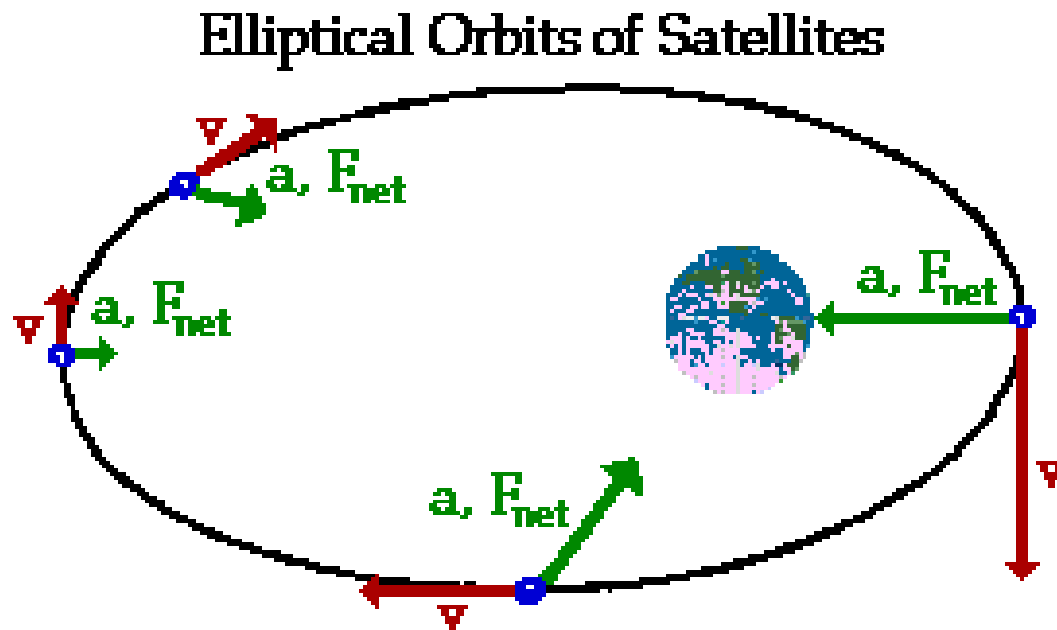
Therefore, an object moving at 8000 m/s will never reach the earth's surface.

Some orbits  
are nearly  
circular.



Satellites encounter inward  
forces and accelerations and  
tangential velocities.

# Some orbits are highly elliptical.



Even moving in elliptical motion, there is a tangential velocity and an inward acceleration and force. In this case, the  $\mathbf{a}$  and  $\mathbf{F}$  vectors are directed towards the central body.



# Centripetal force and gravity

The orbits we analyze mathematically will be nearly circular.

$$F_g = F_c$$

- (centripetal force is provided by gravity)

$$GMm/r^2 = mv^2/r$$

- The mass of the orbiting body cancels out in the expression above.
- One of the  $r$ 's cancels as well

$$GM/r = v^2$$




# Sample Problem

- A. What velocity does a satellite in orbit about the earth at an altitude of 25,000 km have?
- B. What is the period of this satellite?





# Sample Problem

A geosynchronous satellite is one which remains above the same point on the earth. Such a satellite orbits the earth in 24 hours, thus matching the earth's rotation. How high must a geosynchronous satellite be above the surface to maintain a geosynchronous orbit?



Thursday, December 3, 2009

## Gravitational Potential Energy and Escape Velocity



# Gravitational Potential Energy

Remember  $U_g = mgh$ ?

This is also an approximation we use when an object is near the earth.

This formula won't work when we are very far from the surface of the earth. For space travel, we need another formula.





# Gravitational Potential Energy

$$U_g = -Gm_1m_2/r$$

- $U_g$ : Gravitational potential energy (J)
- $G$ : Universal gravitational constant
  - $6.67 \times 10^{-11} \text{N m}^2/\text{kg}^2$
- $m_1$  and  $m_2$ : the two masses (kg)
- $r$ : the distance between the centers of the masses (m)

Notice that the "theoretical" value of  $U_g$  is always negative.

This formula always works for two or more objects.





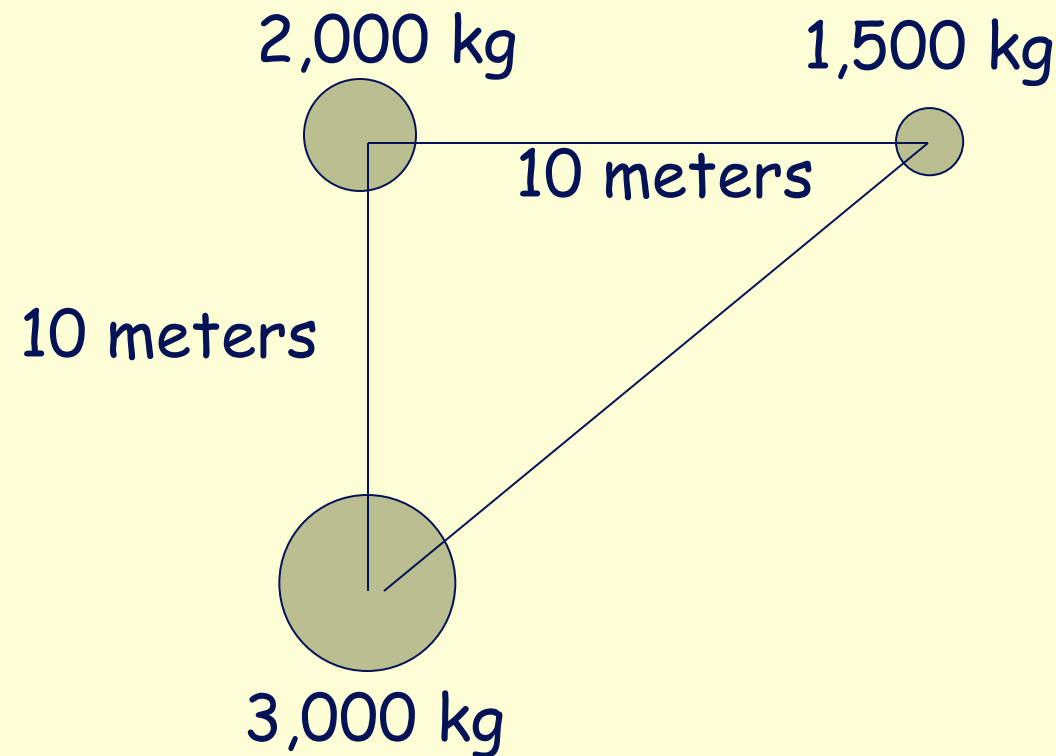
# Sample Problem

What is the gravitational potential energy of a satellite that is in orbit about the Earth at an altitude equal to the earth's radius? Assume the satellite has a mass of 10,000 kg.



# Sample Problem - not in packet

What is the gravitational potential energy of the following configuration of objects?





# Escape Velocity

Calculation of minimum escape velocity from a planet's surface can be done by using energy conservation.

Assume the object gains potential energy and loses kinetic energy, and assume the final potential energy and final kinetic energy are both zero.

$$U_1 + K_1 = U_2 + K_2$$

$$-GMm/r + \frac{1}{2}mv^2 = 0$$

$$v = (2GM/r)^{1/2}$$




## Sample Problem

What is the velocity necessary for a rocket to escape the gravitational field of the earth? Assume the rocket is near the earth's surface.





# Sample Problem

Suppose a 2500-kg space probe accelerates on blast-off until it reaches a speed of 15,000 m/s. What is the rocket's kinetic energy when it has effectively escaped the earth's gravitational field?



Friday, December 4, 2009

Periodic Motion



# Periodic Motion

Motion that repeats itself over a fixed and reproducible period of time is called **periodic motion**.

The revolution of a planet about its sun is an example of periodic motion. The highly reproducible period ( $T$ ) of a planet is also called its year.

Mechanical devices on earth can be designed to have periodic motion. These devices are useful timers. They are called **oscillators**.





# Oscillator Demo

Let's see demo of an oscillating spring using  
LoggerPro and a motion sensor.





# Simple Harmonic Motion

You attach a weight to a spring, stretch the spring past its equilibrium point and release it. The weight bobs up and down with a reproducible period,  $T$ .

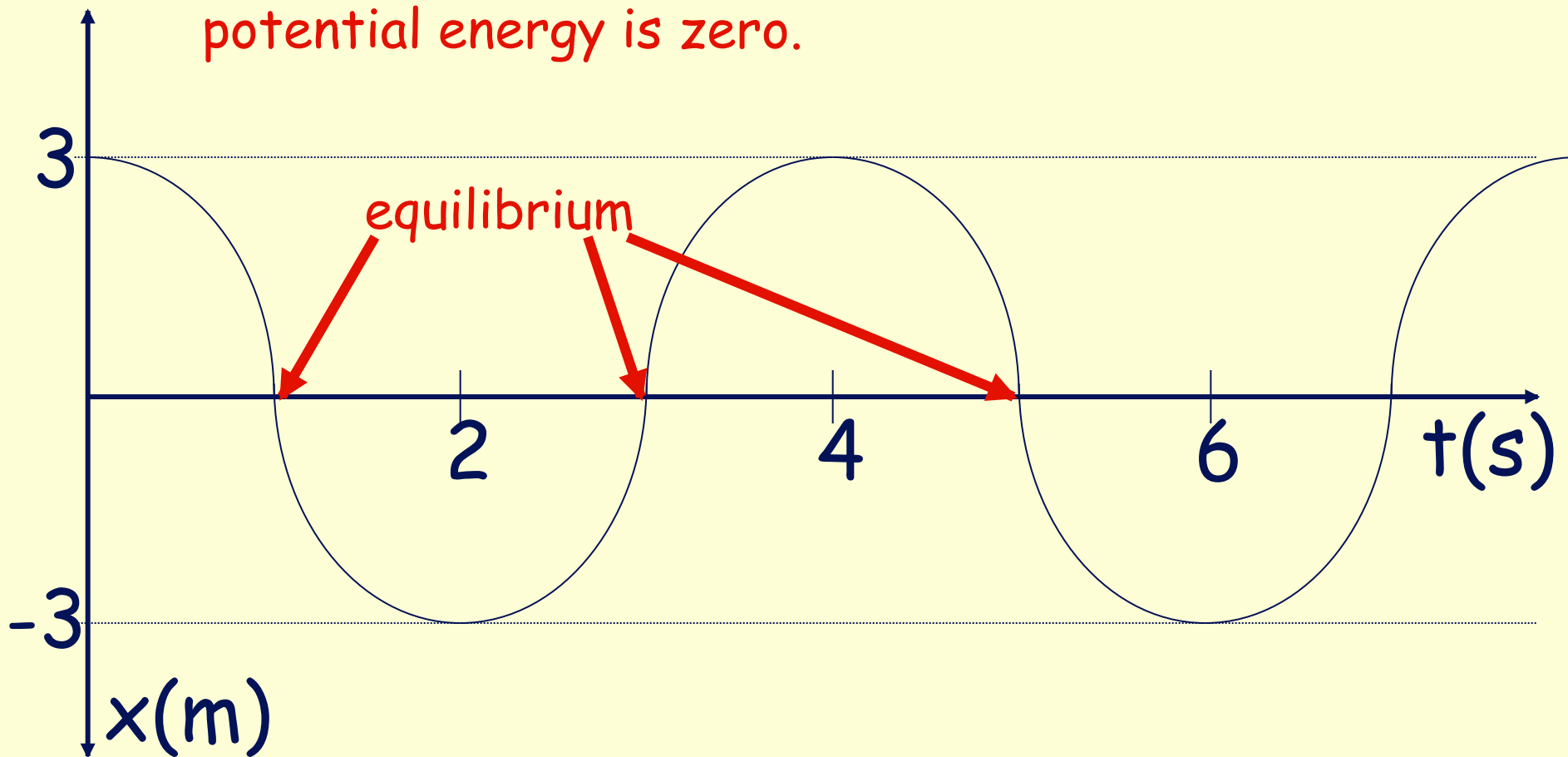
Plot position vs time to get a graph that resembles a sine or cosine function. The graph is "sinusoidal", so the motion is referred to as **simple harmonic motion**.

Springs and pendulums undergo simple harmonic motion and are referred to as **simple harmonic oscillators**.



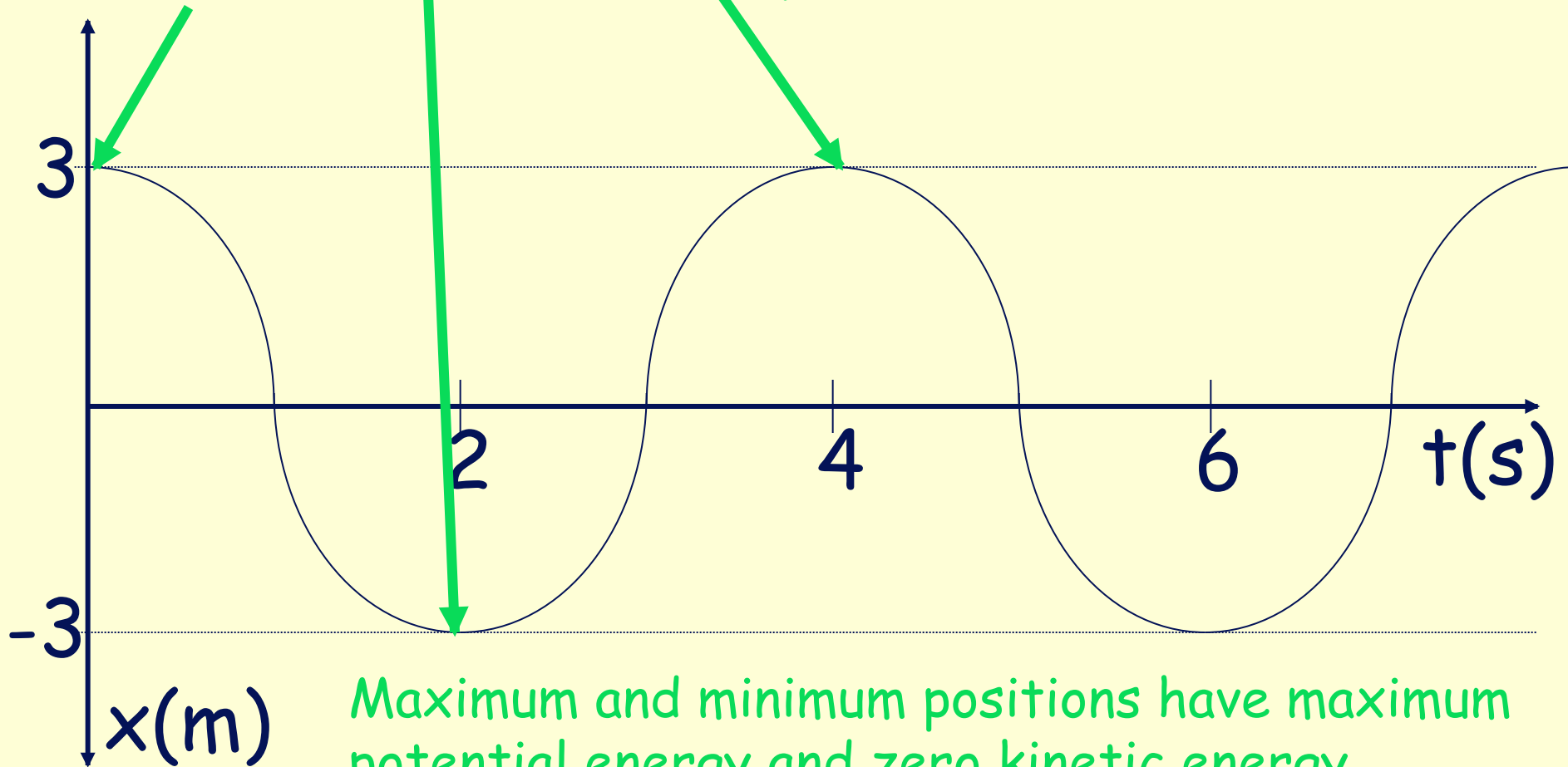
# Analysis of graph

Equilibrium is where kinetic energy is maximum and potential energy is zero.



# Analysis of graph

Maximum and minimum positions



Maximum and minimum positions have maximum potential energy and zero kinetic energy.



# Oscillator Definitions

## Amplitude

- Maximum displacement from equilibrium.
- Related to energy.

## Period

- Length of time required for one oscillation.

## Frequency

- How fast the oscillator is oscillating.
  - $f = 1/T$
  - Unit: Hz or  $s^{-1}$
- 



# Sample Problem

Determine the amplitude, period, and frequency of an oscillating spring using the CBLs and the motion sensors. See how this varies with the force constant of the spring and the mass attached to the spring.



Monday, December 7, 2009

Springs



# Springs

A very common type of Simple Harmonic Oscillator.

Our springs are “ideal springs”.

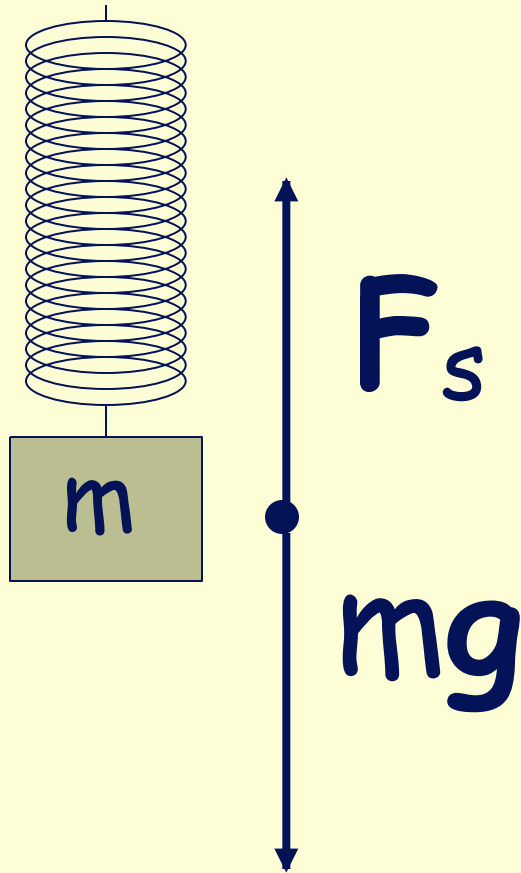
- They are massless.
- They are both compressible and extensible.

They will follow Hooke's Law.

- $F = -kx$



# Review of Hooke's Law



$$F_s = -kx$$

The force constant of a spring can be determined by attaching a weight and seeing how far it stretches.



# Period of a spring

$$T = 2\pi \sqrt{\frac{m}{k}}$$

- $T$ : period (s)
  - $m$ : mass (kg)
  - $k$ : force constant (N/m)
- 



# Sample Problem

Calculate the period of a 200-g mass attached to an ideal spring with a force constant of 1,000 N/m.





# Sample Problem

A 300-g mass attached to a spring undergoes simple harmonic motion with a frequency of 25 Hz. What is the force constant of the spring?





# Sample Problem

An 80-g mass attached to a spring hung vertically causes it to stretch 30 cm from its unstretched position. If the mass is set into oscillation on the end of the spring, what will be the period?





# Sample Problem

You wish to double the force constant of a spring. You

- A. Double its length by connecting it to another one just like it.
  - B. Cut it in half.
  - C. Add twice as much mass.
  - D. Take half of the mass off.
- 



# Sample Problem

You wish to double the force constant of a spring. You

- A. Double its length by connecting it to another one just like it.
  - B. Cut it in half.
  - C. Add twice as much mass.
  - D. Take half of the mass off.
- 



# Conservation of Energy

Springs and pendulums obey conservation of energy.

The equilibrium position has high kinetic energy and low potential energy.

The positions of maximum displacement have high potential energy and low kinetic energy.

Total energy of the oscillating system is constant.





# Sample problem.

A spring of force constant  $k = 200 \text{ N/m}$  is attached to a 700-g mass oscillating between  $x = 1.2$  and  $x = 2.4$  meters. Where is the mass moving fastest, and how fast is it moving at that location?





# Sample problem.

A spring of force constant  $k = 200 \text{ N/m}$  is attached to a 700-g mass oscillating between  $x = 1.2$  and  $x = 2.4$  meters. What is the speed of the mass when it is at the 1.5 meter point?





# Sample problem.

A 2.0-kg mass attached to a spring oscillates with an amplitude of 12.0 cm and a frequency of 3.0 Hz. What is its total energy?





# Mini-Lab

Estimate the force constant of the spring in the plunger cart using conservation of energy.

Equipment:

- Plunger cart (mass 500 g)
- Ramp
- Meter Stick

Hint: consider turning spring potential energy into another form of potential energy.

Turn in one paper per person with your group's data, calculations, and results (that is, the value you think  $k$  has).



Tuesday, December 8, 2009

Pendulums



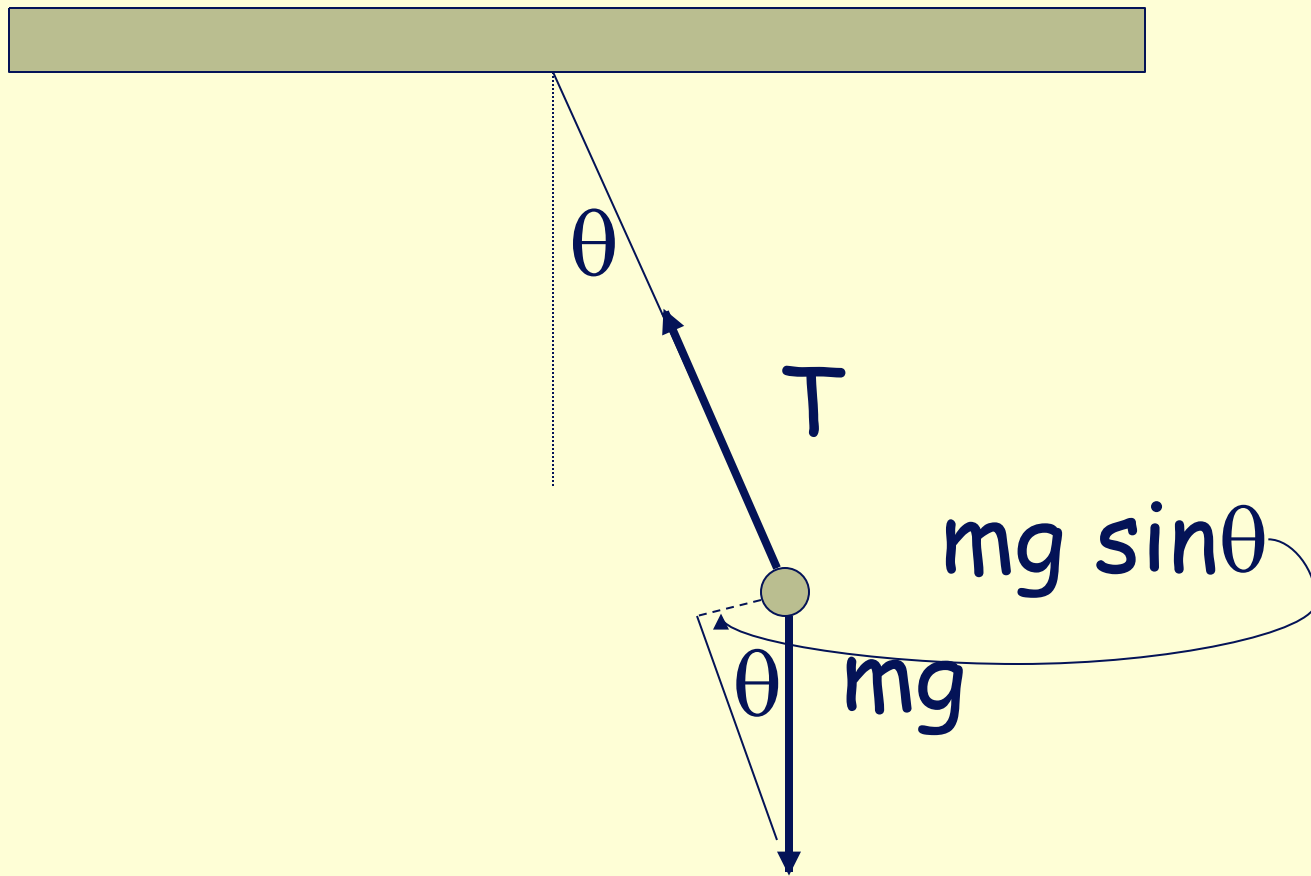
# Pendulums

The pendulum can be thought of as a simple harmonic oscillator.

The displacement needs to be *small* for it to work properly.



# Pendulum Forces





# Period of a pendulum

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$T$ : period (s)

$l$ : length of string (m)

$g$ : gravitational acceleration (m/s<sup>2</sup>)





# Sample problem

Predict the period of a pendulum consisting of a 500 gram mass attached to a 2.5-m long string.





# Sample problem

Suppose you notice that a 5-kg weight tied to a string swings back and forth 5 times in 20 seconds. How long is the string?





# Sample problem

The period of a pendulum is observed to be  $T$ .  
Suppose you want to make the period  $2T$ . What do you do to the pendulum?





# Pendulum Lab

Determine period,  $T$ , and length,  $l$ , of your group's pendulum. For accuracy, time multiple oscillations.

Write your group's data on the board.

Report, due Friday:

- A **table** and **graph** constructed from this data. The graph must be LINEAR such that the slope can be used to obtain  $g$ . In other words, you can't just simply graph  $T$  versus  $l$ . Think of what you must do to produce a linear graph from the data. Axes must be clearly labeled. The graph may be done by hand or in Excel. Show clearly how you get  $g$ , and indicate its value. Perform a percent error calculation.
  - Hint: Consider the formula for the period of a pendulum to decide what to graph.
- 



# 1st Period

| Group | Number of oscillations | Elapsed time (s) | Period (s) | Length (m) |
|-------|------------------------|------------------|------------|------------|
|       |                        |                  |            |            |
|       |                        |                  |            |            |
|       |                        |                  |            |            |
|       |                        |                  |            |            |
|       |                        |                  |            |            |
|       |                        |                  |            |            |
|       |                        |                  |            |            |
|       |                        |                  |            |            |
|       |                        |                  |            |            |





## 2<sup>nd</sup> Period

| Group | Number of oscillations | Elapsed time (s) | Period (s) | Length (m) |
|-------|------------------------|------------------|------------|------------|
|       |                        |                  |            |            |
|       |                        |                  |            |            |
|       |                        |                  |            |            |
|       |                        |                  |            |            |
|       |                        |                  |            |            |
|       |                        |                  |            |            |
|       |                        |                  |            |            |
|       |                        |                  |            |            |
|       |                        |                  |            |            |





# 7<sup>th</sup> Period

| Group | Number of oscillations | Elapsed time (s) | Period (s) | Length (m) |
|-------|------------------------|------------------|------------|------------|
|       |                        |                  |            |            |
|       |                        |                  |            |            |
|       |                        |                  |            |            |
|       |                        |                  |            |            |
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|       |                        |                  |            |            |
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|       |                        |                  |            |            |
|       |                        |                  |            |            |



Tuesday, January 9, 2007

Spring Lab



# Announcements

Rotation, Gravity, Oscillation #9 will be checked tomorrow, which is when you have your next *Homework Quiz*.

Lunch Bunch Photoelectric Effect lab due tomorrow.

US Physics Team exam: Do you have your \$5.00?

Exam is Friday.





# Spring lab

Use **Hooke's Law** to determine the force constant of your spring. Do at least 5 trials. The report will include a graph of the data such that the slope yields  $k$ .

Determine the force constant of your spring from its **period of an oscillation** with various attached masses. The report will include a graph of the data such that the slope yields  $k$ .

Compare the force constants obtained by these two methods.

**Full lab report due next Tuesday, January 16.**



Wednesday, January 10, 2007

Review



# Announcements

## Homework:

- Due today is #9 (folder), pendulum lab (pass up), photoelectric lab (pass up). Your homework folders will be collected after homework quiz.
- Due tomorrow at *beginning* of class -- FR #2 and #3 in your packet.

US Physics Team Exam \$5 is due.

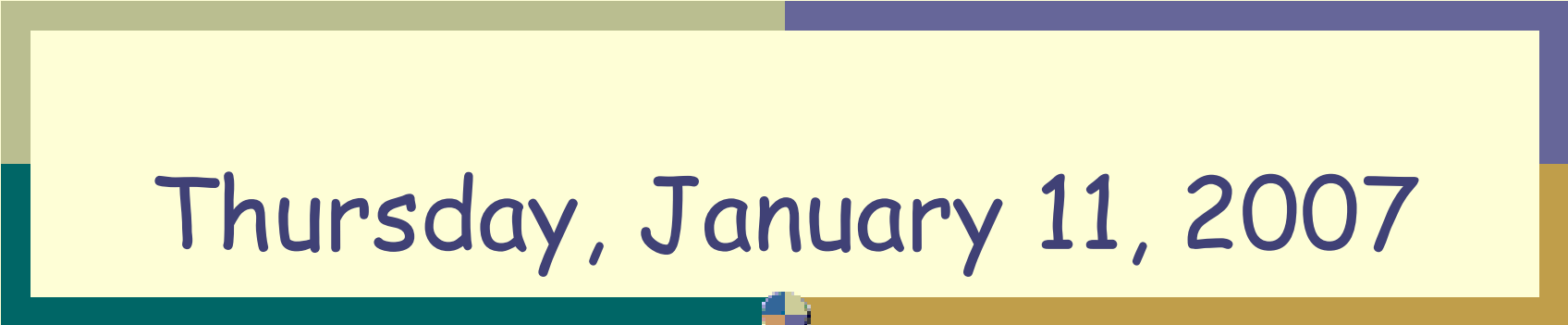
Review today and tomorrow.

Exam Friday.

Makeup lab Friday

Jeopardy





Thursday, January 11, 2007

Review





# Announcements

US Physics Team Exam \$5 is due.

Get out Clickers.

Exam Friday.

Makeup Lab Friday morning before  
school.





# Review: Torque

Torque causes a twist or rotation.

$$\tau = r F \sin \theta$$

- $\tau$  is torque
- $F$  is force
- $r$  is "moment arm"
- $\theta$  is angle between  $F$  and  $r$

Torque units: Nm





# Review: Kepler's Laws

1. Planets orbit the sun in elliptical orbits.
  2. Planets orbiting the sun carve out equal area triangles in equal times.
  3. The planet's year is related to its distance from the sun in a predictable way -- *derivable*
- 



# Review: Gravitation

$$F_g = Gm_1m_2/r^2 \text{ (Magnitude of Force)}$$

$$U_g = -Gm_1m_2/r \text{ (Potential Energy)}$$

Relationships for derivations

- Acceleration due to gravity

$$F_g = mg$$

- Orbital parameters (period, radius, velocity)

$$F_g = mv^2/r$$

- Energy Conservation (escape velocity)

$$U_{g1} + K_1 = U_{g2} + K_2$$


Friday, January 12, 2007

Exam

Friday, January 12, 2007

Exam