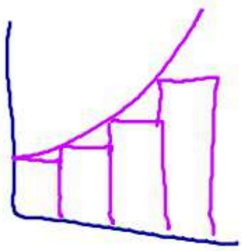


t(minutes)	0	4	9	15	20
W(t) degrees F	55.0	57.1	61.8	67.9	71.0

2012 #1

The temperature of water in a tub at time t is modeled by a strictly increasing, twice differentiable function, W, where W(t) is measured in degrees Fahrenheit and t is measured in minutes. At time t = 0, the temperature of the water is 55° F. The water is heated for 30 minutes, beginning at time t = 0. Values of W(t) at selected times t for the first 20 minutes are given in the table above.

c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

$$\frac{1}{20}\int_0^{20} w(t)\,dt = \frac{1}{20}\left[(4-0)(55) + (9-4)(57.1) + (15-9)(61.8) + (20-15)(67.9)\right]$$

underestimate b/c w(t) is strictly increasing

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	–220	150

2015 BC3: Johanna jogs along a straight path. For $0 \le t \le 40$, Johanna's velocity is given by a differential function v. Selected values of v(t), where t is measured in minutes and v(t) is measured in meters per minute, are given in the table above.

A) Use the data in the table to estimate the value of $v'(16)$

$$v'(16) = \frac{240-200}{20-12}$$

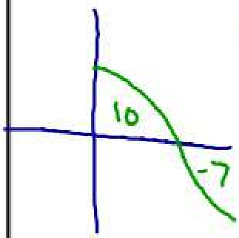
$$\int_0^{40} v(t) = s(t)$$

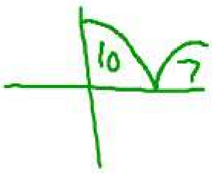
m/min meters

B) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|dt$ in the context of the problem. Approximate the value of $\int_0^{40} |v(t)|dt$ using a right Riemann sum with four sub-intervals indicated in the table.

$\int_0^{40} |v(t)|dt =$ The total distance jogging in meters from $0 \le t \le 40$ minutes.

$$\int_0^{40} |v(t)|dt = (40-24)(150) + \left|(24-20)(-220)\right| + (20-12)(240) + (12-0)(200)$$





C) Bob is riding his bicycle along the same path. For $0 \le t \le 10$. Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and B(t) is measured in meters per minute. Find Bob's acceleration at time t = 5.

$$B'(t) = 3t^2 - 12t$$

$$B'(5) = 3(5)^2 - 12(5)$$

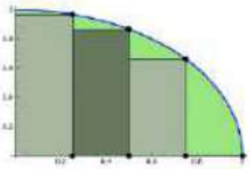
D) Based on the model B from part (c), find Bob's average velocity during the interval $0 \le t \le 10$.

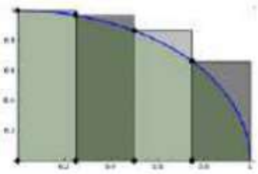
$$B(t) = t^3 - 6t^2 + 300 \qquad \frac{1}{10}\int_0^{10} (t^3 - 6t^2 + 300)\,dt$$

$$\frac{1}{10}\left[\frac{1}{4}t^4 - 2t^3 + 300t\right]_0^{10}$$

$$\frac{1}{10}\left[\frac{1}{4}(10)^4 - 2(10)^3 + 300(10)\right]$$

Left and right Riemann sums


Right Riemann Sum

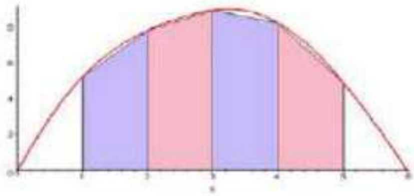

Left Riemann Sum

Correct justification for over and under approximations:

f(x)	Left Riemann Sum	Right Riemann Sum
Increasing ($f'(x) > 0$)	Under approximates the area because f(x) is increasing	Over approximates the area because f(x) is increasing
Decreasing ($f'(x) < 0$)	Over approximates the area because f(x) is decreasing	Under approximates the area because f(x) is decreasing

<u>Incorrect Reasoning:</u> The left Riemann Sum is an under approximation because the rectangles are all underneath or below the graph. Stating that the rectangles are below the function is not acceptable mathematical reasoning. It merely restates that it is an under approximation but does not explain WHY.

Trapezoidal approximations



Over/Under Approximations with Trapezoidal Approximations

f(x)	Trapezoidal Sum
Concave Up ($f''(x) > 0$)	Over approximates the area because $f''(x) > 0$
Concave Down ($f''(x) < 0$)	Under approximates the area because $f''(x) < 0$